AN INVESTIGATION OF THE RELATIONSHIPS BETWEEN MUSICAL TRAINING AND MATHEMATICAL PROBLEM SOLVING

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AN INVESTIGATION OF THE RELATIONSHIPS BETWEEN MUSICAL

TRAINING AND MATHEMATICAL PROBLEM SOLVING

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DEDICATION

To my mom,

for being the harmony to my melody

and for encouraging me to sing

even when I was out of tune.

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ABSTRACT

AN INVESTIGATION OF THE RELATIONSHIPS BETWEEN MUSICAL TRAINING AND MATHEMATICAL PROBLEM SOLVING

by

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The 2007 Trends in International Mathematics and Science Study (TIMSS) revealed that American sixth and eighth grade students are falling significantly behind students from Singapore, Hong Kong, Chinese Taipei, Japan, England, and the Russian Federation in mathematics (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2009). Students' difficulties with mathematics extend into post-secondary education where introductory mathematics courses, such as college algebra, can act as gatekeepers to college success by blocking the academic progress of hundreds of thousands of students each year (Reyes, 2010; Small, 2002). Despite the large body of research

suggesting that music can have beneficial effect on spatial reasoning and mathematics performance, schools continue to cut funding for music programs.

A correlational research design employing chi-square tests, analyses of variance, and ordinal logistic regressions was used to explore the relationships between music background and the mathematical problems-solving strategies utilized by students enrolled in first-year credit-bearing algebra-based university mathematics courses. Participants' music background was measured by the researcher-created Music Background Survey while data regarding the utilization of problem-solving strategies were collected through a problem-solving assessment consisting of three mathematical tasks. Spatial and analytic reasoning ability were also measured and used as control variables.

Analysis of participants' music background revealed that over 25% of the participants, "Low Music" participants, had no music instruction through either school music programs or private music instruction and had not participated in any formal instruction in music theory. In contrast, the participants deemed "High Music" reported means of approximately 11 years of private music instruction, approximately 7 semesters of music participation at the middle school level, and approximately 12 semesters of music participation at the high school level.

The investigation of the relationship between music training and the utilization of mathematical problem-solving strategies revealed one significant difference in the way "Low Music" and "High Music" participants utilize strategies while engaged in mathematical problem-solving tasks. Results indicated that participants with high levels of music training relied more heavily on the use of the construction of tables and lists as a mechanism for finding patterns than participants with low levels of music training.

In general, when comparing problem-solving strategies utilized by participants with high and low levels of analytic reasoning ability and high and low levels of spatial reasoning ability, results were inconclusive. However, when comparing participants with high and low levels of analytic reasoning ability, participants with low analytic reasoning ability reported being more reliant on remembering familiar procedures as a problem-solving strategy than participants with high analytic reasoning ability.

The results of the current research provide an initial look at the relationship between musical training and mathematical problem-solving. Further research investigating the relationship between musical training and mathematical problem-solving should include the collection of demographic data and data related to incidental music participation.

CHAPTER I

INTRODUCTION

Beliefs regarding the interconnectedness of music and mathematics have been prevalent since the time of Pythagoras, around 500 B.C. In fact, Pythagoras and other great historical figures such as Plato, Aristotle, and Leibnitz, have expressed “interest in the mutual affinities between music and mathematics” (Bamberger & Disessa, 2003, p. 123). Beyond the ubiquitous connections made between music and mathematics, “human creativity has always involved the cross-pollination of the disciplines in the generation of new ideas and products” (Bahna-James, 1991, p. 477). For example, Leonardo DaVinci was not only widely known as an inventor but also worked extensively as a painter, sculptor, musician, engineer, astronomer, and geologist. Similarly, Albert Einstein, though perhaps most famous for his theory of relativity, also studied philosophy and music. Nevertheless, as Bahna-James (1991) pointed out, the movement between multiple disciplines has become less prevalent in current education systems and society in general: “The reason for this, perhaps, is that people have begun to perceive the various spheres of intelligence as a good deal more separate that did either DaVinci or Einstein” (p. 477).

Links between mathematical ability, often measured through aptitude tests, standardized test scores, or course grades, and musical training, including private or group instruction on various musical instruments which may or may not include the study of music theory, have been widely researched (Cheek and Smith, 1998; Costa-Giomi, 2004; Johnson and Memmott, 2006; Kinney, 2008; Vaughn, 2000; Whitehead, 2001). Links between mathematical ability and musical training, though, may not be readily apparent or easily proven. As Whitehead (2001) pointed out, “researchers continue to look for ways to connect music to academic achievement and to convince school administrators and board members of the importance of music not only in our lives but also in the curriculum” (p. 2). What may be more obvious are the multiple relationships between mathematics and music: “a numerical pattern of beats, which can be counted, bears direct resemblance to the study of basic arithmetic, which is essential to the study of advanced mathematics” (Bahna-James, 1991, p. 479). Bahna-James continued, “while musical harmony and trigonometry are taught in most schools today, seldom is the connection drawn between the two; yet it is likely that students would recognize a direct relationship between musical pitch and the length of a vibrating string… More striking evidence as to the mathematical nature of music is that entire works of music can be composed on the basis of numerical calculations” (p. 479).

Despite these relationships, it is interesting to note that students who show a strong affinity toward music rarely confess an interest or aptitude in mathematics; however, students who demonstrate strong mathematical ability often demonstrate an affinity toward music (see Gardner, 1985, Frames of Mind). This sentiment was echoed by Bahna-James (1991):

> “Musicians may not be inherently mathematically inclined, yet music is inherently mathematical, even when approached from an intuitive standpoint. Thus, the musician composing for purely aesthetic purposes is aware of and

> confortable with certain mathematical concepts in music, whether consciously or unconsciously; and such cognitive understanding is necessary to employ harmonic, melodic, and rhythmic variations. This leads one to believe that musicians do have some sort of 'mathematical sense,' even if it is overlooked. The question, then, is whether such a 'mathematical sense' exists among musicians, and if so, whether it can be manipulated to increase music students' interest in mathematics and subsequently improve their mathematical skills" (p. 479-480).

Recent research has revealed that mathematical ability in particular is related to multiple intelligences, i.e., the utilization of both spatial-temporal and logical-analytical reasoning (Aldous, 2007). Spatial-temporal reasoning has been defined as "the ability to transform and compare mental images in space and time" (Whitehead, 2001, p. 9) and usually involves a holistic view of a problem. Logical-analytical reasoning, on the other hand, requires that a problem is broken down into parts in order to arrive at a solution (Schildknecht, 1989). As Rauscher and Zupan (2000) asserted, in light of recent research showing that musical training can have a positive effect on spatial cognition, one may "embrace the concept of autonomous intelligences as well as the possibility that experiences in one domain may influence performance in another" (p. 217). While educational practices focus on analytical thinking (Michaelides, 2002), researchers indicate that musical training can increase spatial-temporal reasoning ability (Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997; Rauscher & Zupan, 2000).

Researchers have reported a positive association between spatial-temporal reasoning and mathematical ability as well as between logical-analytical reasoning and mathematical ability (Bishop, 1980; Booth & Thomas, 2000), though researchers also suggest that mathematically gifted students and expert problem solvers have a tendency to move between logical-analytical and spatial-temporal reasoning strategies when engaged in problem solving tasks (Aldous, 2007; Krutetskii, 1976). Thus, it seems apparent that both types of reasoning, logical-analytical and spatial-temporal, are important to successful mathematical problem solving.

Statement of the Problem

The United States continues to lose ground in STEM (Science, Technology, Engineering, and Mathematics) fields when compared to other countries (Raju & Clayson, 2012). Moreover, mathematics continues to lose popularity among American students at all levels. Many students opt not to continue mathematics coursework past minimally required courses, which can prove limiting when selecting a college major or career field (Bahna-James, 1991; Eacott & Holmes, 2010; Raju & Clayson, 2012). As Maltese and Tai (2011) reported, the percentage of bachelor's degrees being awarded in mathematics has rapidly declined from 3.8% in 1966 to only 1.0% in 2006.

Lack of interest in the study of mathematics has also been noted by Rousseau (2009) who has been careful to point out that mathematical knowledge is important for success in the workplace and in a changing world. Also, "as future members of the work force, students will need to be able to interpret and explain structurally complex systems, to reason in mathematically diverse ways, and to use sophisticated tools and resources" (English, 2008, p. 11). Current standards for mathematics education, such as the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM, 2000) and *Crossroads in Mathematics: Standards for*

Introductory College Mathematics before Calculus from the American Mathematical Association of Two-Year Colleges (AMATYC, 1995), have also emphasized the necessity for mathematical understanding in the workplace. As NCTM (2000) stressed, "to meet new challenges in work, school, and life, students will have to adapt and extend whatever mathematics they know" and that "doing so effectively lies at the heart of problem solving" (p. 334). Similarly, AMATYC (1995) contended that students should be able to take the knowledge they gain in introductory college mathematics courses and "transfer this knowledge to problem-solving situations at work or in everyday life" (p. 4). Unfortunately, as Eacott and Holmes (2010) asserted, students' declining interest in mathematics has the potential to cause a "growing shortage of personnel in those careers requiring a strong mathematical foundation" (p. 84).

Moreover, the 2007 Trends in International Mathematics and Science Study (TIMSS) revealed that American students are falling significantly behind students from Singapore, Hong Kong, Chinese Taipei, Japan, England, and the Russian Federation (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2009). As Grandin, Peterson, and Shaw (1998) pointed out, "U.S. eighth grade students are below average in geometry and proportional reasoning, which will harm their understanding of specific science concepts" (p. 13). American students continue to struggle with geometry and proportional reasoning: "U.S. eighth-graders' average score in the geometry domain was lower than the TIMSS scale score by 20 scale score points" (Gonzales et al., 2009, p.10). Students' difficulties with mathematics extend into post-secondary education. Reyes (2010) asserted that introductory mathematics courses, such as college algebra, can act as gatekeepers to college students' success. Furthermore, Small (2002) contended that, nationally, college algebra alone blocks the academic progress of hundreds of thousands of students each year. Michaelides (2002) hypothesized that one possible cause for students' struggles with problem solving is the promotion of analytic reasoning in schools: "In educational practice… rules and formulas, procedures and analytical thinking, are dominant elements in the mathematics curriculum. School geometry is taught in a formal manner, while visualization and intuitive sense about space do not receive much attention" (p. 2). Grandin, Peterson, and Shaw (1998) suggested music education as a possible solution to this problem and recommended such instruction begin in preschool to aid in developing children's spatial-temporal abilities.

Despite the large body of research suggesting that music can have beneficial effect on spatial reasoning and mathematics performance (i.e., Cheek and Smith, 1998; Costa-Giomi, 1999; Grandin, Peterson, & Shaw, 1998; Hetland, 2000; Johnson and Memmott, 2006; Kinney, 2008; Nantais & Schellenberg, 1999; Rauscher, Shaw, & Ky, 1993; Rauscher, Shaw, Levine, Ky, & Wright, 1994; Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997; Rauscher & Zupan, 2000; Tucker and Bushman, 1991; Vaughn, 2000; Whitehead, 2001; Wilson & Brown, 1997; Wolfe, 1983), schools continue to cut funding for music programs. Recent research reveals that "virtually all of schools surveyed offered some music course taught by a specialist" (Abril & Gault, 2008, p. 72) and, "in general, it seems that principals believed that music education programs in the secondary schools were highly successful at helping students meet an array of music and broad educational goals" (p. 78). However, the majority of principals claimed that school music programs were not under their control and cite barriers such as government legislation, funding, and scheduling as reasons that school music programs are not more substantial (Abril & Gault, 2008). According to Beveridge (2010), when funding cuts are made in schools, nontested subjects such as music and arts programs are affected first with consequences that include "everything from the elimination of instrument repair budgets to the loss of entire teaching positions and programs" (p. 5). Funding for music programs is especially scarce in schools with lower socioeconomic status (SES). As Abril and Gault (2008) pointed out, "this is of great concern, especially because students from less affluent backgrounds are unlikely to have the financial resources to study formal music in venues outside of school" (p. 78).

Participation in music, through both private instruction and school sponsored music programs, has been shown to be related to increased academic performance (i.e., Johnson & Memmott, 2006; Kinney, 2008; Vaughn, 2000; Whitehead, 2001), while other researchers have found no differences in academic performance between students who do and do not participate in music programs (i.e., Costa-Giomi, 2004). As Costa-Giomi (2004) indicated, the fundamental argument is that "no study has found that participation in music programs diminishes students' performance at school or their academic achievement" (p. 141). Still, the nature of the relationship between music participation and academic performance continues to be unclear (Costa-Giomi, 2004; Fitzpatrick, 2006). Thus, further research investigating the relationship between music participation and academic performance remains warranted.

Purpose

The purpose of this research was to investigate the relationships between musical training and the utilization of problem-solving strategies on mathematical problem-solving tasks. Many researchers have pointed to music as a means to increase students' mathematical performance and the need to further explore how music education can be fully exploited (Cheek & Smith, 1998; Grandin, Peterson, & Shaw, 1998; Johnson & Memmott, 2006; Rauscher et al., 1997; Rauscher & Zupan, 2000; Vaughn, 2000). Moreover, recent brain research has shown that musical training can help connect the hemispheres of the brain, increasing complex reasoning and mathematics performance (Cheek & Smith, 1998; Grandin, Peterson, & Shaw, 1998).

However, as Zher (2000) noted, musical training has not been fully recognized for its potential as an educational intervention: “Compared with some other educational interventions, the studies on music learning is a thin lot... Little is known, for example, about what kinds of musical training produce results and what kinds don’t, who benefits most, and how long any intellectual gains that result from music learning will last” (p. 6). Rauscher, Shaw, and Ky (1993), in their ground-breaking research of what would later become known as “The Mozart Effect,” found that merely listening to ten minutes of a Mozart sonata resulted in statistically significant increases in students’ performance on a spatial reasoning task. Cheek and Smith (1998) found students who had received 2 or more years of private lessons performed significantly better on mathematics standardized test than students who had not received any music lessons. Furthermore, students who had received private keyboard lessons performed significantly better on mathematics standardized test than students who received private lessons on another instrument. In another study, Kinney (2008) found that there was no statistically significant difference on any subject of a standardized test between students who did and did not participate in a school choir program. However, Kinney did find statistically significant differences between students who did and did not participate in school band.

In addition, many studies have found only temporary performance enhancement rather than lasting effects related to musical training (Costa-Giomi, 1999). Moreover, Bahna-James (1991) suggested that there is a need to look at musical experience with respect to music theory: “The presence of mathematics is evident in musical concepts ranging from the most complicated and technical to the most basic. Yet, while mathematics and music do share certain concepts, the similarities are between mathematics and music *theory*, not between mathematics and music in general” (p. 479). This study investigated the relationships between musical training and the utilization of problem-solving strategies while engaged in mathematical problem-solving tasks. Specific aspects of musical training, including the type of musical training, the duration of the musical training, and whether musical training included the study of music theory, was explored.

It was hypothesized that, due to increased spatial-temporal reasoning abilities, persons with music training would approach mathematical problem-solving tasks differently (i.e., by utilizing different problem-solving strategies) than those without music training. Also, specific types of musical training such as keyboard instruction as well as a greater number of years of study may lead to increased development of problem solving ability than other instruments or limited musical study. In addition, it may become evident that students with musical training who also possess a background in music theory have an advantage over those without this background due to the similarities between mathematics and music theory.

Significance of the Study

Students struggle with mathematical understanding at the elementary and secondary levels (Grandin, Peterson, & Shaw, 1998; Gonzales et al., 2009) as well as at the post-secondary levels (Reyes, 2010; Small, 2002). Researchers have identified spatial reasoning as an integral and necessary skill for mathematics understanding (Aldous, 2007; Bishop, 1980; Booth & Thomas, 2000). Current movements in standards for mathematics education have identified spatial reasoning as a key component for mathematics education. For example, the *Principles and Standards for School Mathematics* (NCTM, 2000) stated students in grades 9 through 12 should be able to "use visualization, spatial reasoning, and geometric modeling to solve problems" (p. 308).

First, a search of recent literature produced few documents investing the effects of musical training on the strategies students employ when engaged in mathematical problem solving (see Appendix A). More specifically, using the search criteria "music" and "problem solving", 53 entries were returned by Education Resource Information Center (ERIC) for January 2005 through December 2011. Of these 53 articles, 27 entries did not explore the relationship between music and problem solving. These entries included state academic standards, teaching resources and handbooks, collections of articles that included papers regarding music or problem solving, and other articles with descriptions that merely included the words music, problems, or solving. Eliminating these 27 articles left 26 entries to consider, of which 11 did not report the results of a research study. Instead, these 11 entries consisted of discussions about pedagogical practices, educational philosophy, and other editorial-style papers. These papers were, therefore, also eliminated. The remaining 15 articles discussed problem-solving strategies utilized during music practice, performance, composition, or teaching. Similar results were obtained when searching the ProQuest Education Journals database. The search criteria “*music*” and “*problem solving*” yielded 53 entries for January 2005 through December 2011. Of the 53 returned items, 41 entries consisted of non-research based articles discussing pedagogical practices, educational philosophy, editorials, and teaching resources. Of the remaining items returned, seven articles discussed problem solving in the context of music composition, practice, performance, or teaching and five articles reported finding or research studies which investigated the effects of music participation on subjects with behavioral problems including Attention Deficit Disorder, aggressive behavior, and other psychological matters. None of the items returned in either search investigated the relationship between music participation and mathematical problem solving; thus, there appears to be a void in research investigating whether persons with musical training employ different problem-solving strategies during mathematical problem solving than those without musical training.

Second, the proposed research hopes to increase the research base, which supports keeping music programs in public schools. Music holds possibilities for increasing mathematics performance, especially as it relates to the development of spatial reasoning (Cheek & Smith, 1998; Grandin, Peterson, & Shaw, 1998; Johnson & Memmott, 2006; Rauscher et al., 1997; Rauscher & Zupan, 2000; Vaughn, 2000). As Grandin, Peterson, and Shaw (1998) recommended, music education should begin as early as preschool in an effort to “develop the ‘hardware’ for [spatial-temporal] reasoning in the child’s brain” (p. 13). Unfortunately, Roberts (2004) reported, “we hear of school districts across North America cutting (or trimming) music programs in their quest to expand curricular time for math or science or language or all three” (p. 5). Roberts continued with the reminder that “typically these math for music exchanges do little or nothing for the improvement of math scores yet the myth remains that isolated attention to these subjects will improve performance” (p. 5).

In the same vein as the potential for music to positively impact spatial reasoning, researchers have also suggested a more general correlation between music training and the development of complex reasoning. Wenger and Wenger (1990) suggested that participation in musical activity can increase children’s performance in complex reasoning tasks. While researchers have made various connections between the study of music and spatial, proportional, and complex reasoning, little research has been conducted to investigate the relationships between musical training and the utilization of problem-solving strategies while engaged in mathematical problem-solving tasks. The proposed study also aims to add to the literature base by examining the relationship between music training and mathematical problem solving, spatial reasoning, and analytical reasoning.

Finally, the research to be conducted in this study aims to add to the research base through the construction of a survey to measure level of musicianship based on musical training and experience, participation in informal musical activities, and other incidental music participation. It has yet to be determined what constitutes an adequate amount, as well as the most effective type, of musical training to realize potential academic benefits. As Rauscher and Zupan (2000) pointed out, the optimal age at which musical training should begin is unknown; also, “little is known regarding the contributions of either the [musical] curriculum or the musical instrument” (p. 224). This discrepancy is evident through the literature in which researchers have attributed academic benefits to various definitions of musical training (see for example: Cheek & Smith, 1998; Kinney, 2008).

Statement of the Research Questions

Schoenfeld and Herrmann (1980) proclaimed "problem-solving skill in mathematics rests in part on the ability to accurately perceive or understand a problem" (p. 2) and "students with similar backgrounds will perceive problems in similar ways" (p. 14). Findings such as these may lead one to question whether students with musical backgrounds will have similar perceptions of mathematical problems and, more specifically, whether these perceptions will differ from students without musical training. The purpose of this study was to investigate the relationship between musical training and the utilization of problem-solving strategies while engaged in mathematical problem-solving tasks. The research question that was addressed through this research is:

What are the relationships between musical training and the utilization of problem-solving strategies while engaged in mathematical problem-solving tasks?

Four sub-questions were identified to guide the investigation of this research question:

1. What is the level of music training of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a music background survey?
2. What is the level of spatial reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by the Spatial Reasoning Test (adapted from the Spatial Visualization Test developed by Michigan State University, 1981) and what are the differences in spatial reasoning ability between students with and without music training?

3. What is the level of analytic reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by an analytic reasoning test and what are the differences in analytic reasoning ability between students with and without music training?
4. What problem-solving strategies are utilized by students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a problem-solving test and what are the differences in strategy selection between students with high and low spatial reasoning ability, between students with high and low analytic reasoning ability, and between student with and without music training?

The following chapter contains a review of the literature relevant to this research study. Two distinct bodies of research, the first focused on the effects of music listening and training and the second on mathematical problem solving, are discussed. Specifically, literature related to the effects of music training and listening on spatial-temporal reasoning, brain development and function, and mathematics performance is reviewed to provide background for research questions 1, 2, and 3 while the literature related to the factors impacting the selection and utilization of various problem-solving strategies was examined to provide a background for research question 4. In addition, the relationship between the study of music theory and mathematics as well as problem solving in the context of music practice and composition is discussed.

CHAPTER II

REVIEW OF LITERATURE

The literature reviewed for this study included research investigating the effects of music training and listening as well as research exploring problem-solving strategy selection and utilization (see Appendix A). Research which investigated the effects of music training and listening was divided into three categories for this review: the effects of music training and listening on spatial-temporal reasoning, the effects of music training and listening on brain development and function, and the effects of music training and listening on mathematics performance. Research focused on the selection and utilization of problem-solving strategies, both in the context of music and in the context of mathematics, was also reviewed.

Definition of Terms

Four key ideas permeate this research project: mathematical problem solving, spatial-temporal reasoning, logical-analytical reasoning, and musically trained or untrained. For reader understanding, definitions from the literature are provided followed by the definition that will be used for the purposes of this study.

Mathematical Problem Solving

As defined by Webb (1977), "for an individual, a problem exists if he desires to obtain a goal but the path leading to the attainment of this goal is not immediately known and cannot be found by just using habitual responses" (p. 2). Webb (1977) continued by asserting that the problem-solving process includes “all behaviors related to the problem-solving procedure performed from the initial step of reading or defining the problem to giving the final solution or the termination of work on the problem” (p. 5). Mayer and Hegarty (1996) expanded on the definitions of Webb by describing mathematical problem solving as consisting of “the cognitive process of figuring out how to solve a mathematics problem that one does not already know how to solve” (p. 31). For the purposes of this study, mathematical problem solving is defined as the behaviors and processes, from the initial step of defining and understanding the problem to the statement of the solution, utilized by an individual while engaged in solving a mathematical problem for which the solution or path to the solution is not immediately known.

Spatial Reasoning

According to Clements and Battista (1992), spatial reasoning “consists of the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated” (p. 420). Spatial reasoning is often also referred to as spatial-temporal or visual-spatial reasoning. Whitehead (2001) defined spatial-temporal reasoning as “the ability to transform and compare mental images in space and time” (p. 9). To add to this definition, Rauscher and Zupan stated that spatial-temporal processes are “used in tasks that require combining separate elements into a single whole by arranging objects in a specific spatial order to match a mental image” (p. 216). According to Sinclair, Mamolo, and Whiteley (2011), visual-spatial reasoning “refers to the application of visual and spatial representations (e.g., diagrams, physical or dynamic graphical models, and mental imagery) and processes (e.g., composing, decomposing, mental and hands-on moving)" (p. 135). For the purposes of this study, spatial reasoning is used interchangeably with visual-spatial and spatial-temporal reasoning and is defined as the cognitive processes involved in physically or mentally transforming, comparing, combining, and arranging objects.

Analytical Reasoning

Zazkis, Dubinsky, and Dautermann (1996) defined analytic thinking as "any mental manipulation of objects or processes with or without the aid of symbols" (p. 442). Analytic reasoning has also been described as being based on propositions (DeShon, Chan, & Weissbein, 1995) or logical operations applied to elements of a problem (Hunt, 1974). Grandin, Peterson, and Shaw (1998) asserted that this type of reasoning is "more involved when we solve equations and obtain a quantitative result" (p. 11). For the purposes of this study, analytical reasoning, often referred to as logical-analytical, verbal-analytical, or verbal-logical reasoning, is defined as reasoning based on the logical and systematic evaluation of a problem often demonstrated by solving equations to obtain a quantitative result.

Musically Trained and Untrained

Madsen and Madsen (2002) identified musically trained individuals as "trained musicians with a minimum of 10 years of formal individual or group music instruction" (p. 115). In contrast, the researchers identified musically untrained individuals as "having had no private instruction and fewer than 3 years of formal group music ensemble study" (p. 115). Madsen and Madsen applied the definition of musically trained individuals to university music majors. Since participants for the present study will be drawn from freshman-level university mathematics courses, the definition for musically trained may be too stringent for the population under investigation. Thus, for the purposes of this study, musically trained individuals are defined as having a minimum of five years of formal individual or group music instruction. Musically untrained individuals are defined as having no private music instruction and less than three years of group music instruction.

Effects of Music Training and Listening

Music has long been believed to have “beneficial effects on the soul” (Costa-Giomi, 2004). Moreover, ancient Greek philosophers and scholars such as Pythagoras, Plato, and Aristotle considered music a branch of mathematics (Papadopoulos, 2002). More recently, Morrison (1994) reminded us, “for years, dating as far back as the early 1900’s, proponents of public school music education have proclaimed that student participation in such activities as band, choir, and orchestra has a positive effect on everything from academic achievement to self-discipline, from citizenship to personal hygiene” (p. 33).

In an attempt to explain the relationship between music and learning, researchers have investigated the effect of music on the brain. Weinberger (1998) pointed out that “learning and performing music actually exercises the brain—not merely by developing specific music skills, but also by strengthening the synapses between brain cells” (p. 38). Furthermore, cognitive systems such as those necessary for symbolic, linguistic, and reading skills depend on synaptic strength and “music making appears to be the most extensive exercise for brain cells and their synaptic interconnectedness” (Weinberger, 1998, p. 39). Some of the most astounding research indicated that music listening and training has an impact on spatial-temporal reasoning (Costa-Giomi, 1999; Hetland, 2000b; Nantais & Schellenberg, 1999; Rauscher, Shaw, & Ky, 1993; Rauscher, Shaw, Levine, Ky, & Wright, 1994; Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997; Rauscher & Zupan, 2000; Wilson & Brown, 1997), brain development and function (Begley, 1996; Booth, 2001; Jakobson, Lewycky, Kilgour, and Stoesz, 2008; Jausovec and Habe, 2004; Pantev, Roberts, and Engelien, 1998; Schlaug, Jancke, Huang, Staiger, and Steinmetz, 1995; Shaw, 2000), and mathematics performance (Cheek and Smith, 1998; Johnson and Memmott, 2006; Kinney, 2008; Tucker and Bushman, 1991; Vaughn, 2000; Wolfe, 1983).

Whitehead (2001) noted that, "within the last century, researchers have sought to understand this connection and forge a meaningful connection between music and academic subjects, especially mathematics" (p. 12). Literature reviewed focused on the effects of music training and listening on the areas of spatial-temporal reasoning, brain development and function, and mathematics achievement. It is interesting to note that experimental studies investigating the effects of musical training have been primarily conducted on children between the ages of 3 and 12 while experimental studies investigating the effects of music listening have been primarily conducted on university level students. Since the present study seeks to investigate the effects of music training on university level students, discussion of both bodies of research have been included to provide background.

Musical Listening and Spatial-Temporal Reasoning: The Mozart Effect

In the ground-breaking publication *Music and spatial task performance*, Rauscher, Shaw, and Ky (1993) reported that listening to the Mozart sonata for two pianos in D Major, K.448, resulted in increased abstract and spatial reasoning performance in college students. Research participants included 36 college students who were exposed to three listening conditions: 10 minutes of the Mozart sonata, 10 minutes of a relaxation tape, or 10 minutes of silence. Following the listening condition, students' spatial reasoning skills were tested using the Stanford-Binet intelligence scale. Rauscher, Shaw, and Ky found that students exposed to the Mozart sonata performed significantly better on the test as compared to the other two listening conditions. Furthermore, there was no difference in performance between students who were exposed to the relaxation tape and students who listened to silence. The researchers noted, though, that the enhancing effects of the Mozart sonata were temporary, not lasting beyond the 10 to 15 minutes students took to take the spatial reasoning test.

As Hetland (2000b) pointed out, the findings reported by Rauscher, Shaw, and Ky (1993) "led to a frenzy of commercial and media attention" (p. 105) and the media coined the phrase "The Mozart Effect" to describe the research findings. Multiple replication studies of the Mozart Effect have been successful (see for example, Nantais & Schellenberg, 1999; Rauscher, Shaw, Levine, Ky, & Wright, 1994; Wilson & Brown, 1997). Even more, numerous researchers have been able to extend the findings of Rauscher, Shaw, and Ky (1993). For example, Rauscher, Shaw, Levine, Ky, and Wright (1994) extended their initial research by including minimalist music and dance/trance electronic music. In this study, the researchers found that "minimalist or rhythmically repetitive music structures do not enhance spatial task performance" (p. 12). Rauscher, Shaw, Levine, Ky, and Wright (1994) also indicated that listening to an audio-taped short-story did not enhance spatial reasoning. Nor did listening to the Mozart sonata improve short-term memory. Furthermore, Wilson and Brown (1997) indicated that participants experienced increased maze completion performance and decreased maze path errors following a listening condition of Mozart's Concerto No. 23 in A Major (K488). The researchers concluded that "listening to the patterned classical music of Mozart can indeed enhance performance on some measures of spatial reasoning" (p. 368). Wilson and Brown cautioned, however: "data suggest that the effect of listening to the music of Mozart on spatial reasoning is not so robust or powerful; it can enhance spatial-task performance under certain circumstances and to a certain point" (p. 369).

Nantais and Schellenberg (1999) conducted two experiments to replicate and extend the findings of Rauscher, Shaw, and Ky (1993). In the first experiment, participants included 56 undergraduate students. Over two sessions, which occurred no more than two weeks apart, each student either listened to music or silence. For half of the participants, the music condition consisted of listening to 10 minutes of the Mozart *Sonata for Two Pianos* in D Major, K. 448 while for the other half of the participants, the music condition consisted of listening to 10 minutes of Schubert's *Fantasia for Piano, Four Hands*, in F minor (D940). The results of this experiment upheld the findings of Rauscher, Shaw, and Ky (1993). Furthermore, Nantais and Schellenberg (1999) contend that, not only was the Mozart Effect "successfully replicated in a completely controlled laboratory setting," but "when a piece by Schubert was substituted for the Mozart piece used by Rauscher and her colleagues, an effect of equivalent magnitude was evident" (p. 371-372).

In a second study, Nantais and Schellenberg (1999) compared the results of students as they listened to 10 minutes of the Mozart sonata or 10 minutes of a recorded version of the short story "The Last Rung on the Ladder" (King, 1994). Data indicated that listening preference (i.e., the listening condition the student identified as preferred) had an interaction effect with the listening condition. More specifically, “participants who preferred the Mozart excerpt scored significantly higher in the Mozart condition than in the story condition” while “participants who preferred the story exhibited marginally better performance in the story than in the Mozart condition” (p. 372). The researchers concluded that, “although listening music composed by Mozart might contribute to improved performance on a subsequently presented spatial-temporal task, our results provide no evidence that the improvement differs from that observed with other engaging auditory stimuli that are equally pleasing to participants” (p. 372).

On the other hand, some researchers have not been able to replicate the Mozart Effect (see for example, Carstens, Huskins, & Hounshell, 1995; Cash, El-Mallakh, Chamberlain, Bratton, & Li, 1997; Newman, Rosenback, Burns, Latimer, Matocha, & Vogt, 1995; Steele, Bass, & Crook, 1999; Stough, Kerkin, Bates, & Mangan, 1994). Rauscher and Shaw (1998) hypothesized that some researchers have not been able to replicate results due to choice of dependent measures, the order of the listening and task conditions, and the choice of musical composition.

For example, researchers attempting to replicate the Mozart Effect using the Raven’s Progressive Matrices Test (Newman, Rosenback, Burns, Latimer, Matocha, & Vogt, 1995; Stough, Kerkin, Bates, & Magnan, 1994) or a backwards digit span task (Steele, Ball, & Runk, 1997) as the dependent measure have not been successful. Rauscher and Shaw (1998) pointed out that the failure to replicate using the Raven’s Progressive Matrices Test is not surprising since this test is considered a measure of general analytic ability rather than spatial ability. Furthermore, Rauscher and Shaw

(1998) insisted that “the Backwards Digit Span task and the matrices task are often not included in subclasses of spatial ability” (p. 837). Steele, Ball, and Runk (1997) admitted that the failure to replicate could be due to the use of a different dependent measure.

With the conflicting evidence for the Mozart Effect, hypotheses citing arousal, mood, and preference have emerged in an attempt to explain the elusive Mozart Effect (Hetland, 2000b). Arousal hypotheses contend that listening to music produces adrenaline in the brain elevating performance on cognitive tasks. On a related note, mood hypotheses assert that positive moods activated by listening to Mozart heighten arousal thus elevating performance. Finally, preference hypotheses allege that cognitive performance is enhanced after listening to something for which preference is shown.

To address these hypotheses, Hetland (2000b) conducted a meta-analysis of studies involving subjects that listened to music and were then tested with spatial measures. Data indicated that the Mozart Effect “is a moderate effect, and it is robust” (p. 136). Moreover, the researcher found evidence against the arousal, mood, and preference hypotheses: “the data to date do not appear to support any of these hypotheses” (p, 136). Specifically, “there is some evidence that levels of arousal do not correlate with levels of spatial-temporal performance” (p. 137). With respect to preference hypotheses, Hetland contended that these hypotheses have not been fully tested. In general, the researcher asserted, “there is still a good deal of unexplained variation in this body of research” (p. 137).

More recent research continues to investigate the Mozart Effect with respect to the hypotheses cited by Hetland (2000b). For example, Thompson, Schellenberg, and Husain (2001) found that it is possible that the Mozart Effect could be explained by mood since “the Mozart effect is associated more with positive than negative mood” (p. 250). These researchers insisted that “enjoyable stimuli induce positive affect and heightened levels of arousal, which lead to modest improvements in performance on a variety of tasks” (p. 251). In another study conducted by Husain, Thompson, and Schellenberg, (2002), it was concluded that, “overall, the findings are consistent with our view that effects of listening to music on cognitive performance are mediated by changes in arousal and mood” (p. 166). However, the researchers pointed out that this study did provide support for the direct link between music and spatial abilities as asserted by Rauscher and colleagues (1993, 1995): “the music manipulations were nonetheless associated with performance in the spatial task when effects of arousal, mood, and enjoyment were held constant” (Husain, Thompson, and Schellenberg, 2002, p. 167).

Other replication studies of the Mozart Effect have investigated whether there are interaction effects with respect to gender or the length of time exposed to stimuli such as the study conducted by Gilleta, Vrbancic, Elias, and Saucier (2003). Participants included 26 women and 26 men ranging from ages 18 to 34 years who listened to either 8 minutes and 24 seconds of the Mozart Sonata (K.448), 4 minutes and 12 seconds of the sonata, or silence. Immediately following the listening condition, participants completed two tasks: the Paper-folding and Cutting task and the Mental Rotations task. Next, participants were exposed to the alternate listening condition (silence if previously exposed to Mozart or the Mozart sonata if previously exposed to silence) immediately followed by alternate Paper-folding and Cutting and Mental Rotation tasks. Data analysis revealed no statistically significant differences between the listening conditions, therefore not replicating the Mozart Effect. Performance on the Mental Rotations task was significantly higher for men than women, though no difference was found for the Paper-folding and Cutting task with respect to gender. In response, the researchers noted that "Mental Rotations may require different spatial abilities than Paper-folding and Cutting, as Mental Rotations requires the ability to correct for changes in the spatial orientation of an object" (p. 1090). Overall, Gilleta, Vrbancic, Elias, and Saucier asserted that gender effects should be more fully investigated with respect to the Mozart Effect.

As Pietsching, Voracek, and Formann (2010) pointed out, results of studies attempting to investigate the Mozart Effect continue to be mixed, "thus rendering primary studies powerless to resolve the issue whether or not the effect exists" (p. 315). Furthermore, previous meta-analyses have failed to assess "influences of possibly confounding publication bias" (p. 315). Pietsching, Voracek, and Formann, therefore, conducted a meta-analysis to investigate the existence of the Mozart Effect hypothesizing publication bias on the overall effect. For this meta-analysis, 39 studies were analyzed giving a total of 38 study effects for which effects of the Mozart K. 488 Sonata were compared to non-musical stimulus or silence, 11 study effects for which effects of the Mozart K. 488 Sonata were compared to any other kind of music stimulus, and 15 study effects for which effects of other of music stimulus were compared to non-musical stimulus or silence.

Overall standardized mean differences for the three paired stimulus conditions revealed that "samples exposed to the Mozart sonata scored significantly higher in spatial tasks that samples exposed to non-musical stimuli of no stimulus at all," "samples exposed to the Mozart sonata KV 448 scored significantly higher on spatial tasks than samples exposed to any other kind of music," and "samples exposed to any other kind of music scored significantly higher on spatial tasks than samples exposed to non-musical stimuli or no stimulus at all" (p. 317). Also, the researchers noted that research conducted in labs affiliated with Rauscher or Rideout indicated significantly larger effect sizes.

With respect to publication bias, Pietsching, Voracek, and Formann (2010) found no evidence of publication bias for studies investigating the effects of the Mozart K. 488 Sonata versus any other kind of music stimulus or in studies investigating the effects of other of music stimulus versus non-musical stimulus or silence. On the other hand, analysis indicated influences of publication bias for studies investigating the effects of the Mozart K. 488 Sonata versus any other kind of music stimulus. In conclusion, "overall effects turned out to be significant but small and not substantially different from effects of other kinds of music" and "on the whole, there is little left that would support the notion of a specific enhancement of spatial task performance through exposure to the Mozart sonata KV 448" (p. 322).

Overall, research on the Mozart Effect has been inconclusive. Factors associated with research design have been identified as plausible causes for discrepancies in findings; however, hypotheses attempting to explain the effect have not been empirically supported, leaving researchers with many questions (Hetland, 2000). Taken as a whole, researchers contend that the Mozart Effect continues to be a "scientific puzzle" (Steele, Ball, & Runk, 1997).

Music Training and Spatial-Temporal Reasoning

In response to the "scientific puzzle" prompted by the Mozart effect, many researchers have attempted to further investigate relationship between music and spatial ability by looking at the effects of music training, rather than merely music listening, on spatial-temporal reasoning (Costa-Giomi, 1999; Rauscher, Shaw, Levine, Ky, & Wright, 1994; Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997; Rauscher & Zupan, 2000). Again, results for these types of studies have been mixed, though research design (i.e., participant age, duration of musical training, type of musical training, and dependent measure) as well as conclusions varied greatly from study to study.

For example, Rauscher, Shaw, Levine, Ky, and Wright (1994) reported results from a study to investigate the effects eight months of weekly 10- to 15-minute private keyboard lessons (taught by two professional piano instructors) and daily 30-minute group singing lessons (taught by a professional vocal instructor) on preschool students' spatial abilities. Using tasks from the Wechsler Preschool and Primary Scale of Intelligence-Revised and the Stanford-Binet Intelligence Scale, the researchers found that children who participated in music training performed better on the Object Assembly task than students who did not receive music training. Scores on the other tasks, including Geometric Design, Block Design, Animal Pegs, and Absurdities, did not differ between those children who had received music training and those children who had not. Lack of significance on Geometric Design, Block Design, Animal Pegs, and Absurdities tasks was attributed to the unique spatial reasoning requirements of the Object Assembly task: "The Object Assembly task was the only task given which required the child to form a mental image, and then orient physical objects to reproduce that image" and they propose that success on this task "is directed by cortical pattern development facilitated by the music lessons" (p. 20).

In a similar study, Rauscher, Shaw, Levine, Wright, Dennis, and Newcomb (1997) conducted a study to investigate the effects of various types of musical training on preschool children’s spatial-temporal reasoning abilities. Using tasks from the Wechsler Preschool and Primary Scale of Intelligence-Revised and the Stanford-Binet Intelligence Scale, the researchers found that students who received keyboard lessons significantly outperformed students who had received singing lessons, computer lessons, or no lessons on spatial-temporal tasks. Moreover, keyboard lessons resulted in long-term (lasting more than one day) enhancements in spatial-temporal skills. There were no significant differences between groups on spatial-recognition tasks.

In a longitudinal study to investigate the effects of three years of piano instruction on the cognitive abilities of 9- to 12-year-olds, Costa-Giomi (1999) found that three years of piano instruction “improved children’s general cognitive abilities and spatial abilities significantly but that these improvements were only temporary” (p. 207). On the other hand, “individual piano instruction did not affect the development of children’s quantitative and verbal cognitive abilities, providing further evidence that the contribution of music instruction to cognitive development might be more limited that has been previously suggested” (p. 207).

In a study conducted by Rauscher and Zupan (2000), kindergarten children who were given bi-weekly 20-minute group keyboard lessons scored significantly higher on spatial-temporal tasks than kindergarten students who were not given keyboard instruction. Along with keyboard training, students in the treatment group participate in other types of music instruction including singing, movement, ear training, music literacy, and solfege. While students in the treatment group did not outperform the control group in pictorial-memory tasks, significant gains in spatial-temporal reasoning were apparent after only four months of keyboard instruction, “a difference that was greater in magnitude after 8 months of lessons" (p. 223). Along with these findings, Rauscher and Zupan (2000) also concluded that this study demonstrated "private lessons are not needed to induce the [spatial-temporal reasoning] enhancement" (p. 223).

In order to address disparities in research design and research findings, Hetland (2000a) conducted what was thought the be "the first quantitative summary of the experimental research exploring whether active music instruction leads to enhanced spatial abilities, and which, if any, variables in students, music programs, or experimental design predict greater transfer to spatial tasks" (Hetland, 2000a, p. 180). The researcher pointed out two types of theories that have been used to attempt to explain the relationship between music training and spatial abilities: neural connection theories and near transfer theories. Neural connection theories posit that musical and spatial abilities may be linked due to neurological connections in the cortex of the brain. Other theories of this type have cited processing in the cerebellum as the connecting factor between music and spatial abilities. Near transfer theories, on the other hand, posit that music making requires the use and coordination of various abilities, including visual-spatial ability, and that music training, therefore, should enhance visual-spatial abilities of all types.

In the first meta-analysis, Hetland (2000a) utilized 15 studies to "test the hypothesis that active instruction in music, for periods ranging from six weeks to two years, enhances performance on a spatial-temporal task during and immediately following" instruction (p. 183). Participants in these 15 studies ranged from the ages of three through 12 and the dependent measures were spatial-temporal tasks, tasks that "require mental rotation and/or multiple solution steps for two-or three-dimensional figures in the absence of a model" (p. 183). In the second meta-analysis, Hetland focused on three studies for which Raven's Standard Progressive Matrices was used as the dependent measure since this assessment is not thought to measure spatial-temporal ability. The third meta- analysis consisted of eight studies, which utilized measures that tested various aspects of spatial ability including spatial recognition, spatial memory, and spatial visualization. Hetland asserted that this was done to test "whether music instruction might enhance spatial abilities beyond the spatial-temporal dimension" (p. 184). In this case, studies which used Raven's Standard Progressive Matrices were excluded "because it is not primarily spatial and was demonstrated in my previous meta-analysis not to be effected by listening to music" (p. 184).

Hetland's (2000a) first analysis revealed that "active music instruction lasting two years or less leads to dramatic improvements in performance on spatial-temporal measures" (p. 203). Hetland contended that, while the reliability of the measures employed in these studies is low, "the consistency across the studies suggests that the measures are indeed systematically indexing an effect" (p. 204). Analysis suggested that younger children are more susceptible to the enhancing effects of music training. Also, the enhancing effects of music training are equivalent across socioeconomic status. Hetland (2000a) also investigated program components to determine if these had an effect on how successful music training is with respect to increasing spatial abilities. Results indicated that lesson format and inclusion of standard notation make significant differences in program effectiveness with respect to improving spatial abilities. More specifically, one-on-one music training and programs that included the use of the standard music notation were shown to be more effective than group music programs and programs that included non-standard notation or no notation at all. Program variables that were not shown to produce significant differences were the length of program, whether the program included keyboard instruction, the presence of expressive movement as a component of the program, and the inclusion of composition and/or improvisation as a program component.

Hetland's second analysis focused on whether or not music instruction enhances performance on nonspatial tasks as measured by Raven's Standard Progressive Matrices test. It was concluded that "while similar kinds of music instruction lead to enhanced performance on spatial-temporal measures, they do not enhance measures that require mainly logical ability" (p. 218). Finally, Hetland's third analysis focused on whether music instruction enhances performance on any kind of spatial task rather than merely spatial-temporal tasks. It was concluded that "analysis provides support for the hypothesis that not only spatial-temporal, but also other spatial tasks requiring spatial memory, spatial recognition, mental rotation, and/or spatial visualization may be enhanced by music instruction" (p. 220).

Overall, Hetland (2000a) concluded that results provide some empirical support for both neural connection theories and near transfer theories as a means to explain why music training has an enhancing effect on spatial reasoning. Furthermore, some support was provided for the view that music instruction is more likely to have effects in younger children, though additional research is warranted. While Hetland concluded that there is a causal relationship between music training and enhanced spatial reasoning, not all music programs have been shown to have such an effect. Private, or one-on-one, instruction and programs that include standard music notation (at least in conjunction with piano instruction) have been shown to be most effective. Moreover, Hetland contended that further research needs to be conducted to determine whether enhancements only occur in the realm of spatial-temporal skills or if music training can also enhance other types of spatial skills.

Results of more current research exploring the effects of music training on spatial task performance remain mixed (see for example Costa-Giomi, 2004; Pietsch and Jansen, 2012). In a study conducted by Pietsch and Jansen (2012), university music majors and sports majors outperformed education majors on mental rotation tasks. Moreover, the researchers found that, while the gender difference favoring males for spatial ability was found for sports majors, this difference was not found for music majors. In sum, research aimed at examining this relationship is still needed. To further understand the unique connection between the enhancing capability of music listening and music training, researchers have explored how music exposure affects brain development and brain function (see for example Jakobson, Lewycky, Kilgour, and Stoesz, 2008; Jausovec and Habe, 2004; Rauscher, Shaw, & Ky, 1993; Rauscher, Shaw, Levine, & Ky, 1994; Schlaug, Jancke, Huang, Staiger, and Steinmetz, 1995).

Effects of Music Training and Listening on Brain Development and Function

Whitehead (2001) asserted that "recent studies of music and learning have grown out of an expanding line of research interested in the development of the human brain" (p. 16). As explained by Booth (2001), "music and math seem to create a connection between the two hemispheres of the brain. Music is considered a right-brain activity, while math is a left-brain activity. When combined, the whole child is engaged not only in the realm of thinking but in all the other domains of social-emotional, creative, language, and physical development. Music and math: together they make a complete developmental package" (p. 50-53). In attendance to this hypothesis, researchers have conducted studies to determine how music actually affects brain development and behavior.

Some researchers have been specifically interested in how listening to music can impact brain behavior, such as how music listening affects cortical firing patterns (Rauscher, Shaw, Levine, & Ky, 1994). Following the ground breaking report from Rauscher, Shaw, and Ky, (1993) that listening to Mozart can enhance performance on spatial-temporal tasks, the researchers asserted that "musical activities help systematize the cortical firing patterns so they can be maintained for other pattern development duties, in particular, the right hemisphere function of spatial task performance" (Rauscher, Shaw, Levine, & Ky, 1994, p. 22). Current research continues to investigate the hypothesis that music listening and training has an impact on cortical firing patterns.

In a study conducted by Jausovec and Habe (2004), electroencephalographic (EEG) imaging was used to investigate brain activity as research participants listened to the Mozart Sonata (K.448) while solving a visual task. Participants' EEG's were recorded as they solved a visual problem under two conditions, while listening to the Mozart sonata or while listening to silence. While listening to the Mozart sonata, the researchers observed an increase in cognitive workload indicating that music does indeed have a specific influence on the brain. The researchers suggested that this increase in cognitive workload may be due to "simultaneously processing the auditory and visual stimuli" (p. 268). Jausovec and Habe provided further explanation:

> “It can be assumed that listening to a certain type of music (e.g. Mozart) increases the coupling of specific brain areas and in that way facilitates the selection and ‘binding’ together of pertinent aspects of sensory stimulus into a perceived whole. It can be further assumed that if such a pattern of activated brain areas coincides with the pattern needed for task completion, an increase in task performance could be the result” (p. 269).

The researchers noted, however, that no evidence was found to support the hypothesis that listening to the Mozart sonata resulted in increased performance on this particular task. They suggested that this could be due to the simplicity of the task. Jausovec and Habe concluded that background music stimuli can have an effect on visual brain activity even if stimuli seems unrelated to the task being performed.

Beyond the potential for music to affect the activity of the brain, researchers have hypothesized that music training has the potential to affect the anatomy of the brain. For example, Schlaug, Jancke, Huang, Staiger, and Steinmetz (1995) conducted a study to compare high-resolution magnetic resonance images of brain anatomy of trained musicians and non-musicians. Their investigation revealed a significant difference in the size of musicians’ corpus callosums, the bundle of neural fibers that connect and facilitate communication between the two hemispheres of the brain, as compared to their matched non-musician counterparts. The researchers asserted that “variation in callosal size is generally considered to be a morphological substrate of interhemispheric connectivity and of hemispheric (a) symmetry, with more symmetrically organized brains having larger callosa,” though the exact cause for increased size has yet to be determined (p.

1050). Furthermore, the increase in corpus callosum size is even more pronounced in musicians who began music training before age seven.

Researchers have also been interested in the effects of music training on specific brain functions such as memory. In a study to investigate memory for verbal and visual material in highly trained musicians, Jakobson, Lewycky, Kilgour, and Stoesz (2008) compared encoding and retrieval skills for verbal and visual material between trained musicians and non-musicians. Data collected indicate that "music training is associated with superior delayed free recall of both verbal and visual information" (p. 50). In the case of verbal memory, the researchers asserted that, rather than merely superior rote memorization skills, musicians' verbal advantage "is associated with a relative strength in the extraction of higher-order, semantic information" (p. 50). Jakobson et al. (2008) hypothesized that music training enhances verbal memory by altering neural circuits used during verbal processing. Similarly, advantage of musicians with respect to visual memory was not attributed to superior rote recall or more effective use of verbal mediation strategies, but could be due to superior visuospatial skills or superior use of visuospatial strategy. While the researchers cautioned against making casual inferences based on the findings, they contend that "there is compelling neuroscientific evidence that music training produces a direct effect on the structure and function of the brain" (p. 52) and that the findings "lend strong support to the claim that active engagement in formal music training 'sculpts' the brain" (p. 53).

Effects of Music Listening on Mathematics Performance

Just as research regarding the effects of listening to music on spatial reasoning has been inconclusive, results regarding the effects of music listening on mathematics performance have been unconvincing. For example, in a study examining whether university students listening to classical music outperformed students listening to silence on a mathematics placement test, Manthei and Kelly (1998) reported no significant difference between students' performance. Similar results for university students were reported by Miller and Schyb (1989) when comparing the listening conditions of silence, Mozart, disco, and rock; and by Wolf and Weiner (1972) when comparing students listening to silence, classical music, recorded speech, and industrial noise. On the other hand, Wolfe (1983) found significant differences between university students assigned to various volume levels when listening to a film soundtrack. Students listening to the film soundtrack at a soft or medium volume level outperformed students listening to silence or the film soundtrack playing at a loud level. Similarly, Tucker and Bushman (1991) found that university students listening to silence outperformed students listening to rock music.

In response to these contradictory results, Vaughn (2000) conducted a meta-analysis to investigate whether listening to music while taking a mathematics test produced higher scores. This meta-analysis consisted of 12 experimental studies (including those studies discussed above). Vaughn concluded "the results of this meta-analysis show that playing music in the background while students are taking math tests has only a small positive effect, at best" (p. 163).

More current research, however, continues to support the notion that music listening can have a positive impact on mathematics performance. For example, Taylor and Rowe (2012) conducted an experiment comparing university students' trigonometry exam performance under two conditions, Mozart music and no music. Under the Mozart music condition, students took six semester exams while Mozart pieces were played in the background. Results revealed that students in the Mozart music condition significantly outperformed students in the no music condition. The researchers concluded that “the Mozart Effect does impact the demonstration of learning in mathematics” and that listening to Mozart during mathematics testing “has the potential to assist students in performing their best on mathematical assessments” (p. 60).

Effects of Music Training on Mathematics Performance

Though research investigating the effects of music training on spatial reasoning skills has been relatively inconclusive, it is agreed upon that spatial reasoning skills are important for mathematics achievement (Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007). There seems to be a natural connection, then, between music and mathematics if music training does indeed have a positive impact on spatial ability. Researchers have investigated the effects of various types of music training on mathematics achievement (Cheek and Smith, 1998; Costa-Giomi, 2004; Johnson and Memmott, 2006; Kinney, 2008; Whitehead, 2001). Again, studies have varied with respect to population, research design, dependent measure, and, perhaps most notable, research results.

For example, in a correlational study investigating the effects of various types of musical training on mathematical achievement levels of ninth-grade students, Cheek and Smith (1998) found evidence that private music lessons, but not school-sponsored music instruction, can enhance mathematical achievement. In general, when participants who had received private music lessons were compared to those who had not received private music lessons, no significant differences were found. However, when students who had received only music lessons through the school were compared to students who had received private music lessons for more than two years, the researchers determined that students who had received private music lessons for two or more years performed significantly better. Furthermore, private keyboard lessons were found to be significantly more effective than private lessons of another type. Cheek and Smith concluded that findings support “the hypothesis that mathematics achievement is enhanced by music training, provided the training is for an extended length of time (two or more years) and provided that the music lessons are private” (p. 4).

Vaughn (2000) further investigated the association between self-selected music study and mathematics achievement by conducting a meta-analysis of eight correlational studies from the years 1950 through 1999. Through the analysis of these studies, a mean effect size of $r = 0.14$ was calculated, which the researcher suggested was a robust effect size, not likely due to chance. Vaughn concluded that research demonstrates “a modest positive association between the voluntary study of music, on the one hand, and mathematical achievement, on the other hand” (Vaughn, 2000, p. 154). However, the researcher reminded readers that “the claim that involvement in music improves math achievement is consistent with – but not proven by – the positive effect size found” (p. 155).

Vaughn’s (2000) conclusions have been further investigated and expanded upon by more recent research indicating that quality of music instruction has a significant impact on music’s ability to enhance mathematics performance. In a study conducted to investigate the relationship between participation in school music programs and standardized test results, Johnson and Memmott (2006) found that students who participated in high-quality school music programs outperformed students who participated in low-quality school music programs on standardized tests for reading and mathematics. Research participants included third- and fourth-grade and eighth- and ninth-grade students from four regions across the United States: South, East Coast, Midwest, and West Coast. The elementary and middle schools chosen to participate in the study were identified as having either high-quality or low-quality music programs by music education professors familiar with each school's music program. While the researchers found a strong positive relationship between the quality of school music programs and students' performance on standardized tests, they pointed out that the existence of this strong relationship is not meant to imply that the high-quality music experience caused the differences in test scores.

Researchers have also expanded Vaughn's (2000) findings by investigating the effect of music instruction in conjunction with other factors believed to impact student performance such as socioeconomic status and home environment. For example, Kinney (2008) conducted a study in which achievement scores of sixth- and eighth-grade students were analyzed to determine whether students' participation in band or choir had an effect on student achievement. Socioeconomic status and home environment (two-parent and single-parent households), were also included in the analysis. Kinney found that sixth-grade students who participated in band performed significantly better on each portion of a standardized achievement test (Reading, Math, Science, and Citizenship) than students who participated in choir or did not participate in school music programs at all. No significant differences were found between students who participated in choir and students who did not participate in either program. Analysis of eighth-grade students' performance showed similar results with band students performing significantly better than students who participated in choir or did not participate in school music programs at all. Again, no significant differences were found between students who participated in choir and students who did not participate in either program. Looking at students' prior standardized test scores revealed significant differences in achievement before enrollment in music programs. Kinney asserted that "higher achieving students may be more attracted to instrumental music instruction from the outset" (p. 157). This was not the case for students who enroll in choir, however: "it is clear from these data that choir students were not higher achievers from the outset, as was the case for band students" (p. 157).

While researchers continue to report positive associations between self-selected music study and mathematics achievement (see for example Baker, 2012; Helmrich, 2010; Prokop, 2011),Vaughn (2000) pointed out "for a test of causal power of music to improve math we must turn to the experimental studies" (p. 155). Vaughn (2000) conducted further analysis on five experimental studies in which students were given instrumental or vocal instruction and then tested on mathematics skills. The dates of these studies ranged from 1959 to 1999. Vaughn calculated a mean effect size for these studies of r = 0.16 after weighting and analysis revealed that is was "highly unlikely that the positive effect size found was due to chance" (p. 157). However, Vaughn indicated that experimental research has been inconclusive.

Experimental research continues to be conducted to investigate the relationship between music training and mathematics performance, as in the study conducted by Whitehead (2001). In this study, 28 participants between the ages of 11 and 17 were randomly assigned to three experimental conditions: full treatment, limited treatment, and no treatment. Full-treatment participants received daily music instruction for 50 minutes per day for 20 weeks while limited-treatment participants received music instruction for 50 minutes once a week for 20 weeks. No-treatment participants did not receive any music instruction during the 20 weeks. Music instruction included lessons on the recorder, instruction in music theory, movement, singing, creativity, and instruction on other instruments, including non-pitched percussion instruments. Using a standardized mathematics test for pre- and post-testing, the researcher found that students who received music instruction (full treatment and limited treatment) performed significantly better than students who received no music instruction (no treatment). Furthermore, the full treatment group significantly outperformed students who received music lessons only once per week.

In contrast, Costa-Giomi (2004) conducted an experimental study to determine the effects of three years of piano instruction on fourth-grade students' self-esteem and academic achievement. While the researcher found that piano instruction had a positive effect on students' self-esteem, private piano lessons did not have a significant effect on mathematics achievement as measured by standardized test results.

Music instruction, along with increasing students' self-esteem, can be used to motivate mathematics learning. Courey, Balogh, Silker, and Paik (2012) conducted a study to determine the effects of a music intervention on third graders' understanding of fractions. The content taught through the music intervention included basic music notation, the connection between fractions and music notes, the addition and subtraction of fractions using music notes to represent the fractions, and the addition and subtraction of fractions using other representations including fraction circles, fraction tiles, and the number line. While the researchers found no significant difference between the experimental and comparison groups in students' performance on the fraction concepts posttest, the intervention was particularly effective for students who were initially behind with respect to fraction understanding. The researchers also discovered that students in the experimental group were less likely to make common computation errors related to fractions including adding across numerator and denominator.

When taken in its entirety, the research base for studies investigating the effects of music training on mathematics performance leaves many questions unanswered. Vaughn (2000) asserted, "there is a dearth of existing evidence testing the hypothesis that music training enhances performance in mathematics, and I conclude that the hypothesis has not yet been adequately put to the test" (p. 163). Not only has a causal relationship between music training and mathematics performance not been fully demonstrated, but the reasons for such a relationship remain vague, at best.

Music Theory and Mathematics

Educators are quick to point out the various mathematics connections between music theory and mathematics. As Bahna-James (1991) suggested "while mathematics and music do share certain concepts, the similarities are between mathematics and music *theory*, not between mathematics and music in general" (p. 479). However, a review of the literature revealed a gap in research that investigates the relationship between the study of music theory and mathematics learning.

Bahna-James (1991) conducted a study to investigate whether students attending a New York high school of the arts with certain musical strengths also showed strength in related mathematics areas. Students enrolled in six sections of music theory, a course required for all students studying music, constituted the sample population. A questionnaire designed specifically for the study was administered to research participants to determine music and mathematics background. “Significant relationships between sight-singing and arithmetic, algebra, geometry, and graphing; rhythmic dictation and arithmetic, algebra, geometry, and logic; pitch and arithmetic; key signatures and arithmetic, algebra, geometry, trigonometry, graphing, and calculus; and chords and arithmetic, algebra, geometry, graphing, and calculus” were uncovered (p. 483-484).

In a study to determine the best predictor of music theory grades, Harrison (1990) found that measures of academic ability and achievement, such as the SAT, piano study, principal instrument, and performance on more than one instrument were the best predictors of achievement in freshman level music theory courses, even more so than musical aptitude. Moreover, SAT Math scores were found to be significant predictors of achievement in second semester music theory classes.

Research investigating the relationship between the study of music theory and mathematics performance is limited. Correlational studies have indicated that there exists a relationship between music theory scores and mathematics scores, however no experimental research investigating a causal relationship between the study of music theory and increased mathematics performance was located. Thus, there is need for additional research to investigate whether such a relationship exists and, if so, to what extent music theory has the ability to support mathematics performance.

Music Training and Problem Solving

Music training has been shown to increase performance on spatial-temporal tasks (Costa-Giomi, 1999; Hetland, 2000a; Rauscher, Shaw, Levine, Ky, & Wright, 1994;

Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997; Rauscher & Zupan, 2000), which researchers contend is an integral component of problem solving, but has not been shown to increase performance on other types of tasks, such as tasks included in the Raven's Progressive Matrices Test. Rauscher and Shaw (1998) asserted that the Raven's Progressive Matrices Test is considered a measure of general analytic ability rather than spatial ability. This leads the current researcher to wonder whether music training has the capacity to increase performance on more general mathematical problem-solving tasks.

While a review of literature revealed a gap in research investigating the impact of music training on mathematical problem-solving, the review of literature did uncover several studies investigating the problem-solving process musicians use when learning and composing new music. For example, McAdams (2004) discussed the problems encountered by a composer during the creation of a new composition and how he addresses such problems while Chafin, Imreh, Lemieux, and Chen (2003) investigated how an expert pianist addresses problems during her preparation for the performance of a new piece of music.

McAdams (2004) conducted an observational study to investigate the problem-solving strategies used by a composer during his work on a new piece of music written for piano, orchestra, and computer-processed sound. Three compositional problems that needed to be solved were identified as part of the study: (a) complexities due to the various instruments that the piece would need to be written for, (b) the parallel structure used to organize the two parts of the piece, which needed to be written so that the parts could be played in either order, and (c) problems attributed to the computerized version needing to be played during the second half of the piece, no matter the order that the two parts were played in. McAdams analyzed the composer's notebooks and sketches, the final score of music, and approximately 20 hours of interviews with the composer. Several problem-solving strategies used by the composer during his writing process were identified. First, the composer was identified as a pianist and was found to use his knowledge of orchestral instrument ranges to coordinate what was "initially conceived as pianistic gestures to an orchestral realization" (p. 412).

Another problem-solving strategy utilized by the composer was the construction of various graphic representations (i.e., sketches and maps) to communicate "a spatial, architectonic conception of the global form" (p. 418). Finally, the third strategy identified by the researcher involved the composer breaking the piece down into discrete subsections based on basic thematic material to develop points of transition within the piece. McAdams asserted that the composer's method involved the use of different kinds of visual aids as well as "the ability to break down a problem into components that can be addressed individually" (p. 427).

Chafin, Imreh, Lemieux, and Chen (2003) followed a professional concert pianist for 10 months as she prepared for the performance of a new piece of music. Practice sessions were videotaped and analyzed to determine behavior that indicated the main focus of the pianist's attention and the musical dimensions which affected her practice. The researchers identified four "dimensions of a composition that a pianist must attend to and make decisions about while learning and performing" (p. 469): (a) basic dimensions such as patterns, fingering, and technical difficulties; (b) interpretive dimensions such as phrasing, tempo, and pedaling; (c) performance dimensions such as expressive and interpretive cues; and (d) the formal structure of the musical composition. Analysis indicated that the pianist began with an artistic image, or big picture, of the piece of music and, as practice progressed, the pianist's image of the piece progressed. Furthermore, the pianist exhibited expert problem-solving characteristics in her ability to anticipate later developments such as tempo, phrasing, and technical difficulties. Overall, Chafin et al. asserted that developing an artistic image of the piece is a vital initial step when learning a new piece of music.

Chafin et al. (2003) further contended that expert musicians process new music in a similar manner as experts in other fields address problems, by focusing on the big picture rather than on superficial characteristics: "If forming an artistic image of a piece before starting work on technique is akin to identifying the underlying principles involved, then expert musicians appear to approach the learning of a new piece in much the same way that experts in other domains approach new problems, by starting with the big picture" (p. 467). Schoenfeld and Herrmann (1980) came to similar conclusions regarding the domain of mathematics asserting that expert mathematical problem solvers perceive mathematics problems based on the deep structure of the problem rather than surface structure. McAdams (2004) identified specific strategies utilized by a composer during the writing of a new composition including the use of graphic representations and the strategy of breaking a problem down into subproblems. Analogous strategies are commonly discussed as viable strategies for solving mathematical problems (see for example Johnson, Herr, & Kysh, 2004; Polya, 1945). As Schoenfeld and Hermann have alluded, experience with problem solving can impact the way in which mathematics problems are approached and solved. Researchers have identified other factors that affect mathematical problem solving.

Factors Impacting Mathematical Problem Solving

Mathematical problem solving is an area that has been highly researched. For example, recent research has focused on various aspects of mathematical problem solving including the impact of problem-solving instruction on problem-solving ability and the relationship between metacognitive behaviors and problem-solving ability.

Both aspects of problem solving have been explored by Schoenfeld and Herrmann in their seminal research *Problem perception and knowledge structure in expert and novice mathematical problem solvers* (1980). In an experiment to investigate how novice and expert problem solvers perceive certain tasks, Schoenfeld and Herrmann (1980) asked expert problem solvers (nine mathematics professors) and novice problem solvers (19 undergraduate students) to sort a set of 32 mathematics problems into groups so that groups consisted of problems that were "similar mathematically in that they would be solved the same way" (p. 5). Data revealed that experts perceived problems based on deep structure (e.g., based on mathematical structure) whereas novice problem solvers perceived problems based on surface structure (e.g., based on wording). In another experiment, Schoenfeld and Herrmann (1980) investigated whether the perceptions of novice problem solvers would come to resemble that of experts after receiving problem-solving training. The same 19 undergraduate students who participated in the first experiment also participated in the second experiment. The participants were divided into an experimental group and a control group. Students in the experimental group took a course in "Techniques of Problem Solving" (p. 9) taught by Schoenfeld. Results indicated that, not only did the problem-solving ability of students in the experimental group significantly improve where the control group did not, but participants in the experimental group also experienced a dramatic shift in in the way they perceived problems moving their perception of problems closer to that of expert problem solvers. Thus, “as the students acquire problem-solving expertise… not only their performance, but also their perceptions, become more like experts’” (p. 14).

Many researchers, like Schoenfeld and Herrmann (1980), have been able to show that problem-solving instruction has the potential to increase problem-solving ability. For example, Mousoulides, Christou, and Sriraman (2008) found that including modeling activities in the classroom improved students’ ability to solve problems using modeling strategies. In another study to compare the effects of problem-solving instructional strategies, researchers found that schema-based instruction was more effective than general strategy instruction in increasing third-grade students’ problem-solving skills and performance on a statewide mathematics achievement test (Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007). Additionally, through the use of an interdisciplinary program incorporating reading, music, and mathematical problem solving, Rousseau (2009) discovered students became more successful problem solvers though their attitudes toward problem solving did not change.

Other instructional interventions that have been shown to increase students’ problem-solving performance are productive failure (Kapur & Bielaczyc, 2012) and problem-solving journaling (Hensberry & Jacobbe, 2012). Kapur and Bielaczyc (2012) found that exposing seventh grade students in Singapore to productive failure situations, where students work collaboratively to solve complex mathematics problems without instructor support, resulted in increased problem-solving performance and greater flexibility in representing problem situations graphically. Hensberry and Jacobbe (2012) investigated whether structured problem-solving diaries impacted the problem-solving performance of African-American elementary students. Structured problem-solving diary entries asked students to respond to prompts related to understanding the problem, devising a plan, and carrying out the plan. The researchers found that, while the majority of the students who participated in the research study improved in their use of problem-solving strategies, the heuristic included in the diary entries were not internalized. Hensberry and Jacobbe concluded that journaling about problem solving in a way that follows Polya's heuristic of understand, plan, and carry-out has the potential to increase students' problem-solving ability.

Metacognitive knowledge has also been found to impact problem-solving performance. Swanson (1990) conducted a study to determine whether metacognitive knowledge could compensate for low aptitude when children are engaged in mathematical problem-solving tasks. Findings indicated that the fourth- and fifth-grade children ranked as having high metacognitive knowledge outperformed children having low-metacognitive knowledge "regardless of their overall aptitude level" (p. 312). Swanson concluded that problem-solving performance is more closely related to high metacognitive knowledge than high aptitude.

In a study which combined the investigation of problem-solving training and metacognition, Ozsoy and Ataman (2009) investigated the impact of metacognitive strategy training and problem-solving achievement. Like problem-solving instruction and increased metacognitive knowledge, metacognitive strategy training was found to be an effective means for increasing students' problem-solving performance.

Along with metacognitive factors, cognitive factors such as level of spatial understanding have been shown to be related to mathematical problem solving (Edens & Potter, 2007). In a study of fourth and fifth grade students, Edens and Potter observed that "students at the highest level of spatial understanding more successfully problem-solved than those students at the lowest level of spatial understanding" (p. 292).

While "mathematical problem solving is a complex cognitive activity" (Zhu, 2007, p. 188) and there are many aspects of problem solving worth investigating, the problem solving literature reviewed for this study is focused on factors that researchers have identified as impacting the selection and utilization of problem-solving strategies.

Factors Impacting the Selection of Problem-Solving Strategies

Siegler (2003) indicated that there are substantial differences in the kinds of strategy choices that students make. Furthermore, "these involve differences in knowledge and differences in cognitive style" (Siegler, 2003, p. 224). In fact, researchers have identified many factors that impact the way students select problem-solving strategies when engaged in a problem-solving task: gender and stereotype threat (Battista, 1990; Gallagher & DeLisi, 1994; Quinn & Spencer, 2001), problem-solving experience (Schoenfeld & Herrmann, 1980; Stylainou, 2011), the type of problems or the format in which a problem is presented (Katz, Bennett, & Berger, 2000; Michaelides, 2002; Nathan & Koedinger, 2000; Walkington, Petrosino, & Sherman, 2013), metacognitive behavior (Cifarelli, Goodson-Espy, & Chae, 2010; Hoffman & Spatariu, 2008; Mamona-Downs & Downs, 2008; Mitchum & Kelley, 2010), cognitive factors such as working memory (Beilock & DeCaro, 2007) or spatial versus analytic preferences (Battista, 1990; Booth & Thomas, 2000; Campbell, Collis, & Watson, 1995; Hegarty & Kozhevnikov, 1999; Lean

& Clements, 1981), and problem-solving instruction (Rittle-Johnson, Star, & Durkin, 2012). Factors that impact the way students select and utilize problem-solving strategies are discussed below.

Gender Issues

The reality that men outperform women in mathematics has been well documented in literature (see for example the report compiled by Hill, Corbett, & St. Rose, 2010). Even more, spatial reasoning scores for males have consistently been found to be higher than that for females (Battista, 1990; Johnson & Meade, 1987; Lynn, Allik, & Irwing, 2004). However, as Battista asserted, "knowing that such gender differences exist… provides little insight into how instructional practices can accommodate these differences of how spatial visualization affects the processes students use when doing mathematics" (p. 47).

Battista provided insight into how these differences can be accommodated by conducting a study to investigate the nature of gender differences in geometry performance. Data on 128 high school geometry students was collected and analyzed to determine how spatial visualization, logical reasoning, and the discrepancy between the two affect performance in geometry. Students' ability to mentally visualize rotation of objects in space was measured through the use of a modified version of the Purdue Spatial Visualization Test: Rotation (Guay, 1977). Students were given 20 of the 30 original multiple-choice questions from the test and were asked to complete the test in eight minutes. Battista gave students this shortened amount of time "to better measure students' ability to transform mental visual images of figures as organized wholes, which can be done quickly, rather than to analytically process relationships between different parts of figures, which is more time consuming" (p. 50). An existing measurement, the Cooperative Mathematics Test (Geometry, Part 1, Form B), was also used to measure geometry achievement with respect to basic geometry skills. Researcher-constructed instruments were used to measure students' logical reasoning and students' geometric problem solving and strategy utilization.

When analyzing problem-solving strategies, students' responses were classified as drawing, visualization without drawing, nonspatial, or none of the above. Results indicated that male students scored significantly higher than female students on spatial visualization, geometry achievement, and geometric problem solving, but not on logical reasoning. In addition, there was not a significant difference between the problem-solving strategies used by male and female students. Battista concluded that "the discrepancy between spatial visualization and verbal-logical reasoning ability may be a contributing factor to gender-related performance differences on mathematics with a visual component" (p. 57). Furthermore, the researcher asserted that, while both spatial visualization and logical reasoning were important factors in geometry problem solving and achievement, low-achieving geometry students tend to take a more visual approach to the subject.

While Battista indicated that there was not a significant difference between the problem-solving strategies used by male and female students, other researchers have made contradictory conclusions. For example, in a study investigating whether problem-solving strategies utilized by male and female students could be used to explain gender differences observed on the mathematics portion of the Scholastic Aptitude Test (SAT-M), Gallagher and DeLisi (1994) found that female students have a tendency to rely on conventional problem-solving strategies while male students are more apt to use unconventional strategies. Participants of the study consisted of 25 male and 22 female junior and senior high school students who had scored 670 or above on a recent administration of the SAT-M. To investigate students' problem-solving strategies, the researchers constructed a problem set consisting of 27 items taken from five forms of the SAT-M and asked participants to think aloud while solving the problems. Interviews were recorded, transcribed, and categories were developed based on solution strategies. Through this process, eight categories were identified: (1) algorithm, (2) insight with algorithm, (3) logic, estimation, or insight, (4) assign values to variables, (5) plug in options, (6) guessing, (7) no strategy, and (8) misinterpretation. Strategies were then classified as conventional strategies, strategies that represented "standard computational strategies" such as (1), (4), and (5), or unconventional strategies, those "strategies or application of strategies that generally are not taught in school" such as (2) and (3) (p. 207). Results of this study indicated that "female students are more likely than male students to use solution strategies provided by the teacher are, as a result, less likely to do well on novel problems for which they have not learned a specific solution strategy" (p. 210). However, Gallagher and DeLisi also pointed out that "female students are generally better at tasks that require rapid retrieval of information from memory; whereas male students are usually better at tasks that require the manipulation of information that is already represented in memory" (p. 210).

More recent research has expanded on this idea by exploring whether preference differences based on gender also influences the use of retrieval-based problem-solving processes. In a longitudinal study, Bailey, Littlefield, and Geary (2012) determined that girls and boys in first grade were on equal footing with respect to the accuracy of retrieval of arithmetic facts. However, as students progressed to sixth grade, boys' retrieval accuracy steadily increased, while girls' accuracy remained relatively constant, resulting in boys outperforming girls in retrieval accuracy. The researchers also found differences in preference as related to gender. While boys tended to prefer retrieval method to alternative problem-solving strategies, girls often resorted to more time-consuming, but accurate counting strategies. The researchers hypothesize that boys prefer retrieval strategies because "they seek to answer single-digit addition problems faster and more often than girls, perhaps because they are more competitive or less perfectionist" (p. 90).

Che, Wiegert, and Threlkeld (2011) also noted difference in the problem-solving strategy preferences for boys and girls. When investigating the strategies used by sixth graders, the researchers concluded that boys are more likely to use unconventional problem-solving strategies while girls tend to focus on familiar strategies. Che, Wiegert, and Threlkeld hypothesized that the difference in strategy selection of the boys and girls and girls' struggle to solve open-ended problems in the study may be related to a socialization process in which females are "manipulated by the stereotype of 'good girls follow the rules'" (p. 324).

Quinn and Spencer (2001) also asserted that women are affected stereotypes, particularly the belief that men are better at math. Furthermore, "these stereotypes are transmitted in the culture in a variety of ways, including through mass media, books, parents, peers and teachers" (p. 56). With this in mind, Quinn and Spencer (2001) conducted a replication study of the Gallagher and DeLisi (1994) study to investigate how “the heightened anxiety and diminished cognitive capacity that accompanies stereotype threat” affect women’s ability to solve mathematics problems (p. 59). The participants of the study consisted of 36 students enrolled at the University of Michigan. The researchers administered a test consisting of difficult word-problems pulled from the SAT-M. Participants were split into two groups: a testing situation with high stereotype threat and a testing situation with reduced stereotype threat. Each participant took part in a tape-recorded testing session. Before the testing session, participants in the reduced stereotype threat group were told that prior use of the test had shown that men and women performed equally well on the problems contained in the test. Results of the study indicate women in the high stereotype threat condition did not perform as well as men and were less able to formulate problem-solving strategies than men; however, the women in the reduced stereotype threat condition performed as well as men and were just as able to formulate strategies to solve the problems on the test. The researchers concluded that the depression in women’s performance in testing situations “can be partly explained by the interference of stereotype threat with the ability to formulate problem-solving strategies” (p. 67).

Other areas of research have focused on gender differences due to differences in spatial reasoning ability. Males have an advantage in spatial ability and this advantage emerges at least as early as the fourth grade, around the age of ten (Johnson & Meade, 1987). Furthermore, the male advantage in spatial ability is present across racial differences. Johnson and Meade (1987) asserted that it is possible that “verbal precocity of young girls gives them an advantage in testing situations” and that “verbal precocity may predispose girls to adopt a verbal strategy for solving spatial problems” (p. 736).

Cognitive Factors

It may seem obvious that cognition plays a role in students' selection and utilization of problem-solving strategies. However, researchers have focused on specific areas of cognition believed to impact problem-solving behavior, for example, working memory (Beilock & DeCaro, 2007) and reasoning preferences (Booth & Thomas, 2000; Campbell, Collis, & Watson, 1995; Hegarty & Kozhevnikov, 1999).

Working memory is a cognitive factor that can impact students' selection of a problem-solving strategy. Beilock and DeCaro (2007) conducted two experiments to "demonstrate how differences in working memory (WM) impact the strategies used to solve complex math problems" (p. 983). In the first experiment, the working memory of 92 undergraduate students was measured using established measurement instruments. These students were then assigned to either a low-pressure or high-pressure testing condition. Beilock and DeCaro hypothesized that "if pressure-induced consumption of WM denies individuals the resources necessary to compute demanding rule-based computations, then those individuals who rely most heavily on such processes to begin with (e.g., higher WM individuals) should be most likely to choke under pressure" (p 985). The researchers validated this hypothesis: "Pressure prompted higher WM individuals to use simpler (and less efficacious) problem-solving strategies of the type typically used by those lower in WM" (p. 994). In the second experiment, 45 low-pressure participants and 46 high-pressure participants were asked to complete tasks that could be accomplished by using either a difficult, working memory demanding rule-based algorithm or by using a much simpler formula. This experiment revealed that "under low-pressure conditions, individuals lowest in WM were most likely to recognize the shortcut strategy when available. Under high-pressure, higher WM individuals recognized the shortcut strategy at a level equal to their lower WM counterparts" (p. 994). Together, these experiments demonstrated that working memory and environment affect problem-solving approach. In general, "to the extent that higher WM individuals are especially good both at focusing their attention on select task properties and at ignoring others, these individuals may actually be worse at detecting alternate problem solutions" (p. 995).

Working memory has also been found to impact problem solving in students of a younger age. Research conducted by van der Ven, Boom, Kroesbergen, and Leseman (2012) revealed that second grade children with high working memory abilities surpass their counterparts with low working memory abilities in the maturity of strategy selection (using multiple strategies) as well as in accuracy.

A specific subset of cognitive factors that affects the selection of problem-solving strategies is an individual's preference for using spatial versus analytic means to solve problems (Battista, 1990): "Students classified as analytic, for instance, tend to use a more complicated logical-analytic method of solution even for problems that would yield to a relatively simple visual approach, whereas students classified as visual-pictorial would attempt to use visual schemes even for problems more easily solved with analytic means" (Battista, 1990, p. 47). Zhu (2007) claimed "higher ability students tend to solve problems using more spatial processes, while others tried to solve problems in a more analytical way" (p 190). However, Campbell, Collis, and Watson (1995) have reported contradictory results. Clearly, research had not been able to clarify "whether persons who prefer to use visual imagery, with little verbal coding, when processing mathematical information are likely to do better on certain mathematical tasks than persons who prefer a verbal-logic processing mode” (Lean and Clements, 1981, p. 278). In fact, Aldous (2007) indicates that expert problem solvers move back and forth between visual-spatial and analytic-verbal reasoning when engaged in a problem-solving task. More recently, researchers have begun to investigate how spatial and analytic reasoning work together during problem solving instead of considering the two types of reasoning separately.

Campbell, Collis, and Watson (1995) conducted a study to investigate “the different contributions made to visual thinking by abstract logical reasoning abilities and by the ability to form rich specific imagery” (p. 180). Two screening assessments were administered to the group of participants, which consisted of 100 tenth-grade students. The first assessment measured mathematical operational ability and the second assessment determined the vividness of participants’ visual imagery. Participants with the highest and lowest scores on the two assessments were selected to participate in individual problem-solving sessions resulting in four groups of participants: low operational and low vividness, low operational and high vividness, high operational and low vividness, and high operational and high vividness. Analysis of data revealed that “success in problem solving was related to a logical reasoning component of mathematical ability rather than to an ability to form vivid visual images” (p. 191). Furthermore, Campbell et al. insisted that these results are “largely related to students’ preferences to use particular visual strategies when solving specific problems” and that “further research is needed to investigate the relationship between preferences and ability” (p. 191).

However, Hegarty and Kozhevnikov (1999) hypothesized that many studies, such as the study conducted by Campbell et al. (1995), may not find significant relationships between visual-spatial reasoning and problem solving because “characterizing students as visualizers and verbalizers is too general a classification” (p. 688). The researchers contended that there are actually two types of visualizers: schematic visualizers, who are good at “representing the spatial relationship between objects and imagining spatial transformations,” and pictorial visualizers, who are especially good at “constructing vivid and detailed visual images” (p. 685). In a study of 33 sixth-grade boys, Hegarty and Kozhevnikov administered a Mathematical Processing Instrument to measure level of visuality, the Drumcondra Verbal Reasoning Test (Educational Research Center, 1968), the Raven’s Progressive Matrices Test to measure nonverbal reasoning (Raven, 1958), and two measures of spatial reasoning: the Block subtest of the Wechsler Intelligence Scale for Children (Wechsler, 1976) and the Space subtest for the Primary Mental Abilities Test (Thurstone & Thurstone, 1947). Through this study, the researchers deduced that the use of some visual-spatial representations, such as schematic representations, promote problem-solving performance while the use of other visual-spatial representations, such as pictorial representations, can actually be a barrier to problem-solving success.

Booth and Thomas (2000) asserted that, while “it is generally accepted that spatial ability is involved in the learning and understanding of mathematical concepts” (p. 169), studies have “failed to demonstrate an advantage of high spatial ability for mathematical problem solving” (p. 170). In a study to examine the relationship between spatial ability and arithmetic problem-solving performance, Booth and Thomas measured participants’ mathematics skills, spatial ability, and arithmetic problem-solving ability. Participants completed a standardized mathematics achievement test, a “battery of seven spatial tests” (p. 174), and then participated in arithmetic problem-solving interviews during which they were encouraged to talk about their approach to finding solutions to multiple mathematics problems. Through this investigation, the researchers found that participants with higher spatial ability performed significantly better on the arithmetic problems. Furthermore, these participants performed better regardless of whether a picture, diagram, or neither was provided with the question. With respect to preferred method of solution, Booth and Thomas revealed that, although participants with high spatial ability and low spatial ability alike had a tendency to not utilize a given or self-constructed picture or diagram, high spatial ability participants were more successful with this method. According to the researchers, this may be possible because high spatial ability participants “have the skills available to perform calculations mentally” or because “their superior spatial skills are facilitating an improved use of a mental image of the diagram to help them solve the problem” (p. 180).

Metacognitive Factors

Cifarelli, Goodson-Espy, and Chae (2010) acknowledged that “it is important to consider the role that mathematical beliefs play in students’ problem-solving behaviors” (p. 206). This consideration not only allows instructors to design curriculum that “promotes positive mathematical beliefs in support of productive problem solving” (p. 206), but also provides insight into how students approach problem solving. Similarly, Mamona-Downs and Downs (2008) identified monitoring and control as a key aspect of problem solving and asserted that “self-evaluation of your work and practices” is perhaps the most critical aspect in advanced mathematical thinking (p. 162). Mitchum and Kelley (2010) extended this idea by contending that "accurate monitoring is a key component of cognition" (p. 699). In fact, Ozsoy and Ataman (2009) claimed that "metacognition plays an important role during each level of mathematical problem solving" (p. 71).

Cifarelli, Goodson-Espy, and Chae (2010) conducted a study to investigate the relationship between students' self-regulated problem-solving activity and their views toward mathematics. Students' views of mathematics were measured with respect to self-efficacy beliefs and mathematics understanding (relational versus instrumental). Participants included 12 university college algebra students who volunteered to participate in interviews. Of the 12 participants, 6 were identified as having an instrumental view of mathematics and 6 were identified as having a somewhat instrumental view of mathematics as measured by an instrumental versus relation understanding of mathematics survey (Skemp, 1976). With respect to mathematics self-efficacy, 5 participants were identified as having negative attitudes, 3 as having somewhat negative attitudes, 2 as somewhat positive attitudes, and 2 were identified as having positive attitudes as measured by Yackel's (1984) Mathematical Belief Systems Survey. Through individual participant problem-solving interviews, the researchers observed that "participants who are somewhat instrumental in their beliefs used a more conceptual strategy to solve problems than participants who are instrumental in their beliefs" (p. 218). Furthermore, the two participants identified as having high mathematics self-efficacy beliefs used more complex solution strategies and demonstrated persistence in difficult problem-solving situations. However, the researchers pointed out that high self-efficacy did not always help students progress toward a solution.

Experience—Novice and Expert Problem Solvers

Schoenfeld and Herrmann (1980) have demonstrated that there exist differences in how expert and novice problem-solvers perceive problems. Moreover, they have been able to show that problem-solving training has the potential to move novice problem solvers' abilities and perceptions toward that of expert problem solvers. However, other researchers (i.e., Stylainou, 2011) have been interested in exploring how strategies utilized by expert and novice problem solvers differ.

Stylainou (2011) conducted a study to investigate how expert mathematics problem solvers and novice mathematics problem solvers utilize representation during problem-solving activity. Research participants included ten practicing mathematicians (expert problem solvers) and 24 sixth-grade students. Each expert problem solver was asked to solve non-standard Calculus problems during an individual interview. Data from novice problem solvers was gathered during video-taped classroom lessons and individual interviews. Stylainou found that both expert and novice problem solvers use representation as tools to understand information, to record, to facilitate exploration, to monitor and evaluate problem-solving progress, and to present their work. However, novice problem solvers used fewer representations and, unlike expert problem solvers, may not realize when a representation takes on a different role due to context or need. Moreover, novices' lack of ability to flexibly move between representations can be a barrier to problem-solving success.

Problem Type and Format

Nathan and Koedinger (2000) insisted that, when investigating problem solving, two important factors need to be considered: "(a) the position of the unknown quantity in the problem and (b) the linguistic presentation of the problem" (p. 169). Thus, problem format can also impact students' problem-solving approach.

Katz, Bennett, and Berger (2000) conducted a study to investigate whether the problem-solving strategies utilized by students solving algebra problems differed depending on the format of the problem and whether the strategies utilized had a significant impact on students' performance. In this study, students' problem-solving strategies were categorized as either "traditional" or "nontraditional." Strategies were classified as "traditional" strategies, those strategies in which mathematical operations implied by the question stem are directly carried out or in which equations are written to represent the relationship described in the question stem and then solved, or "nontraditional" strategies those, strategies in which students make an estimate of the solution, however determined, and then check this estimate against the information presented in the problem. Participants consisted of 55 high-school students who were selected based on their scores on the mathematics portion of the Scholastic Aptitude Test (SAT-M), gender, and scheduling availability. Participants were grouped by ability level, low, medium, and high, based on their SAT-M score. Each group consisted of an approximately equal numbers of males and females. The researchers administered a 20-question which consisted of 20 items, 10 multiple-choice items and 10 constructed-response items. Multiple-choice and constructed-response items were designed to be stem-equivalent so that the same mathematical knowledge was being tested regardless of question format, with the intention that the only difference between corresponding multiple-choice and constructed-response items was the availability of answer options. To determine problem-solving strategies used, participants were asked to verbalize as they solved each problem. Since constructed-response items tend to be more challenging for students to solve, the researchers hypothesized that students would be more likely to utilize traditional solution strategies to solve constructed-response items where, when solving multiple-choice items, students would be more apt to utilize nontraditional solution strategies such as guess-and-check since possible answers are present. However, contradictory to their hypothesis, the researchers concluded that "instead of solution strategy mediating the effects of format on difficulty, the results suggest that comprehension factors mediate the effect of format on both strategy choice and difficulty" (p. 53). Moreover, some strategies previously thought to be applicable only to multiple-choice items actually have corresponding constructed response strategies. In fact, for some items, "participants adopted nontraditional strategies more often for constructed response items than for multiple-choice counterparts" (p. 54).

Current research efforts continue to explore the impact of task format on problem-solving strategy selection and utilization. Ibrahim and Rebello (2012) conducted a study to determine the impact of task format on problem-solving strategies in a university calculus-based physics course. Tasks included for this research were related to kinematics and work. Results indicated including an equation or formula within the statement of the problem task limited students thinking to quantitative approaches. On the other hand, including graphical representations along with the statement of the problem task caused problems with comprehension. When students were unsure how to interpret a graphical representation, they reverted to equations as a problem-solving strategy.

Investigation regarding the impact of problem format on strategy selection has also included the exploration of personalized problem scenarios. Walkington, Petrosino, and Sherman (2013) conducted a study to examine whether presenting students with mathematical problem-solving tasks in a context personalized to the students' interests effected performance and strategy usage. Using a sample of 24 high school students enrolled in Algebra I, the researchers found personalized problem scenarios resulted in improved problem-solving performance, especially when students worked on challenging problems. Also, data analysis indicated that the personalized problem scenarios "seemed to elicit informal strategies, cueing these students that this type of reasoning was acceptable and valued" (p. 106). Informal strategies, including repeated addition, trial-and-error, and unwinding, were compared to more formal strategies such as the writing of formal algebraic equations. Overall, the researchers concluded "personalizing instruction to out-of-school interests seemed to provide an additional resource for some students who were faced with a challenging algebra story problem" (p. 109). On the other hand, personalized problem scenarios did not appear to be an effective scaffold for high-scoring students when working on easier problems and were found to cause possible distractions as students worked on problems.

Michaelides (2002) conducted a study with 107 randomly selected fifth through eighth grade students to investigate problem-solving strategies children utilized when solving spatial rotation problems. Through a multiple-choice test and follow-up interviews with a sample of 31 students, the researcher found that students did not show a preference for visual versus non-visual strategies on the spatial rotation tasks. However, Michaelides noted that when students applied visual problem-solving strategies, the approach seemed to be more holistic and intuitive and difficult for students to articulate during interviews. The researcher also found that participants were more likely to use analytic problem-solving strategies when the figures to be analyzed were presented using a three-dimensional representation instead of a two-dimensional representation. More specifically, “the nature of each task was critical for the choice of strategy and was probably a more influential factor than gender, age and level of achievement” (p. 21).

Problem-Solving Instruction

Researchers have also investigated the effects of various types of problem-solving instructional interventions on problem-solving performance and strategy approach. Rittle-Johnson, Star, and Durkin (2012) compared the problem-solving ability and strategy use of eighth grade students under three conditions: immediate comparison procedure, delayed comparison procedure, and delayed exposure. Students in the immediate comparison procedure group were asked to identify similarities and differences when presented with two solution procedures for solving a mathematics problem. Immediate comparison students were also asked to identify whether one procedure was more efficient than the other. Students in the delayed comparison procedure group received instruction on the use of one solution procedure before comparing it to an alternative procedure. In the delayed exposure condition, students received instruction on the use of one solution procedure followed instruction on the use of a second solution procedure, and the two procedures were not compared. The researchers found that students in the immediate comparison procedure group “had greater flexibility knowledge, flexible use, and retention of procedural knowledge” than students in the delayed comparison procedure group (p. 450). Flexibility knowledge included the ability to use multiple solution procedures as well as the ability to evaluate non-standard solution procedures while flexible use required frequent use of appropriate shortcut procedures on procedural knowledge items. Rittle-Johnson, Star, and Durkin concluded that including the comparison of solution procedures effectively supports problem-solving flexibility in novice problem solvers.

Utilization of Problem-Solving Strategies

Specific problem-solving strategies students use give insight into how students understand mathematical problem situations and also illustrate how students use this understanding to problem solve (Llinares & Riog, 2008). The following section describes research regarding some of the most commonly researched strategies students use during problem solving: the use of familiar procedures, the utilization of visual images, students' verbal mediation or private speech, the use of guess and check, and the construction of mathematical models including mathematical equations.

Familiar Procedures

Lithner (2000) conducted a study to investigate undergraduate students' difficulties in solving two calculus problems. Participants, consisting of four first-year undergraduate students, were asked to take part in individual video-taped problem-solving sessions in which they thought aloud as they solved the two problems. Each participant also met with the research for a follow-up interview where they reviewed the researcher's written analysis of the problem-solving session and participants were allowed to make comments and clarifications. Through this investigation, Lithner discovered the four participants' problem-solving success was hindered by their focus on remembering familiar procedures and that "this focus is so dominating that it prevents other approaches from being initiated and implemented" (p. 93-94). Moreover, Lithner asserted that focusing on attempting to remember familiar procedures causes students to make careless mistakes during the problem-solving process: "Their focus is to use familiar procedures, and there are essentially no checking comparisons with other type of reasoning that might have detected the errors" (p. 92). Lithner stressed that when students' problem solving is based on a limited type of familiar exercises, "this base is likely to lead them in the wrong direction as soon as the task is not completely familiar" (p. 95).

Drawings, Diagrams, and Mental Imagery

Researchers have investigated students' use of drawings, diagrams, and mental imagery during mathematical problem solving (Pantziara, Gagatsis, & Elia, 2009; Presmeg & Balderas-Canas, 2001). Diagrams can be an effective problem-solving strategy: "In problem solving, diagrams can serve to represent the structure of a problem; thus, it can be a useful tool in the solution of the problem" (Pantziara, Gagatsis, & Elia, 2009, p. 40). As Presmeg and Balderas-Canas (2001) pointed out, though, "there have been claims in the literature that students, particularly at the college level, are reluctant to visualize when they do mathematics" (p. 290) and that "the stressing of algebra at several levels may be the reason for the reluctance to visualize" (p. 307). Therefore, to understand a student's problem-solving process, there is a need "to investigate whether or not a solver visualizes, when he or she does so, and what kinds of imagery are used" (Presmeg & Balderas-Canas, 2001, p. 290).

Pantziara, Gagatsis, and Elia (2009) conducted a study to investigate the impact of diagrams on students' problem-solving process. Using two researcher-created tests, 194 grade six students were asked to solve non-routine mathematics problem. For the first test, students were asked to solve the problems any way they chose. However, in the second test, students were provided diagrams to accompany each question and were required to use the diagrams to solve the problems. The first test was administered one week before the second test. Results from the data analysis indicated that, for some students, the inclusion of diagrams along with the problem scenario made the problem easier to solve, but for other students, diagrams made solving the problem more difficult. Analysis indicated that "pupils with different visual-spatial abilities responded differently in the test" (p. 55). When diagrams were not included with problems, students used a variety of strategies such as trial-and-error, drawing a picture or diagram, working backwards, using an equation, and using logical reasoning.

In a study to investigate when, how, and why students use visualization to solve problems, Presmeg and Balderas-Canas (2001) conducted task-based interviews with four mathematics education graduate students. Each participant solved two sets of nonroutine mathematics problems, each consisting of three problems, during the course of two interviews. For the first interview, three problems were selected from the Preference for Visuality test (Presmeg, 1985) while, for the second interview, three problems were adapted from previous research conducted by Balderas (1998). Through the interviews, Presmeg and Balderas-Canas discovered that the participants used visual imagery, including diagrams and mental imagery, for four reasons: (1) when participants encountered a cognitive obstacle "they resorted to visual imagery, often expressed in diagrams, in their attempts to break the impasse" (p. 307), (2) in preparing to solve a problem, (3) during the solution phase when doubt arose, and (4) during the checking phase, where "diagrams and imagery also played a large role in making sense of their solutions" (p. 308).

Verbal Mediation

A common problem-solving strategy used by students of various ages is verbal mediation, often referred to as private speech (Winsler and Naglieri, 2003). Private speech can be overt, where students talk out loud during problem solving; covert, in which students whisper or make inaudible sounds during problem solving; or fully covert inner speech.

Winsler and Naglieri (2003) conducted a study to investigate the verbal problem-solving strategies utilized by children between the ages of 5 and 17. Participants consisted of 2,156 children from across the United States and were a representative sample of the national population. The researchers administered the Planned Connections subset of the Cognitive Assessment System to test participants' planning ability and expected the use of verbal problem-solving strategies since "the self-regulatory demands of the task were high" (p. 662). Through this study, Winsler and Naglieri found that children move from overt verbal strategies to covert verbal strategies with age: "For 5- to 8-year-olds, overt self-talk was most prominent but by age 17 it was the least common form of verbal regulation. Similarly, inner speech, or fully covert verbal thinking, although present among the youngest age groups, was relatively rare for the youngest children and rose to become the most common form of verbal strategy use among children ages 14 and beyond" (p. 672). With respect to whether the use of verbal strategies is related to children's performance on the task, the researchers concluded that the use of overt and covert verbal strategies was unrelated to performance. Nonetheless, Winsler and Naglieri asserted that "awareness of verbal strategy use on this task might be a prerequisite for strategy effectiveness" since, "children who reported talking to themselves did better than those who did not, and those who were aware of (i.e., reported) covert speech, had higher standardized achievement scores than those who did not" (p. 675).

Guess and Check

Capraro, An, Ma, Rangel-Chavez, and Harbaugh (2012) maintained that "guess and check is one of the heuristic strategies that students use to solve mathematical problems and is an important component of strategic competence" (p. 106). However, "in order to successfully guess and check, students must be able to identify the relationships between quantities that are presented in a problem context" (Johanning, 2007, p. 124). While guess and check is often considered a less sophisticated approach to problem solving, "it is important to look closely at students' systematic guess and check reasoning" (Johanning, 2007, p. 132).

Capraro, et al., (2012) conducted a study to investigate pre-service teachers' use of guess and check. Participants included junior-level pre-service math and science teachers enrolled in a problem-solving course consisting of 64 students. Students were given a mathematics puzzle so the researchers could evaluate how problem-solving strategies were employed by the participants. The puzzle asked students to place numbers in a triangular arrangement so that the sum of each of the sides of the triangle was nine. Of the 64 students enrolled in the course, eight were selected to participate in interviews based on their performance on the problem-solving task. Using a grounded theory approach, the researchers found that the primary strategy used to complete the puzzle was the guess and check strategy. However, each participant was found to utilize one of two different types of guess and check strategies: unsystematic guessing and checking or systematic guessing and checking. Interviews with participants revealed those who utilized an unsystematic guessing and checking strategy "encountered an anxious situation where they repeatedly attempted different erroneous answers" (p. 110). Thus, the researchers concluded that "guessing without organized, rational thinking was not an effective problem-solving strategy and did not always lead to correct answers" and participants who tried to solve the problem using unsystematic guessing and checking "did not understand the entire scope of the problem" (p. 112-113).

Johanning (2007) made a similar conclusion in an investigation of middle school students' use of the guess and check method. Participants included 31 sixth-, seventh-, and eighth-grade students who were asked to solve an algebra word problem involving either an additive or multiplicative relationship between variables. The researcher collected field notes, written solutions to the problems, and conducted interviews with select participants. Conclusions were based on the data collected from 11 participants who utilized the systematic guess and check method. Data analysis revealed that "understanding the structure of the problem is a key component in being able to create a system to use when guessing and checking" (p. 130).

Research indicates that in order for the use of guess and check to be an effective problem-solving strategy, it is essential that students utilize systematic guessing and checking "where the problem solver works with the situational context and applies relational reasoning to solve the problem" (Johanning, 2007, p. 123). Systematic guessing and checking requires that problem solvers "understand the underlying structure of the problem and articulate this into a formal plan or system to use when guessing and checking" (Johanning, 2007, p. 126).

Mathematical Modeling

Building on Johanning's (2007) assertion that effective problem solving requires an understanding of the underlying structure of the problem, Llinares and Riog (2008) contended, "research indicates that students use informal strategies for solving problems in their attempts to give meaning to stated situations" (p. 506). One of the strategies students use to develop understanding of and solutions for mathematical problems is mathematical modeling (Llinares & Riog, 2008).

Llinares and Riog (2008) conducted a study to investigate how secondary students construct and use mathematical models when solving word problems. The sample population consisted of 511 students between the ages of 15 and 18 years old. Using a five-question test comprised of problems that would be included in standard secondary-education curriculum, the researchers asked the students to solve problems and justify their solutions. Analysis of students' responses was conducted on three of the five questions. Also, interviews were led with 71 randomly selected students. Llinares and Riog alleged that, when faced with these types of problems, "a student should somehow represent its structure by identifying the quantities and the relationships between them in order to make a decision and justify it, whereby involving a process of mathematical modeling" (p. 526). Findings revealed that students utilized mathematical modeling in two ways: to translate and to organize. In using models to translate, "once the different quantities involved in the situation and the relationships between them have been recognized, the student is able to find a mathematical content or procedure which models the revealed structure" (p. 526). On the other hand, in using models to organize, models may "emerge as a result of an organizing activity… [reflecting] the different relationships which underlie the situation” (p. 526). Moreover, students utilized the method of investigating specific cases for determining and understanding the structure of the situation: “perceiving the general in the particular and perceiving the particular from the general (making the general meaningful by proposing particular instances) is a characteristic of students’ modeling processes and as such is an important aspect of their abstraction process” (p. 529) Yet, the researchers found that the participants in this study had difficulty recognizing the underlying structure of the problems presented to them and, therefore, had difficulty modeling and solving the problem situations. Llinares and Riog concluded “the difficulties that students were seen to experience in using their mathematical knowledge as a conceptual tool for problem solving may be due to educational factors” (p. 529). Furthermore, students’ lack of understanding of the underlying structure of mathematics problems also affects students’ ability to solve problems using symbolic manipulation and equations: “Researchers have revealed that many students merely carry out arithmetical operations on the quantities specified in the problem-question without taking into account the conditions of the situation” (Llinares & Riog, 2008, p. 505). The researchers suggested students have the opportunity to practice mathematical modeling as a means to make sense of different situations.

Summary

A review of literature has provided an overview of the effects of music training and listening as well as an overview of problem-solving strategy utilization. Research regarding the effects of music training on spatial reasoning and mathematics performance remains mixed and relatively inconclusive. Although there is strong evidence for both correlational and causal relationships between music training and the development of spatial reasoning, results of studies investigating the relationships between music training and mathematics performance have been less convincing.

Scholars and researchers agree, however, that spatial reasoning is an important component of success in mathematics. Moreover, researchers have asserted that factors including spatial and analytic reasoning impact how students select and use problem-solving strategies during mathematical problem solving. Still, multiple searches of current literature revealed no studies investigating the relationships between music training and mathematical problem solving. Researchers have demonstrated that music training has a causal impact on the development of spatial reasoning and that spatial reasoning, in turn, has an effect on how students solve mathematics problems; therefore, researchers have provided a basis for an investigation of the possible relationships between music training and mathematics problem solving.

CHAPTER III

METHODOLOGY

The following chapter provides a background on the methods that were used during this research investigation as well as details regarding the methodological framework of the study. First, the research questions along with null and alternative hypotheses are discussed. Second, sampling procedures and research design are outlined along with a brief discussion of the pilot study conducted as an initial entry point for the research. Finally, the instrumentation and data collection and analysis procedures are discussed.

Purpose of the Study

The purpose of this study was to investigate the relationships between musical training and the utilization of problem-solving strategies on mathematical problem-solving tasks. Both quantitative and qualitative methods were utilized for this investigation.

Research Questions

Four research questions were selected to guide this study. Null and alternative hypotheses are included, where appropriate.

1. What is the level of music training of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a music background survey?

2. What is the level of spatial reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by the Spatial Reasoning Test (adapted from the Spatial Visualization Test developed by Michigan State University, 1981) and what are the differences in spatial reasoning ability between students with and without music training?

   H_0: There is no statistically significant difference in spatial reasoning ability between students with and without music training.

   H_A: There is a statistically significant difference in spatial reasoning ability between students with and without music training.

3. What is the level of analytic reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by an analytic reasoning test and what are the differences in analytic reasoning ability between students with and without music training?

   H_0: There is no statistically significant difference in analytic reasoning ability between students with and without music training.

   H_A: There is a statistically significant difference in analytic reasoning ability between students with and without music training.

4. What problem-solving strategies are utilized by students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a problem-solving test and what are the differences in strategy selection between students with high and low spatial reasoning ability, between students with high and low analytic reasoning ability, and between student with and without music training?

H_{01}: There is no difference in the problem-solving strategies selected by students with high and low spatial reasoning ability.

H_{A1}: There is a difference in the problem-solving strategies selected by students with high and low spatial reasoning ability.

H_{02}: There is no difference in the problem-solving strategies selected by students with high and low analytic reasoning ability.

H_{A2}: There is a difference in the problem-solving strategies selected by students with high and low analytic reasoning ability.

H_{03}: There is no difference in the problem-solving strategies selected by students with and without music training.

H_{A3}: There is a difference in the problem-solving strategies selected by students with and without music training.

Research Design

A correlational research design utilizing chi-square tests and analysis of variance was used to investigate the relationships between music background and mathematical problem solving. Data was collected through four instruments: the Music Background Survey, the Analytic Reasoning Assessment, the Spatial Reasoning Assessment, and the Problem-Solving Assessment. (For instruments, see Appendices B through E).

Pilot Study

As an initial step in this research, a pilot study was conducted to investigate the validity and reliability of the four instruments that were used in the dissertation research: the Music Background Survey, the Analytic Reasoning Assessment, the Spatial Reasoning Assessment, and the Problem-Solving Assessment. The pilot study provided the researcher an opportunity to ensure the instruments were appropriate for the population under investigation and to ensure the instruments were appropriate for collecting the necessary data for the dissertation study (see Appendix F for details regarding the pilot study). These goals were accomplished through (a) development of items to be included on the Music Background Survey, the Analytic Reasoning Assessment, and the Problem-Solving Assessment; (b) redesign of the Spatial Visualization Test (Michigan State University, 1981) for online administration as the Spatial Reasoning Assessment; (c) expert content validation of the Music Background Survey; (d) administration of the three assessments and one survey using methods proposed for the dissertation study to ensure method feasibility; and (e) exploratory factor analysis to explore latent structures in the Music Background Survey, the Analytic Reasoning Assessment, and the Spatial Reasoning Assessment.

In addition, the pilot study allowed the researcher to make appropriate revisions to each instrument for the purposes of the dissertation research. During the pilot study, the reliability of a researcher-created assessment of analytic reasoning could not be verified. Further research was conducted to locate an appropriate assessment of analytic reasoning resulting in the Analytic Reasoning Assessment. Additional piloting was then conducted in order to ensure the validity and reliability of the Analytic Reasoning Assessment.

Instrumentation

For the purposes of this study, all research participants were asked to complete one survey and three assessments. The Music Background Survey was used to determine participants' music training background as well as their experience with music theory. Participants' spatial and analytic reasoning abilities were measured using the Spatial

Reasoning Assessment and Analytic Reasoning Assessment, respectively. The Problem-Solving Assessment was administered in order to investigate the problem-solving strategies participants utilize when engaged in mathematical problem-solving tasks.

Music Background Survey

An appropriate existing measure of participants' musical background that included participants' experience with music theory could not be located during the review of literature. Therefore, the Music Background Survey instrument was created by the researcher for the purpose of discerning research participants' music background and music theory experience (see Appendix B).

A total of 24 questions were included in this assessment: 10 multiple choice and open response questions in which participants were asked to identify or describe musical activities they had participated in including school-sponsored music programs and private music instruction and 14 Likert-type questions in which participants rated their confidence with respect to performing music theory related tasks. Music theory related questions were written based on the text book used at the university and chosen to be representative of each of the four levels of music theory courses.

The validity of the Music Background Survey was confirmed using two forms of expert validation. First, a professor from the university's Department of Music was asked provide feedback as an expert regarding the content of the survey as well as any discrepancies with respect to terminology, question construction, and general understandability. Second, university music theory students acted as an expert population for the validation of the music theory questions contained in the Music Background

Survey. A complete discussion of the validation of the Music Background Survey is included with the results from the pilot study (see Appendix F).

SPSS was used to conduct a reliability analysis for the music theory portion of the Music Background Survey (questions 8 through 21) for the population of interest, students enrolled in first-year credit-bearing algebra-based mathematics courses. Analysis of reliability indicated the music theory portion of the assessment had high internal consistency (Chronbach's alpha = .961).

In addition, exploratory factor analysis was used to explore the latent structure of the music theory portion of the Music Background Survey as well as to further investigate the internal consistency on the survey. Results of the exploratory factor analysis indicated that the music theory portion of the Music Background Survey consisted of a single primary factor, illustrated by a scree plot (see Figure 1).

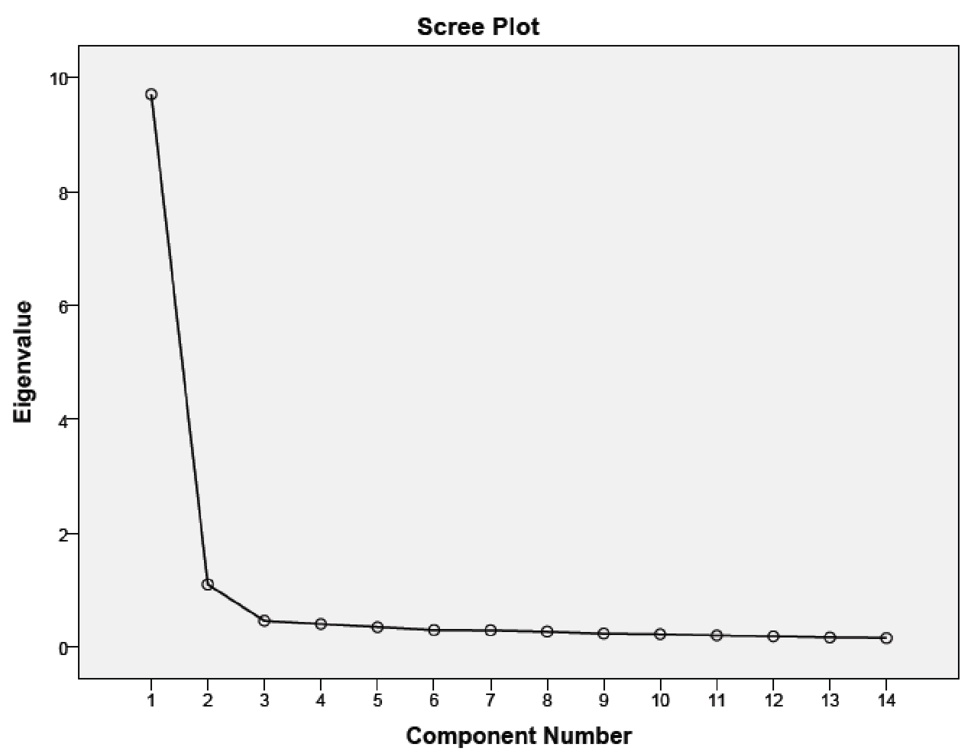


Figure 1. Scree Plot Illustrating Components of the Music Background Survey

Analytic Reasoning Assessment

The Analytic Reasoning Assessment (see Appendix C) used for this study was modeled after the Employee Aptitude Survey (Grimsley, Ruch, Warren, and Ford, 1986) used by McFarlane (1989) to measure verbal reasoning. (For details regarding permission of use, see Appendix G.) The Analytic Reasoning Assessment contained five sections, each consisting of seven items. Each section of the Analytic Reasoning Assessment included statements that were assumed to be true followed by items which could be true, false, or for which the validity could not be determined based on the given information.

Before reliability analysis was conducted, data were trimmed by excluding data associated with assessment times in the lower 5th percentile and in the upper 5th percentile (times less than 230 seconds or greater than 1302 seconds). SPSS was used to conduct a reliability analysis of the assessment. Results of the reliability analysis indicated the Analytic Reasoning Assessment was internally consistent for the population of interest, students enrolled in first-year credit-bearing algebra-based mathematics courses (Chronbach's alpha = .816).

Exploratory factor analysis was used to examine the factor structure of the Analytic Reasoning Assessment as well as to further investigate the internal consistency of the assessment. Results of the exploratory factor analysis indicated that the Analytic Reasoning Assessment consisted of a single primary factor, illustrated by a scree plot (see Figure 2).

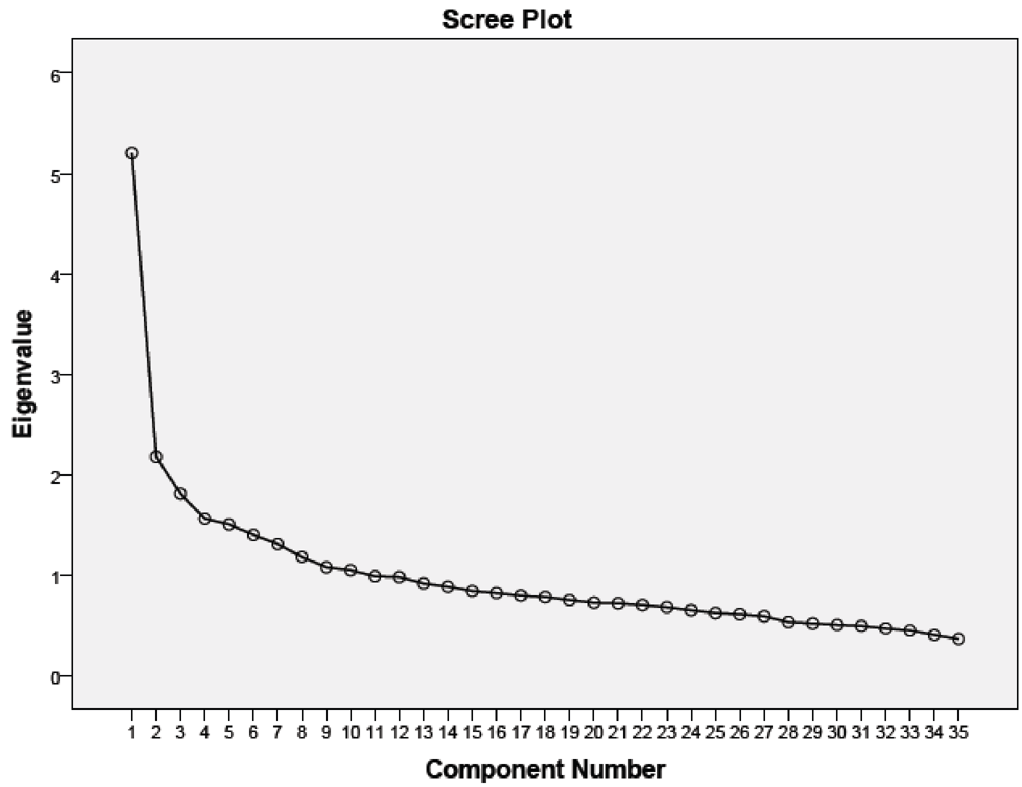


Figure 2. Scree Plot Illustrating Components of the Analytic Reasoning Assessment

Spatial Reasoning Assessment

The Spatial Reasoning Assessment utilized in this study was an electronic reproduction of the Spatial Visualization Test developed by the Mathematics Department at Michigan State University (1981). The test was reproduced using Microsoft Word®. (For details regarding permission of use, see Appendix G.) The Spatial Visualization Test (MSU, 1981) was selected since researchers have asserted that block design items of this type "are the most complex tests of spatial ability, involving a sequence of spatial transformations of a spatial representation" (Hegarty & Kozhevnikov, 1999, p. 688). The Spatial Visualization Test (MSU, 1981) consists of 32 multiple-choice items of 10 different types (see Table 1) intended to measure "different aspects of spatial visualization skills" (Ben-Chaim, Lappan, & Houang, 1986, p. 660).

Table 1

Types of Questions Included on the Spatial Visualization Assessment

Type	Description	Assessment Item(s)
I	Given a three-dimensional corner view of a building, the task is to find the two-dimensional front, right, back, or left view of the same building.	1, 2, 4
II	Given the mat plan of a building, the task is to find a specified two-dimensional view of the same building.	3, 6, 7
III	Given a two-dimensional view of a building, the task is to find the two-dimensional view of the opposite side of the same building.	5, 8, 9
IV	Given a three-dimensional corner view of a rectangular solid built from smaller cubes, the task is to find the number of cubes needed to build the solid.	10, 12
V	Given the base, and the two-dimensional front and right views of a building, the task is to find the mat plan that can be completed to fit the building or that fits the building with a maximum or minimum number of cubes.	11, 13, 15, 16
VI	Given a three-dimensional corner view of a building, the task is to find the number of cubes that touch an exposed cube face-to-face.	14, 17
VII	Given a three-dimensional corner view of a building, the task is to find the three-dimensional corner view of the same building if one or more indicated cubes are added or removed.	18, 19, 23, 26
VIII	Given the mat plan of a building, the task is to identify a three-dimensional corner view of the same building.	20, 25, 27
IX	Given three-dimensional corner views of two different solids, the task is to identify a three-dimensional corner view of a building made from the two solids.	22, 28
X	Given a three-dimensional corner view of a building, the task is to find another corner view of the same building.	21, 24, 29, 30, 31, 32

Note. Adapted from "Development and Analysis of a Spatial Visualization Test for Middle School Boys and Girls," by D. Ben-Chaim, G. Lappan, and R. T. Houang, 1986, *Perceptual and Motor Skills*, *63*, p. 662-663.

Before reliability analysis was conducted, data were trimmed by excluding data associated with assessment times in the lower 5th percentile and in the upper 5th percentile (times less than 340 seconds or greater than 3230 seconds). SPSS was used to conduct a reliability analysis of the Spatial Reasoning Assessment for the population of interest, students enrolled in first-year credit-bearing algebra-based mathematics courses. Analysis revealed that two of the items, item 9 and item 23, contained errors. Therefore, items 9 and 23 were removed from data analysis and reliability analysis was rerun. Results of reliability analysis indicated the Spatial Reasoning Assessment was internally consistent for the population of interest, students enrolled in first-year credit-bearing algebra-based mathematics courses (Chronbach's alpha = .864).

Exploratory factor analysis was used to examine the factor structure of the Spatial Reasoning Assessment as well as to further investigate the internal consistency of the assessment. Results of the exploratory factor analysis indicated that the Spatial Reasoning Assessment consisted of a single primary factor, illustrated by a scree plot (see Figure 3).

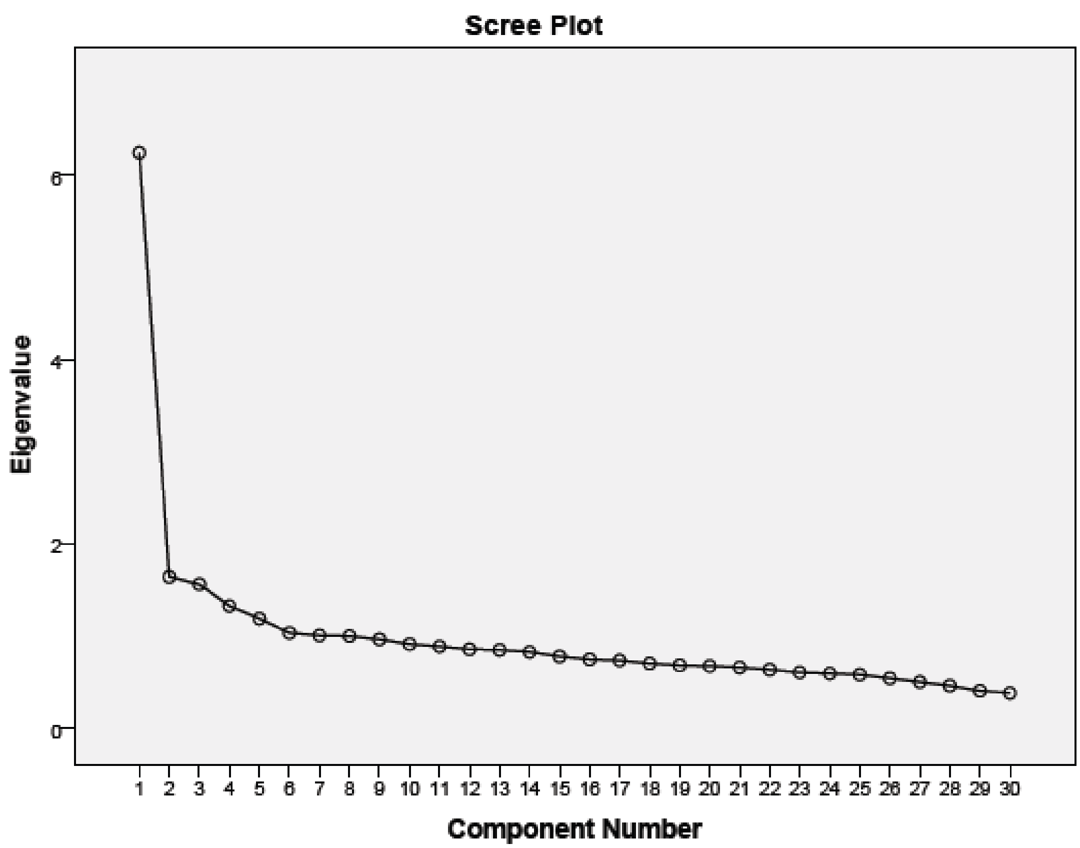


Figure 3. Scree Plot Illustrating Components of the Spatial Reasoning Assessment

Problem-Solving Assessment

The Problem-Solving Assessment was constructed by the researcher to investigate the problem-solving strategies utilized by participants. Piloting of six classic problem-solving tasks resulted in the selection of three tasks that provided the greatest insight into the different strategies students use (for results of the pilot study, see Appendix F). The three classic word problems included in the Problem-Solving Assessment (see Appendix E) were adapted from Johnson, Herr, & Kysh (2004).

In addition to the three problems-solving tasks included in the Problem-Solving Assessment, each problem was followed by a three-question survey designed to illicit data related to prior exposure to the problem-solving task, planning involved in the solution process, and perceived difficulty of the task. The final page of the Problem-Solving Assessment consisted of a survey in which participants were asked to reflect on their problem-solving process to ensure strategies that may not be apparent to the researcher were not overlooked. Student self-reporting is especially important when investigating the visualization strategies students utilize: “Unless the [researcher] asks about imagery used in mathematical problem solving, it may not be reported, even when it is present and constitutes an integral part of the problem-solving process” (Presmeg & Balderas-Canas, 2001, p. 293).

Interview Protocol

Semi-structured, task-based problem-solving interviews were conducted in order to validate participants’ self-reported problem-solving strategy utilization from the Problem-Solving Assessment. The Interview Protocol (see Appendix H) consisted of three mathematical problem-solving tasks adapted from Johnson, Herr, and Kysh, (2004) and Presmeg and Balderas-Canas (2001). Problems included in the interview were selected since these problems "could be solved by many different methods, some of which involved visual thinking and some of which did not" (Presmeg & Balderas-Canas, 2001, p. 294).

Population and Sampling

The research took place at a four-year university in central Texas. The university had an enrollment of over 30,000 students of which approximately 56% were female and 74% were under the age of 25.

Population

The population of interest in this research study included all university students enrolled in the three first-year credit-bearing algebra-based mathematics courses. During the Fall 2012 semester, over 3,000 students enrolled in the three first-year credit-bearing algebra-based mathematics courses. Of these students, 40 students were music majors, 16 were music minors, 15 were mathematics majors, and 9 were mathematics minors.

Sampling

Convenience and volunteer sampling was used to recruit research participants. Instructors of the 44 sections of first-year credit-bearing algebra-based mathematics courses were contacted at the beginning of the semester and asked if they would be willing to schedule 30 minutes of class time for a classroom visit by the researcher. Instructors from a total of 21 sections agreed to participate in the research. Classroom visits were conducted in which students were informed about the research project and asked to participate. Each potential participant was given a copy of the Information Handout for Potential Participants (see Appendix I) and Online Instructions for

Participants (see Appendix J). The Problem-Solving Assessment was also administered during the classroom visit and was collected for all students who were present during the classroom visit.

Potential participants were informed that participation was voluntary and that they could agree to participate by completing any of the online assessments and surveys. A total of 1071 students volunteered to participate in the research. Students who did not complete either of the online assessments or the online survey were deemed as non-consenting. The Problem-Solving Assessments associated with non-consenting students were destroyed.

The vast majority of students who participated in the research fully (i.e., completed the in-class Problem-Solving Assessment as well as the online Analytic Reasoning and Spatial Reasoning Assessments and the online Music Background Survey) were given extra credit by the instructors for their participation. The amount of extra credit offered to the participants varied by instructor.

Data Collection and Preliminary Analysis

Data from the Music Background Survey, the Analytic Reasoning Assessment, and the Spatial Reasoning Assessment were collected online through TRACS, a university-supported online course and project management system which allows for the administration of assessments and collection of assessment data. Participants were provided access to the research site for a minimum of two weeks during which time they were able to complete the online survey and the two online assessments.

At the end of the access period, results of the survey and the two assessments were exported from TRACS as a Microsoft Excel® file. For the Spatial Reasoning and

Analytic Reasoning assessments, data exported into the Microsoft Excel® files included individual participants' responses to each item, the total number of correct items for each individual participant, the time the individual participant had the assessment open, and item analysis for the assessment as a whole. For the Music Background Survey, data exported into the Microsoft Excel® files included only individual participants' responses to each item.

Music Background Survey

Participants' responses to the Music Background Survey were collected through TRACS and exported to a Microsoft Excel® document. Participants' responses to open-ended items, in particular questions 1, 6, and 10, were coded. Questions 2 through 5 and 7 through 9 were quantitative in nature. Therefore, coding was not required for these questions. Responses to Likert-type questions (questions 11 through 24) were given a quantitative score: a response of "Not Confident At All" received a score of 1 while responses of "Very Confident" received a score of 5.

Question 1 of the Music Background Survey asked participants to indicate any instruments (including voice) on which they had received private music instruction. They were also asked to disclose the number of years they had taken private lessons and the age at which these private lessons had begun. Instrument of study was coded according to the following instrument classification scheme: brass, woodwind, string, piano, voice, or percussion. Piano was created as a separate category due to the frequency of participants' study of piano. The number of years and age of commencement of study for each instrument class was also recorded.

Question 6 of the Music Background Survey asked participants to describe any other formal music training that was not addressed in prior questions. Responses to this question included participants' description of various types of incidental music participation and dance training, which was not considered formal music training for the purposes of this study. Therefore, the data collected through this question was not used when determining participants' levels of music training.

Question 10 of the Music Background Survey asked participants to describe additional experience with music theory that was not addressed in prior questions. Again, responses included descriptions of various types of incidental experience with music theory, which was not considered formal music training for the purposes of this study. Hence, the data collected through this question was not used when determining participants' levels of music training.

Questions 11 through 24 were Likert-type questions in which participants rated their confidence in respect to performing certain music theory related tasks. For each participant, responses were coded and the mean of the responses for questions 11 through 24 was calculated resulting in a music theory confidence level for each participant. The music theory confidence level was then included in the calculation of each participants' level of music training.

Data in the form of a comma delimited file were uploaded into SPSS. The Music Background Survey included questions measuring various aspects of music training. In order to create a single variable that could be used to describe level of music training, principal component analysis was conducted on music background data to determine the factor loadings that could be given to individual variables to assign a music background index for each participant. The Music Index represented the optimal linear combination of music background variables in that it explained the most variation in the data from the participants of this study. Variables included in the factor analysis along with the resulting extraction weights are provided in Appendix K. It is important to note that the results of the factor analysis used in this study are relative to the participants who completed the Music Background Survey. Hence, similar analysis conducted on a different sample may yield different weights since the music index is a relative scale rather than an absolute scale.

Music Indices for each participant were computed by SPSS using the factor loadings computed during the factor analysis. The music index was then used to group participants based on low and high levels of musical training. Participants with music indices in the lower 25th percentile, indices less than or equal to -0.756, were coded as being "Low Music" while participants in the top 90th percentile, indices greater than 1.344, were coded as being "High Music" (see Table 2).

Table 2

Percentiles Based on the Music Index

	Percentiles						
	5	10	25	50	75	90	95
Music Index	-0.756	-0.756	-0.756	-0.357	0.344	1.344	1.961

Participants in the lower 25th percentile had the same music index of -0.75621 which represented little to no music training (see Table 3). Participants with music indices near the middle were not included in Low/High Music groupings to ensure that the two groups, "Low Music" and "High Music," were clearly distinct.

Table 3

Illustrative Examples of the Interpretation of the Music Index

Index	Measure	Score	Interpretation
-0.756	PI	0	No private music instruction
	MS	0	No participation in music programs at the middle school level
	HS	0	No participation in music programs at the high school level
	CU	0	No participation in music programs at the college or university level
	MTh	1.00	Not at all confident with respect to the music theory content contained in the Music Background Survey
0.001	PI	0	No private music instruction.
	MS	6	Participated in 6 semesters of school music at the middle school level
	HS	2	Participated in 2 semesters of school music at the high school level
	CU	0	No participation in music programs at the college or university level
	MTh	1.71	Fairly unsure with respect to the music theory content contained in the Music Background Survey
4.015	PI	7	Private music instruction for a total of 7 years
	MS	4	Participated in 4 semesters of school music at the middle school level
	HS	16	Participated in 16 semesters of school music at the high school level; Involved in multiple music programs
	CU	2	Participated in 2 semesters of school music at the college or university level
	MTh	4.79	Fairly confident with respect to the music theory content contained in the Music Background Survey
8.548	PI	21	Private music instruction for a total of 21 years; Participant received private music instruction on multiple instruments
	MS	14	Participated in 14 semesters of school music at the middle school level; Involved in multiple music programs
	HS	32	Participated in 32 semesters of school music at the high school level; Participant was involved in multiple music programs for multiple semesters
	CU	15	Participated in 15 semesters of school music at the college or university level; Involved in multiple music programs
	MTh	5.00	Very confident with respect to the music theory content contained in the Music Background Survey

Note. PI = total number of years of private music instruction; MS = total number of semesters of participation in school music programs at the middle school level; HS = total number of semesters of participation in school music programs at the high school level; CU = total number of semesters of participation in school music programs at the college or university level; MTh = mean music theory confidence level.

Analytic Reasoning Assessment

Participants' responses to the Analytic Reasoning Assessment were collected through TRACS and exported to a Microsoft Excel® document. Responses were coded as either correct or incorrect and each participant was given an overall score for analytic reasoning based on the number of correct responses. Data in the form of a comma delimited file was uploaded into SPSS.

As an initial step in the analysis of participants' scores, time for completing the assessment was investigated. Data were trimmed by excluding data associated with assessment times in the lower 5^{th} percentile and in the upper 5^{th} percentile (times less than 230 seconds or greater than 1302 seconds). Corresponding assessment data were not included in subsequent analyses. In addition, the researcher along with one other reader was timed while reading through the assessment. It was determined that it would take a minimum of 200 seconds to read through the assessment. Therefore, trimming data corresponding to assessment times less than 230 seconds was deemed valid.

Following data trimming, participants' overall scores were used to group participants based on low and high levels of analytic reasoning ability. Participants with overall scores in the lower 25^{th} percentile (overall scores less than or equal to 23) were coded as being "Low Analytic" while participants in the top 75^{th} percentile (overall scores greater than 30) were coded as being "High Analytic" (see Table 4). Participants with scores in the middle 50^{th} percentile were not included to ensure the distinction between the "Low Analytic" and "High Analytic" groups.

Table 4

Percentiles Based on the Results of the Analytic Reasoning Assessment

	Percentiles						
	5	10	25	50	75	90	95
Score	17	19	23	27	30	32	34

Spatial Reasoning Assessment

Participants' responses to the Spatial Reasoning Assessment were collected through TRACS and recorded using Microsoft Excel®. Responses were coded as either correct or incorrect. Each participant was given an overall score for spatial reasoning based on the number of correct responses. Questions 9 and 23 were removed from data analysis due to internal errors in the questions.

Again, as an initial step in the analysis of participants' scores, time for completing the assessment was investigated. Data were trimmed by excluding data associated with assessment times in the lower 5th percentile and in the upper 5th percentile (times less than 340 seconds or greater than 3230 seconds). Corresponding assessment data were not included in subsequent analyses. In addition, the researcher, along with one other reader, was timed while reading through the assessment. It was determined that it would take a minimum of 360 seconds to read through the assessment. Therefore, trimming data corresponding to assessment times less than 340 seconds was deemed valid.

Following data trimming, participants' overall scores were used to group participants based on low and high levels of spatial reasoning ability. Participants with overall scores in the lower 25th percentile (overall scores less than or equal to 13) were coded as being "Low Spatial" while participants in the top 75th percentile (overall scores greater than 23) were coded as being "High Spatial" (see Table 5). Participants with scores in the middle 50th percentile were not included to ensure the distinction between the "Low Spatial" and "High Spatial" groups.

Table 5

Percentiles Based on the Results of the Spatial Reasoning Assessment

	Percentiles						
	5	10	25	50	75	90	95
Score	7.75	9	13	18	23	26	28

Problem-Solving Assessment

The Problem-Solving Assessment was paper-based and administered in-class by the researcher. Participants were given 25 minutes to complete the assessment and were allowed to use calculators. Data was collected through participants' hand written responses to the three problem-solving tasks as well as the responses to the survey questions following each task and at the end of the assessment.

Data collected from the three survey questions that followed each problem-solving task to illicit prior exposure to the problem-solving task, planning involved in the solution process, and perceived difficulty of the task were recorded and coded. Data related to prior exposure to the problem-solving task were given a numeric code of either 0, 1, or 2 for responses of "No, I have not seen this kind of problem before," "Yes, I have seen a similar problem before," or "Yes, I have seen this exact problem before," respectively. Regarding the planning involved in the solution process, responses were given a numeric code of either 0, 1, or 2 for responses of "I jumped in," "I planned bit," or "I thought it out first," respectively. Responses to questions related to the perceived difficulty of the task were given a numeric code of either 0, 1, or 2 for responses of "Easy," "Approachable," or "Hard," respectively.

The final page of the Problem-Solving Assessment consisted of a survey in which participants were asked to reflect on their problem solving process. Participants were provided with a list of common problem-solving strategies and asked to select the strategy that most closely resembled the strategy they used in solving each problem. The list of common problem-solving strategies included 10 strategies:

A. Drawing a picture or diagram to help think about the problem.
B. Trying to remember a formula that would help solve the problem.
C. Trying to set up an equation.
D. Trying to make a table or list of possible answers.
E. Trying to make a list of possible answers to see if there was a pattern.
F. Trying to visualize the scenario while thinking about the problem.
G. Trying to remember how I've seen the problem solved before.
H. Trying to work backwards.
I. Trying to guess and then checking to see if the guess was right.
J. Trying to start with an easier problem and looking for a pattern.

For the three problem-solving tasks, each problem-solving strategy was given a numeric code of 0, 1, or 2. If a participant reported using one of the strategies as "Primary Strategy," the strategy was given a code of 2. A strategy identified by the participant as "Other Strategy" was given a code of 1. If a participant reported using a particular strategy as both "Primary Strategy" and "Other Strategy," the strategy was considered a primary strategy and was given a code of 2. If the strategy was not identified by the participant as either a "Primary Strategy" or "Other Strategy," the strategy was given a code of 0 to represent non-use.

To validate participants' self-reported utilization of problem-solving strategies, a random selection of 300 completed Problem-Solving Assessments were coded. Participants' hand-written work for each of the three problem-solving tasks was reviewed. Problem-solving strategy codes identified during the pilot study were employed during the coding of the Problem-Solving Assessment:

PD. Drawing a picture or diagram

F. Using a formula to solve the problem

E. Setting up an equation to solve the problem

TL. Making a table or list to solve the problem

P. Noticing a pattern to solve the problem

WB. Working backwards to solve the problem

GC. Using guess and check to solve the problem

EP. Starting with an easier problem to solve the problem

Problem-solving strategies such as "Trying to visualize the scenario while thinking about the problem" and "Trying to remember how I've seen the problem solved before" were not coded by the researcher as these strategies can only be known by the participants completing the problem-solving task.

The Problem-Solving Assessment Coding Rubric (see Appendix L) was used during the coding each of the 300 randomly selected Problem-Solving Assessments. For each assessment, problems were coded as either correct or incorrect and the problem-solving strategies utilized were identified. Strategies identified by the researcher were coded as either a primary problem-solving strategy or a secondary problem-solving strategy. A primary problem-solving strategy was considered a strategy that either (1) led the participant to the solution of the problem or (2) was the most evident strategy utilized by the participant. Secondary problem-solving strategies were considered those strategies that either were (1) abandoned by the participant for another strategy or (2) not the most evident strategy utilized by the participant. In general, only one problem-solving strategy was coded as primary while multiple strategies could be identified as secondary. In some cases, a participant used two problem-solving strategies in conjunction with each other. In these cases, more than one problem-solving strategy was coded as primary.

In order to confirm the validity and reliability of the researcher's coding of the Problem-Solving Assessments, inter-rater coding was employed. A mathematics education professional was asked to act as an inter-rater and code a random sample (n=100) of the researcher-coded Problem-Solving Assessments using the Problem-Solving Assessment Coding Rubric. The strategies identified by the researcher were not revealed to the inter-rater. The inter-rater was given exemplars created by the researcher to illustrate various problem-solving strategies that could be used for each of the three problem-solving tasks. In addition, the researcher and inter-rater looked at several examples of participant work and discussed the problem-solving strategies used.

Data collected through the Problem-Solving Assessment, including participant responses, researcher codes, and inter-rater codes, were recorded using Microsoft Excel®. First, to investigate the validity and reliability of the researcher's coding of the Problem-Solving Assessments, Cohen's Kappa was computed between the researcher and the inter-rater for each problem-solving task (see Appendix M). Inter-rater reliability for Problem 1, Problem 2, and Problem 3 were calculated to be 0.78, 0.65, and 0.88, respectively. The overall inter-rater reliability between the researcher's codes and inter-rater's codes was calculated to be 0.79. According to Landis and Koch (1977), a value of 0.79 indicates substantial agreement. Therefore, the researcher's codes were determined to be a reliable interpretation of the problem-solving strategies evident in participants' work.

In order to investigate the validity of the participants' reported strategy utilization on the Problem-Solving Assessments, Cohen's Kappa was computed between the researcher and the participants for each problem-solving task (see Appendix M). Inter-rater reliability for Problem 1, Problem 2, and Problem 3 were calculated to be 0.35, 0.25, and 0.37, respectively. The overall inter-rater reliability was calculated to be 0.33. According to Landis and Koch (1977), a value of 0.33 indicates fair agreement. During task-based interviews, however, participants were able to identify and describe the problem-solving strategies they were employing validating participants' self-reported strategy utilization. Therefore, the use of participants' reported problem-solving strategy utilization on the Problem-Solving Assessment was determined to be valid even though the inter-rater reliability between participants' codes and researcher's codes for the Problem-Solving Assessment indicated only fair agreement (Cohen's Kappa of 0.33).

Data Analysis

Analysis of quantitative data was performed using SPSS with statistical significance set at $p \leq .05$. However, significance levels less than .10 ($.05 < p \leq .10$) were also considered for marginal significance.

Research Question 1

The purpose of the first research question was to ascertain the level of formal music training of the general population of students enrolled in first-year credit-bearing algebra-based mathematics courses. Descriptive statistics were used to determine the level of formal music training of the general population of students enrolled in first-year credit-bearing algebra-based mathematics courses. In addition, factor analysis was used to compute music indices for each participant to represent overall level of music training.

Private instruction. Question one of the Music Background Survey was used to investigate participants' experience with private music instruction as well as musical instrument(s) of study. Descriptive statistics, including participants' total number of years of private music instruction, were used to measure one facet of participants' level of music training. Participants' total number of years of private music instruction was computed by finding the sum of the years of study for each individual class of instrument, brass, woodwind, string, piano, voice, and percussion.

School music participation. Participants' involvement in school music programs at the elementary school, middle school, high school, and college or university levels was investigated through questions two, three, four, and five of the Music Background Survey. Descriptive statistics included binary data related to school music participation (participation or non-participation) for each of the education levels, elementary, middle school, high school, or college/university, as well as the number of semesters of participation at each level.

Music theory confidence. Participants' confidence in the discipline of music theory was measured by 14 Likert-type questions (questions 11 through 24 of the Music Background Survey). Each of the 14 questions asked participants to report how confident they were that they could perform specific music theory related tasks. Responses of "Not Confident At All" were given a score of 1 while responses of "Very Confident" were given a score of 5. For each participant, the mean score for questions 11 through 24 was computed resulting in the mean confidence for music theory. The highest mean confidence level that could be reported for any participant was 5 (corresponding to responses of “Very Confident” for all 14 of the Likert-type questions) while the lowest mean level of confidence that could be reported was 1 (corresponding to responses of “Not Confident At All” for all 14 of the Likert-type questions).

Music Index. Each participant was given a Music Index based on a factor analysis conducted using the data collected from the Music Background Survey. (See Appendix K for variables included in the factor analysis as well as weights assigned.)

Research Question 2

The purpose of the second research question was to ascertain the level of spatial reasoning of the university students enrolled in first-year credit-bearing algebra-based mathematics courses as well as to determine whether statistically significant differences in the levels of spatial reasoning existed when controlling for level of music training. Data collected through the Spatial Reasoning Assessment was used to determine participants’ level of spatial reasoning ability.

In order to determine whether a statistically significant difference in spatial reasoning level existed when controlling for level of music training, participants placed into the “Low Music” group were compared to participants placed in the “High Music” group as determined by Music Indices. An analysis of variance was conducted with overall spatial reasoning score acting as the dependent variable and music level (Low Music or High Music) acting as a fixed factor.

Research Question 3

The purpose of the third research question was to ascertain the level of analytic reasoning of university students enrolled first-year credit-bearing algebra- based mathematics courses as well as to determine whether statistically significant differences in the levels of analytic reasoning existed when controlling for level of music training. Data collected through the Analytic Reasoning Assessment was used to determine participants' level of analytic reasoning ability.

In order to determine whether a statistically significant difference in analytic reasoning level existed when controlling for level of music training, participants placed into the "Low Music" group were compared to participants placed in the "High Music" group as determined by music indices. An analysis of variance was conducted with overall analytic reasoning score acting as the dependent variable and music level (Low Music or High Music) acting as a fixed factor.

Research Question 4

The purpose of the fourth research question was to investigate the problem-solving strategies utilized by university students enrolled in first-year credit-bearing algebra-based mathematics courses. In addition, the goals for the fourth research question included determining whether statistically significant differences existed with respect to the problem-solving strategies utilized by students while controlling for spatial reasoning ability ("Low Spatial" or "High Spatial"), analytic reasoning ability ("Low Analytic" or "High Analytic"), and level of music formal music training ("Low Music" or "High Music").

Chi-square tests of homogeneity were employed to investigate whether statistically significant differences in strategy usage existed between participants deemed “Low Spatial” and “High Spatial.” For each of the three problems included on the problem-solving assessment, 10 chi-square tests were conducted: one for each strategy. For each test conducted, usage (Primary, Secondary, or Not Used) was compared based on grouping (“Low Spatial” or “High Spatial”). Ordinal logistic regression was employed to further investigate the relationship between participants’ scores on the Spatial Reasoning Assessment and problem-solving strategies used on the Problem-Solving Assessment.

Similarly, chi-square tests of homogeneity were employed to investigate whether statistically significant differences in strategy usage existed between participants grouped by analytic reasoning ability. For each test conducted, usage (Primary, Secondary, or Not Used) was compared based on analytic reasoning ability (“Low Analytic” or “High Analytic”). Again, ordinal logistic regression was employed to further investigate the relationship between participants’ scores on the Analytic Reasoning Assessment and problem-solving strategies used on the Problem-Solving Assessment.

Finally, chi-square tests of homogeneity were employed to investigate whether statistically significant differences in strategy usage existed between participants grouped by level of music training. Strategy usage (Primary, Secondary, or Not Used) was compared based on level of music training (“Low Music” or “High Music”).

Summary

This chapter provided a description of the methods that were used during this research investigation as well as details regarding the methodological framework of the study. Research design and sampling procedures were also outlined. The development and piloting of the instruments was discussed along with the data collection and analysis procedures.

CHAPTER IV

RESULTS

The purpose of this research was to investigate the relationships between musical training and the utilization of problem-solving strategies on mathematical problem-solving tasks. Participants of this research study consisted of students enrolled in first-year credit-bearing algebra-based mathematics courses offered at a four-year university in central Texas.

The results of data collection and analysis are described in this chapter. Each of the four research questions will be addressed.

Research Question 1

What is the level of music training of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a music background survey?

The Music Background Survey was administered in order to determine research participants' music background and music theory experience (see Appendix B). A total of 24 questions were included in this assessment: 10 multiple choice and open response questions in which participants were asked to identify or describe musical activities they had participated in including school-sponsored music programs and private music instruction and 14 Likert-type questions in which participants rated their confidence with respect to performing music theory related tasks. Participants could rate their confidence between 1 (“Not Confident At All”) and 5 (“Very Confident”). Hence, the survey consisted of three components of formal music instruction: private music instruction, school music instruction, and music theory instruction and confidence. In order to answer the first research question, an analysis of each component was conducted.

Formal Music Instruction

Private music instruction. Participants were asked to report private music instruction they had received for any instruments including brass instruments, woodwind instruments, stringed instruments, piano, voice, and percussion instruments. Overall, approximately 52% of participants attested to having private music instruction of some type. The most common type of private music instruction was for stringed instruments. However, the duration of private instruction was highest for private voice lessons with an average of 5.21 years of private instruction (see Table 6).

Table 6

Frequency and Duration of Private Music Instruction by Instrument Class

	% Participation	Mean* (years)	Standard Deviation* (years)
Brass	7.5	3.85	2.68
Woodwind	16.3	3.97	2.45
String	19.1	3.10	2.72
Piano	17.9	3.25	2.72
Voice	9.6	5.21	3.84
Percussion	4.2	3.51	3.46
Overall	51.7	5.32	4.54

Note. The mean and standard deviation of years of private music instruction was computed for the participants who attested to having private music instruction.

School music instruction. Overall, 74.2% of participants attested to experiencing school music instruction at the elementary school level. However, results indicate that school music participation declines rapidly as the level of schooling increases. At the middle school level, 61.3% of participants took part in school music programs. School music involvement decreases sharply at the high school and university levels with only 32.2% and 5.9% of participant attesting to school music involvement at these levels, respectively. Band is the most popular school music program at all levels of education as illustrated by Table 7.

Table 7

Frequency and Mean Duration of School Music Instruction by Program Type and Level of Schooling

Level	Program Type	% Participation	Mean* (semesters)	Standard Deviation* (semesters)
M.S.	Band	34.4	4.10	1.88
	Orchestra	5.7	4.39	1.69
	Instrumental Ensemble	4.8	3.52	2.20
	Choir	24.3	3.49	1.98
	Vocal Ensemble	4.9	3.14	1.83
	Overall	61.3	4.64	2.78
H.S.	Band	14.8	6.40	2.34
	Orchestra	4.2	5.33	2.46
	Instrumental Ensemble	6.2	4.88	2.83
	Choir	13.2	4.67	2.59
	Vocal Ensemble	4.1	4.98	2.43
	Overall	32.2	7.14	5.04
Univ.	Band	3.1	1.81	1.64
	Orchestra	0.7	2.14	1.46
	Instrumental Ensemble	1.8	1.84	1.17
	Choir	2.0	2.05	1.80
	Vocal Ensemble	1.1	3.00	3.07
	Overall	5.9	3.02	3.69

Note. M.S. = Middle School, H.S. = High School, Univ. = College or University
*The mean and standard deviation of years of school music involvement was computed for the participants who attested to having school music instruction.

Music theory instruction and confidence. Private music instruction was the most common means of music theory instruction with 11.9% of participants reporting that they received music theory instruction through private music instruction. Moreover, the number of semesters of music theory instruction was highest for those receiving private music instruction with an average of 4.52 semesters of study (see Table 8).

Table 8

Frequency and Mean Duration of Music Theory Instruction by Type of Instruction

	% Participation	Mean* (semesters)	Standard Deviation* (semesters)
High School Course	10.4	1.76	0.9794
University Course	8.1	1.37	0.8328
Private Instruction	11.9	4.52	2.4540
Overall	22.9	3.64	3.0530

**Note.* The mean and standard deviation of semester of music theory instruction was computed for the participants who attested to having music theory instruction.

Overall, participants who attested to some form of music theory instruction reported an average music theory confidence of 2.56. Using the Likert-type rating associated with the coding of responses, a score of 2.56 falls between the responses “Somewhat Unsure” and “Neutral.” In comparison, participants who reported no music theory instruction reported an average music theory confidence of 1.58 which would correspond to a response between “Not Confident At All” and “Somewhat Unsure” (see Table 9).

Table 9

Music Theory Confidence by Type of Instruction

	% of Participants	Mean (confidence)	Standard Deviation (confidence)
High School Course	10.4	2.87*	1.2807*
University Course	8.1	2.56*	1.4331*
Private Instruction	11.9	2.77*	1.1840*
Overall	22.9	2.56*	1.2746*
No Music Theory	77.1	1.58	0.8333
Entire Sample	100.0	1.81	1.0356

**Note.* The mean and standard deviation of music theory confidence was computed for the participants who attested to having music theory instruction.

Participants’ Levels of Music Training

Results of the analyses of the three components of the Music Background Survey were used to calculate a music index for each participant. Factor analysis was conducted using data collected through the Music Background Survey. (See Appendix K for a list of the variables used to compute the music index as well as the factor weights assigned to each variable.) The music index computed for each participant represented the optimal linear combination that could be used to explain the largest amount of variation in participants' responses. All factor weights that resulted from the factor analysis were positive, indicating that each variable added to a participants' overall Music Index. Music theory confidence received the largest factor weight revealing that participants' mean music theory confidence explained the largest amount of variation in music indices.

The Music Index was used to determine the level of music training for each of the participants. Overall, the mean music index for all participants was 0.00 with a standard deviation of 1. Music indices were skewed right demonstrating that the majority of participants had low levels of music training (see Figure 4).

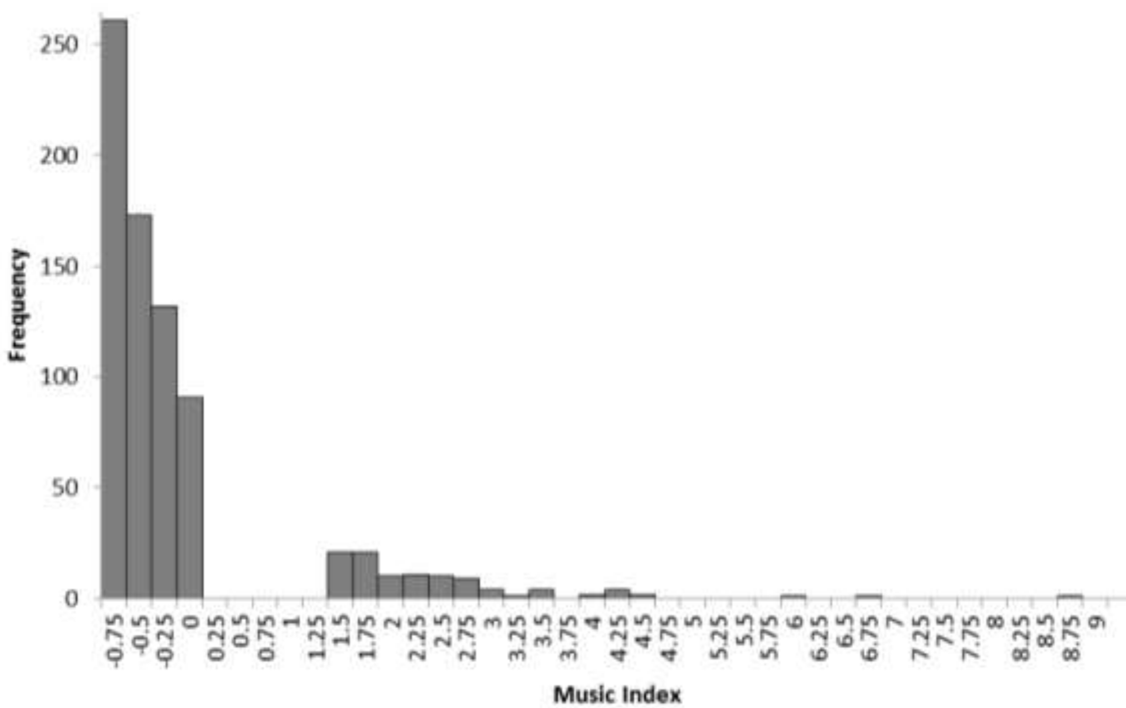


Figure 4. Histogram of Music Indices for Entire Sample of University Students Enrolled in First-Year Credit-Bearing Algebra-Based Mathematics Courses

All participants in the lower 25th percentile had the same music index, -0.75621, representing little to no music background and, therefore, were categorized as "Low Music" participants. Participants in the top 90th percentile had music indices greater than 1.344 and were categorized as "High Music" participants.

As illustrated in Table 10, "Low Music" participants, participants with music indices of -0.75621, reported no school music instruction at the middle school, high school, or university levels and no private music instruction. In addition, "Low Music" participants reported a mean music theory confidence of 1.09, the lowest of which could be 1.00. In contrast, "High Music" participants reported substantial music instruction both through private instruction and school music instruction. In addition, the mean music theory confidence for "High Music" participants was 3.53, which corresponds to a response between "Neutral" and "Somewhat Confident."

Table 10

Descriptive Statistics for "Low Music" and "High Music" Participants

	Low Music		High Music	
Music Background Variable	Mean	S.D.	Mean	S.D.
Years of Private Music Instruction	0.00	0.0000	11.02	5.2430
Semesters of Middle School Music Instruction	0.00	0.0000	7.39	3.9103
Semesters of High School Music Instruction	0.00	0.0000	11.91	5.6465
Semesters of University Music Instruction	0.00	0.0000	1.57	3.1513
Music Theory Confidence	1.09	0.2450	3.53	0.9447

"Low Music" participants represented one-quarter of total participants indicating that over 25% of the participants had no music instruction through either school music programs or private music instruction and had very low music theory confidence. In contrast, approximately 10% of the participants (the participants deemed "High Music") reported high levels of involvement in music instruction through school music programs and/or private music instruction and a substantially higher level of mean music theory confidence.

Research Question 2

What is the level of spatial reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by the Spatial Reasoning Assessment (adapted from the Spatial Visualization test developed by Michigan State University, 1981) and what are the differences in spatial reasoning ability between students with low and high levels of music training?

The second research question was investigated through the Spatial Reasoning Assessment, an electronically adapted version of the Spatial Visualization Test developed by Michigan State University (1981). The Spatial Reasoning Assessment consisted of 30 multiple-choice items intended to measure participants' ability to mentally manipulate and construct various block buildings. Each participant was given a score equal to the number of correct responses. Analysis of spatial reasoning ability was conducted through the use of descriptive statistics drawn from participants' scores on the Spatial Reasoning Assessment and analysis of variance to compare spatial reasoning scores between students with high and low music indices.

Number of correct responses on the Spatial Reasoning Assessment ranged from 1 correct response to 30 correct responses. The mean number of correct responses for the entire sample was 17.70 out of 30 total questions with a standard deviation of 6.203 questions (see Figure 5). As illustrated in Figure 6, participants' scores were approximately normally distributed.

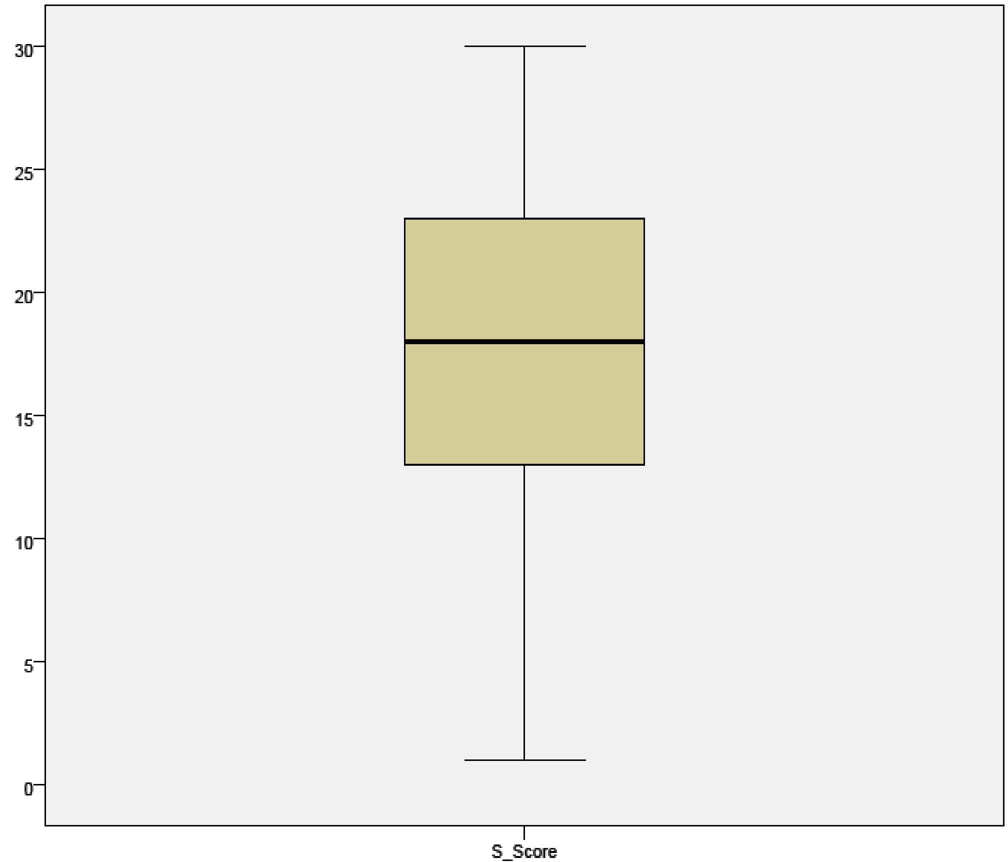


Figure 5. Boxplot of Participants' Scores on the Spatial Reasoning Assessment

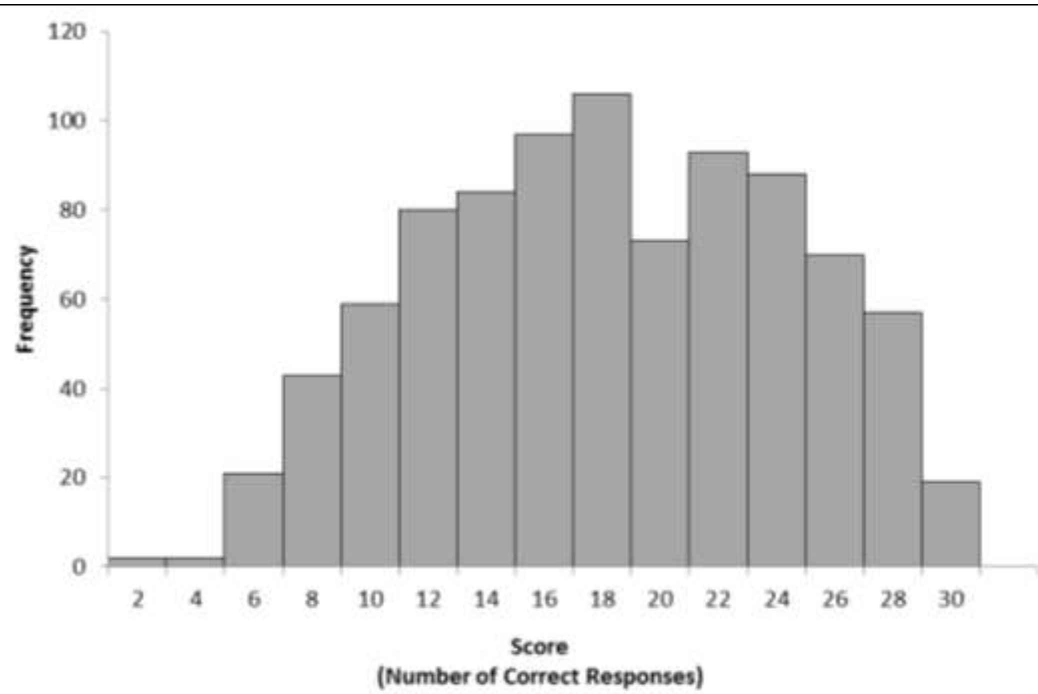


Figure 6. Histogram of Participants' Scores on the Spatial Reasoning Assessment

In addition, descriptive statistics were used to conduct an initial investigation of whether there existed a difference in analytic reasoning based on participants' music background as illustrated by their music index. Participants deemed "Low Music" were compared to participants deemed "High Music" (see Table 11).

Table 11

Comparison of Mean and Standard Deviation for Spatial Reasoning Assessment Scores for "Low Music" and "High Music" Participants

	n	Mean	Std. Deviation
"Low Music" Participants	222	17.16	5.926
"High Music" Participants	81	18.40	6.222
Cumulative	303	17.49	6.021

As illustrated in Table 11 and in Figure 7, "High Music" participants scored higher on the Spatial Reasoning Assessment. However, scores for "High Music" participants showed more variation than scores for "Low Music" participants.

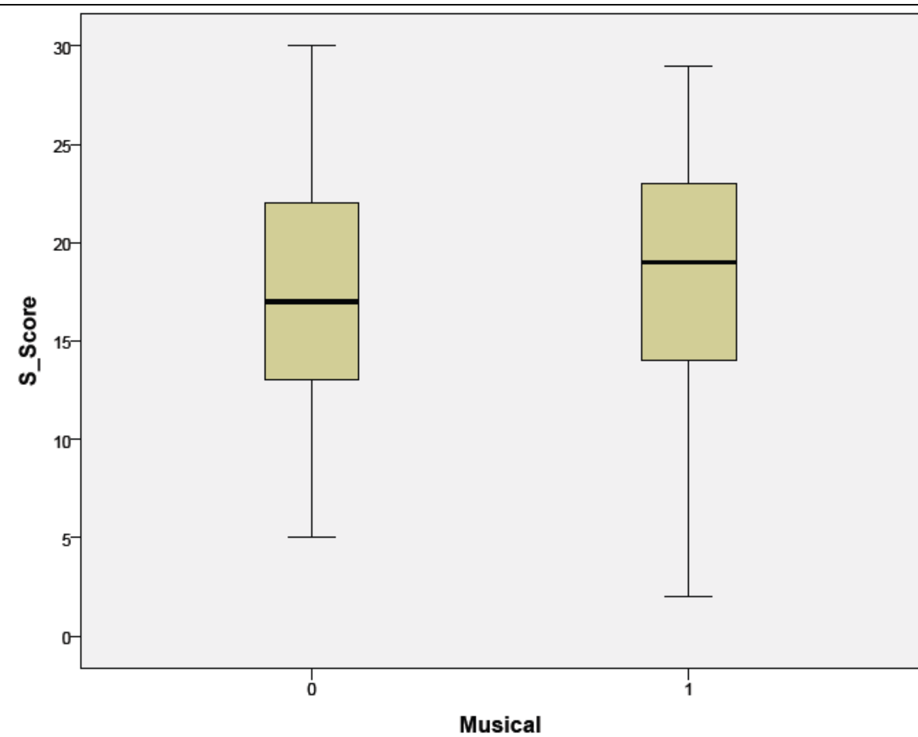


Figure 7. Boxplot Comparing Spatial Reasoning Assessment Scores for "Low Music" Participants (Musical=0) and "High Music" Participants (Musical=1)

An analysis of variance was conducted to determine whether the spatial reasoning scores for "Low Music" and "High Music" participants were significantly different. As outlined in Table 12, the difference between scores for "Low Music" and "High Music" participants was not significant (p = .115). Therefore, "Low Music" and "High Music" participants have comparable levels of spatial reasoning ability as measured by the Spatial Reasoning Assessment.

Table 12

ANOVA for Spatial Reasoning Assessment Score× Low and High Music Index

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	90.209	1	90.209	2.510	.115
Within Groups	10857.520	301	36.071		
Total	10947.729	302			

In anticipation of the analyses to be conducted to answer the fourth research question, participants' scores on the Spatial Reasoning Assessment were used to create distinct groups based on low and high levels of spatial reasoning ability. Participants with a score in the lower 25th percentile (a score less than or equal to 13) were coded as being "Low Spatial" while participants in the top 75th percentile (a score greater than 23) were coded as being "High Spatial."

Research Question 3

What is the level of analytic reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by an analytic reasoning assessment and what are the differences in analytic reasoning ability between students with and without music training?

The third research question was investigated through the Analytic Reasoning Assessment, a web-based assessment modeled after the Employee Aptitude Survey

(Grimsley, Ruch, Warren, and Ford, 1986) used by McFarlane (1989). The Analytic Reasoning Assessment consisted of 35 multiple-choice items intended to measure participants' verbal reasoning ability. Each participant was given a score based on the number of correct responses.

Analysis of analytic reasoning ability was conducted through the use of descriptive statistics and analysis of variance to compare analytic reasoning between the students with high and low Music Indices. Number of correct responses on the Analytic Reasoning Assessment ranged from 7 correct responses to 35 correct responses. The mean number of correct responses for the entire sample was 26.34 out of 35 total questions with a standard deviation of 5.150 questions (see Figure 8).

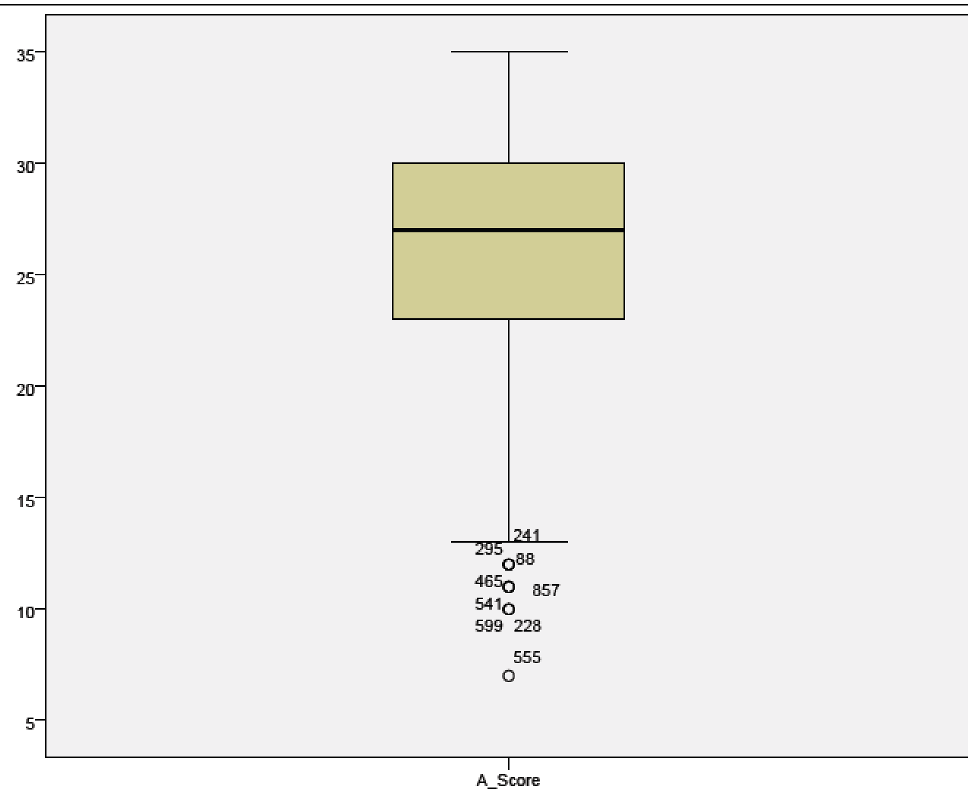


Figure 8. Boxplot of Participants' Scores on the Analytic Reasoning Assessment

Participants' scores were skewed left indicating the median score on the assessment was higher than the mean score on the assessment (see Figure 9). Hence, the majority of participants received high scores on the assessment.

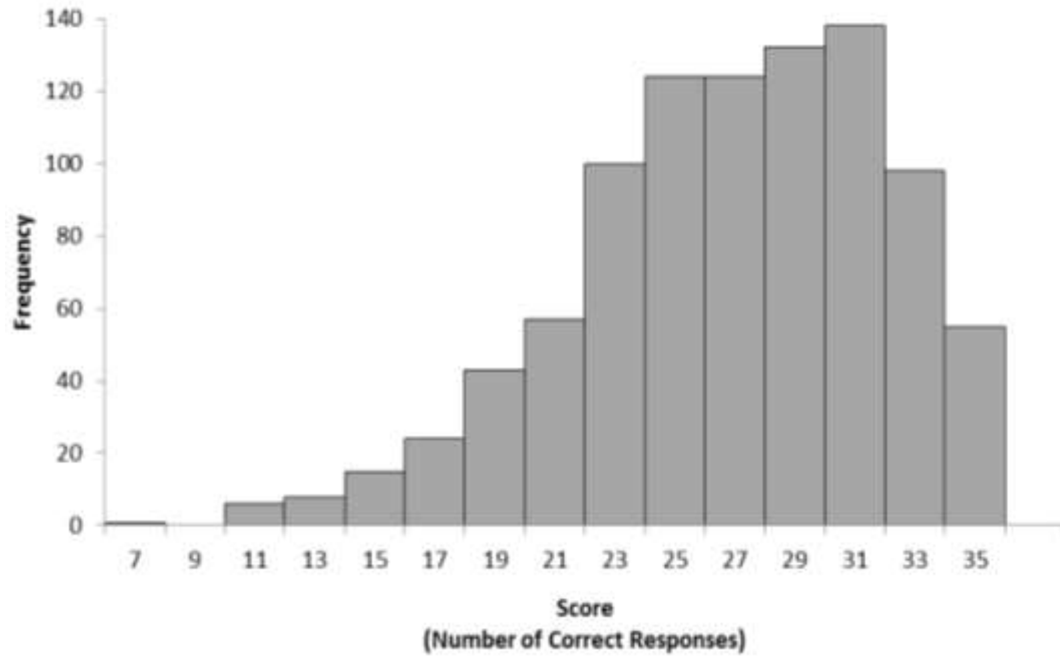


Figure 9. Histogram of Participants' Scores on the Analytic Reasoning Assessment

Descriptive statistics were used to conduct an initial investigation of whether there existed a difference in analytic reasoning based on participants' music background as illustrated by their music index. Participants deemed "Low Music" were compared to participants deemed "High Music" (see Table 13).

Table 13

Comparison of Mean and Standard Deviation for Analytic Reasoning Assessment Scores for "Low Music" and "High Music" Participants

	n	Mean	Std. Deviation
"Low Music" Participants	231	26.48	5.279
"High Music" Participants	89	26.18	4.845
Cumulative	320	26.40	5.154

As illustrated in Table 13 and Figure 10, “High Music” participants scored lower on the Analytic Reasoning Assessment. However, scores for “High Music” participants showed less variation than scores for “Low Music” participants.

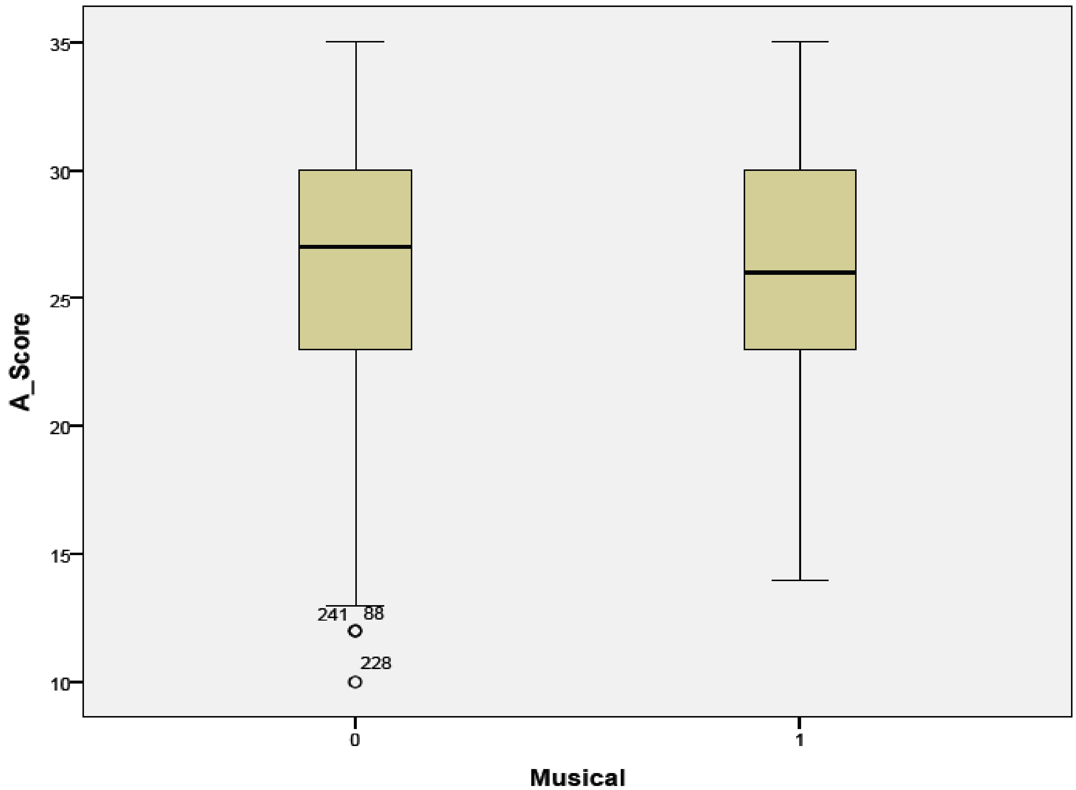


Figure 10. Boxplot Comparing Analytic Reasoning Assessment Scores for “Low Music” Participants (Musical=0) and “High Music” Participants (Musical=1)

An analysis of variance was conducted to determine whether the Analytic Reasoning Assessment scores for “Low Music” and “High Music” participants were significantly different. As outlined in Table 14 the difference between scores for High Music and Low Music participants was not significant (p = .636). Therefore, “Low Music” and “High Music” participants have comparable levels of analytic reasoning ability as measured by the Analytic Reasoning Assessment.

Table 14

ANOVA for Analytic Reasoning Assessment Score× Low and High Music Index

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	5.979	1	5.979	.225	.636
Within Groups	8466.821	318	26.625		
Total	8472.800	319			

For the purposes of answering the fourth research question, participants' Analytic Reasoning Assessment scores were used to create distinct groups based on low and high levels of spatial reasoning ability. Participants with a score in the lower 25^{th} percentile (a score less than or equal to 23) were coded as being "Low Analytic" while participants in the top 75^{th} percentile (a score greater than 30) were coded as being "High Analytic."

Research Question 4

What problem-solving strategies are utilized by university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a problem-solving test and what are the differences of strategy selection between students with high and low spatial reasoning ability, between students with high and low analytic reasoning ability, and between student with and without music training?

The final research question was investigated through the Problem-Solving Assessment. The investigation was conducted using the various categories developed through research questions 1 through 3, "Low Music" versus "High Music," "Low Spatial" versus "High Spatial," and "Low Analytic" versus "High Analytic." The Problem-Solving Assessment consisted of three mathematical word problems, each of which could be approached and solved using a variety of problem-solving strategies, and survey items in which participants indicated the problem-solving strategies used for each problem (see Appendix E).

Analysis of problem-solving strategy utilization was conducted through the collection of frequency data based on participants' reported use of strategies (see Appendix N). Chi-square tests were used to compare strategy utilization between participants with low and high analytic reasoning scores, low and high spatial reasoning scores, and low and high music background indices.

The first problem included on the Problem-Solving Assessment, a problem about helping Farmer Ben remember the number of cows and chickens he has, was a traditional problem typically approached in the mathematics classroom using a system of equations. This word problem was selected because it could be approached in many different ways. In general, the most common strategy participants reported employing for Problem 1 was "Trying to guess and then checking to see if the guess was right" followed closely by the strategy "Trying to set up an equation." The least common strategy employed was the utilization of the strategy "Trying to start with an easier problem and looking for a pattern" (see Figure 11).

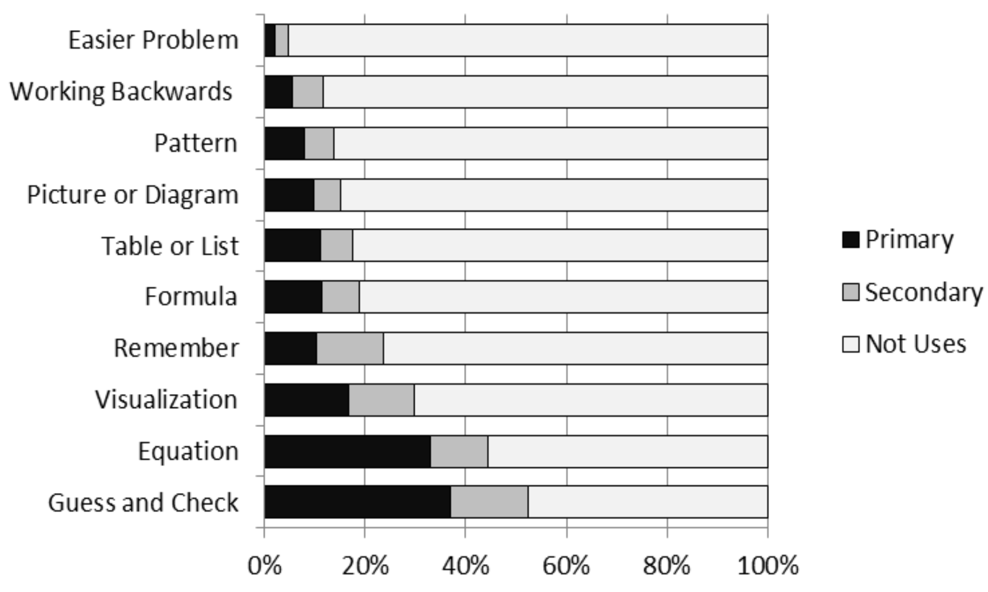


Figure 11. Ribbon Chart Illustrating Frequency of Participants' Reported Use of Each Problem-Solving Strategy for Problem 1

The second problem included on the Problem-Solving Assessment, a problem about the number of games played in an intermural softball league, was a word problem typically approached in mathematics classrooms when learning about counting principles, specifically, when using the formula for computing the number of combinations. This problem was included on the assessment because it could be approached using various strategies including a diagram, formula, table, or by identifying a pattern. For Problem 2, participants reported using a picture or diagram most often followed closely by the use of a table or list and the use of visualization. Again, the utilization of an easier problem was reportedly used least frequently (see Figure 12).

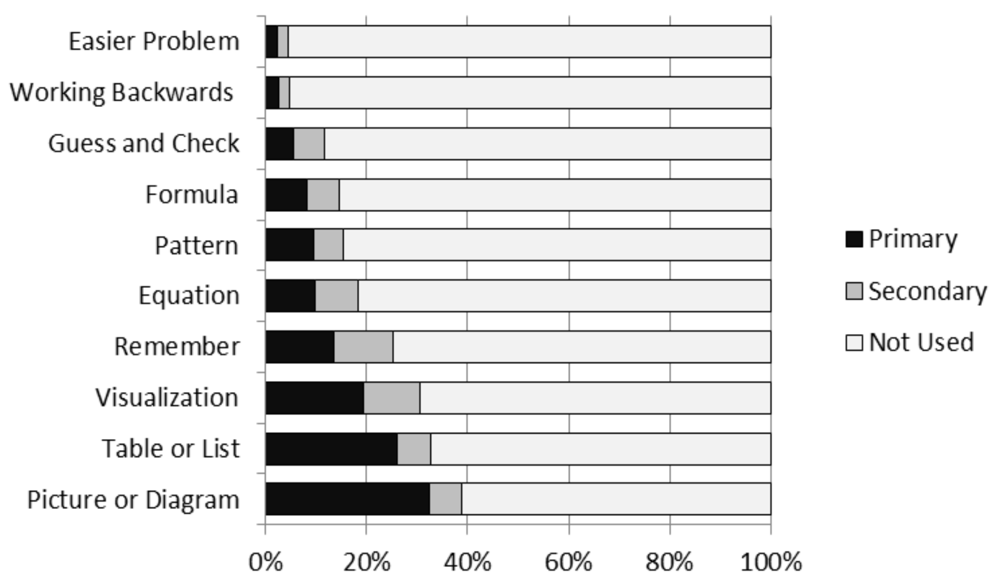


Figure 12. Ribbon Chart Illustrating Frequency of Participants' Reported Use of Each Problem-Solving Strategy for Problem 2

The third problem included on the Problem-Solving Assessment, a problem about finding the amount of rain that fell on a certain day, was a relatively simple word problem that is traditionally approached using a linear equation in one variable or a system of equations. However, it can easily be approached using a picture or diagram or guess and check. Participants reported using an equation most often to solve Problem 3 while the utilization of an easier problem was once more reportedly used the least often. Once again, the utilization of an easier problem was reportedly used least frequently (see Figure 13).

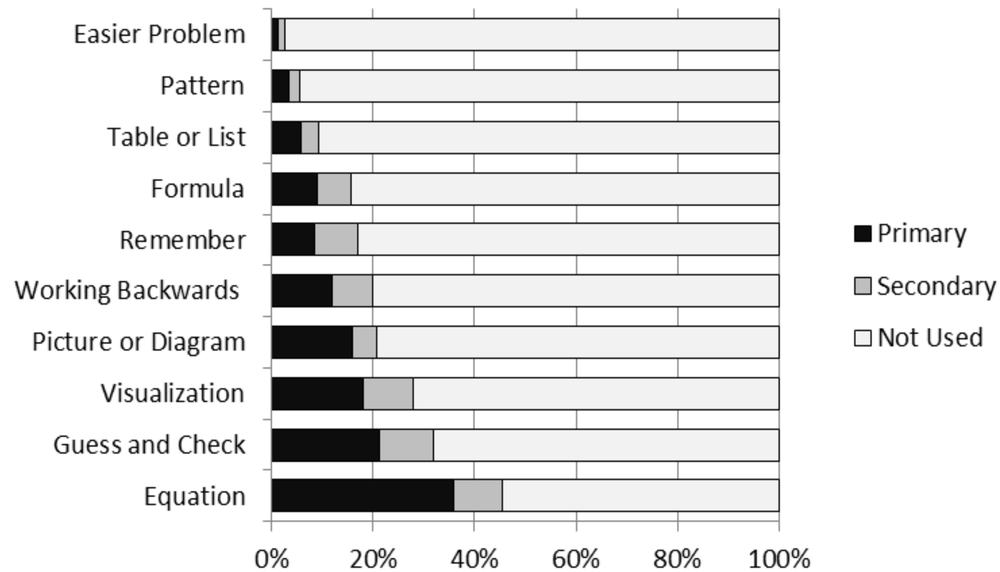


Figure 13. Ribbon Chart Illustrating Frequency of Participants' Reported Use of Each Problem-Solving Strategy for Problem 3

Comparing "Low Analytic" and "High Analytic" Participants

The frequency of participants' use of problem-solving strategies was further investigated by comparing participants coded as either "Low Analytic" or "High Analytic" based on participants' scores on the Analytic Reasoning Assessment. Chi-square tests were conducted for each of the three questions included in the Problem-Solving Assessment in order to investigate whether there were significant differences in participants' reported strategy usage when controlling for "Low Analytic" and "High Analytic."

Problem 1. For Problem 1, the frequency of reported strategies "Trying to make a list of possible answers to see if there was a pattern," "Trying to remember how I've seen the problem solved before," and "Trying to guess and then checking to see if the guess was right" appear to be dependent upon analytic reasoning ability. Participants ranked as "Low Analytic" reported looking for a pattern more frequently than participants ranked as "High Analytic" whereas participants ranked as "High Analytic" reported using the strategy of guess-and-check more frequently than participants ranked as "Low Analytic." The strategy of "Trying to remember how I've seen the problem solved before" was reported equally frequently between participants ranked as "Low Analytic" or "High Analytic." However, participants ranked as "Low Analytic" were more likely to identify this strategy as a primary strategy whereas participants ranked as "High Analytic" more frequently identified this strategy as a secondary strategy (see Table 15).

Table 15

Significant Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 1 by "High Analytic" and "Low Analytic"

	Low Analytic (n = 266)		High Analytic (n = 339)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Pattern						
Primary	27	10.8	8	4.0	7.759	.021
Secondary	15	6.0	10	5.0		
Not Used	207	83.1	183	91.0		
Remember						
Primary	29	11.6	17	8.5	8.794	.012
Secondary	21	8.4	35	17.4		
Not Used	199	79.9	149	74.1		
Guess and Check						
Primary	97	39.0	81	40.3	12.150	.002
Secondary	26	10.4	42	20.9		
Not Used	126	50.6	76	37.8		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Analytic" and "High Analytic" assessment scores, see Appendix O.

Significant results from the chi-square analysis were further investigated by conducting ordinal logistic regressions using the data collected from the entire set of participants. Ordinal logistic regression did not produce statistically significant results for the strategy “Trying to remember how I’ve seen the problem solved before.”

Results of the ordinal logistic regression investigating the use of the strategy “Trying to make a list of possible answers to see if there was a pattern” for Problem 1 (see Table 16) revealed that participants’ scores on the Analytic Reasoning Assessment and participants’ reported use of the strategy were negatively correlated (*p* = .021). This result indicated that as Analytic Reasoning Assessment score increased, participants’ reliance on the strategy “Trying to make a list of possible answers to see if there was a pattern” decreased.

Table 16

Results of Ordinal Logistic Regression for Problem 1 and Strategy “Pattern”

Parameter	β	e^{β}	Std. Error	Sig.
Pattern = 0 (Not Used)	0.770	2.160	0.4745	.104
Pattern = 1 (Secondary)	1.424	4.154	0.4800	.003
Analytic Score	-0.042	0.959	0.0182	.021

Results of the ordinal logistic regression investigating the use of the strategy “Trying to guess and then checking to see if the guess was right” for Problem 1 (see Table 17) revealed that participants’ scores on the Analytic Reasoning Assessment and participants’ reported use of the strategy were positively correlated, though this result was not statistically significant (*p* = .073). Positive correlation indicated that as Analytic Reasoning Assessment score increased, participants’ reliance on the strategy “Trying to guess and then checking to see if the guess was right” also increased.

Table 17

Results of Ordinal Logistic Regression for Problem 1 and Strategy "Guess and Check"

Parameter	β	e^{β}	Std. Error	Sig.
Guess/Check = 0 (Not Used)	0.442	1.556	0.3342	.186
Guess/Check = 1 (Secondary)	1.071	2.918	0.3358	.001
Analytic Score	0.022	1.022	0.0124	.073

Chi-square tests revealed that the strategy "Drawing a picture or diagram to help think about the problem" was interesting, as it yielded a nearly significant p-value (p = .054). Hence, to further investigate this strategy, a chi-square test was run to compare participants' report of use (either as a primary or secondary strategy) or non-use while controlling for analytic reasoning ability. This test returned a p-value indicating whether the strategy was used or not used was dependent on course enrollment (p = .045). In this case, participants ranked as "Low Analytic" reported using the strategy "Drawing a picture or diagram to help think about the problem" more frequently that participants ranked as "High Analytic" (see Table 18).

Table 18

Additional Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 1 by "High Analytic" and "Low Analytic"

	Low Analytic (n = 266)		High Analytic (n = 339)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Picture or Diagram						
Used	41	16.5	20	10.0	4.029	.045
Not Used	208	83.5	181	90.0		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Analytic" and "High Analytic" assessment scores see Appendix O.

Problem 2. For Problem 2, the frequency of reported strategy "Drawing a picture or diagram to help think about the problem" was found to be dependent upon analytic reasoning ability. In this case, participants ranked as "High Analytic" reported using the strategy “Drawing a picture or diagram to help think about the problem” more frequently than participants ranked as “Low Analytic” (see Table 19).

Table 19

Significant Chi-Square Results for Participants’ Reported Use of Problem-Solving Strategy for Problem 2 by “High Analytic” and “Low Analytic”

	Low Analytic (n = 266)		High Analytic (n = 339)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Picture or Diagram						
Primary	68	27.3	76	37.8	6.352	.042
Secondary	15	6.0	14	7.0		
Not Used	166	66.7	111	55.2		

Note. For the results of all chi-square tests conducted for participants’ reported use of problem-solving strategies while controlling for “Low Analytic” and “High Analytic” assessment scores see Appendix O.

Results of the ordinal logistic regression investigating the use of the strategy “Drawing a picture or diagram to help think about the problem” for Problem 2 (see Table 20) revealed that participants’ scores on the Analytic Reasoning Assessment and participants’ reported use of the strategy were positively correlated (p = .001). This result indicates that as Analytic Reasoning Assessment score increased, participants’ reliance on the strategy “Drawing a picture or diagram to help think about the problem” also increased.

Table 20

Results of Ordinal Logistic Regression for Problem 2 and Strategy “Picture or Diagram”

Parameter	β	e^{β}	Std. Error	Sig.
Picture/Diagram = 0 (Not Used)	1.683	5.382	0.3669	.000
Picture/Diagram = 1 (Secondary)	1.956	7.071	0.3684	.000
Analytic Score	0.046	1.047	0.0135	.001

Problem 3. Results of the chi-square test conducted for Problem 3 did not initially yield any significant results. However, the chi-square test revealed that the strategy “Trying to remember how I’ve seen the problem solved before” warranted

further investigation yielding a nearly significant p-value (p = .055). A chi-square test conducted comparing whether participants reported this strategy as primary or secondary indicated participants' reported use of this strategy was dependent on ranking as "High Analytic" or "Low Analytic" with participants ranked as ""Low Analytic" more likely to identify this strategy as a primary strategy and participants ranked as "High Analytic" more likely to identify this strategy as a secondary strategy (see Table 21).

Table 21

Additional Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 3 by "High Analytic" and "Low Analytic"

	Low Analytic (n = 266)		High Analytic (n = 339)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Remember						
Primary	25	10.0	14	7.0	5.132	.023
Secondary	15	6.0	24	11.9		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Analytic" and "High Analytic" assessment scores see Appendix O.

Trends. When comparing problem-solving strategies utilized by "Low Analytic" and "High Analytic" participants, utilization of the strategy "Drawing a picture or diagram to help think about the problem" was found to be dependent on analytic reasoning level for both Problem 1 and Problem 2. However, for Problem 1, the strategy was more frequently reported by "Low Analytic" participants whereas, for Problem 2, the strategy was more frequently reported by "High Analytic" participants.

In addition, the use of the strategy "Trying to remember how I've seen the problem solved before" was found to be dependent on analytic reasoning level for both Problem 1 and Problem 3. For each of these problems, it was determined that "High Analytic" participants who report using this strategy are more likely to identify it as a secondary strategy whereas “Low Analytic” participants who report using this strategy are more likely to identify it as a primary strategy.

Comparing “Low Spatial” and “High Spatial” Participants

The frequency of participants’ use of problem-solving strategies was also investigated by comparing participants coded as either “Low Spatial” or “High Spatial” based on participants’ scores on the Spatial Reasoning Assessment. Chi-square tests were conducted for each of the three questions included in the Problem-Solving Assessment in order to investigate whether there were significant differences in participants’ reported strategy usage when controlling for “Low Spatial” and “High Spatial.”

Problem 1. For Problem 1, the frequency at which participants reported the strategy “Trying to start with an easier problem and looking for a pattern” was found to be dependent upon participants’ ranking as “Low Spatial” or “High Spatial.” When comparing participants who reported using the strategy, those ranked as “Low Spatial’ reported using the strategy as a primary strategy more frequently while participants ranked as “High Spatial” reported using the strategy as a secondary strategy more frequently (see Table 22).

Table 22

Significant Chi-Square Results for Participants’ Reported Use of Problem-Solving Strategy for Problem 1 by “High Spatial” and “Low Spatial”

	Low Spatial (n = 227)		High Spatial (n = 250)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Easier Problem						
Primary	12	4.9	2	1.1	6.939	.031
Secondary	2	0.8	5	2.8		
Not Used	233	94.3	172	96.1		

Note. For the results of all chi-square tests conducted for participants’ reported use of problem-solving strategies while controlling for “Low Spatial” and “High Spatial” assessment scores see Appendix P.

Significant results from the chi-square analysis were further investigated by conducting ordinal logistic regressions using data collected from the entire set of participants. Ordinal logistic regression investigating the use of the strategy “Trying to start with an easier problem and looking for a pattern” for Problem 1 did not produce statistically significant results.

Additional exploration of the chi-square test results for Problem 1 revealed three strategies, “Trying to remember a formula that would help solve the problem,” “Trying to make a table or list of possible answers,” and “Trying to work backwards,” that warranted additional investigation. For the strategy “Trying to remember a formula that would help solve the problem,” the proportion of participants ranked as “Low Spatial” who identified the strategy as a primary strategy was greater than the proportion of participants ranked as “High Spatial” who identified the strategy as primary and the proportion of participants ranked as “High Spatial” who identified the strategy as a secondary strategy was greater than the proportion of participants ranked as “Low Spatial” who identified the strategy as secondary. Additional analysis conducted for the strategy “Trying to make a table or list of possible answers” did not yield any statistically significant results. Additional analysis conducted for the strategy “Trying to work backwards,” however, revealed the proportion of participants ranked as “Low Spatial” who identified the strategy as a primary strategy was greater than the proportion of participants ranked as “High Spatial” who identified the strategy as primary. Also, the proportion of participants ranked as “High Spatial” who identified the strategy as a secondary strategy was greater than the proportion of participants ranked as “Low Spatial” who identified the strategy as secondary (see Table 23).

Table 23

Additional Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 1 by "High Spatial" and "Low Spatial"

	Low Spatial (n = 227)		High Spatial (n = 250)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Formula						
Primary	30	12.1	17	9.5	4.034	.045
Secondary	15	6.1	21	11.7		
Working Backwards						
Primary	17	6.9	7	3.9	5.265	.022
Secondary	10	4.0	16	8.9		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Spatial" and "High Spatial" assessment scores see Appendix P.

Problem 2. With respect to Problem 2, the frequency at which participants reported the strategy "Trying to guess and then checking to see if the guess was right" was found to be dependent upon spatial reasoning ability. The proportion of "Low Spatial" participants who reported using this strategy was nearly identical to the proportion of "High Spatial" participants who reported using the strategy. However, there was a statistically significant difference in the way participants reported the use of the strategy with a larger proportion of "Low Spatial" participants identifying the strategy as primary and a larger proportion of "High Spatial" participants identifying the strategy as secondary (see Table 24).

Table 24

Significant Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 2 by "High Spatial" and "Low Spatial"

	Low Spatial (n = 227)		High Spatial (n = 250)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Guess and Check						
Primary	20	8.1	3	1.7	15.384	.001
Secondary	6	2.4	15	8.4		
Not Used	221	89.5	161	89.9		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Spatial" and "High Spatial" assessment scores see Appendix P.

Significant results from the chi-square analysis were further investigated by conducting ordinal logistic regressions using data collected from the entire set of participants. Ordinal logistic regression investigating the use of the strategy "Trying to guess and then checking to see if the guess was right" for Problem 2 did not produce statistically significant results.

Additional investigation using chi-square analysis regarding the strategy "Trying to work backwards" was conducted and revealed, when comparing whether the strategy was used or not used, a larger proportion of participants ranked "High Spatial" attested to using the strategy than participants ranked as "Low Spatial" (see Table 25).

Table 25

Significant Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 2 by "High Spatial" and "Low Spatial"

	Low Spatial (n = 227)		High Spatial (n = 250)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Working Backwards						
Use	6	2.4	12	6.7	4.687	.030
Not Used	241	97.6	167	93.3		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Spatial" and "High Spatial" assessment scores see Appendix P.

Problem 3. Results of the chi-square tests conducted for Problem 3 did not initially yield any significant results. However, several strategies warranted further investigation yielding nearly significant p-values (see Table 26). Further chi-square analysis for the strategy "Drawing a picture or diagram to help think about the problem" revealed statistically significant differences in whether participants identified the strategy as primary or secondary with a larger proportion of "Low Spatial" participants identifying the strategy as primary and a larger proportion of "High Spatial" participants identifying the strategy as secondary.

Table 26

Additional Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 3 by "High Spatial" and "Low Spatial"

	Low Spatial (*n* = 227)		High Spatial (*n* = 250)			
Strategy	*n*	Percent	*n*	Percent	χ^2	Sig.
Picture or Diagram						
Primary	42	17.0	20	11.2	5.102	.078
Secondary	10	4.0	14	7.8		
Not Used	195	78.9	145	81.0		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Spatial" and "High Spatial" assessment scores see Appendix P.

Further investigation regarding the strategy "Drawing a picture or diagram to help think about the problem" was conducted using data collected from the entire set of participants. Ordinal logistic regression did not yield significant results.

Additional chi-square analysis regarding the strategy "Trying to work backwards" did not reveal any significant differences between "Low Spatial" and "High Spatial" participants. For the strategy "Trying to start with an easier problem and looking for a pattern," there appeared to be differences in whether participants identified the strategy as primary or secondary with a larger proportion of "Low Spatial" participants identifying the strategy as primary and a larger proportion of "High Spatial" participants identifying the strategy as a secondary strategy. However, chi-square analysis could not be conducted due to the small counts associated with the data.

Trends. When comparing problem-solving strategies utilized by "Low Spatial" and "High Spatial" participants, the utilization of the strategy "Trying to work backwards" was found to be dependent on spatial reasoning level for both Problem 1 and Problem 2. For Problem 1, it was determined that "High Spatial" participants who report using this strategy are more likely to identify it as a secondary strategy whereas "Low Spatial" participants who report using this strategy are more likely to identify it as a primary strategy. On the other hand, for Problem 2, "High Spatial" participants reported using the strategy "Trying to work backwards" more frequently overall than "Low Spatial" participants.

Comparing "Low Music" and "High Music" Participants

The frequency of participants' use of problem-solving strategies was also investigated by comparing participants coded as either "Low Music" or "High Music" based on participants' responses to the Music Background Survey and the Music Index computed by preforming a factor analysis on various music background variables. Again, chi-square tests were conducted for each of the three questions included in the Problem-Solving Assessment in order to investigate whether there were significant differences in participants' reported strategy usage when controlling for "Low Music" and "High Music."

Problem 1. For Problem 1, the frequency at which participants reported the strategies "Trying to make a table or list of possible answers" and "Trying to make a list of possible answers to see if there was a pattern" were found to be dependent upon participants' ranking as "Low Music" or "High Music." With respect to the strategy "Trying to make a table or list of possible answers," a statistically significantly larger proportion of "High Music" participants reported using the strategy. Similarly, a statistically significantly larger proportion of "High Music" participants reported using the strategy "Trying to make a list of possible answers to see if there was a pattern." For this strategy, however, a larger proportion of "High Music" participants identified this strategy as a secondary strategy with "Low Music" and "High Music" participants identifying the strategy as a primary strategy equally frequently (see Table 27).

Table 27

Significant Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 1 by "High Music" and "Low Music"

	Low Music (n = 258)		High Music (n = 97)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Table or List						
Primary	27	10.5	22	22.7	9.476	.009
Secondary	19	7.4	4	4.1		
Not Used	212	82.2	71	73.2		
Pattern						
Primary	23	8.9	8	8.2	17.804	.000
Secondary	8	3.1	15	15.5		
Not Used	227	88.0	74	76.3		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Music" and "High Music" background indices see Appendix Q.

Problem 2. Results of the chi-square test conducted for Problem 2 did not yield any significant results indicating that, for Problem 2, "Low Music" and "High Music" participants reported using the various problem-solving strategies equally frequently. However, it is interesting to note that frequency data for the strategy "Trying to make a list of possible answers to see if there was a pattern" indicate that "Low Music" participants who reported using this strategy had a tendency to identify the strategy as a primary strategy where as "High Music" participants who reported using this strategy had a tendency to identify the strategy as a secondary strategy (see Table 28).

Table 28

Additional Chi-Square Results for Participants' Reported Use of Problem-Solving Strategy for Problem 2 by "High Music" and "Low Music"

	Low Music (n = 258)		High Music (n = 97)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Pattern						
Primary	27	10.5	6	6.2	3.843	.146
Secondary	16	6.2	11	11.3		
Not Used	215	83.3	80	82.5		

Note. For the results of all chi-square tests conducted for participants' reported use of problem-solving strategies while controlling for "Low Music" and "High Music" background indices see Appendix Q.

Problem 3. Results of the chi-square test conducted for Problem 3 did not yield any significant results. However, the strategy "Trying to make a list of possible answers to see if there was a pattern" was further investigated since significant or interesting results related to this strategy were found for both Problem 1 and Problem 2. Further investigation did not yield any significant results.

Trends. When comparing problem-solving strategies utilized by "Low Music" and "High Music" participants, the utilization of the strategy "Trying to make a list of possible answers to see if there was a pattern" was found to be dependent on level of music training for Problem 1. Further investigation of the results for Problem 2 indicated that "Low Music" and "High Music" participants reported using the strategy differently with "Low Music" participants reporting the strategy as a primary strategy and "High Music" participants reporting the strategy as a secondary strategy.

Summary of Results from the Problem Solving Assessment

In order to summarize the results from the Problem-Solving Assessment, three tables were created, one for each problem included on the assessment. Frequency data in the form of percentages were entered into a table organized by strategy and comparison categories (overall, “Low Music” and “High Music,” “Low Spatial” and “High Spatial,” and “Low Analytic” and “High Analytic”). For each strategy and each comparison category, the most frequently reported primary strategy as well as the most frequently reported secondary strategy was highlighted (see Tables 29, 30, and 31).

Analysis of reported strategy use by comparison categories for Problem 1 revealed two interesting deviations from problem-solving trends (see Table 29). First, the most common secondary strategy reported by participants deemed “High Music” was “Trying to make a list of possible answers to see if there was a pattern” which deviated from the overall most commonly reported secondary strategy of “Trying to guess and then checking to see if the guess was right.” Second, the most common secondary strategy reported by participants deemed “Low Analytic” was “Trying to visualize the scenario while thinking about the problem” which again deviated from the overall most commonly reported secondary strategy of “Trying to guess and then checking to see if the guess was right.”

Table 29

Most Frequently Reported Primary and Secondary Strategies for Problem 1 by Comparison Categories

Strategy	Use	Overall (%)	LM (%)	HM (%)	LS (%)	HS (%)	LA (%)	HA (%)
Picture/Diagram	Prim.	9.8	7.8	10.3	10.9	7.3	9.2	7.5
	Sec.	5.3	6.2	7.2	5.3	3.9	7.2	2.5
	Non.	84.9	86.0	82.5	83.8	88.8	83.5	90.0
Formula	Prim.	11.4	11.6	12.4	12.1	9.5	9.6	10.9
	Sec.	7.5	5.0	8.2	6.1	11.7	5.2	10.0
	Non.	81.2	83.3	79.4	81.8	78.8	85.1	79.1
Equation	Prim.	33.0	31.0	29.9	32.4	36.9	32.9	36.8
	Sec.	11.5	12.8	6.2	10.5	14.5	10.8	14.9
	Non.	55.5	56.2	63.9	57.1	48.6	55.4	48.3
Table/List	Prim.	11.0	10.5	22.7	10.1	8.9	11.2	10.9
	Sec.	6.4	7.4	4.1	4.5	10.1	5.6	7.0
	Non.	82.6	82.2	73.2	85.4	81.0	83.1	82.1
Pattern	Prim.	7.8	8.9	8.2	6.9	6.7	10.8	4.0
	Sec.	6.0	3.1	15.5	2.8	6.7	6.0	5.0
	Non.	86.1	88.0	76.3	90.3	86.6	83.1	91.0
Visualization	Prim.	16.7	15.9	15.5	16.2	17.9	18.1	15.9
	Sec.	13.1	9.3	13.4	10.1	14.5	12.9	14.9
	Non.	70.2	74.8	71.1	73.7	67.6	69.1	69.2
Remember	Prim.	10.4	11.6	14.4	12.1	8.4	11.6	8.5
	Sec.	13.2	8.1	12.4	11.7	16.8	8.4	17.4
	Non.	76.4	80.2	73.2	76.1	74.9	79.9	74.1
Work Backwards	Prim.	5.5	5.8	2.1	6.9	3.9	8.0	4.5
	Sec.	6.2	7.8	8.2	4.0	8.9	7.6	7.0
	Non.	88.3	86.4	89.7	89.1	87.2	84.3	88.6
Guess & Check	Prim.	37.0	39.5	32.0	37.7	39.7	39.0	40.3
	Sec.	15.3	15.9	14.4	12.6	18.4	10.4	20.9
	Non.	47.7	44.6	53.6	49.8	41.9	50.6	37.8
Easier Problem	Prim.	2.1	3.1	1.0	4.9	1.1	3.2	1.5
	Sec.	2.6	3.5	5.2	0.8	2.8	2.8	4.0
	Non.	95.3	93.4	93.8	94.3	96.1	94.0	94.5

Note. LM = "Low Music"; HM = "High Music"; LS = "Low Spatial"; HS = "High Spatial"; LA = "Low Analytic"; HA = "High Analytic."

Analysis of reported strategy use by comparison categories for Problem 2 revealed one particularly interesting deviation from problem-solving trends (see Table 30). The most common primary strategy reported by participants deemed "Low Spatial" was "Trying to visualize the scenario while thinking about the problem" which deviated from the overall most commonly reported primary strategy of "Drawing a picture or diagram to help think about the problem."

Table 30

Most Frequently Reported Primary and Secondary Strategies for Problem 2 by Comparison Categories

Strategy	Use	Overall (%)	LM (%)	HM (%)	LS (%)	HS (%)	LA (%)	HA (%)
Picture/Diagram	Prim.	32.4	31.8	36.1	28.3	33.0	27.3	37.8
	Sec.	6.3	8.5	4.1	7.3	8.9	6.0	7.0
	Non.	61.3	59.7	59.8	64.4	58.1	66.7	55.2
Formula	Prim.	8.1	8.5	10.3	10.5	8.4	9.6	7.5
	Sec.	6.4	5.4	3.1	4.0	7.3	5.6	10.0
	Non.	85.5	86.0	86.6	85.4	84.4	84.7	82.6
Equation	Prim.	9.8	13.2	13.4	13.4	9.5	12.0	9.0
	Sec.	8.6	7.4	7.2	8.5	7.8	9.6	8.0
	Non.	81.6	79.5	79.4	78.1	82.7	78.3	83.1
Table/List	Prim.	25.9	24.0	24.7	28.3	26.8	25.7	22.9
	Sec.	6.9	7.0	6.2	3.2	3.9	5.6	7.5
	Non.	67.2	69.0	69.1	68.4	68.7	68.7	69.7
Pattern	Prim.	9.6	10.5	6.2	10.5	10.6	10.0	11.4
	Sec.	5.7	6.2	11.3	3.2	7.8	4.4	7.5
	Non.	84.7	83.3	82.5	86.2	81.6	85.5	81.1
Visualization	Prim.	19.3	19.8	14.4	29.0	19.6	20.1	19.9
	Sec.	11.2	12.4	14.4	8.5	13.4	12.4	12.9
	Non.	69.5	67.8	71.1	72.5	67.0	67.5	67.2
Remember	Prim.	13.4	12.4	10.3	12.1	8.9	14.5	12.9
	Sec.	11.8	10.1	11.3	13.0	17.9	9.2	14.9
	Non.	74.9	77.5	78.4	74.9	73.2	76.3	72.1
Work Backwards	Prim.	2.6	2.7	3.1	1.2	3.9	2.4	2.5
	Sec.	2.2	1.9	5.2	1.2	2.8	1.6	3.0
	Non.	95.2	95.3	91.8	97.6	93.3	96.0	94.5
Guess & Check	Prim.	5.6	6.2	7.2	8.1	1.7	6.8	3.5
	Sec.	6.0	5.0	8.2	2.4	8.4	5.2	5.5
	Non.	88.3	88.8	84.5	89.5	89.9	88.0	91.0
Easier Problem	Prim.	2.2	3.9	1.0	1.6	1.7	2.8	2.5
	Sec.	2.2	3.1	3.1	2.0	3.9	1.6	3.5
	Non.	95.6	93.0	95.9	96.4	94.4	95.6	94.0

Note. LM = "Low Music"; HM = "High Music"; LS = "Low Spatial"; HS = "High Spatial"; LA = "Low Analytic"; HA = "High Analytic."

Analysis of reported strategy use by comparison categories for Problem 3 also revealed interesting deviations from problem-solving trends (see Table 31). Specifically, the most common secondary strategy reported by participants deemed “High Music” as well as participants deemed “High Analytic” was “Trying to visualize the scenario while thinking about the problem.”

Table 31

Most Frequently Reported Primary and Secondary Strategies for Problem 3 by Comparison Categories

Strategy	Use	Overall (%)	LM (%)	HM (%)	LS (%)	HS (%)	LA (%)	HA (%)
Picture/Diagram	Prim.	15.8	13.6	15.5	17.0	11.2	16.9	13.9
	Sec.	4.9	5.4	5.2	4.0	7.8	3.2	7.0
	Non.	79.3	81.0	79.4	78.9	81.0	79.9	79.1
Formula	Prim.	9.0	8.1	8.2	11.7	7.8	11.2	7.5
	Sec.	6.5	5.8	8.2	4.0	7.3	4.4	8.0
	Non.	84.5	86.0	83.5	84.2	84.9	84.3	84.6
Equation	Prim.	35.8	36.0	37.1	34.0	34.1	33.3	40.8
	Sec.	9.8	9.7	6.2	8.1	12.3	10.0	9.0
	Non.	54.5	54.3	56.7	57.9	53.6	56.6	50.2
Table/List	Prim.	5.7	5.8	6.2	4.5	6.7	6.8	6.0
	Sec.	3.5	4.3	2.1	2.4	3.4	3.2	3.5
	Non.	90.7	89.9	91.8	93.1	89.9	90.0	90.5
Pattern	Prim.	3.3	2.7	3.1	4.9	2.2	2.0	3.5
	Sec.	2.2	1.6	4.1	2.4	2.2	1.6	2.5
	Non.	94.6	95.7	92.8	92.7	95.5	96.4	94.0
Visualization	Prim.	18.0	18.6	21.6	16.2	19.6	21.7	16.4
	Sec.	9.8	8.5	10.3	6.1	10.1	9.6	11.9
	Non.	72.3	72.9	68.0	77.7	70.4	68.7	71.6
Remember	Prim.	8.4	9.3	7.2	8.1	5.6	10.0	7.0
	Sec.	8.6	10.1	7.2	9.3	11.2	6.0	11.9
	Non.	83.0	80.6	85.6	82.6	83.2	83.9	81.1
Work Backwards	Prim.	11.8	12.0	12.4	11.3	14.0	12.9	11.9
	Sec.	8.2	8.9	8.2	6.1	11.2	9.6	10.9
	Non.	80.0	79.1	79.4	82.6	74.9	77.5	77.1
Guess & Check	Prim.	21.1	25.2	20.6	21.9	20.1	18.5	19.9
	Sec.	10.9	11.6	6.2	11.3	8.4	10.0	11.4
	Non.	68.0	63.2	73.2	66.8	71.5	71.5	68.7
Easier Problem	Prim.	1.3	2.7	1.0	2.8	0.0	2.4	0.5
	Sec.	1.3	0.8	2.1	0.4	1.1	0.8	1.5
	Non.	97.3	96.5	96.9	96.8	98.9	96.8	98.0

Note. LM = “Low Music”; HM = “High Music”; LS = “Low Spatial”; HS = “High Spatial”; LA = “Low Analytic”; HA = “High Analytic.”

Summary

This chapter discussed the results of data collection and analysis. Each of the four research questions was addressed. Results from both quantitative and qualitative data were provided. The following chapter presents a complete summary of the results for each research question and a discussion of implications of the research study.

CHAPTER V

DISCUSSION

Researchers have investigated the effects of music training on mathematics performance (Cheek and Smith, 1998; Costa-Giomi, 2004; Johnson and Memmott, 2006; Kinney, 2008; Whitehead, 2001). Various instructional interventions such as schema-based instruction and problem-solving strategy instruction have been shown to increase students' problem-solving performance and impact how students select and utilize specific problem-solving strategies (see for example Hensberry & Jacobbe, 2012; Kapur & Bielaczyc, 2012; Mousoulides, Christou, & Sriraman, 2008; Rousseau, 2009; Schoenfeld and Herrmann, 1980). The purpose of this research study was to combine these two ideas to investigate whether there is a relationship between music training and students' utilization of problem-solving strategies on mathematical problem-solving tasks.

This chapter presents a summary of the significant results followed by a discussion of the conclusions that can be drawn from the results. In addition, limitations and assumptions of the research study are provided along with recommendations and suggestions for future research. Finally, implications of the research are discussed and concluding remarks are made.

Summary of Results

Research Question 1

What is the level of music training of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a music background survey?

Approximately 52% of participants attested to having private music instruction of some type. The most common type of private music instruction was for stringed instruments followed by piano and woodwind instruments. For those participants who attested to having private music instruction, the mean of the reported years of private music instruction was 5.32 years.

School music participation declined rapidly as the level of schooling increased with 74.2% of participants experiencing school music instruction at the elementary school level, 61.3% of participants experiencing school music instruction at the middle school level, 32.2% of participants experiencing school music instruction at the high school level, and a meager 5.9% of participants experiencing school music instruction at the college or university level. In addition, band was the most popular school music program at all levels of education followed by choir.

Over 25% of the participants, "Low Music" participants, had no music instruction through either school music programs or private music instruction and had not participated in any formal instruction in music theory. In contrast, the participants deemed "High Music" reported means of approximately 11 years of private music instruction, approximately 7 semesters of music participation at the middle school level, and approximately 12 semesters of music participation at the high school level.

Research Question 2

What is the level of spatial reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by the Spatial Reasoning Assessment (adapted from the Spatial Visualization test developed by Michigan State University, 1981) and what are the differences in spatial reasoning ability between students with low and high levels of music training?

Participants' scores on the Spatial Reasoning Assessment were approximately normally distributed with a mean of 17.70 out of 30 questions and a standard deviation of 6.203 questions. "High Music" participants scored higher on the Spatial Reasoning Assessment, though the scores for "High Music" participants showed more variation than the scores for "Low Music" participants. No significant difference was found in performance on the Spatial Reasoning Assessment between participants ranked as "Low Music" and "High Music."

Research Question 3

What is the level of analytic reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by an analytic reasoning assessment and what are the differences in analytic reasoning ability between students with and without music training?

The distribution of participants' scores on the Analytic Reasoning Assessment was bell-shaped and skewed left with a mean of 26.34 out of 35 questions and a standard deviation of 5.150 questions. "High Music" participants scored slightly lower on the Analytic Reasoning Assessment, though the scores for "High Music" participants showed less variation than the scores for "Low Music" participants. No significant difference was found in performance on the Analytic Reasoning Assessment between participants ranked as “Low Music” and “High Music.”

Research Question 4

What problem-solving strategies are utilized by university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a problem-solving test and what are the differences of strategy selection between students with high and low spatial reasoning ability, between students with high and low analytic reasoning ability, and between student with and without music training?

When controlling for Analytic Reasoning, statistically significant differences in reported problem-solving strategy use were found for all three questions of the Problem-Solving Assessment. For problem 1, participants ranked as “High Analytic” reported using the strategy “Trying to guess and then checking to see if the guess was right” more frequently than participants ranked as “Low Analytic” whereas participants ranked as “Low Analytic” were more likely to report using the strategies “Drawing a picture or diagram to help think about the problem” and “Trying to make a list of possible answers to see if there was a pattern” than participants ranked as “High Analytic.” With respect to problem 2, participants ranked as “High Analytic” were more likely to report using the strategy “Drawing a picture or diagram to help think about the problem” than participants ranked as “Low Analytic.”

Controlling for Spatial Reasoning resulted in statistically significant differences in strategy utilization for Problems 1 and 2. For problem 1, participants ranked as “Low Spatial” who reported using the strategy “Trying to start with an easier problem and looking for a pattern” were more likely to report using the strategy as primary whereas participants ranked as “High Spatial” who reported using the strategy were more likely to report using the strategy as secondary. Similarly, for Problem 1, participants ranked as “Low Spatial” who reported using the strategy “Trying to guess and then checking to see if the guess was right” were more likely to report the strategy as primary whereas participants ranked as “High Spatial” who reported using the strategy were more likely to report the strategy as secondary.

When controlling for Music Background, statistically significant or marginally significant differences were found with respect to the utilization of the strategies “Trying to make a table or list of possible answers” and “Trying to make a list of possible answers to see if there was a pattern.” For Problem 1, the use of both strategies was found to be dependent on level of music training with participants deemed “High Music” reporting more frequent use of the strategies than “Low Music” participants. While no statistically significant results were found for Problem 2 when comparing “Low Music” and “High Music” participants, further investigation of the results for Problem 2 indicated that “Low Music” and “High Music” participants reported using the strategy “Trying to make a list of possible answers to see if there was a pattern” differently with “Low Music” participants reporting the strategy as primary and “High Music” participants reporting the strategy as secondary.

Additionally, analysis of comparison categories revealed that various groups of participants reported using strategies differently than the majority of the other participants. Specifically, for Problem 1, the most common secondary strategy reported by participants deemed “High Music” was “Trying to make a list of possible answers to see if there was a pattern” and the most common secondary strategy reported by participants deemed “Low Analytic” was “Trying to visualize the scenario while thinking about the problem,” both of which deviated from the overall most commonly reported secondary strategy of “Trying to guess and then checking to see if the guess was right.” With respect to Problem 2, the most common primary strategy reported by participants deemed “Low Spatial” was “Trying to visualize the scenario while thinking about the problem” which deviated from the overall most commonly reported primary strategy of “Drawing a picture or diagram to help think about the problem.” Finally, for Problem 3, the most common secondary strategy reported by participants deemed “High Music” as well as participants deemed “High Analytic” was “Trying to visualize the scenario while thinking about the problem” both of which deviated from the overall most commonly reported secondary strategy of “Trying to guess and then checking to see if the guess was right.”

Conclusions

The results of this research study provided evidence that students with musical training employ different mathematical problem-solving strategies than students without musical training. Conclusions are made based on the results of statistical analyses. Connections are also made between the results of this study and current research as discussed in the Review of Literature.

Research Question 1

What is the level of music training of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a music background survey?

In general, participants enrolled in first-year credit-bearing algebra-based mathematics courses have a wide variety of music training levels. However, music indices were skewed right indicating the majority of participants are on the low end of music training level.

Participants' self-reported involvement in school music programs indicated that school music participation declined rapidly as the level of schooling increased. Over 25% of the participants reported no music instruction at the middle school, high school, or university level. In a national survey of public elementary and secondary schools conducted during the 2008-2009 school year, 91% of the 1014 selected public secondary schools nationally that responded to the survey reported that some type of music course was offered during the 2008-2009 school year (Parsad and Spiegelman, 2011, p. 2-3). Hence, the fact that over one-third of participants were not involved in school music programs at the middle school or high school level is not likely due to access.

The Texas Education Agency (TEA) requires only one Fine Arts credit for high school graduation. Fine Arts credit can be obtained from courses including art, dance, music, theatre, or floral design (Texas Education Agency, 2013). So it is not surprising that only 32.2% of participants reported involvement in school music programs at the high school level since participation in school music programs at this level is optional.

Research Question 2

What is the level of spatial reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by the Spatial Reasoning Test (adapted from the Spatial Visualization Test developed by Michigan State

University, 1981) and what are the differences in spatial reasoning ability between students with and without music training?

Participants' scores on the Spatial Reasoning Assessment were approximately normally distributed with a mean of 17.70 out of 30 questions, corresponding to 59.0% correct, and a standard deviation of 6.203 questions. Participants in the present study have comparable spatial reasoning ability as compared to other university students who have been administered a similar analytic reasoning assessment (see for example Ben-Chaim & Lappan, 1986).

While, on average, participants with a strong music background ("High Music" participants) attained higher scores than participants with no music background ("Low Music" participants), the difference in mean score between "Low Music" and "High Music" participants was not statistically significant. Therefore, with respect to spatial reasoning ability, mathematics students with high levels of music training have no advantage or disadvantage when compared to students with little or no music training.

Research Question 3

What is the level of analytic reasoning ability of university students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by an analytic reasoning test and what are the differences in analytic reasoning ability between students with and without music training?

Participants' scores on the Analytic Reasoning Assessment (adapted from the Employee Aptitude Survey, Grimsley, Ruch, Warren, and Ford, 1986 as published by McFarlane, 1989) were skewed left with a mean of 26.34 out of 35 questions, corresponding to 75.3% correct responses, and a standard deviation of 6.203 questions.

Participants in the present study have comparable analytic reasoning ability as compared to other university students who have been administered a similar analytic reasoning assessment (see for example McFarlane, 1989; Reeve and Lam, 2005).

Analysis of participants' scores on the Analytic Reasoning Assessment revealed no significant difference in analytic reasoning ability between participants deemed "High Music" and participants deemed "Low Music." Hence, with respect to analytic reasoning ability, mathematics students with high levels of music training have no advantage or disadvantage when compared to students with little or no music training.

Research Question 4

What problem-solving strategies are utilized by students enrolled in first-year credit-bearing algebra-based mathematics courses as measured by a problem-solving test and what are the differences of strategy selection between students with high and low spatial reasoning ability, between students with high and low analytic reasoning ability, and between students with and without music training?

The primary purpose of this research was to investigate the relationships between musical training and the utilization of problem-solving strategies on mathematical problem-solving tasks. Current research has revealed that mathematics performance and mathematical problem-solving is related to both spatial-temporal and logical-analytical reasoning (Aldous, 2007; Bishop, 1980; Booth & Thomas, 2000). Moreover, researchers have pointed to music as a means to increase performance on assessments of spatial reasoning (Rauscher, Shaw, & Ky, 1993) as well as mathematics performance (Cheek & Smith, 1998). Therefore, all three aspects, music training, spatial reasoning ability, and analytic reasoning ability, were utilized as a means to investigate the utilization of problem-solving strategies.

Differences based on analytic reasoning ability. When comparing the problem-solving strategies utilized by “Low Analytic” and “High Analytic” participants, few meaningful differences were found. However, when comparing problem-solving strategies employed by “Low Analytic” and “High Analytic” participants, the utilization of the strategy “Trying to remember how I’ve seen the problem solved before” was found to be dependent on analytic reasoning ability. Results for both Problem 1 and Problem 3 revealed that “Low Analytic” participants more frequently reported using the strategy as primary whereas “High Analytic” participants more frequently reported using the strategy as secondary. Hence, “Low Analytic” participants demonstrated a greater reliance on remembering familiar procedures as a problem-solving strategy than “High Analytic” participants.

Differences based on spatial reasoning ability. When comparing problem-solving strategies utilized by “Low Spatial” and “High Spatial” participants, results were inconclusive. Meaningful conclusions based on the comparison of strategy utilization between “Low Spatial” and “High Spatial” participants could not be extrapolated.

Differences based on music background. When controlling for Music Background, significant results were found only for Problem 1. With respect to Problem 1, the utilization of the strategies “Trying to make a table or list of possible answers” and “Trying to make a list of possible answers to see if there was a pattern” was found to be dependent on level of music training. “High Music” participants reported using the strategies more frequently than “Low Music” participants. The strategies “Trying to make a table or list of possible answers" and "Trying to make a list of possible answers to see if there was a pattern" are related through the construction of a list. When solving Problem 1, participants with high levels of music training relied more heavily on the use of the construction of tables and lists as a mechanism for finding patterns than participants with low levels of music training.

Limitations and Assumptions

When considering the results of this study, four limitations of the research should be taken into account. Limitations include issues related to sampling procedure and the instrumentation used in the research study.

First, participation in this research study was voluntary. Sampling bias must be accounted for since participants could opt not to participate. Furthermore, not all participants complete the online survey and both online assessments; therefore, participant mortality was a limitation of this research.

Second, there exist students with both high levels of music training and high levels of mathematical ability. Many students with high levels of mathematical ability do not enroll in first-year credit-bearing mathematics courses because they can receive credit for these courses in alternative ways. For example, many high schools offer dual-credit courses in which students can receive college credit while completing high school coursework. Therefore, since the sampling procedure for this study focused on students enrolled in first-year credit-bearing algebra-based mathematics courses, students with both high levels of music training and high levels of mathematical ability were excluded from the research.

Third, with respect to instrumentation, issues arose during the implementation of the research study. The Spatial Reasoning Assessment contained two questions with internal errors that were not identified until after the research had been conducted. These errors could have affected the results from the Spatial Reasoning Assessment.

It is not known whether or not problems included in the Problem-Solving Assessment were free of cultural bias. Wilburne, Marinak, and Strickland (2011) pointed out “the language and context of many word problems may be familiar to U.S. students who speak English, but they are often unfamiliar to the ever-growing population of students from different cultures or those who are not fluent in English” (p. 461). Since demographic data related to race, ethnicity, and native language were not collected, it is impossible to rule out bias in the wording of the mathematical problems.

In addition to the limitations of the research study, two assumptions were made during the analysis of the data collected. First, it was assumed that participants were honest in their response to questions included in the Music Background Survey. Also, the assumption was made that, while completing the in-class Problem-Solving Assessment and the web-based Spatial and Analytic Reasoning Assessments, participants answered all questions to the best of their ability.

Suggestions

Results of the current study have given rise to three suggestions. First, strategies related to the creation of a list or table for the purposes of discerning a pattern was found to be dependent on participants’ musical background. Since pattern recognition is an important aspect of mathematical understanding (Bahna-James, 1991), educators should capitalize on music students’ inclination for pattern recognition to make mathematics meaningful and approachable, especially since students who show an affinity for music rarely confess an aptitude in mathematics (Gardner, 1985; Bahna-James, 1991).

Second, based on the results of this study, and echoed throughout current research, it is evident that students are reliant on algebraic methods of solving mathematical problem-solving tasks (see for example Lithner, 2000; Ross, Reys, Chavez, McNaught, & Grouws, 2011). Since mathematics encompasses more than just algebra, other mathematical reasoning (i.e., measurement, geometric, probabilistic, and statistical reasoning) should be included to provide a more well-rounded mathematics curriculum (Texas Higher Education Coordinating Board & Texas Education Agency, 2009).

Finally, Tillman (2002) defined culture as “a group’s individual and collective ways of thinking, believing, and knowing” (p. 4). Since the results of this study indicated that participants with high levels of music training favor the utilization of pattern recognition during mathematical problem solving, it appears that students with high levels of music training have a collective way of thinking and should be considered a cultural group. Mireles, Rahrovi, and Vasquez (2013) recommend that educators “attempt to utilize various cultural connections” (p.181). Hence, instructors at all levels should endeavor to recognize specific cultural groups, especially musically inclined students, and entice them by planning culturally relevant activities for the classroom.

Recommendations

Researchers interested in replicating or expanding upon the research conducted in the present study should consider making several improvements to instrumentation and procedure. Perhaps most pertinent to the investigation or problem-solving strategy utilization is the selection of tasks to be included in the problem-solving assessment.

While the selection of tasks for the Problem-Solving Assessment utilized in this research study was based on tasks presented in current literature, drawing substantial conclusions became difficult since connections could rarely be made between the various problem-solving tasks. More specifically, the current research would have benefitted from multiple questions of the similar type.

With respect to the problems included as mathematical problem-solving tasks for the present research, Problem 1 revealed the greatest variations in strategy usage between the different comparison groups. Problem 1 was a system of equations word problem traditionally approached in high school mathematics classes. On the other hand, Problem 3 revealed the least variation in strategy usage between the different comparison groups. Problem 1 received the highest difficulty rating from the participants while Problem 3 received the lowest difficulty rating. Hence, problem-solving tasks included on an assessment designed to answer question related to problem-solving strategy utilization should be approachable for participants but still challenging.

Results of self-reported strategy usage for Problem 1 revealed differences in the frequency in which “High Music” and “Low Music” participants reported using strategies related to pattern recognition. Since Problem 1 was the only problem to uncover differences in strategies related to pattern recognition, future research investigating the relationship between mathematical problem solving and music training would benefit from the inclusion of mathematical problem-solving tasks aimed at gauging participants’ use of strategies related to pattern recognition.

The present research study did not take into account certain demographic data that should be accounted for in future research studies, specifically, gender and socioeconomic status. Gender has been shown to be related to spatial reasoning ability (Battista, 1990; Johnson & Meade, 1987; Lynn, Allik, & Irwing, 2004) as well as mathematics performance (Hill, Corbett, & St. Rose, 2010). With respect to both spatial reasoning ability and mathematics performance, males have been shown to outperform females. Moreover, literature suggests that socioeconomic status has an impact on musical understanding and access to music programs (Abril & Gault, 2008; Keiper, Sandene, Persky, & Kuang, 2009).

The focus of this research was the relationship between mathematical problem solving and formal musical training; therefore, incidental music participation outside of formal musical training was also not taken into consideration for the purpose of this study. However, numerous participants attested to taking part in informal musical activities such as playing in a rock band or being a member of a dance team. Future research should shed light in whether or not informal music participation has an impact on spatial or analytic reasoning ability or even mathematical problem-solving.

The current study did not ask participants to report whether or not they were currently involved in any formal music training. Researchers have pointed out that spatial enhancement due to music instruction does not have a lasting effect (see for example Costa-Giomi, 1999). Therefore, an important factor to investigate in future research exploring the relationship between music training and mathematical problem solving is current involvement in music instruction.

Task-based interviews were attempted for this research project; however, only a limited number of participants agreed to take part in such interviews. Future research would benefit from a more comprehensive approach to collecting qualitative data.

Interviews could prove to be especially enlightening when considering when and how students utilize problem-solving strategies as well as the ways in which students use multiple strategies concurrently.

Task-based interviews employing concurrent verbal protocols and retrospective debriefing should be considered to provide additional insights into how students utilize problem-solving strategies while engaged in mathematical problem-solving tasks. While task-based interviews were attempted for this research project, a limited number of participants were willing to take part in interviews. Therefore, data collected from participant interviews was not sufficient. Future research should employ a more comprehensive approach to collecting qualitative data.

Finally, online administration of the Spatial Reasoning Assessment and the Analytic Reasoning Assessment was successful in attaining a high participant response rate. However, participants were able to access these assessments at their leisure and, therefore, assurances could not be made that participants were not receiving additional help from outside sources, including, for example, other people or the internet, while completing the assessments. In the future, researchers may wish to administer such assessments in a more controlled environment in order to ensure the assessments give an accurate measurement of spatial and analytic ability.

Summary and Implications

The results of participants' self-reported music background indicated that the overall level of music training of students enrolled in first-year credit-bearing algebra-based mathematics courses varies greatly. However, over 25% of participants had very little to no music instruction including no private music instruction and no music instruction at the middle school, high school, or university level. Movements in education, in an attempt to provide students with the best possible education, should continue to strive to ensure that all students have access to strong music programs. Furthermore, while many researchers have attested to the potential academic benefits of music involvement, perhaps music instruction should be championed for music's sake rather than for potential benefits in other academic areas since, as Demorest and Morrison (2000) pointed out, "musical intelligence and achievement is its own reward" (p. 38).

The idea of music for music's sake is further supported by the results indicating participants with strong music backgrounds have spatial and analytic reasoning abilities equivalent to those of participants with little to no music background. From these results, it can be concluded that music participation does not detract from students' development of reasoning skills or from students' academic achievement. Demorest and Morrison (2000) echoed this sentiment when they pointed out that research citing higher than average academic achievement of music students is a "direct contradiction to the 'back to basics' mentality that views music and other arts as frills that distract students from more important subjects" (p. 38-39). Therefore, movements toward encouraging the elimination of arts programs, specifically music programs, in hopes of giving students additional time for "core" courses should be reconsidered.

When considering mathematical problem-solving, there is a general belief that there are two distinct types of thinking or reasoning, analytic and spatial, and that people are inclined to use one over the other based on ability (Battista, 1990). However, the inconclusive results of this research study indicated selected problem-solving approach may have less to do with ability and more to do with general preference as asserted by

Campbell, et al. (1995). As the researchers suggested, further research needs to be conducted to investigate the effects that preference and ability have on students' selection and use of various problem-solving strategies.

Results from the investigation of the relationship between music training and the utilization of mathematical problem-solving strategies indicated that participants with high levels of music training have a tendency to use tables and lists to identify patterns when solving mathematical problems. When faced with an approachable yet challenging mathematical problem-solving task, participants with high levels of music training rely more heavily on the construction of tables and lists and the recognition of patterns than participants with low levels of music training. Pattern recognition and generalization plays an important role in the development of mathematical understanding and problem solving (see for example Bahna-James, 1991). Mathematics educators could capitalize on these students' affinities for music and pattern recognition to make mathematics meaningful for these students, especially since students who show a strong affinity toward music rarely confess an interest or aptitude in mathematics (Gardner, 1985; Bahna-James, 1991).

Finally, the overall strategy utilization results of Problems 1 and 3 suggested that students rely on problem-solving strategies they have seen demonstrated in the mathematics classroom. For each of these problems, the construction of an equation was reported as a primary problem-solving strategy by over one-third of the entire group of participants. Since both Problem 1 and Problem 3 were word problems commonly included in school algebra curricula, it may be concluded that students are conditioned to solve mathematical problems using the algebraic methods they experience in school curriculum. Lithner (2000) attested to the dangers of students' focus on familiar procedures pointing out that such a focus can lead to the oversight of alternative strategies, careless mistakes, and struggles when problem-solving tasks are not completely familiar. Participants' reliance on the use of equations to solve mathematical problems can also be attributed to the promotion of analytic reasoning in schools (Michaelides, 2002). However, current curriculum standards for mathematics have pointed out that mathematical knowledge is not only about algebraic reasoning, but includes numerical reasoning, geometric reasoning, and even statistical reasoning. For example, the Texas Career and College Readiness Standards stated, "Mathematics cannot be viewed solely as a series of courses or a set of specific skills" (Texas Higher Education Coordinating Board & Texas Education Agency, 2009, p. 7). Mathematics educators should take a more holistic approach to teaching mathematics in which both analytic and spatial problem-solving strategies are embraced. Moreover, students should be given the opportunity to develop their problem-solving ability through tasks and projects where they can explore various traditional and non-traditional problem-solving techniques.

In order to help students develop critical thinking skills and true problem-solving prowess, mathematics educators should strive to embrace different reasoning preferences and problem-solving styles as well as encourage divergent and inventive problem-solving approaches in the mathematics classroom. In addition, if educators can capitalize on students' interests and strengths, students may be able to develop into effective and innovative problem solvers.

APPENDIX A

LITERATURE REVIEW PROCESS

The literature reviewed for this research study was pulled from two distinct bodies of research, the first focused on the effects of music listening and training and the second on mathematical problem solving. First, literature related to the effects of music training and listening on spatial-temporal reasoning, brain development and function, and mathematics performance was reviewed to provide background regarding the effects of music listening and training. Second, literature related to the factors impacting the selection and utilization of various problem-solving strategies was examined to provide a background mathematical problem solving. Finally, in an attempt to link these two disparate bodies of research, a search of literature was conducted for literature related to problem-solving strategies employed when creating and performing music. This appendix outlines the process used to locate the literature used to provide the background for this research study.

Effects of Music Training and Listening

An initial search of peer-reviewed literature regarding the effects of music training and listening on spatial reasoning, brain function and development, and mathematics performance was conducted to establish the first iteration of the literature review and to inform the pilot study. Education Resource Information Center (ERIC) was used to collect relevant literature published between January 2005 and December 2011.

Three searches were conducted using the following search terms: “music” and “spatial,” “music” and “brain,” and “music” and “mathematics” was used for the initial search. ERIC returned 59, 123, and 180 results, respectively, for a total of 362 documents. A similar search using ProQuest Education Journals database yielded a total of 403 documents. Documents acquired from ERIC and ProQuest were then narrowed using a process similar to the process described below.

An additional search for literature related to the effects of music training and listening on spatial reasoning, brain function and development, and mathematics performance was conducted prior to the publication of this dissertation. Education Resource Information Center (ERIC) was used to collect relevant literature published between January 2012 and June 2013. Three searches were conducted using the following search terms: “music” and “spatial,” “music” and “brain,” and “music” and “mathematics.”

The first search, using the search criteria “music” and “spatial,” yielded five results, three of which did not report results of research related to the impact of music training or listening on spatial reasoning ability. Therefore, ERIC returned two articles regarding the impact of music training or listening on spatial reasoning ability that were relevant to the current study.

The second search, using the search criteria “music” and “brain,” yielded 17 results, 12 of which did not report results of research related to the impact of music training or listening on brain development or function. Of the remaining five items, four items were editorial in style and/or discussed pedagogical practices rather than the results of research. The final article reported research relating to the effects of music on children with autism. Therefore, ERIC did not return any articles relevant to the current study regarding the impact of music training or listening on brain development or function.

The third search, using the search criteria “music” and “mathematics,” yielded 21 results, 14 of which did not report results of research related to the impact of music training or listening on mathematics learning or ability. Of the remaining seven items, three items were editorial in style and/or discussed pedagogical practices rather than the results of research. Therefore, ERIC returned four articles regarding the impact of music training or listening on mathematics learning or ability that were relevant to the current study.

The ProQuest Education Journals database was also used to locate relevant literature published between January 2011 and June 2013. Again, three searches were conducted using the following search terms: “music” and “spatial,” “music” and “brain,” and “music” and “mathematics” was used for the initial search. The search using ProQuest yielded seven documents relevant to the current study; however, the four documents had already been obtained during previous searches. Hence, the search using ProQuest contributed three additional documents regarding the impact of music training or listening on mathematics learning or ability that were relevant to the current study.

Selection and Utilization of Problem-Solving Strategies

An initial search of peer-reviewed literature regarding the selection and utilization of problem-solving strategies was conducted to establish the first iteration of the literature review and to inform the pilot study. ERIC was used to collect relevant literature published between January 2005 and December 2011. The search criteria “mathematics” and “problem solving” was used for the initial search. Nearly 2000 documents were returned by ERIC. These documents were then narrowed using a process similar to the process described below.

An additional search for literature related to the factors impacting the selection and utilization of problem-solving strategies was conducted prior to the publication of this dissertation. Using the search criteria “mathematics” and “problem solving”, 401 entries were returned by Education Resource Information Center (ERIC) for January 2012 through June 2013. Of these 401 documents, 252 entries were not related to the selection or utilization of problem-solving strategies. These entries included state academic standards, teaching resources and handbooks, articles describing pedagogical practices, discussions of educational philosophy, editorial-style papers, mathematics papers outlining various approaches to solving specific mathematical problems, collections of articles that included papers regarding mathematics or problem solving, and other articles with descriptions that merely included the words mathematics, problems, or solving. The elimination of these 252 documents left 149 entries to consider. An additional 29 documents were not considered for the purposes of this research study due to the emphasis on special student populations. Specifically, 19 articles focused on interventions intended to enhance the problem-solving performance of English language learners, students with disabilities, and students with behavioral problems and 10 articles involved research related to the development of mathematics ability and knowledge in very young children (i.e., children at the pre-school or kindergarten levels). Of the remaining 93 entries, 75 articles reported the results of research related to assessment of instrumentation, research methodology practices, or interventions intended to increase mathematics performance. Therefore, a search using ERIC for relevant literature published between January 2012 and June 2013 yielded 18 articles reporting the results of research related to factors impacting the selection and utilization of problem-solving strategies.

The ProQuest Education Journals database was also used to locate relevant literature published between January 2011 and June 2013. Using the search criteria "mathematics" and "problem solving," ProQuest yielded 263 documents. Only 12 documents reporting the results of research related to factors impacting the selection and utilization of problem-solving strategies were located. Of these 12 documents, four documents had already been obtained during previous searches. Hence, the search using ProQuest contributed eight additional documents regarding factors impacting the selection and utilization of problem-solving strategies that were relevant to the current study.

Music and Problem Solving

An initial search of literature produced few documents investing the effects of musical training on the strategies students employ when engaged in mathematical problem solving. Using the search criteria "music" and "problem solving," 53 entries were returned by ERIC for January 2005 through December 2011. Of these 53 articles, 27 entries did not explore the relationship between music and problem solving, 11 items did not report the results of a research study, and the remaining 15 articles discussed problem-solving strategies utilized during music practice, performance, composition, or teaching. Similar results were obtained when searching the ProQuest Education Journals database. The search criteria "music" and "problem solving" yielded 53 entries for January 2005 through December 2011. Of the 53 returned items, 41 entries consisted of non-research based articles, seven articles discussed problem solving in the context of music composition, practice, performance, or teaching, and five articles reported finding or research studies which investigated the effects of music participation on subjects with behavioral problems. Hence, none of the items returned in either search investigated the relationship between music training and mathematical problem solving. However, the items related to problem solving in the context of music composition, practice, and performance were included in the literature review to provide insight regarding the problem-solving strategy utilization of musicians.

An additional search for literature related to the effects of musical training on the strategies students employ when engaged in mathematical problem solving was conducted prior to the publication of this dissertation. Using the search criteria "music" and "problem solving," five entries were returned by ERIC for January 2012 through June 2013. Of these five entries, four entries were related to curriculum issues (not reporting the results of research related to music and problem solving) or merely contained the terms "music" and "problem solving." The final document discussed the results of research regarding problem solving in the context of music learning. Therefore, ERIC did not return any articles relevant to the current study regarding the effects of musical training on the strategies students employ when engaged in mathematical problem solving or the problem-solving process employed by musicians when composing, practicing, or performing.

The ProQuest Education Journals database was also used to locate relevant literature published between January 2011 and June 2013. Using the search criteria "music" and "problem solving," ProQuest yielded 6 documents. None of the entries returned from this search were related to the effects of musical training on the strategies students employ when engaged in mathematical problem solving. In addition, no entry returned discussed the problem-solving process employed by musicians when composing, practicing, or performing. Therefore, ProQuest did not return any results relevant to the current research study.

APPENDIX B

MUSIC BACKGROUND SURVEY

Participation in this pilot study is voluntary. By completing this assessment you are giving your consent to the researcher to have free access to your assessment results. Also, the researcher may request information from your instructor to further validate the assessments under investigation. However, it is important to note that **your course grade will in no way be negatively affected by the results of the assessment you complete as part of this pilot study.**

This assessment has been constructed by the researcher to learn about your background in music.

Instructions:
- Read each question carefully.
- Please respond honestly to each question.

1. Have you ever taken private music lessons (including voice lessons)?

   If yes, please indicate in the space provided:
   - the instrument(s) you studied
   - the number of years you studied each instrument
   - the age at which you started private lessons on each instrument

Instrument	Years Studies	Age Started
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

2. During **elementary school**, how many years did you participate in school music programs such as school band or school choir?

   ☐ A. None
   ☐ B. 1 year of school music
   ☐ C. 2 years of school music
   ☐ D. 3 or more years of school music

3. While in **middle school**, did you participate in any school music programs?

   If so, to the right of the program(s) in which you participated, indicate the number of **semesters** you participated in the program.

Middle School Band	______ semesters
Middle School Orchestra	______ semesters
Other Instrumental Ensemble	______ semesters
Middle School Choir	______ semesters
Other Vocal Ensemble	______ semesters

4. While in **high school**, did you participate in any school music programs?

   If so, to the right of the program(s) in which you participated, indicate the number of **semesters** you participated in the program.

High School Band	______ semesters
High School Orchestra	______ semesters
Other Instrumental Ensemble	______ semesters
High School Choir	______ semesters
Other Vocal Ensemble	______ semesters

5. While in **college**, did you participate in any school music programs?

   If so, to the right of the program(s) in which you participated, indicate the number of **semesters** you participated in the program.

College Band	______ semesters
College Orchestra	______ semesters
Other Instrumental Ensemble	______ semesters
College Choir	______ semesters
Other Vocal Ensemble	______ semesters

6. Please describe any other formal music training or activity (private or group) not noted in the previous questions that you feel may be relevant.

7. Have you taken any music theory courses at the **university** level?
   If yes, please indicate the number of semesters of music theory you have taken. If no, please select "None."

   ☐ A. 1 semester of Music Theory
   ☐ B. 2 semesters of Music Theory
   ☐ C. 3 semesters of Music Theory
   ☐ D. 4 or more semesters of Music Theory
   ☐ E. None

8. Did you take any music theory courses when you were in **high school**?
   If yes, please indicate the number of semesters of music theory you have taken. If no, please select "None."

   ☐ A. 1 semester of Music Theory
   ☐ B. 2 semesters of Music Theory
   ☐ C. 3 semesters of Music Theory
   ☐ D. 4 or more semesters of Music Theory
   ☐ E. None

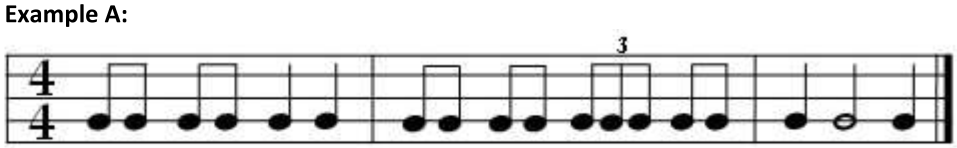
9. Did you study music theory as a component of **private music lessons**?
   If yes, please indicate the number of semesters of music theory you have taken. If no, please select "None."

   ☐ A. 1 year
   ☐ B. 2 years
   ☐ C. 3 years
   ☐ D. 4 or more years
   ☐ E. None

10. Please describe any other experience you have with Music Theory that you feel may be relevant.

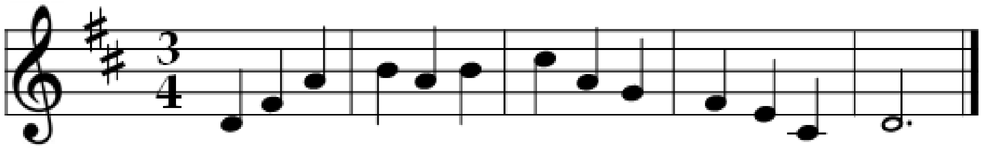
On a scale from 1 to 5, with 5 being the highest, indicate how confident you are that you could perform the task described.

Example A:



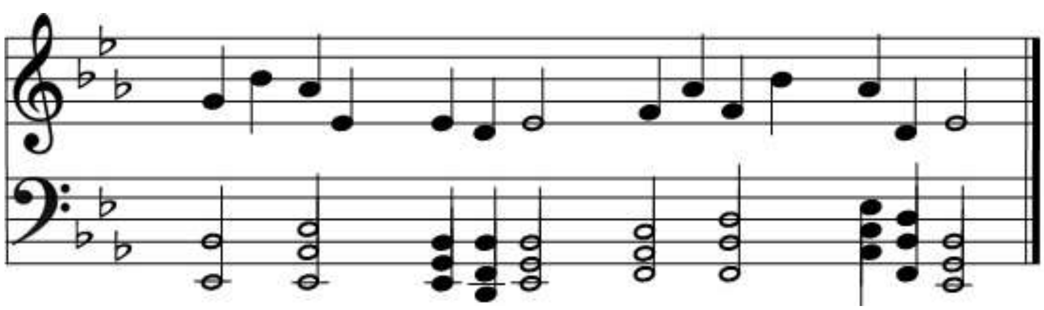
	Not Confident 1	**2**	**3**	**4**	**Very Confident 5**
11. Given **Example A**, how confident are you that you could correctly clap this rhythmic pattern?	☐	☐	☐	☐	☐

Example B:



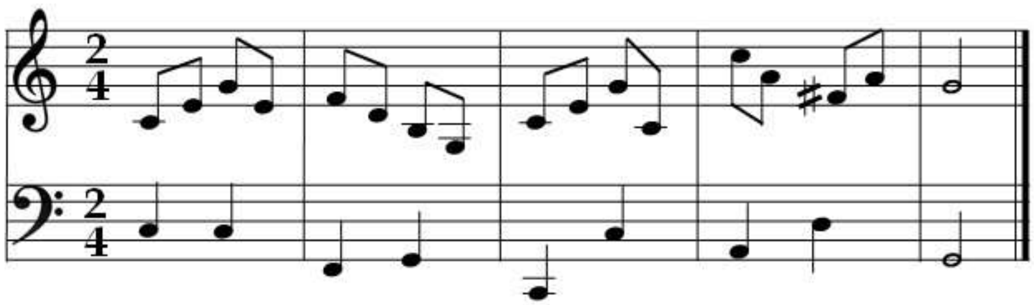
	Not Confident 1	2	3	4	Very Confident 5
12. Given **Example B**, how confident are you that you could correctly identify the note names written in the music?	☐	☐	☐	☐	☐
13. Given **Example B**, how confident are you that you could correctly determine the key the music is written in?	☐	☐	☐	☐	☐
14. Given **Example B**, how confident are you that you could correctly distinguish between major and minor intervals?	☐	☐	☐	☐	☐

Example C:



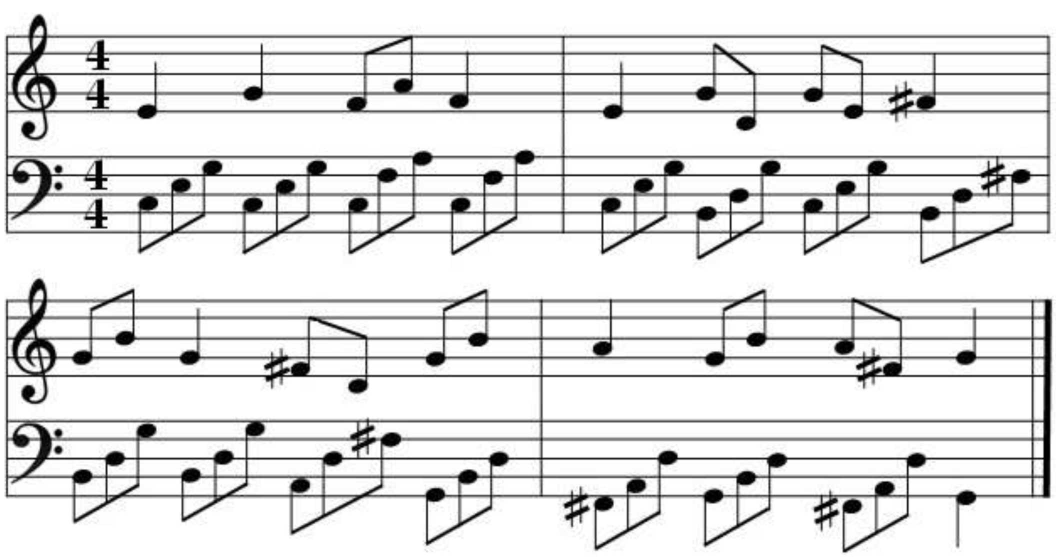
	Not Confident 1	2	3	4	Very Confident 5
15. Given **Example C**, how confident are you that you could correctly distinguish between triads in root position and inverted triads?	☐	☐	☐	☐	☐
16. Given **Example C**, how confident are you that you could correctly transpose the music into the key of G Major?	☐	☐	☐	☐	☐
17. Given **Example C**, how confident are you that you could correctly identify the harmonic progressions (i.e., I, i, II, ii, etc.) throughout the music?	☐	☐	☐	☐	☐

Example D:



	Not Confident 1	2	3	4	Very Confident 5
18. Given **Example D**, how confident are you that you could correctly identify the chromatic tone(s)?	☐	☐	☐	☐	☐
19. Given **Example D**, how confident are you that you could correctly identify the diatonic seventh chords?	☐	☐	☐	☐	☐
20. Given **Example D**, how confident are you that you could correctly identify the secondary dominant chords in the composition?	☐	☐	☐	☐	☐

Example E:



	Not Confident 1	2	3	4	Very Confident 5
21. Given **Example E**, how confident are you that you could correctly determine whether or not the composition modulates to a new key within the piece of music?	☐	☐	☐	☐	☐

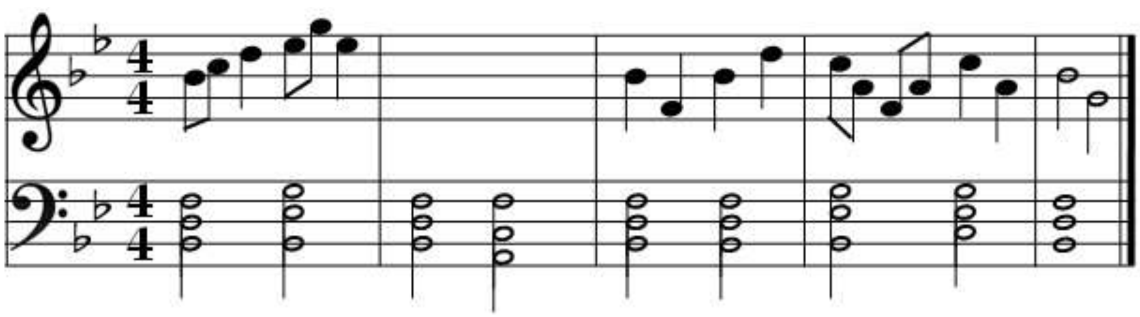
Example F:

	Not Confident 1	2	3	4	Very Confident 5
22. Given the melody in **Example F**, how confident are you that you could produce an appropriate chord progression to accompany the melody?	☐	☐	☐	☐	☐

Example G:

	Not Confident 1	2	3	4	Very Confident 5
23. Given **Example G**, how confident are you that you could determine a sensible place to start a canon within the four measures?	☐	☐	☐	☐	☐

Example H:



	Not Confident 1	2	3	4	Very Confident 5
24. Given **Example H**, how confident are you that you could fill in the missing measure in the melody in a way that would make melodic sense?	☐	☐	☐	☐	☐

APPENDIX C

ANALYTIC REASONING ASSESSMENT

Participation in this study is voluntary. By completing this assessment you are giving your consent to the researcher to have free access to your assessment results. Also, the researcher may request information from your instructor to further validate the assessments under investigation. However, it is important to note that **your course grade will in no way be negatively affected by the results of the assessment you complete as part of this pilot study**.

INSTRUCTIONS
This assessment contains a series of 10 items. For each item, you will be given several facts followed by five statements. Your task is to determine whether each statement is true given **only the facts that you are presented with**.

If the statement is true given the facts, select **TRUE**.
If the statement is false given the facts, select **FALSE**.
If you cannot determine whether the statement is true or false given the facts, select **CANNOT BE DETERMINED**.

1. Here are the facts:
   Antiques are older than classics.
   Classics are younger than heirlooms.
   Classics are the same age as relics.
   Relics are older than collectibles.

	TRUE	FALSE	CANNOT BE DETERMINED
a. Relics are older than antiques.	☐	☐	☐
b. Antiques and heirlooms are the same age.	☐	☐	☐
c. Classics are older than collectibles.	☐	☐	☐
d. Classics are not the oldest.	☐	☐	☐
e. Relics are the oldest.	☐	☐	☐
f. Collectibles are the youngest.	☐	☐	☐
g. Antiques are the oldest.	☐	☐	☐

2. Here are the facts:
All seats in section A are box seats.
Hal has box seat season tickets.
Peter does not have bleacher seats.
Jake's season tickets are in section A.
All seats in section A are padded.

	TRUE	FALSE	CANNOT BE DETERMINED
a. Jake sits on padded seats.	☐	☐	☐
b. Peter sits in box seats.	☐	☐	☐
c. Hal's season tickets are in section A.	☐	☐	☐
d. Some seats in section A are bleacher seats.	☐	☐	☐
e. Jake is a Blue Jay fan.	☐	☐	☐
f. All box seats in section A are padded.	☐	☐	☐
g. Jake does not have box seats.	☐	☐	☐

3. Here are the facts:
Sam is taller than Tony.
Jim is taller than Alex.
Carl is shorter than Jim.
Tony is the same height as Jim.

	TRUE	FALSE	CANNOT BE DETERMINED
a. Carl is shorter than Tony.	☐	☐	☐
b. Alex is taller than Carl.	☐	☐	☐
c. Sam is taller than Alex.	☐	☐	☐
d. Jim is taller than Tony.	☐	☐	☐
e. Sam is shorter than Jim.	☐	☐	☐
f. Alex is the shortest.	☐	☐	☐
g. Sam is the tallest.	☐	☐	☐

4. Here are the facts:
   Kathleen does not ski.
   Elizabeth and all of her co-workers do ski.
   Elizabeth is not a computer programmer.
   Elizabeth has a co-worker who is a computer programmer.

	TRUE	FALSE	CANNOT BE DETERMINED
a. Kathleen is not Elizabeth's co-worker.	☐	☐	☐
b. Kathleen is a skier.	☐	☐	☐
c. Elizabeth is a skier.	☐	☐	☐
d. All computer programmers ski.	☐	☐	☐
e. Some computer programmers ski.	☐	☐	☐
f. No computer programmers ski.	☐	☐	☐
g. Kathleen is a computer programmer.	☐	☐	☐

5. Here are the facts:
   City A is further than City B.
   City C is nearer than City B.
   City D is further than City C.
   City B is the same distance as City E.

	TRUE	FALSE	CANNOT BE DETERMINED
a. City E is further than City A.	☐	☐	☐
b. City A and City D are the same distance.	☐	☐	☐
c. City A is nearer than City C.	☐	☐	☐
d. City C is nearer than City E.	☐	☐	☐
e. City B is further than City E.	☐	☐	☐
f. City A is the furthest.	☐	☐	☐
g. City C is the closest.	☐	☐	☐

Adapted from Grimsley, Ruch, Warren, and Ford (1986) as published by McFarlane (1989).

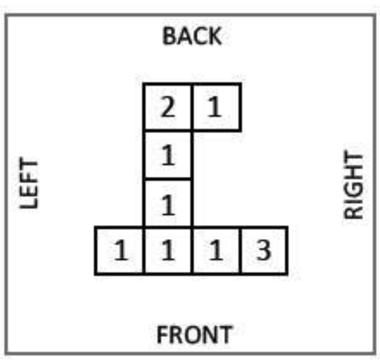
APPENDIX D

SPATIAL REASONING ASSESSMENT

Participation in this pilot study is voluntary. By completing this assessment you are giving your consent to the researcher to have free access to your assessment results. Also, the researcher may request information from your instructor to further validate the assessments under investigation. However, it is important to note that **your course grade will in no way be negatively affected by the results of the assessment you complete as part of this pilot study**.

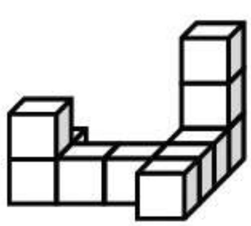
Sample Items

This is an example of a mat plan of a building. The number in each square tells how many cubes are to be placed on that square.



Use the information in the mat plan to answer the two sample items.

Sample Item 1:
This is a corner view of the building above. Which corner was it drawn from?



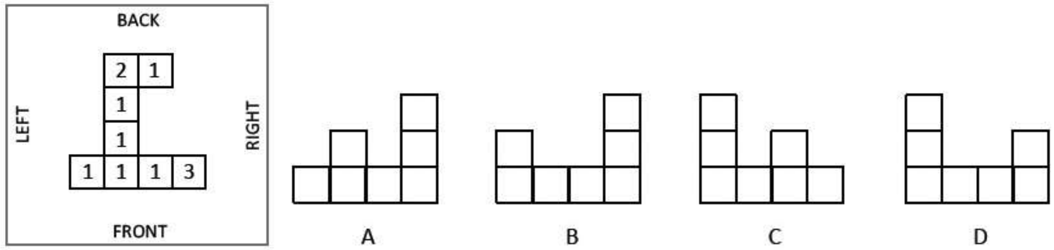
A. FRONT-RIGHT

B. BACK-RIGHT

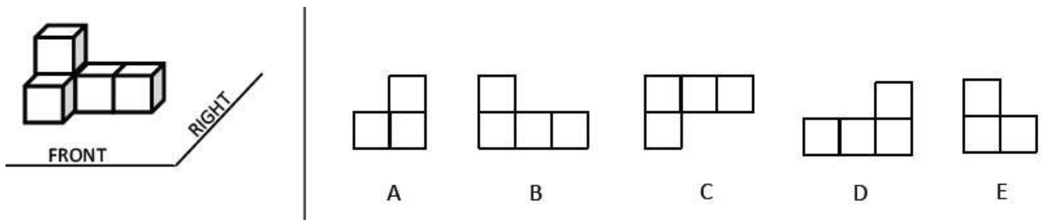
C. BACK-LEFT

D. FRONT-LEFT

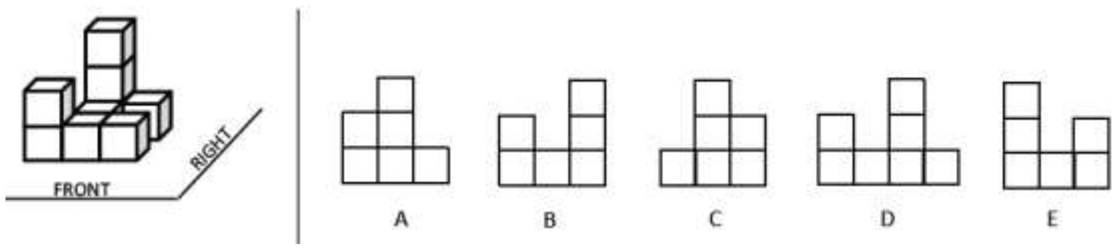
Sample Item 2:
These are the views of the same building, when seen straight on from the sides.
Which is the FRONT-VIEW?



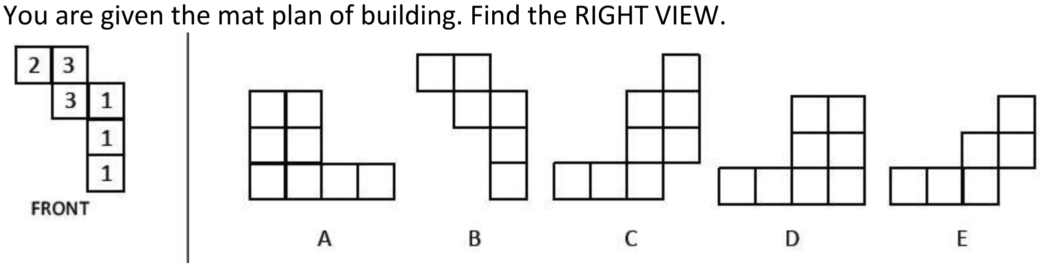
1. You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the RIGHT VIEW.



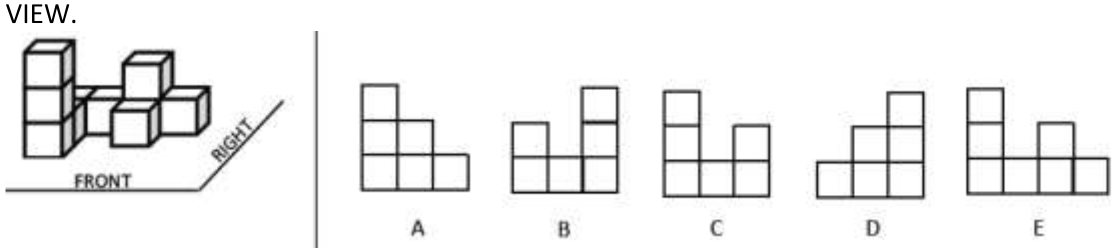
2. You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the BACK VIEW.



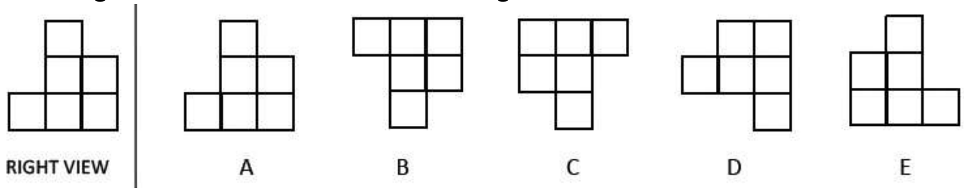
3. You are given the mat plan of building. Find the RIGHT VIEW.



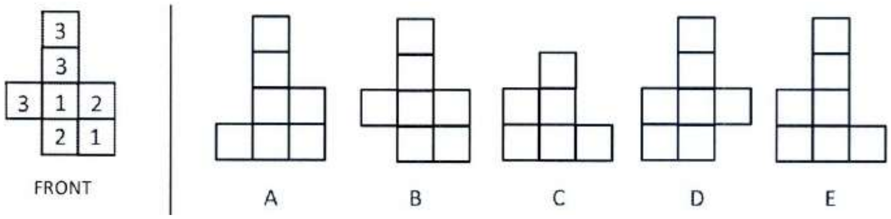
4. You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the LEFT VIEW.



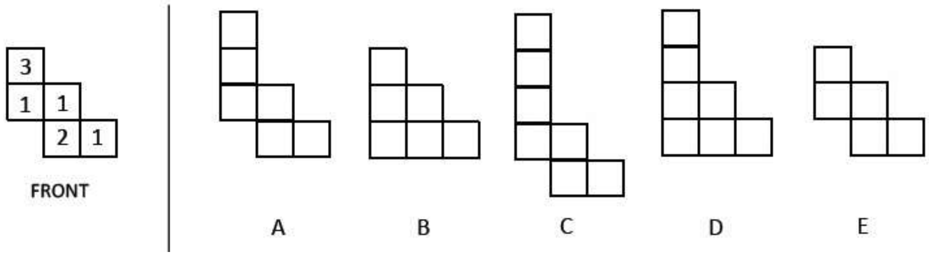
5. You are given the RIGHT VIEW of a building. Find the LEFT VIEW.



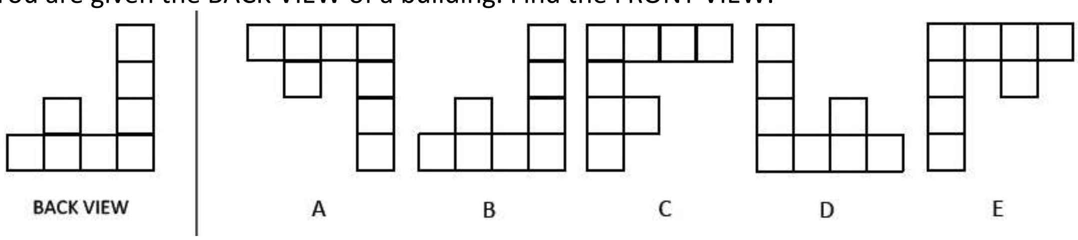
6. You are given the mat plan of a building. Find the BACK VIEW.



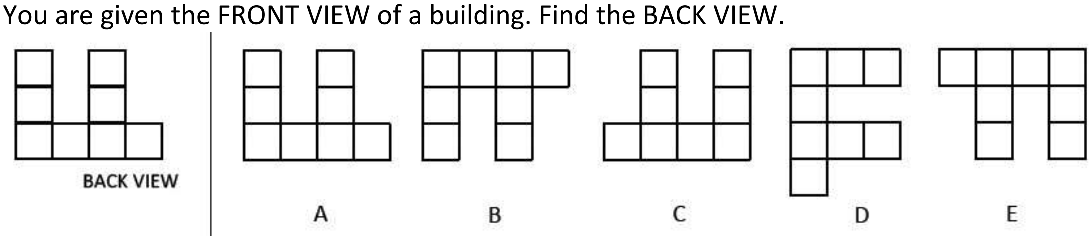
7. You are given a mat plan of a building. Find the FRONT VIEW.



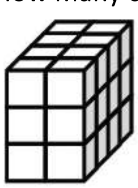
8. You are given the BACK VIEW of a building. Find the FRONT VIEW.



9. You are given the FRONT VIEW of a building. Find the BACK VIEW.

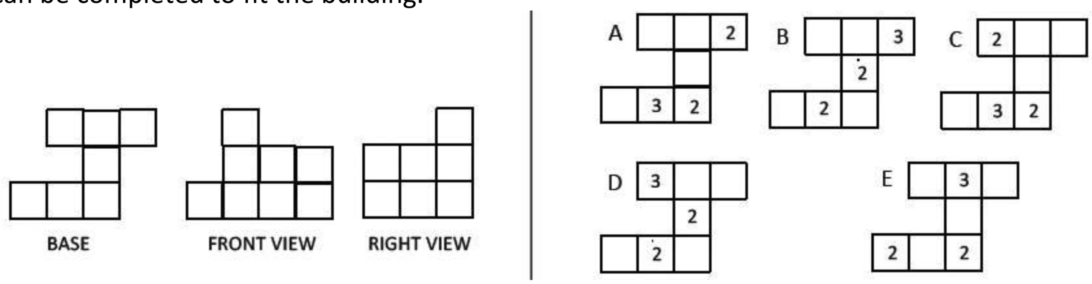


10. How many cubes are needed to build this rectangular solid?

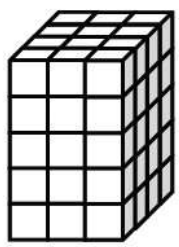


A. 18
B. 24
C. 26
D. 36
E. 52

11. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan that can be completed to fit the building.

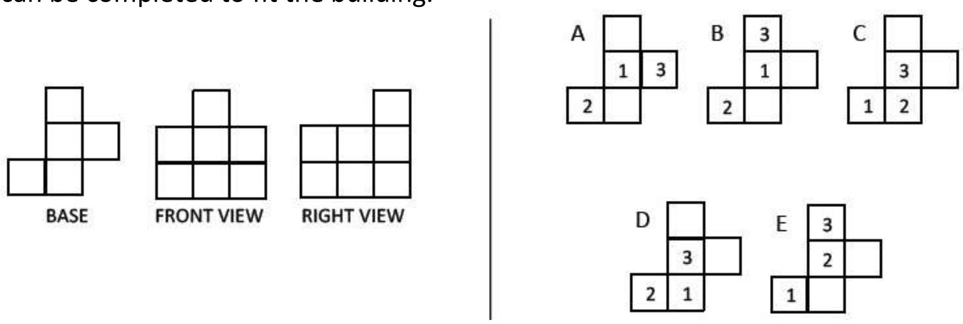


12. How many cubes are needed to build this rectangular solid?

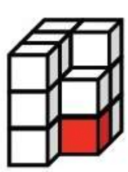


A. 36
B. 47
C. 60
D. 72
E. 94

13. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan that can be completed to fit the building.

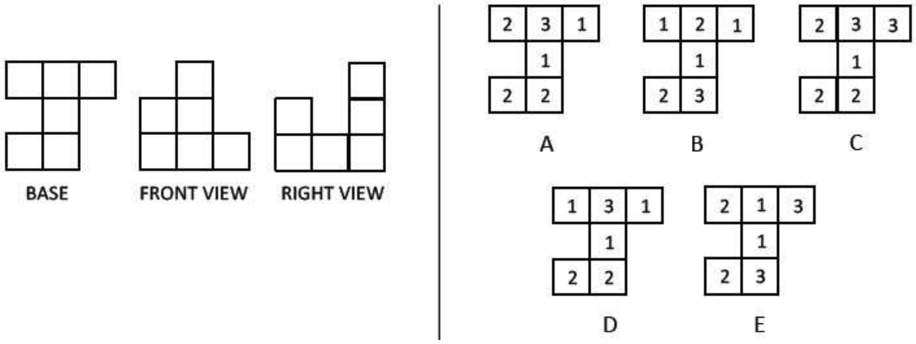


14. How many cubes touch the red cube face to face?

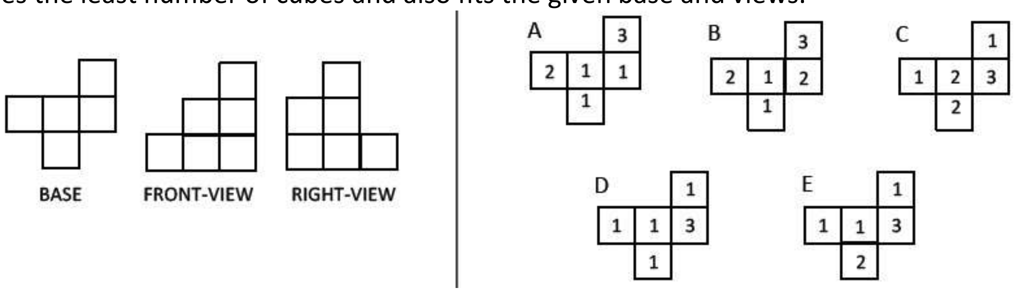


A. 1
B. 2
C. 3
D. 4
E. 5

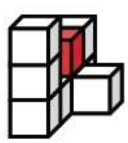
15. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan that uses the greatest number of cubes and also fits the given base and views.



16. You are given the BASE, FRONT VIEW, and RIGHT VIEW of a building. Find the mat plan that uses the least number of cubes and also fits the given base and views.



17. How many cubes touch the red cube face to face?



A. 1

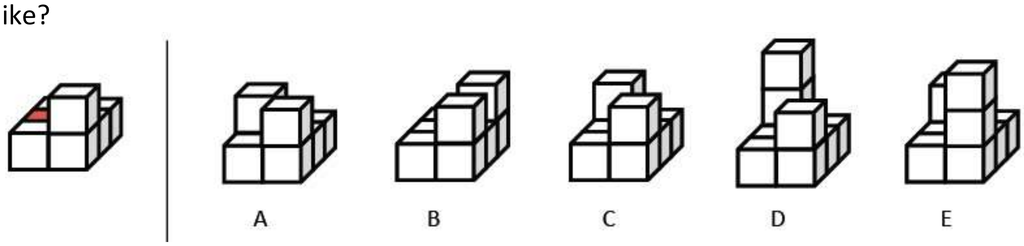
B. 2

C. 3

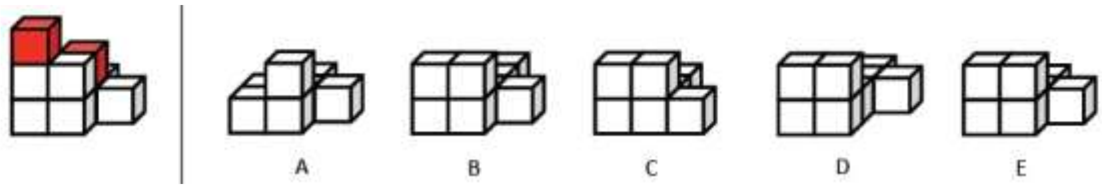
D. 4

E. 5

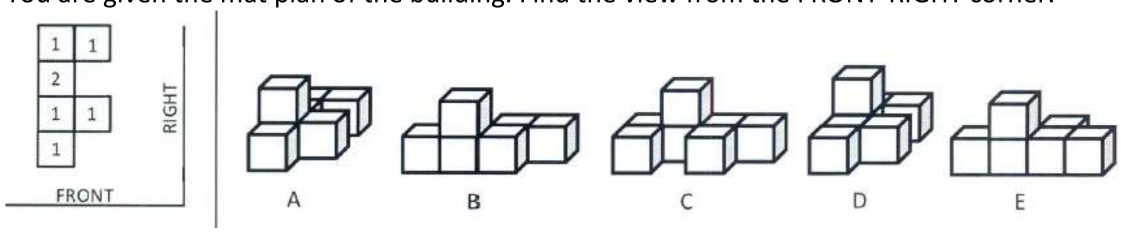
18. If a cube were added to the red face of the given building, what would the new building look like?



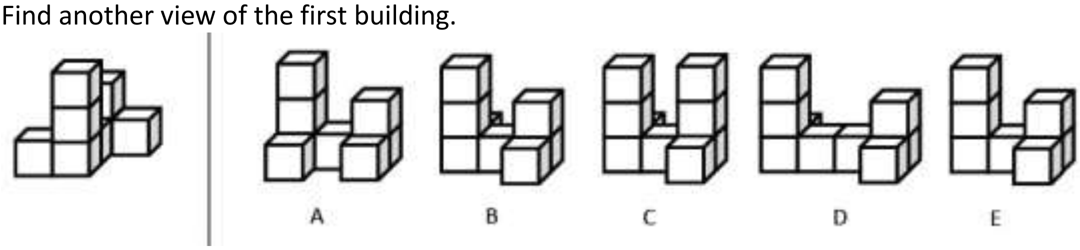
19. If the red cubes were removed from the given building, what would the new building look like?



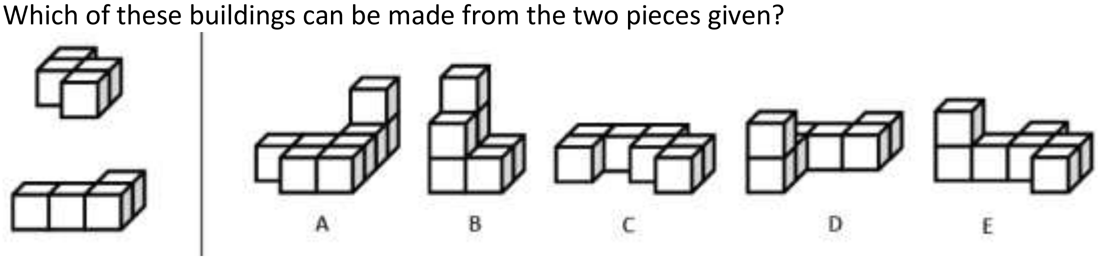
20. You are given the mat plan of the building. Find the view from the FRONT-RIGHT corner.



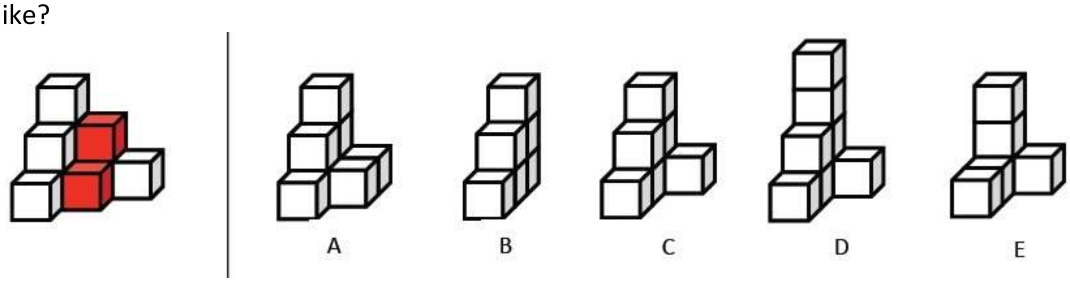
21. Find another view of the first building.



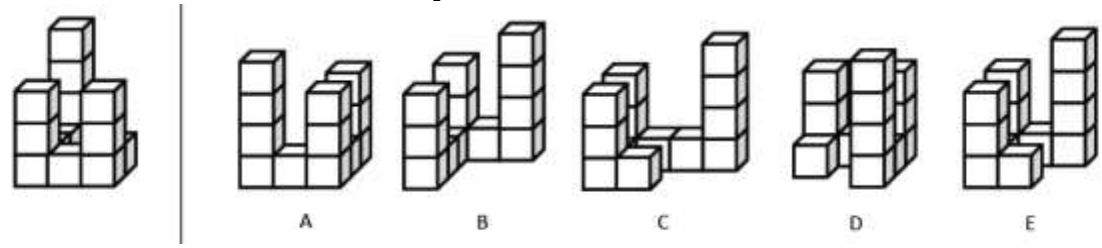
22. Which of these buildings can be made from the two pieces given?



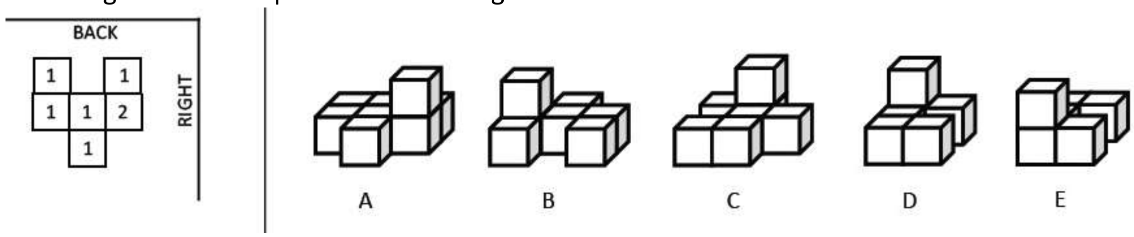
23. If the red cubes were removed from the given building, what would the new building look like?



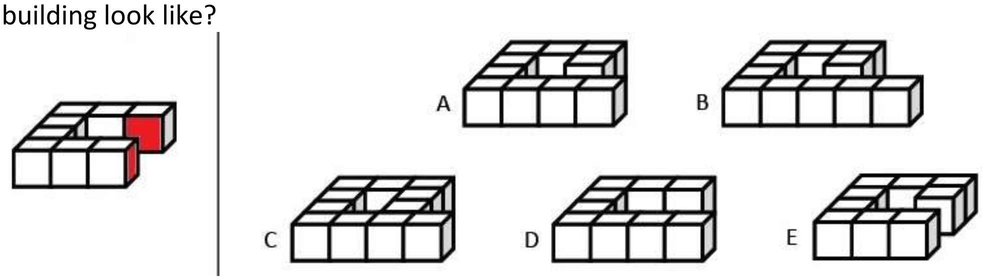
24. Find another view of the first building.



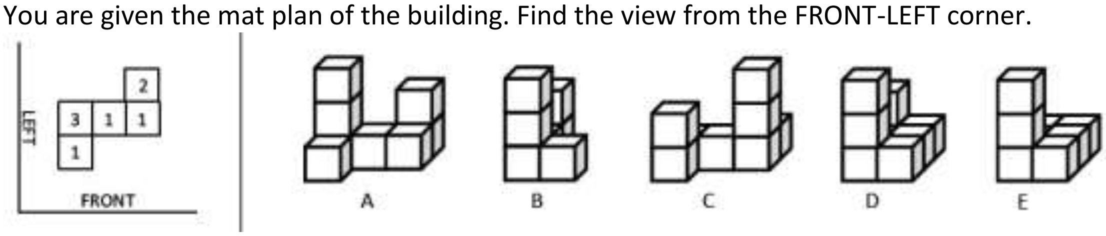
25. You are given the mat plan of the building. Find the view from the BACK-RIGHT corner.



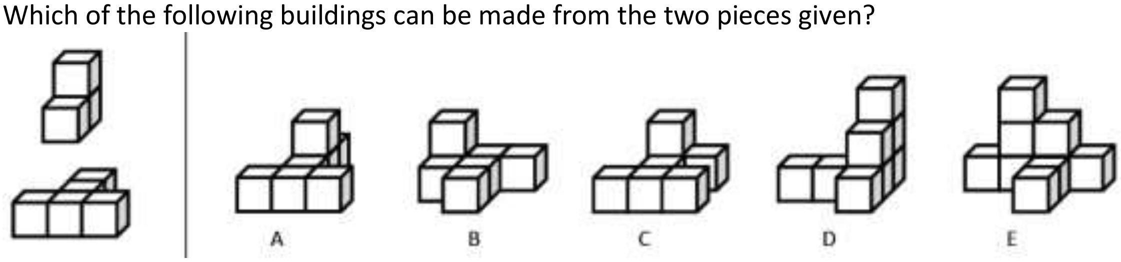
26. If a cube were added to <u>each</u> shaded face of the given building, what would the new building look like?



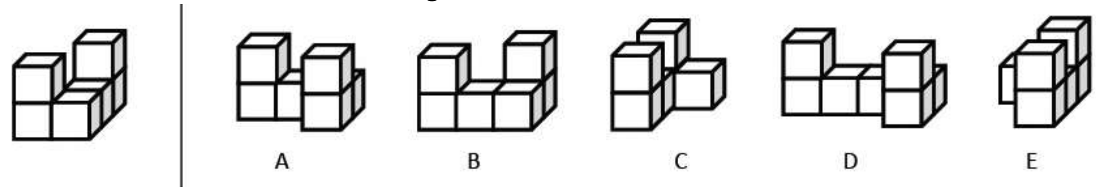
27. You are given the mat plan of the building. Find the view from the FRONT-LEFT corner.



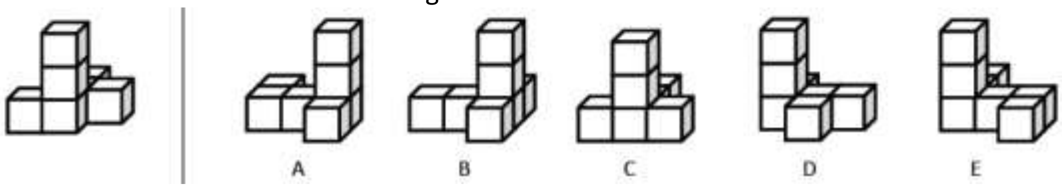
28. Which of the following buildings can be made from the two pieces given?



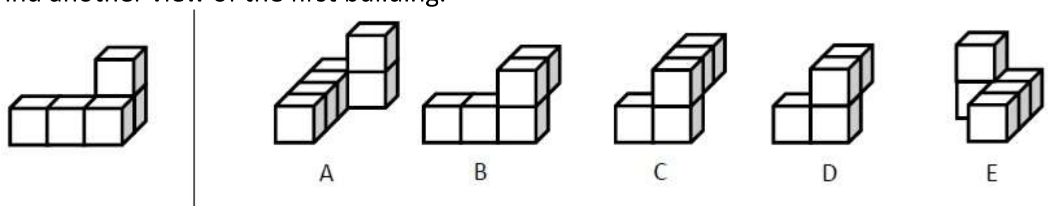
29. Find another view of the first building.



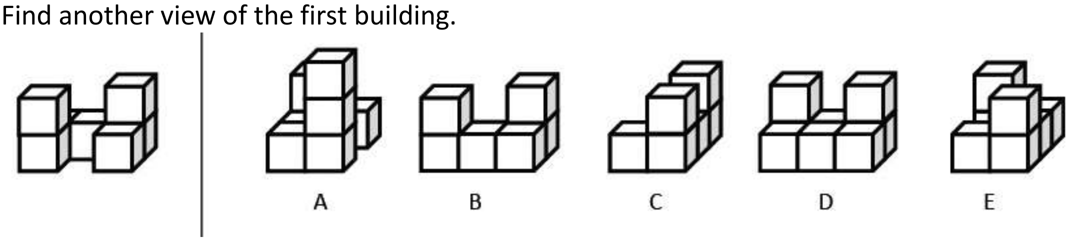
30. Find another view of the first building.



31. Find another view of the first building.



32. Find another view of the first building.



Adapted from Lappan (1983).

APPENDIX E

PROBLEM-SOLVING ASSESSMENT

Name: ____

ID Number: ________________________________

Dear Student,

Thank you for taking time to solve these three problems. Your effort is greatly appreciated!

I look forward to understanding more about how students, like you, solve math problems!

Instructions:

Solve each problem to the best of your ability and circle your final answer.

Please write down everything you think about the problems as you work to solve them. The examiner is interested in everything you think about, including (a) things you try which don't work, (b) approaches to the problem you think might work but don't have time to try, and (c) the reasons why you did try what you did.

You are not being graded on the correctness of your responses.

Please use pen and please do not scratch out any work!

Problem 1
Farmer Ben has only cows and chickens. He can't remember how many of each he has, but he doesn't need to remember because he knows he has 22 animals and that 22 is also his age. He also knows that the animals have a total of 56 legs, because 56 is also his father's age. Assuming that each animal has all legs intact and no extra limbs, how many of each animal does Farmer Ben have?

Have you seen this problem or a closely related problem before?

___ Yes, I have seen this exact problem before
___ Yes, I have seen a similar problem before
___ No, I have not seen this kind of problem before

Did you plan your solution or "plunge" into it?

___ I jumped in ___ I planned a bit ___ I thought it out first

Please rate the difficulty of the problem:

___ Easy ___ Approachable ___ Hard

Problem 2

Leonard and his friends play in an intramural softball league in which each team in the league will play against each of the other teams once. There are nine teams: the Amazings, the Bombers, the Catastrophes, the Destroyers, the Emperors, the Fighters, the Goliaths, the Hard-hitters, and the Incredibles. How many games will be played in all?

Have you seen this problem or a closely related problem before?

___ Yes, I have seen this exact problem before

___ Yes, I have seen a similar problem before

___ No, I have not seen this kind of problem before

Did you plan your solution or "plunge" into it?

___ I jumped in ___ I planned a bit ___ I thought it out first

Please rate the difficulty of the problem:

___ Easy ___ Approachable ___ Hard

Problem 3

A bucket was left outside to measure how much rain we have had in the last 2 days. After two days the water level was 10 cm deep. Four more centimeters of rain fell on the first day than on the second day. How deep was the water after the first day?

Have you seen this problem or a closely related problem before?

___ Yes, I have seen this exact problem before

___ Yes, I have seen a similar problem before

___ No, I have not seen this kind of problem before

Did you plan your solution or “plunge” into it?

___ I jumped in ___ I planned a bit ___ I thought it out first

Please rate the difficulty of the problem:

___ Easy ___ Approachable ___ Hard

Some strategies that people commonly use when solving mathematical problems are listed below:

A. Drawing a picture or diagram to help think about the problem.
B. Trying to remember a formula that would help solve the problem.
C. Trying to set up an equation.
D. Trying to make a table or list of possible answers.
E. Trying to make a list of possible answers to see if there was a pattern.
F. Trying to visualize the scenario while thinking about the problem.
G. Trying to remember how I've seen the problem solved before.
H. Trying to work backwards.
I. Trying to guess and then checking to see if the guess was right.
J. Trying to start with an easier problem and looking for a pattern.

Look back at your work on Problems 1, 2 and 3.

1. For each problem, try to determine the **primary problem-solving strategy** (or the main problem-solving strategy) that you used to solve the problem. In the column labeled "Primary Strategy," identify the strategy you used by writing the letter of the strategy above that most closely corresponds to your strategy. If the strategy you used is not listed above, please describe the strategy you used.

2. In the column labeled "Other Strategies," identify any other strategies that you used to think about or solve the problem.

	Primary Strategy	**Other Strategies**
Problem 1		
Problem 2		
Problem 3		

APPENDIX F

RESULTS FROM THE PILOT STUDY

The purpose of the pilot study was to investigate feasibility of the proposed research as well as to examine the validity and reliability of the four instruments that were used in the dissertation research: the Music Background Survey, the Analytic Reasoning Assessment, the Spatial Reasoning Assessment, and the Problem-Solving Assessment. More specifically, three instruments were designed by the researcher for the purposes of the dissertation study: the Music Background Survey, the Analytic Reasoning Assessment, and the Problem-Solving Assessment. The Spatial Reasoning Assessment was developed by the Department of Mathematics at Michigan State University. The pilot study also allowed the research to refine and re-design the instrumentation as needed.

Population and Sampling

The pilot study utilized two different populations: university mathematics students enrolled in first-year credit-bearing algebra-based mathematics courses and university music majors. The pilot study utilized a sample of convenience along with voluntary consent. Only data from students who had provided their informed consent to participate in the research (see Appendix A) were analyzed. No students participated both as a mathematics student and a music student.

University mathematics students. The sample of university mathematics students consisted of students enrolled in 19 sections of first-year credit-bearing algebra-based mathematics courses offered at the university. Of the 19 sections of first-year credit-bearing algebra-based mathematics courses, only 18 were available for piloting. Random assignment was used to assign each section two of the four instruments under investigation. A total of 624 mathematics students agreed to participate in the pilot study while only 458 students actually participated: 361 students completed the music background survey, 379 students completed the analytic reasoning assessment, 301 students completed the spatial reasoning assessment, and 109 students completed the problem-solving assessment. All of the mathematics students who agreed to participate in the pilot study were added as research participants to a research project site on TRACS, a university supported online course management system which allows for the administration of assessments and collection of assessment data.

University music majors. A total of 80 university music majors enrolled in Music Theory participated in the pilot study: 24 Music Theory I students, 15 Music Theory II students, and 41 Music Theory IV students. The university music majors completed only the music background survey.

Instrument Design and Administration

Three instruments were developed for the purposes of this research study: the Music Background Survey, the Analytic Reasoning Assessment, and the Problem-Solving Assessment. The fourth instrument, the Spatial Reasoning Assessment, was developed by the Department of Mathematics at Michigan State University as the Spatial Visualization Test.

Music Background Survey. The Music Background Survey instrument was created by the researcher for the purpose of discerning research participants' music background and music theory experience. Questions included multiple choice and open response questions in which students were asked to identify or describe musical activities they had participated in including school-sponsored music programs and private music instruction as well as Likert scale questions in which students rated their confidence with respect to performing certain music theory related tasks. Music theory related questions were written based on the text book used in the music theory courses offered by the university. Content included in the music theory portion of the Music Background Survey was chosen to be representative of each of the four levels of music theory courses offered at the university.

Following the initial development of the Music Background Survey, a professor from the university's Department of Music was asked provide feedback as an expert regarding the content of the survey as well as any discrepancies with respect to terminology, question construction, and general understandability. The expert also provided feedback on possible music background variables that had been overlooked in the initial construction of the instrument.

Samples from both populations of students, university mathematics students and university music theory students, were administered the Music Background Survey. University mathematics students were asked to complete an online version of the Music Background Survey through TRACS while the university music majors enrolled in music theory were given paper-based survey so that they could comment on question structure, terminology, and question accuracy. The Music Background Survey given to the university music majors included one additional question to determine the most recent mathematics class he or she had completed. Since the purpose of including music theory students as a population of interest was primarily for determining the validity and reliability of the Music Background Survey, this survey was the only instrument administered to the music theory students who participated in the pilot study.

Analytic Reasoning Assessment. The Analytic Reasoning Assessment utilized in the initial pilot study was developed by the researcher and included 20 questions: 10 syllogisms in which students were given 2 statements assumed to be true and asked to determine a reasonable or unreasonable conclusion and 10 syllogisms in which students are given a scenario and asked to determine a reasonable or unreasonable conclusion. Syllogism were initially chosen as the basis of the Analytic Reasoning Assessment since, as Battista (1990) asserted, “ logical reasoning of the type required to solve verbal syllogisms seems more directly related to mathematics performance than does verbal ability as measured by a vocabulary test” (p. 48). Questions were modeled after those used by Battista (1990) and questions commonly found on the Law School Admissions Test (LSAT). Each of the 20 questions was uploaded to the TRACS research site which collected students’ responses to each test item. Participants were given approximately two weeks to complete the online assessment.

Spatial Reasoning Assessment. The Spatial Reasoning Assessment utilized in this study was a reproduction of the Spatial Visualization Test developed by the Mathematics Department at Michigan State University (1981). The test was reproduced using Microsoft Word® and converted to images that were then uploaded onto the TRACS research site which collected students’ responses to each test item. Each of the

32 questions was uploaded to the TRACS research site which collected students' responses to each test item. Participants were given approximately two weeks to complete the online assessment.

Problem-Solving Assessment. For the pilot study, four versions of the Problem-Solving Assessment were created. The purpose of creating four versions of this assessment was to determine the problem-solving items that provided the greatest variations in problem-solving strategies utilized by students. Problems included on the Problem-Solving Assessment consisted of mathematics word problems which could be solved using a variety of strategies. Following each problem was a six question survey in which students were asked to reflect on their problem solving process. Students were asked to reflect on their problem solving process to ensure strategies that may not be apparent to the researcher were not overlooked. Student self-report is especially important when looking at visualization: "Unless the [researcher] asks about imagery used in mathematical problem solving, it may not be reported, even when it is present and constitutes an integral part of the problem-solving process" (Presmeg & Balderas-Canas, 2001, p. 293).

Analysis of Instruments

Music Background Survey. Participants' responses to items included in the Music Background Survey were entered into a Microsoft Excel® spreadsheet. Responses to questions 1 through 7 were coded for data analysis. A mean theory confidence score was computed for each participant by averaging Likert scale responses for questions 8 through 21. Separate analyses were conducted for the two populations under investigation: university music theory students and university mathematics students.

Instrument validation. Following the initial development of the Music Background Survey, a professor from the university's Department of Music was asked provide feedback as an expert regarding the content of the survey as well as any discrepancies with respect to terminology, question construction, and general understandability. University music theory students acted as an expert population for the validation of the music theory questions contained in the Music Background Survey. Mean music theory confidence between three groups (participants enrolled in Music Theory I, participants enrolled in Music Theory II, and participants enrolled in Music Theory IV) were compared. Analysis indicated that mean theory confidence increased along with the level of music theory participants were enrolled in, as shown in Table 1.

Table 1

Results of Music Background Survey Pilot by Item and Participant Group

	Mathematics (n = 361)		Music Theory I (n = 24)		Music Theory II (n = 15)		Music Theory IV (n = 41)	
Survey Item	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Item 8	2.75	1.62	4.79	0.51	5.00	0.00	5.00	0.00
Item 9	2.31	1.51	4.92	0.28	5.00	0.00	5.00	0.00
Item 10	2.03	1.41	4.96	0.20	5.00	0.00	5.00	0.00
Item 11	1.99	1.31	4.58	0.65	4.93	0.26	4.98	0.16
Item 12	1.62	1.11	4.54	0.72	5.00	0.00	5.00	0.00
Item 13	1.43	0.97	3.88	1.12	4.40	0.63	4.78	0.42
Item 14	1.49	1.00	3.88	1.03	4.73	0.59	4.83	0.44
Item 15	1.58	1.12	3.08	1.47	4.93	0.26	4.98	0.16
Item 16	1.39	0.89	3.17	1.55	4.53	0.64	4.73	0.50
Item 17	1.40	0.89	2.58	1.35	4.00	0.93	4.66	0.57
Item 18	1.60	1.19	3.58	1.21	4.10	0.54	4.73	0.50
Item 19	1.70	1.20	3.83	1.13	4.40	0.63	4.68	0.61
Item 20	1.70	1.23	3.04	1.46	3.93	1.33	4.39	0.95
Item 21	1.69	1.20	3.75	1.07	4.47	0.64	4.63	0.54
Overall	1.76	0.98	3.90	0.65	4.60	0.23	4.81	0.20

Note. M = mean; SD = standard deviation.

The Music Background Instrument was also validated through expert review. A professor of Music Theory from the university's Department of Music was asked to review the Music Background survey and report at what level of music theory each question would be initially taught. Furthermore, for each music theory related question, the professor was asked to hypothesize how confident a student in Music Theory I, II, III, or IV would be in performing the task indicated in the question. The results of the expert's analysis are included in Table 2.

Table 2

Results of Review of Music Background Survey by Music Theory Expert

		Hypothesized Student Confidence by Level			
Item	Level Content is Taught	Theory I	Theory II	Theory III	Theory IV
8	Fundamental Music Theory	5	5	5	5
9	Fundamental Music Theory	5	5	5	5
10	Fundamental Music Theory	5	5	5	5
11	Fundamental Music Theory	4	5	5	5
12	Music Theory I	4	5	5	5
13	Not Addressed in Curriculum	3	4	5	5
14	Music Theory I	4	5	5	5
15	Music Theory II	4	5	5	5
16	Music Theory II	2	4	5	5
17	Music Theory II	2	4	5	5
18	Music Theory III	2	3	5	5
19	Music Theory I	4	5	5	5
20	Graduate Level Theory	2	3	4	5
21	Music Theory II	3	4	5	5

Instrument reliability. SPSS was used to conduct a reliability analysis for the population of interest, university mathematics students. A Chronbach's alpha value of .956 was returned indicating the Music Background Survey is a reliable measure of students' music theory confidence.

Factor analysis. Exploratory factor analysis was conducted on the data collected from both populations to investigate the existence of latent factors within the music theory related questions contained in the Music Background Survey. Analysis of the data collected from university music students indicated one primary factor with two possible underlying latent variables: composition and identification (see Figure 2).

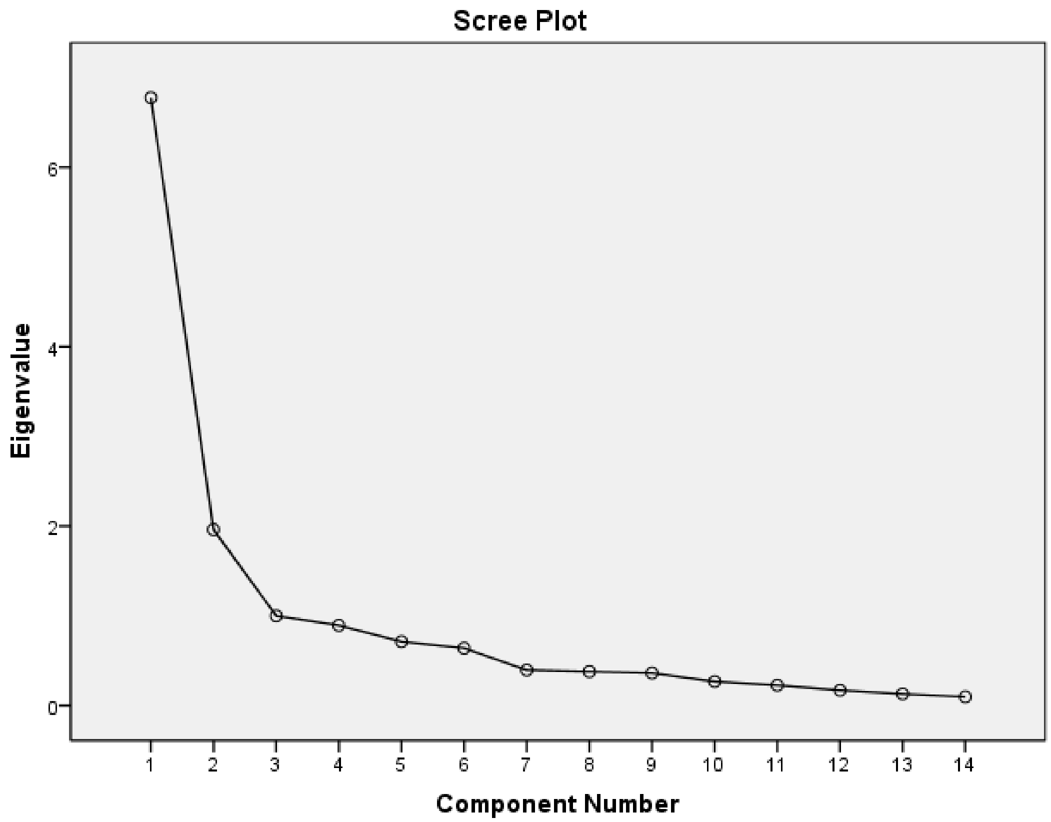


Figure 2. Scree plot representing the eigenvalue associated with each identified music theory component for the Music Background Survey using results collected from university music majors.

On the other hand, analysis of data from university mathematics students did not indicate the existence of latent variables associated with music theory related questions, as shown in Figure 3.

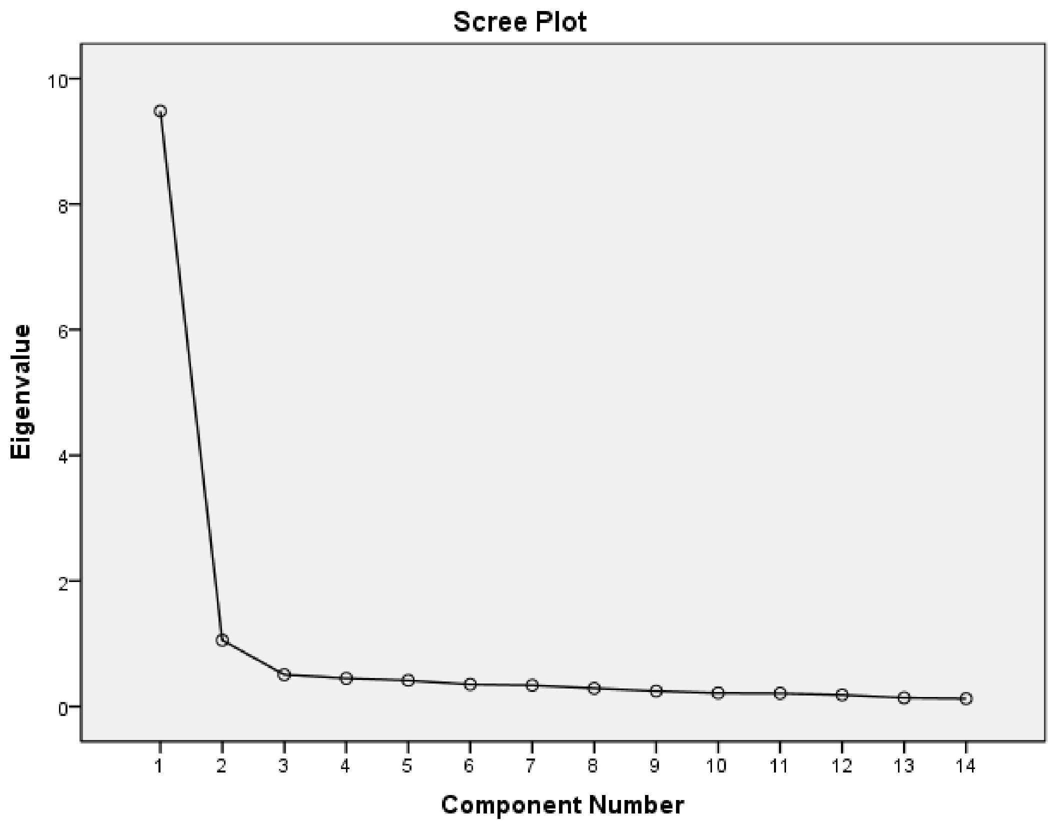


Figure 3. Scree plot representing the eigenvalue associated with each identified music theory component for the Music Background Survey using results collected from university freshman-level mathematics students.

***Other results*.** The feasibility of the proposed research was also investigated through the analysis of the results from the pilot of the Music Background Survey. Analysis of pilot results indicates the proposed sample population would be able to provide the desired data for the dissertation study. More specifically, the sample population for the pilot study included mathematics students with varying levels of musical training. Of the 361 participants who completed the Music Background survey, 136 participants attested to having participated in private music instruction (see Table 3 for specific demographic information regarding instrument and duration of private instruction).

Table 3

Music Background Demographic Data Including Instrument and Duration of Private Instruction (n = 361)

Instrument of Study	Participants Having Private Instruction	Participants Having 2 or More Years of Private Instruction	Participants Having 10 or More Years of Private Instruction
Brass	14	12	1
Percussion	12	7	0
Piano	54	32	2
String	27	20	2
Voice	24	19	3
Woodwind	30	22	0

Moreover, 204 participants indicated participation in school music programs during either middle school or high school (see Table 4 for specific demographic information regarding program and level of participation).

Table 4

School Music Participation Data Including Program of Study (n = 361)

Program of Study	Middle School	High School
Band or Orchestra	100	56
Choir or Vocal Ensemble	85	43

Analytic Reasoning Assessment. Participant responses to items included in the researcher-created assessment of analytic reasoning were recorded in a Microsoft Excel® spreadsheet. Responses were coded as either correct or incorrect and each participant was assigned an overall score based on the number of correct responses. Item analysis revealed that items contained in the assessment did not effectively discriminate between students with high and/or low analytic reasoning ability; Item discrimination values ranged from -0.07 to 0.52. Moreover, analysis of instrument reliability returned a Chronbach's alpha of 0.532 prior to item deletion and a maximum of 0.580 following item deletion. Therefore, the reliability of the measurement instrument could not be substantiated. Further investigation was required to obtain a valid and reliable measure of analytic reasoning ability.

Further research regarding the valid measurement of analytic reasoning resulted in the Analytic Reasoning Assessment modeled after the assessment used by McFarlane (1989) to measure verbal reasoning. The Analytic Reasoning Assessment was piloted in the Summer of 2012. A total of 44 students participated in the re-piloting of the Analytic Reasoning Assessment. The revised assessment included 5 parts, each containing 7 items. Each of the 35 items was uploaded to the TRACS research site which collected students' responses to each test item. Participant responses to items included in the Analytic Reasoning Assessment were recorded in a Microsoft Excel® spreadsheet. Responses were coded as either correct or incorrect and each participant was assigned an overall score based on the number of correct responses. Reliability analysis returned a Chronbach's alpha of 0.857, indicating the revised Analytic Reasoning Assessment was a reliable instrument.

Spatial Reasoning Assessment. Participant responses to items included in the Spatial Reasoning Assessment were recorded in a Microsoft Excel® spreadsheet. Responses were coded as either correct or incorrect and each participant was assigned an overall score based on the number of correct responses. Item analysis indicated that the assessment contained three items, 6, 25, and 27, that warranted further investigation. Subsequent analysis of these three items revealed internal errors. Hence, data for these items were removed for additional analysis. Table 5 illustrates participants' performance on each item included in the pilot of the Spatial Reasoning Assessment.

Table 5

Response Results of Spatial Reasoning Assessment Pilot by Item

	Participant Responses (*n* = 286)					
Assessment Item	A	B	C	D	E	Percent Correct
Item 1	**164**	45	54	12	10	57
Item 2	86	25	**122**	24	29	42
Item 3	13	58	45	**134**	35	47
Item 4	14	**151**	48	47	26	52
Item 5	42	33	20	16	**174**	61
Item 6	30	65	**56**	114	20	19
Item 7	46	**97**	26	20	96	34
Item 8	49	38	15	**147**	37	51
Item 9	122	23	**102**	10	29	35
Item 10	6	**242**	9	17	12	84
Item 11	63	38	73	**87**	24	30
Item 12	6	8	**239**	21	12	83
Item 13	33	**82**	57	61	53	28
Item 14	2	27	**199**	44	13	69
Item 15	**69**	46	87	55	29	24
Item 16	33	52	77	51	**72**	25
Item 17	22	**190**	50	20	3	66
Item 18	**243**	8	14	13	7	85
Item 19	11	128	29	15	**102**	35
Item 20	37	15	29	**196**	8	68
Item 21	27	**163**	29	31	36	56
Item 22	**98**	39	73	36	38	34
Item 23	15	11	**218**	21	20	76
Item 24	62	41	**89**	42	52	31
Item 25	35	138	54	27	**32**	11
Item 26	**173**	29	51	18	14	60
Item 27	141	69	29	**37**	9	12
Item 28	26	41	42	**142**	32	50
Item 29	**146**	64	19	37	19	51
Item 30	35	33	39	**160**	18	56
Item 31	52	16	29	26	**162**	56
Item 32	19	77	30	24	**130**	46

Note. For each item, the number of correct responses has been bolded.

SPSS was used to conduct a reliability analysis as well as an exploratory factor analysis. Reliability analysis returned a Chronbach's alpha of 0.855. As illustrated by a scree plot (see Figure 4), exploratory factor analysis indicated one primary factor.

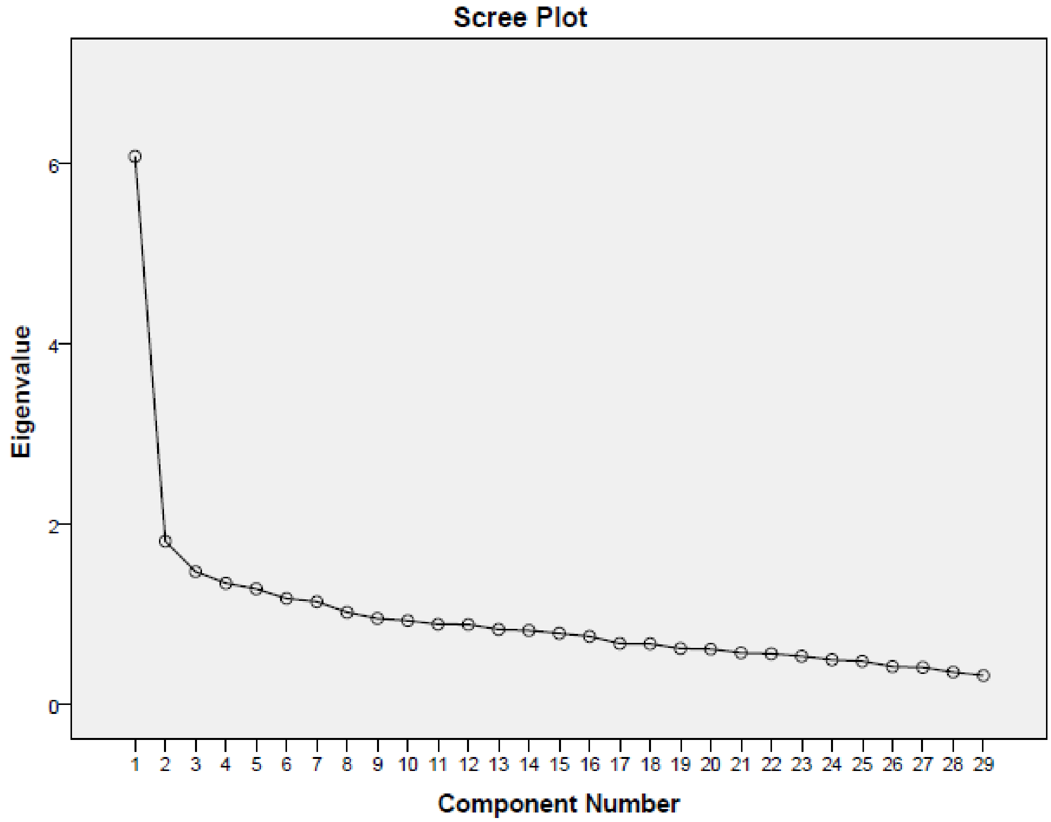


Figure 4. Scree plot representing the eigenvalue associated with each identified component for the Spatial Reasoning Assessment. The scree plot indicates the assessment consists of a single primary factor.

Problem-Solving Assessment. A random selection of 30 completed Problem-Solving Assessments were reviewed by the researcher. Through this review, a coding scheme was developed based on the common strategies identified, Polya's (1945) problem-solving heuristics (see Appendix P), and strategies reported in the review of literature.

After the initial development of the coding scheme, a new random selection of 30 assessments was made and coded by the researcher. Following the researcher's coding, another mathematics education professional was asked to code the assessments using the previously developed coding scheme in the form of a coding rubric. The coding rubric consisted of a checklist of problem-solving strategies for each problem. The codes assigned by the researcher were not revealed to the second coder until after coding was complete.

Cohen's Kappa was computed for each question to determine inter-rater reliability. Inter-rater reliability for questions 1 through 6 was found to be 0.49, 0.76, 0.91, 0.82, 0.64, and 0.85, respectively, with an overall inter-rater reliability for all six questions of 0.68. According to Landis and Koch (1977), a value of 0.68 indicates substantial agreement.

Through the analysis of inter-rater reliability, it was determined that the majority of coding discrepancies occurred due to a lack of direction given by the researcher. Instructions given to the additional coder were insufficient at best and problem-solving strategies contained in the coding rubric were not well defined for the coder. These findings were taken into consideration for the purposes of the dissertation research.

The remainder of the completed Problem-Solving Assessments was coded by the researcher to identify the problem-solving strategies most commonly used by participants. A total of 121 assessments were coded: 33 Form A assessments which contained Problems 1, 2, and 3; 29 Form B assessments which contained Problems 1, 2, and 4; 28 Form C assessments which contained Problems 1, 2, and 5; and 31 Form D assessments which contained Problems 1, 2, and 6. Table 6 describes the percentage of participants who used various problem-solving strategies while working on the six piloted problem-solving items.

Table 6

Percent of Participants Who Utilized Each Problem-Solving Strategy by Question

	Problem-Solving Strategy Identified					
Item	PD	F	E	TL	GC	VS
Problem 1 (*n* = 121)	6.6	0.8	43.8	18.2	41.3	9.1
Problem 2 (*n* = 121)	49.6	20.7	8.3	32.2	0.8	7.4
Problem 3 (*n* = 33)	78.8	0.0	6.1	0.0	3.0	6.1
Problem 4 (*n* = 29)	100.0	0.0	20.7	0.0	0.0	6.9
Problem 5 (*n* = 28)	42.9	0.0	14.3	10.7	7.1	3.6
Problem 6 (*n* = 31)	16.1	0.0	64.5	3.2	25.8	3.2

Note. PD = use of picture or diagram; F = use of formula; E = set up equation(s); TL = construction of table or list; GC = use of guess and check; VS = use of verbal strategy.

Results of the analysis of problem-solving strategy utilization for the six items piloted resulted in three problems deemed unsuitable for the dissertation research. First, Problem 5 was deemed unsuitable since none of the students who participated in the piloting of this problem were able to answer the question correctly. Problems 3 and 4 were also deemed unsuitable for the dissertation research as the vast majority of the students who participated in the piloting of these problems used the same problems solving strategy, diagram or picture, to solve the problem providing little insight into the different strategies students use.

APPENDIX G

PERMISSION FOR ASSESSMENT USE

Spatial Reasoning Assessment

The Spatial Reasoning Assessment was a direct adaptation of the Middle Grades Mathematics Project Spatial Visualization Test. The Middle Grades Mathematics Project was directed by Glenda Lappan and funded by the National Science Foundation–Development in Science Education, Grant #SED08-18025. Permission to adapt and use the Spatial Visualization Test for the purpose of this research study was granted by Glenda Lappan via email correspondence.

Analytic Reasoning Assessment

The Analytic Reasoning Assessment was adapted from the Employee Aptitude Survey, Test 7 – Verbal Reasoning (Grimsley, Ruch, Warren, & Ford, 1986).Permission to adapt and use the Employee Aptitude Survey, Test 7 – Verbal Reasoning for the purpose of this research study was granted by Psychological Services, Inc. via telephone and email correspondence.

APPENDIX H

INTERVIEW PROTOCOL

(Before recording starts) Do you mind if I record this interview?

This is a task-based interview. What this means is that I am going to give you some problems to solve and we are going to talk about what you are doing as you work through each problem.

While you are working on each problem, I would like you to talk about what you are thinking about as you work toward the solution. If at any point, you stay quiet for an extended period of time, I will ask you to tell me what you are thinking. I will use questions like “What are you thinking about?” or statements such as “Please tell me what you are thinking.”

While you are working on the problems, I am not going to give you any advice or hints about how to solve the problems.

I am going to leave this calculator here on the table if you would like to use it, but when you do use it, I may ask you what you are using it for, so please don’t be alarmed. I just want to make sure I capture everything you do as you are solving these problems.

I have also laid out extra paper with some extra pens and markers for you to use if you want but you can also use the chalk board of you’d like.

At the start of each question, I will give you a minute to read the question before I start asking you about what you are thinking. Once you are done with the problem, we will take a few minutes to talk about how you solved the problem before we go on to the next problem.

Do you have any questions before we start?

Problem 1

A train leaves Roseville heading east at 6:00 a.m. at 40 miles per hour. Another eastbound train leaves on a parallel track at 7:00 a.m. at 50 miles per hour. What time will it be when the two trains are the same distance from Roseville?

Problem 2

An older brother said to a younger, “Give me eight walnuts, then I will have twice as many as you do.” But the younger brother said to the older one, “You give me eight walnuts, then we will have an equal number.” How many walnuts did each have?

Problem 3

The night before their debut at Carnegie Hall, the dancers of a ballet company stayed at a hotel on 57th street. Fourteen members of the ballet company went to an all-night card room to play poker. Half of the remaining dancers went to Madison Square Garden for a special midnight professional wrestling show featuring Buff Bargle. After about an hour, 6 of the dancers who had gone to play poker came back to the hotel broke. The 11 dancers now at the hotel went to bed and got enough sleep, but the rest of the dancers were tired for their debut the next day. How many dancers were in the ballet company?

Additional Interview Questions:

For people with High Spatial Visualization scores:

1. Your score on the Spatial Visualization test (the one with the block designs) was pretty high. Can you think of anything in your background (even your childhood) that could have contributed to your development of spatial reasoning? Or do you think you were just born with a sharper sense of spatial reasoning?

For people with High Analytic Reasoning scores:

1. Your score on the Analytic Reasoning assessment (the one where you decided if a statement was true or false) was pretty high. Can you think of anything in your background (even your childhood) that could have contributed to your development of analytic (logical) reasoning? Or do you think you were just born with the ability to think analytically or logically?

For people with a music background:

1. Do you think your music background has had any impact on how you approach mathematical problems? If so, how?

2. Instrument: ____________________________________
   Do you think that your background with this instrument has contributed to your development of spatial reasoning?
3. Do you think that your background with this instrument has contributed to your development of analytic (logical) reasoning?

4. Music Theory Background: ____
5. Do you think that your background in music theory has contributed to your development of spatial reasoning?
6. Do you think that your background music theory has contributed to your development of analytic (logical) reasoning?

APPENDIX I

INFORMATION HANDOUT FOR POTENTIAL PARTICIPANTS

August 27, 2012

Dear Student,

My name is Debra Ward and I am a doctoral student at [redacted] working on my dissertation "An Investigation of the Relationships between Musical Training and Mathematical Problem Solving." I am interested in learning how students with various backgrounds think about mathematics.

You will be asked to complete four assessments: one assessment will be done during class and three will be completed online through a TRACS project site. The first assessment is a Problem-Solving Test which will be administered during class and will take about 30 minutes to complete. The second and third assessments, both of which will be administered online through TRACS, will be used to measure spatial and analytical reasoning. Each of these assessments will take approximately 20 minutes to complete. The final assessment is a Music Background Survey which will take approximately 10 minutes to complete thorough TRACS.

This study has received Institutional Review Board (IRB) exemption from the Office of Research Compliance at [redacted]. Federal regulations describe six types of research that may qualify for exemption. This pilot study is categorized as

> (1.ii) Research conducted in established or commonly accepted educational settings, involving normal educational practices, such as research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.

For more information regarding this pilot's exemption you may contact the Office of Research Compliance at [redacted] using the exemption approval number [redacted].

By participating in this study you are agreeing that I will have free access to your results on TRACS. However, it is important to note that **your course grade will in no way be negatively affected by your participation in this study**. Also, individual results of the different assessment are confidential!

I appreciate your consideration and participation!

Sincerely,
Debra D. Ward
Department of Mathematic

APPENDIX J

ONLINE INSTRUCTIONS FOR PARTICIPANTS

Thank you for taking the time to participate in this study. The following assessments can be completed online through TRACS:

____ Music Background Survey (10 minutes)
____ Spatial Visualization Assessment (20-30 minutes)
____ Analytic Reasoning Assessment (20-30 minutes)

You will be given access to the TRACS project site "**Music-Math Research**" where you will be able to complete assessments online at your convenience! Once you have access to this site (in about two days), you will receive an email notification in your [redacted] email account. Please follow these instructions to access and complete the assessments:

- From the [redacted] homepage, click the link to TRACS near to top of the webpage.
- Log into TRACS using your Texas State user name and password (the same ones you use to check your [redacted] email).
- Click on the "**Music- Math Research**" tab at the top of your homepage.
  If "**Music-Math Research**" is not a tab at the top of your TRACS page:
  - Click on "**My Active Worksites**" tab at top of your home page
  - Under PROJECTS, click on "**Music-Math Research**"
- Click on "**Assessments**" on the left-hand toolbar
- Under "**Take an Assessment**" click on the assessment you want to take.
- You will be given information about the assessment you are taking.
  Click on "**Begin Assessment**" button.
- Follow the on-screen directions and answer each question to the best of your ability. You will 1 hour to complete each assessment, though you will most likely not need this much time. **Please take your time on the assessment. Your effort is greatly appreciated!**
- At the end of the assessment click the "**Submit for Grading**" button. If you do not click on this button, your responses to the assessment will not be submitted and TRACS will not report that you ever took the assessment! Remember, **your performance on the assessments will in no way impact your course grade!**

You will have access to the online assessments until **10:00 p.m. on Friday, October 26, 2012**.

Please feel free to contact me if you have any questions!!!

Debra D. Ward
Department of Mathematics

APPENDIX K

MUSIC BACKGROUND VARIABLES FACTOR ANALYSIS

Table

Results of the Factor Analysis Including Music Background Variables and Weights

Music Background Variable	Weight
Brass, years of study	.395
Brass, age at commencement of study	.281
Woodwind, years of study	.467
Woodwind, age at commencement of study	.372
String, years of study	.263
String, age at commencement of study	.189
Piano, years of study	.346
Piano, age at commencement of study	.342
Voice, years of study	.410
Voice, age at commencement of study	.367
Percussion, years of study	.225
Percussion, age at commencement of study	.211
Elementary school, years of study	.172
Middle school band, semesters of study	.493
Middle school orchestra, semesters of study	.092
Middle school instrumental ensemble, semesters of study	.493
Middle school choir, semesters of study	.115
Middle school vocal ensemble, semesters of study	.367
High school band, semesters of study	.599
High school orchestra, semesters of study	.298
High school instrumental ensemble, semesters of study	.559
High school choir, semesters of study	.367
High school vocal ensemble, semesters of study	.386
College band, semesters of study	.483
College orchestra, semesters of study	.255
College instrumental ensemble, semesters of study	.456
College choir, semesters of study	.393
College vocal ensemble, semesters of study	.298
Music Theory Confidence	.767

APPENDIX L

PROBLEM-SOLVING ASSESSMENT CODING RUBRIC

Test ID: ___________________________ **Coder:** _____________________________

		Picture or Diagram	Use Formula	Set up Equation	Construct Table or List	Look for a Pattern	Work Backwards	Guess and Check	Start with an Easier Problem
Problem 1 ____ Correct ____ Incorrect	**Primary Strategy**								
	Secondary Strategy								
Problem 2 ____ Correct ____ Incorrect	**Primary Strategy**								
	Secondary Strategy								
Problem 3 ____ Correct ____ Incorrect	**Primary Strategy**								
	Secondary Strategy								

Notes:

APPENDIX M

INTER-RATER RELIABILITIES

Table

Inter-rater Reliabilities of Problem-Solving Strategy Utilization

Reliability Between Participants' Self-Reported Use of Problem-Solving Strategies and Researcher Coding of Problem-Solving Strategy Use			
	Strategy	Cohen's Kappa	Interpretation
Problem 1	Picture or Diagram	0.38	Fair agreement
	Formula	0.01	Slight agreement
	Equation	0.46	Moderate agreement
	Table or List	0.23	Fair agreement
	Pattern	0.04	Slight agreement
	Working Backwards	0.03	Slight agreement
	Guess and Check	0.37	Fair agreement
	Easier Problem	0.00	No agreement
	Overall	0.35	Fair agreement
Problem 2	Picture or Diagram	0.23	Fair agreement
	Formula	0.13	Slight agreement
	Equation	0.10	Slight agreement
	Table or List	0.25	Fair agreement
	Pattern	-0.01	No agreement
	Working Backwards	0.00	Slight agreement
	Guess and Check	0.06	Slight agreement
	Easier Problem	-0.01	No agreement
	Overall	0.25	Fair agreement
Problem 3	Picture or Diagram	0.42	Moderate agreement
	Formula	0.00	Slight agreement
	Equation	0.53	Moderate agreement
	Table or List	0.18	Slight agreement
	Pattern	0.00	Slight agreement
	Working Backwards	0.09	Slight agreement
	Guess and Check	0.27	Fair agreement
	Easier Problem	0.00	Slight agreement
	Overall	0.37	Fair agreement

Table Continued

Reliability Between Researcher Coding of Problem-Solving Strategy Use and Inter-Rater Coding of Problem-Solving Strategy Use			
	Strategy	Cohen's Kappa	Interpretation
Problem 1	Picture or Diagram	0.34	Fair agreement
	Formula	1.00	Near perfect agreement
	Equation	0.78	Substantial agreement
	Table or List	0.07	Slight agreement
	Pattern	0.00	Slight agreement
	Working Backwards	0.41	Moderate agreement
	Guess and Check	0.37	Fair agreement
	Easier Problem	1.00	Near perfect agreement
	Overall	0.78	Substantial agreement
Problem 2	Picture or Diagram	0.27	Fair agreement
	Formula	0.76	Substantial agreement
	Equation	0.72	Substantial agreement
	Table or List	0.23	Fair agreement
	Pattern	0.16	Slight agreement
	Working Backwards	1.00	Near perfect agreement
	Guess and Check	-0.02	No agreement
	Easier Problem	0.00	Slight agreement
	Overall	0.65	Substantial agreement
Problem 3	Picture or Diagram	0.46	Moderate agreement
	Formula	1.00	Near perfect agreement
	Equation	0.85	Near perfect agreement
	Table or List	0.82	Near perfect agreement
	Pattern	1.00	Near perfect agreement
	Working Backwards	0.43	Moderate agreement
	Guess and Check	0.16	Slight agreement
	Easier Problem	1.00	Near perfect agreement
	Overall	0.88	Near perfect agreement

APPENDIX N

REPORTED USE OF STRATEGIES FOR ALL PARTICIPANTS

Table

Participants' Reported Use of Problem-Solving Strategies by Problem (n=1046)

	Strategy	Primary (%)	Secondary (%)	Not used (%)
Problem 1	Picture or Diagram	9.8	5.3	84.9
	Formula	11.4	7.5	81.2
	Equation	33.0	11.5	55.5
	Table or List	11.0	6.4	82.6
	Pattern	7.8	6.0	86.1
	Visualization	16.7	13.1	70.2
	Remember	10.4	13.2	76.4
	Working Backwards	5.5	6.2	88.3
	Guess and Check	37.0	15.3	47.7
	Easier Problem	2.1	2.6	95.3
Problem 2	Picture or Diagram	32.4	6.3	61.3
	Formula	8.1	6.4	85.5
	Equation	9.8	8.6	81.6
	Table or List	25.9	6.9	67.2
	Pattern	9.6	5.7	84.7
	Visualization	19.3	11.2	69.5
	Remember	13.4	11.8	74.9
	Working Backwards	2.6	2.2	95.2
	Guess and Check	5.6	6.0	88.3
	Easier Problem	2.2	2.2	95.6
Problem 3	Picture or Diagram	15.8	4.9	79.3
	Formula	9.0	6.5	84.5
	Equation	35.8	9.8	54.5
	Table or List	5.7	3.5	90.7
	Pattern	3.3	2.2	94.6
	Visualization	18.0	9.8	72.3
	Remember	8.4	8.6	83.0
	Working Backwards	11.8	8.2	80.0
	Guess and Check	21.1	10.9	68.0
	Easier Problem	1.3	1.3	97.3

APPENDIX O

RESULTS OF CHI-SQUARE TESTS FOR PARTICIPANTS' REPORTED USE OF

PROBLEM-SOLVING STRATEGIES BY ANLYTIC REASONING SCORE

Table

Chi-square Results for Participants' Reported Use of Problem-Solving Strategy by "High Analytic" and "Low Analytic"

	Low Analytic (n = 249)		High Analytic (n = 201)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
			Problem 1			
Picture or Diagram						
Primary	23	9.2	15	7.5	5.853	.054
Secondary	18	7.2	5	2.5		
Not Used	208	83.5	181	90.0		
Formula						
Primary	24	9.6	22	10.9	4.707	.131
Secondary	13	5.2	20	10.0		
Not Used	212	85.1	159	79.1		
Equation						
Primary	82	32.9	74	36.8	3.030	.220
Secondary	27	10.8	30	14.9		
Not Used	138	55.4	97	48.3		
Table or List						
Primary	28	11.2	22	10.9	0.346	.841
Secondary	14	5.6	14	7.0		
Not Used	207	83.1	165	82.1		
Pattern						
Primary	27	10.8	8	4.0	7.759	.021
Secondary	15	6.0	10	5.0		
Not Used	207	83.1	183	91.0		
Visualization						
Primary	45	18.1	32	15.9	0.648	.723
Secondary	32	12.9	30	14.9		
Not Used	172	69.1	139	69.2		

Table Continued

Remember						
Primary	29	11.6	17	8.5	8.794	.012
Secondary	21	8.4	35	17.4		
Not Used	199	79.9	149	74.1		
Working Backwards						
Primary	20	8.0	9	4.5	2.477	.290
Secondary	19	7.6	14	7.0		
Not Used	210	84.3	178	88.6		
Guess and Check						
Primary	97	39.0	81	40.3	12.150	.002
Secondary	26	10.4	42	20.9		
Not Used	126	50.6	76	37.8		
Easier Problem						
Primary	8	3.2	3	1.5	1.806	.405
Secondary	7	2.8	8	4.0		
Not Used	234	94.0	190	94.5		
Problem 2						
Picture or Diagram						
Primary	68	27.3	76	37.8	6.352	.042
Secondary	15	6.0	14	7.0		
Not Used	166	66.7	111	55.2		
Formula						
Primary	24	9.6	15	7.5	3.426	.180
Secondary	14	5.6	20	10.0		
Not Used	211	84.7	166	82.6		
Equation						
Primary	30	12.0	18	9.0	1.665	.435
Secondary	24	9.6	16	8.0		
Not Used	195	78.3	167	83.1		
Table or List						
Primary	64	25.7	46	22.9	0.961	.619
Secondary	14	5.6	15	7.5		
Not Used	171	68.7	140	69.7		
Pattern						
Primary	25	10.0	23	11.4	2.253	.324
Secondary	11	4.4	15	7.5		
Not Used	213	85.5	163	81.1		
Visualization						
Primary	50	20.1	40	19.9	0.024	.988
Secondary	31	12.4	26	12.9		
Not Used	168	67.5	135	67.2		

Table Continued

Remember						
Primary	36	14.5	26	12.9	3.502	.174
Secondary	23	9.2	30	14.9		
Not Used	190	76.3	145	72.1		
Working Backwards						
Primary	6	2.4	5	2.5	0.979	.613
Secondary	4	1.6	6	3.0		
Not Used	239	96.0	190	94.5		
Guess and Check						
Primary	17	6.8	7	3.5	2.465	.292
Secondary	13	5.2	11	5.5		
Not Used	219	88.0	183	91.0		
Easier Problem						
Primary	7	2.8	5	2.5	1.674	.433
Secondary	4	1.6	7	3.5		
Not Used	238	95.6	189	94.0		
Problem 3						
Picture or Diagram						
Primary	42	16.9	28	13.9	3.829	.147
Secondary	8	3.2	14	7.0		
Not Used	199	79.9	159	79.1		
Formula						
Primary	28	11.2	15	7.5	3.992	.136
Secondary	11	4.4	16	8.0		
Not Used	210	84.3	170	84.6		
Equation						
Primary	83	33.3	82	40.8	2.668	.264
Secondary	25	10.0	18	9.0		
Not Used	141	56.6	101	50.2		
Table or List						
Primary	17	6.8	12	6.0	0.155	.925
Secondary	8	3.2	7	3.5		
Not Used	224	90.0	182	90.5		
Pattern						
Primary	5	2.0	7	3.5	1.403	.496
Secondary	4	1.6	5	2.5		
Not Used	240	96.4	189	94.0		
Visualization						
Primary	54	21.7	33	16.4	2.289	.318
Secondary	24	9.6	24	11.9		
Not Used	171	68.7	144	71.6		

Table Continued

Remember						
Primary	25	10.0	14	7.0	5.814	.055
Secondary	15	6.0	24	11.9		
Not Used	209	83.9	163	81.1		
Working Backwards						
Primary	32	12.9	24	11.9	0.262	.877
Secondary	24	9.6	22	10.9		
Not Used	193	77.5	155	77.1		
Guess and Check						
Primary	46	18.5	40	19.9	0.450	.798
Secondary	25	10.0	23	11.4		
Not Used	178	71.5	138	68.7		
Easier Problem						
Primary	6	2.4	1	0.5	3.107	.212
Secondary	2	0.8	3	1.5		
Not Used	241	96.8	197	98.0		

APPENDIX P

CHI-SQUARE RESULTS TESTS FOR PARTICIPANTS' REPORTED USE OF

PROBLEM-SOLVING STRATEGIES BY SPATIAL REASONING SCORE

Table

Chi-square Results for Participants' Reported Use of Problem-Solving Strategy by "High Spatial" and "Low Spatial"

	Low Spatial (n = 247)		High Spatial (n = 179)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Problem 1						
Picture or Diagram						
Primary	27	10.9	13	7.3	2.197	.333
Secondary	13	5.3	7	3.9		
Not Used	207	83.8	159	88.8		
Formula						
Primary	30	12.1	17	9.5	4.710	.095
Secondary	15	6.1	21	11.7		
Not Used	202	81.8	141	78.8		
Equation						
Primary	80	32.4	66	36.9	3.363	.186
Secondary	26	10.5	26	14.5		
Not Used	141	57.1	87	48.6		
Table or List						
Primary	25	10.1	16	8.9	5.179	.075
Secondary	11	4.5	18	10.1		
Not Used	211	85.4	145	81.0		
Pattern						
Primary	17	6.9	12	6.7	3.649	.161
Secondary	7	2.8	12	6.7		
Not Used	223	90.3	155	86.6		
Visualization						
Primary	40	16.2	32	17.9	2.396	.302
Secondary	25	10.1	26	14.5		
Not Used	182	73.7	121	67.6		

Table Continued

Remember						
Primary	30	12.1	15	8.4	3.303	.192
Secondary	29	11.7	30	16.8		
Not Used	188	76.1	134	74.9		
Working Backwards						
Primary	17	6.9	7	3.9	5.737	.057
Secondary	10	4.0	16	8.9		
Not Used	220	89.1	156	87.2		
Guess and Check						
Primary	93	37.7	71	39.7	3.895	.143
Secondary	31	12.6	33	18.4		
Not Used	123	49.8	75	41.9		
Easier Problem						
Primary	12	4.9	2	1.1	6.939	.031
Secondary	2	0.8	5	2.8		
Not Used	233	94.3	172	96.1		
Problem 2						
Picture or Diagram						
Primary	70	28.3	59	33.0	1.748	.417
Secondary	18	7.3	16	8.9		
Not Used	159	64.4	104	58.1		
Formula						
Primary	26	10.5	15	8.4	2.496	.287
Secondary	10	4.0	13	7.3		
Not Used	211	85.4	151	84.4		
Equation						
Primary	33	13.4	17	9.5	1.646	.439
Secondary	21	8.5	14	7.8		
Not Used	193	78.1	148	82.7		
Table or List						
Primary	70	28.3	48	26.8	0.218	.897
Secondary	8	3.2	7	3.9		
Not Used	169	68.4	123	68.7		
Pattern						
Primary	26	10.5	19	10.6	4.489	.106
Secondary	8	3.2	14	7.8		
Not Used	213	86.2	146	81.6		
Visualization						
Primary	47	29.0	35	19.6	2.816	.245
Secondary	21	8.5	24	13.4		
Not Used	179	72.5	120	67.0		

Table Continued

Remember						
Primary	30	12.1	16	8.9	2.703	.259
Secondary	32	13.0	32	17.9		
Not Used	185	74.9	131	73.2		
Working Backwards						
Primary	3	1.2	7	3.9	4.789	.091
Secondary	3	1.2	5	2.8		
Not Used	241	97.6	167	93.3		
Guess and Check						
Primary	20	8.1	3	1.7	15.384	.001
Secondary	6	2.4	15	8.4		
Not Used	221	89.5	161	89.9		
Easier Problem						
Primary	4	1.6	3	1.7	1.354	.508
Secondary	5	2.0	7	3.9		
Not Used	238	96.4	169	94.4		
Problem 3						
Picture or Diagram						
Primary	42	17.0	20	11.2	5.102	.078
Secondary	10	4.0	14	7.8		
Not Used	195	78.9	145	81.0		
Formula						
Primary	29	11.7	14	7.8	3.572	.168
Secondary	10	4.0	13	7.3		
Not Used	208	84.2	152	84.9		
Equation						
Primary	84	34.0	61	34.1	2.187	.335
Secondary	20	8.1	22	12.3		
Not Used	143	57.9	96	53.6		
Table or List						
Primary	11	4.5	12	6.7	1.401	.496
Secondary	6	2.4	6	3.4		
Not Used	230	93.1	161	89.9		
Pattern						
Primary	12	4.9	4	2.2	2.007	.367
Secondary	6	2.4	4	2.2		
Not Used	229	92.7	171	95.5		
Visualization						
Primary	40	16.2	35	19.6	3.540	.170
Secondary	15	6.1	18	10.1		
Not Used	192	77.7	126	70.4		

Table Continued

Remember						
Primary	20	8.1	10	5.6	1.290	.525
Secondary	23	9.3	20	11.2		
Not Used	204	82.6	149	83.2		
Working Backwards						
Primary	28	11.3	25	14.0	4.645	.098
Secondary	15	6.1	20	11.2		
Not Used	204	82.6	134	74.9		
Guess and Check						
Primary	54	21.9	36	20.1	1.383	.501
Secondary	28	11.3	15	8.4		
Not Used	165	66.8	128	71.5		
Easier Problem						
Primary	7	2.8	0	0.0	5.869	.053
Secondary	1	0.4	2	1.1		
Not Used	239	96.8	177	98.9		

APPENDIX Q

RESULTS OF CHI-SQUARE TESTS FOR PARTICIPANTS' REPORTED USE OF PROBLEM-SOLVING STRATEGIES BY MUSIC BACKGROUND INDEX

Table

Chi-square Results for Participants' Reported Use of Problem-Solving Strategy by "High Music" and "Low Music"

	Low Music (n = 258)		High Music (n = 97)			
Strategy	n	Percent	n	Percent	χ^2	Sig.
Problem 1						
Picture or Diagram						
Primary	20	7.8	10	10.3	0.763	.683
Secondary	16	6.2	7	7.2		
Not Used	222	86.0	80	82.5		
Formula						
Primary	30	11.6	12	12.4	1.394	.498
Secondary	13	5.0	8	8.2		
Not Used	215	83.3	77	79.4		
Equation						
Primary	80	31.0	29	29.9	3.548	.170
Secondary	3	12.8	6	6.2		
Not Used	145	56.2	62	63.9		
Table or List						
Primary	27	10.5	22	22.7	9.476	.009
Secondary	19	7.4	4	4.1		
Not Used	212	82.2	71	73.2		
Pattern						
Primary	23	8.9	8	8.2	17.804	.000
Secondary	8	3.1	15	15.5		
Not Used	227	88.0	74	76.3		
Visualization						
Primary	41	15.9	15	15.5	1.274	.529
Secondary	24	9.3	13	13.4		
Not Used	193	74.8	69	71.1		

Table Continued

Remember						
Primary	30	11.6	14	14.4	2.251	.324
Secondary	21	8.1	12	12.4		
Not Used	207	80.2	71	73.2		
Working Backwards						
Primary	15	5.8	2	2.1	2.180	.336
Secondary	20	7.8	8	8.2		
Not Used	223	86.4	87	89.7		
Guess and Check						
Primary	102	39.5	31	32.0	2.400	.301
Secondary	41	15.9	14	14.4		
Not Used	115	44.6	52	53.6		
Easier Problem						
Primary	8	3.1	1	1.0	1.689	.430
Secondary	9	3.5	5	5.2		
Not Used	241	93.4	91	93.8		
			Problem 2			
Picture or Diagram						
Primary	82	31.8	35	36.1	2.262	.323
Secondary	22	8.5	4	4.1		
Not Used	154	59.7	58	59.8		
Formula						
Primary	22	8.5	10	10.3	1.053	.591
Secondary	14	5.4	3	3.1		
Not Used	222	86.0	84	86.6		
Equation						
Primary	34	13.2	13	13.4	0.005	.998
Secondary	19	7.4	7	7.2		
Not Used	205	79.5	77	79.4		
Table or List						
Primary	62	24.0	24	24.7	0.080	.961
Secondary	18	7.0	6	6.2		
Not Used	178	69.0	67	69.1		
Pattern						
Primary	27	10.5	6	6.2	3.843	.146
Secondary	16	6.2	11	11.3		
Not Used	215	83.3	80	82.5		
Visualization						
Primary	51	19.8	14	14.4	1.432	.489
Secondary	32	12.4	14	14.4		
Not Used	175	67.8	69	71.1		

Table Continued

Remember						
Primary	32	12.4	10	10.3	0.375	.829
Secondary	26	10.1	11	11.3		
Not Used	200	77.5	76	78.4		
Working Backwards						
Primary	7	2.7	3	3.1	2.722	.256
Secondary	5	1.9	5	5.2		
Not Used	246	95.3	89	91.8		
Guess and Check						
Primary	16	6.2	7	7.2	1.483	.477
Secondary	13	5.0	8	8.2		
Not Used	229	88.8	82	84.5		
Easier Problem						
Primary	10	3.9	1	1.0	1.903	.386
Secondary	8	3.1	3	3.1		
Not Used	240	93.0	93	95.9		
Problem 3						
Picture or Diagram						
Primary	35	13.6	15	15.5	0.213	.899
Secondary	14	5.4	5	5.2		
Not Used	209	81.0	77	79.4		
Formula						
Primary	21	8.1	8	8.2	0.699	.705
Secondary	15	5.8	8	8.2		
Not Used	22	86.0	81	83.5		
Equation						
Primary	93	36.0	36	37.1	1.090	.580
Secondary	25	9.7	6	6.2		
Not Used	140	54.3	55	56.7		
Table or List						
Primary	15	5.8	6	6.2	0.976	.614
Secondary	11	4.3	2	2.1		
Not Used	232	89.9	89	91.8		
Pattern						
Primary	7	2.7	3	3.1	2.172	.338
Secondary	4	1.6	4	4.1		
Not Used	247	95.7	90	92.8		
Visualization						
Primary	48	18.6	21	21.6	0.814	.666
Secondary	22	8.5	10	10.3		
Not Used	188	72.9	66	68.0		

Table Continued

Remember						
Primary	24	9.3	7	7.2	1.182	.554
Secondary	26	10.1	7	7.2		
Not Used	208	80.6	83	85.6		
Working Backwards						
Primary	31	12.0	12	12.4	0.044	.978
Secondary	23	8.9	8	8.2		
Not Used	204	79.1	77	79.4		
Guess and Check						
Primary	65	25.2	20	20.6	3.749	.153
Secondary	30	11.6	6	6.2		
Not Used	163	63.2	71	73.2		
Easier Problem						
Primary	7	2.7	1	1.0	1.922	.382
Secondary	2	0.8	2	2.1		
Not Used	249	96.5	94	96.9		

APPENDIX R

PROBLEM-SOLVING HEURISTICS

Polya (1945) advocates the utilization of various problem-solving strategies, which provided the basis for the coding scheme used to analyze problem-solving assessments. A selection of the problem-solving strategies discussed by Polya is included here.

- Draw a Picture or Diagram – involves the construction of a visual representation illustrating the structure of a problem in a spatial layout (Pantziara, Gagatis, & Elia, 2009)
- Set up an Equation – "to express in mathematical symbols a condition that is stated in words… translate from ordinary language into the language of mathematical formulas" (Polya, 1957, p. 174)
- Make a Table or List – "if the reader is sufficiently acquainted with the list and can see, behind the suggestion, the action suggested, he may realize the list enumerates, indirectly, mental operations typically useful for the solution of problems" (Polya, 1957, p. 2)
- Look for a Pattern – "passing from the consideration of a restricted set to that of a more comprehensive set containing the restricted one" (Polya, 1957, p. 108)
- Try to Remember – "when solving a problem, we always profit from previously solved problems, using their results, their methods, or the experience we acquired solving them" (Polya, 1957, p. 98)
- Work Backwards – process in which problem solvers concentrate on the desired end, retracing steps until what is required is derived (Polya, 1957)
- Guess and Check – "an examinee estimates a potential solution to an arithmetic or algebra item then propagates that estimate through the mathematical relations of the item, calculating the value of other unknown quantities as possible" (Katz, Bennett, & Berger, 2000, p. 42)
- Solve a Related Problem – solve a problem that is considered "not for its own sake, but because we hope that its consideration may help us to solve another problem, our original problem" (Polya, 1957, p. 51)

REFERENCES

Abril, C. R. & Gault, B. M. (2008). The state of music in secondary schools: The principal’s perspective. *Journal of Research in Music Education*, *56*(1), 68-81.

Aldous, C. (2006). Attending to feeling: Productive benefit to novel mathematics problem solving. *International Education Journal*, *7*(4), 410-422.

Aldous, C. R. (2007). Creativity, problem solving, and innovative science: Insights from history, cognitive psychology and neuroscience. *International Education Journal*, *8*(2), 176-186.

American Mathematical Association of Two-Year Colleges (AMATYC). (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus*. Memphis, TN: AMATYC.

Bahna-James, T. (1991). The relationship between mathematics and music: Secondary school student perspectives. *The Journal of Negro Education*, *60*(3), 477-485.

Bailey, D. H., Littlefield, A., & Geary, D. C. (2012). The codevelopment of skill at and preference for use of retrieval-based process for solving addition problems: Individual and sex differences from first to sixth grades. *Journal of Experimental Psychology*, *113*, 78-92.

Baker, R. A. (2012). The effects of high-stakes testing policy and arts education. *Arts Education Policy Review*, *113*, 17-25.

Balderas, P. E. (1998). *Representation and visual reasoning in mathematics teaching*. Doctoral Dissertation, Universidad Nacional Autonoma de Mexico, Mexico City.

Bamberger, J. & Disessa, A. (2003). Music as embodied mathematics: A study of a mutually informing affinity. *International Journal of Computers for Mathematical Education*, *8*, 123-160.

Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. *Journal for research in Mathematics Education*, *21*(1), 47-60.

Begley, S. (1996). Your child's brain. *Newsweek*, *127*(8), 55-57.

Beilock, S. L. & DeCaro, M. S. (2007). From poor performance to success under stress: Working memory, strategy selection, and mathematical problem solving under pressure. *Journal for Experimental Psychology*, *33*(6), 983-998.

Ben-Chaim, D., Lappan, G., & Houang, R. T. (1986). Development and analysis of a spatial visualization test for middle school boys and girls. *Perceptual and Motor Skills*, *63*, 659-669.

Beveridge, T. (2010). No Child Left Behind and fine arts classes. *Arts Education Policy Review*, *111*(1), 4-7.

Bishop, A. J. (1980). Spatial abilities and mathematics education: A review. *Educational Studies in Mathematics*, *11*(3), 257-269.

Blatto-Vallee, G., Kelly, R. R., Gaustad, M. G., Porter, J., & Fonzi, J. (2007). Visual-spatial representation in mathematical problem solving by deaf and hearing students. *Journal of Deaf Studies and Deaf Education*, *12*(4), 432-448.

Booth, E. (2001). Music and math: The magical connection. *Scholastic*, *8*(3), 50-54.

Booth, R., & Thomas, M. (2000). Visualization in mathematics learning: Arithmetic problem-solving and student difficulties. *Journal of Mathematical Behavior*, *18*(2), 169-190.

Campbell, K. J., Collis, K. F., & Watson, J. M. (1995). Visual processing during mathematical problem solving. *Educational Studies in Mathematics*, *28*(2), 177-194.

Capraro, M. M., An, S. A., Ma, T., Rangel-Chavez, A. F., & Harbaugh, A. (2012). An investigation of preservice teachers' use of guess and check in solving a semi open-ended mathematics problem. *Journal of Mathematical Behavior*, *31*, 105-116.

Carstens, C. B., Huskins, E., & Hounshell, G. W. (1995). Listening to Mozart may not enhance performance on the Revised Minnesota Paper Form Board Test. *Psychological Reports*, *77*, 111-114.

Cash, A. H., El-Mallakh, R. S., Chamberlain, K., Bratton, J. Z., & Li, R. (1997). Structure of music may influence cognition. *Perceptual and Motor Skills, 84*, 66.

Chaffin, R., Imreh, G., Lemieux, A. F., & Chen, C. (2003). "Seeing the big picture": Piano practice as expert problem solving. *Music Perception: An Interdisciplinary Journal*, *20*(4), 465-490.

Che, M., Wiegert, E., & Threlkeld, K. (2012). Problem solving strategies of girls and boys in single-sex mathematics classrooms. *Educational Studies in Mathematics*, *79*, 311-326.

Cheek, J. M. & Smith, L. R. (1998). *Music training and mathematics achievement of ninth graders*. Washington, D.C.: U.S. Department of Education.

Cifarelli, V., Goodson-Espy, T., & Chae, J. L. (2010). Associations of students' beliefs with self-regulated problem solving in college algebra. *Journal of Advanced Academics*, *21*, 204-232.

Clements, D. H. & Battista, M. T. (1992). Geometry and Spatial Reasoning. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*, (p. 420-464). New York, NY: NCTM.

Costa-Giomi, E. (2004). Effects of three years of piano instruction on children's academic achievement, school performance and self-esteem. *Psychology of Music*, *32*(2), 139-152.

Costa-Giomi, E. (1999). The effects of three years of piano instruction on children's cognitive development. *Research in Music Education*, *47*(3), 198-212.

Courey, S. J., Balogh, E., Silker, J. R., & Paik, J. (2012). Academic music: Music instruction to engage third-grade students in learning basic fraction concepts. *Educational Studies in Mathematics*, *81*, 251-278.

Demorest, S. M. & Morrison, S. J. (200). Does music make you smarter? *Music Educators Journal*, *87*(2), 33-39.

DeShon, R. P., Chan, D., & Weissbein, D. A. (1995). Verbal overshadowing effects on Raven's Progressive Matrices: Evidence for multidimensional performance determinants. *Intelligence*, *21*, 135-155.

Eacott, S. & Holmes, K. (2010). Leading reform in mathematics education: Solving a complex equation. *Mathematics Teacher Education and Development*, *12*(2), 84-97.

Edens, K. & Potter, E. (2007). The relationship of drawing and mathematical problem solving: Draw for Math tasks. *Studies in Art Education*, *48*(3), 282-298.

Educational Research Center. (1968). *Drumcondra verbal reasoning test*. Dublin, Ireland.

English, L. D. (2008). Setting an agenda for international research in mathematics education. In L. D. English (Ed.), Handbook of International Research in Mathematics Education (2nd edition) (pp. 3-19). New York: Taylor and Francis Group.

Gallagher, A. M. & DeLisi, R. (1994). Gender differences in Scholastic Aptitude Test—Mathematics problem solving among high-ability students. *Journal of Educational Psychology*, *86*(2), 204-211.

Gardner, H. (1985). *Frames of Mind: The Theory of Multiple Intelligences*. New York: Basic Books.

Gilleta, K. S., Vrbancic, M. I., Elias, L. J., & Saucier, D. M. (2003). A Mozart Effect for women on a mental rotations task. *Perceptual and Motor Skills*, *96*, 1086-1092.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2009). *Highlights from TIMSS 2007: Mathematics and science achievement of U.S. fourth- and eighth-grade students in an international context* (Report No. NCES 2009-001 Revised). Washington, DC: National Center for Education Statistics, Institute for Education Sciences, U.S. Department of Education.

Grandin, T., Peterson, M., & Shaw, G. (1998). Spatial-temporal versus language-analytic reasoning: The role of music training. *Arts Education Policy Review*, *99*(6), 11-14.

Grimsley, G., Ruch, F. L., Warren, N. D., & Ford, J. S. (1986). Employee aptitude survey, test 7 – Verbal reasoning. Los Angeles, CA: Psychological Services, Inc.

Guay, R. (1977). *Purdue Spatial Visualization Test: Rotations*. West Lafayette, IN: Purdue Research Foundation.

Harrison, C. S. (1990). Predicting music theory grades: The relative efficiency of academic ability, music experience, and musical aptitude. *Journal of Research in Music Education, 38*(2), 124-137.

Hegarty, M. & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology*, *91*(4), 684-689.

Helmrich, B. H. (2010). Window of opportunity? Adolescence, music, and algebra. *Journal of Adolescent Research*, *25*(4), 557-577.

Hensberry, K. K. R. & Jacobbe, T. (2012). The effects of Polya's heuristic and diary writing on children's problem solving. *Mathematics Education Research Journal*, *24*, 59-85.

Hetland, L. (2000a). Learning to make music enhances spatial reasoning. *Journal of Aesthetic Education*, *34*(3/4), 179-238.

Hetland, L. (2000b). Listening to music enhances spatial-temporal reasoning: Evidence for the "Mozart Effect." *Journal of Aesthetic Education*, *34*(3/4), 105-148.

Hill, C., Corbett, C., & St. Rose, A. (2010). *Why so few? Women in science, technology, engineering, and mathematics*. Washington, DC: American Association of University Women.

Hunt, E. (1974). Quote the Raven? Nevemore! In L. W. Gregg (Ed.), *Knowledge and Cognition*. Hillsdale, NJ: Erlbaum.

Ibrahim, B. & Rebello, N. S. (2012). Representational task formats and problem solving strategies in kinematics and work. *Physical Review Special Topics - Physics Education Research*, *8*(1), 010126-1-010126-19.

Jakobson, L. S., Lewycky, S. T., Kilgour, A. R., & Stoesz, B. M. (2008). Memory for verbal and visual material in highly trained musicians. *Music Perception: An Interdisciplinary Journal*, *26*(1), 41-55.

Jausovec, N. & Habe, K. (2004). The influence of auditory background stimulation (Mozart's sonata K. 448) on visual brain activity. *International Journal of Psychophysiology*, *51*, 261-271.

Jitendra, A. K., Griffin, C. G., Haria, P., Leh, J., Adams, A., & Kaduvettoor, A. (2007). A comparison of single and multiple strategy instruction on third-grade students' mathematical problem solving. *Journal of Educational Psychology*, *99*(1), 115-127.

Johanning, D. I. (2007). Is there something to be gained from guessing? Middle school students' use of systematic guess and check. *School Science and Mathematics*, *107*(4), 123-133.

Johnson, C. M. & Memmott, J. E. (2006). Examination of relationships between participation in school music programs of differing quality and standardized test results. *Journal of Research in Music Education*, *54*(4), 293-307.

Johnson, E. A. & Meade, A. C. (1987). Developmental patterns of spatial ability: An early sex difference. *Child Development*, *58*(3), 725-740.

Johnson, K., Herr, T., & Kysh, J. (2004). *Crossing the River with Dogs: Problem Solving for College Students*. Hoboken, NJ: John Wiley and Sons, Inc.

Kapur, M. & Bielaczyc, K. (2012). Designing for Productive Failure. *The Journal of Learning Sciences*, *21*(1), 45-83.

Katz, I. R., Bennett, R. E., & Berger, A. E. (2000). Effects of response format on difficulty of SAT-Mathematics items: It's not the strategy. *Journal of Educational Measurement*, *37*(1), 39-57.

Keiper, S., Sandene, B.A., Persky, H.R., and Kuang, M. (2009). The Nation's Report Card: Arts 2008 Music & Visual Arts (NCES 2009–488). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education, Washington, D.C.

King, S. (1994). The last rung on the ladder (narrated by J. Glover). In *Graveyard Shift and other stories from Night Shift* [cassette]. New York: BDD Audio Publishing.

Kinney, D. W. (2008). Selected demographic variables, school music participation, and achievement test scores of urban middle school students. *Journal of Research in Music Education*, *56*(2), 145-161.

Koedinger, K. R. & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *Journal of the Learning Sciences*, *13*(2), 129-164.

Krutetskii, V. A. (1976). *The Psychology of Mathematical Abilities in the Schoolchildren*. Chicago: The University of Chicago Press.

Landis, J.R. & Koch, G. G. (1977). The measure of observer agreement for categorical data. *Biometrics*, *33*(1), 159-174.

Lithner, J. (2000). Mathematical reasoning and familiar procedures. *International Journal of Mathematical Education in Science and Technology*, *31*(1), 83-95.

Llinares, S. & Roig, A. I. (2008). Secondary school students' construction and use of mathematical models in solving word problems. *International Journal of Science and Mathematics Education*, *6*, 505-532.

Lynn, R., Allik, J., & Irwing, P. (2004). Sex differences on three factors identified in Raven's Standard Progressive Matrices. *Intelligence*, *32*, 411-424.

Madden, S. P. & Diaz, R. (2008). Spatial reasoning and Polya's Five Planes Problem. *Mathematics Teacher*, *102*(2), 128-137.

Madsen, C. K. & Madsen, K. (2002). Perception and cognition in music: Musically trained and untrained adults compared to sixth-grade and eighth-grade children. *Journal of Research in Music Education*, *50*(2), 111-130.

Maltese, A. V. & Tai, R. H. (2011). Pipeline persistence: Examining the association of educational experiences with earned degrees in STEM among U.S. students. *Science Education*, *95*(5), 877-907.

Mamona-Downs, J. & Downs, M. L. N. (2008). Advanced mathematical thinking and the role of mathematical structure. In L. D. English (Ed.), Handbook of International Research in Mathematics Education (2nd edition) (pp. 154-175). New York: Taylor and Francis Group.

Manthei, M. & Kelly, S. N. (1998). *Effects of Popular and Classical Background Music on the Math Test Scores of Undergraduate Students*. Retrieved from http://music.arts.usf.edu/rpme/effects.htm

Mayer, R. E. & Hegarty, M. (1996). The process of understanding mathematics problems. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The Nature of Mathematical Thinking* (pp. 29-53). Hilldale, NJ: Erlbaum.

McAdams, S. (2004). Problem-solving strategies in music composition: A case study. *Music Perception: An Interdisciplinary Journal*, *21*(3), 391-429.

McFarlane, K. A. (1989). *Social drinking and sober cognitive functioning* (Doctoral dissertation). University of Windsor, Windsor, Ontario, Canada.

Michaelides, M. P. (2002). *Students' solution strategies in spatial rotation tasks* (Master's thesis). Available from ERIC database. (ED 468817)

Miller, L. K. & Schyb, M. (1989). Facilitation and interference by background music. *Journal of Music Therapy*, *26*(1), 42-54.

Mireles, S. V., Rahrovi, S. R., & Vasquez, F. (2013). Culturally relevant mathematics. In S. Mayo & P. J. Larke (Eds.), *Integrating multiculturalism into the curriculum* (167-190). New York, NY: Peter Lang.

Mitchum, A. L. & Kelley, C. M. (2010). Solve the problem first: Constructive solution strategies can influence the accuracy of retrospective confidence judgments. *Journal of Experimental Psychology*, *36*(3), 699-710.

Morrison, S. J. (1995). Music students and academic growth. *Music Educators Journal*, *81*(2), 33-36.

Mousoulides, N. G., Christou, C., & Sriraman, B. (2008). A modeling perspective on the teaching and learning of mathematical problem solving. *Mathematical Thinking and Learning*, *10*, 293-304.

Nantais, K. M. & Schellenberg, E. G. (1999). The Mozart Effect: An artifact of preference. *Psychological Science, 10*(4), 370-373.

Nathan, M. J. & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education*, *31*(2), 168-190.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Newman, J., Rosenback, J. H., Burns, I. L., Latimer, B. C., Matocha, H. R., & Vogt, E. R. (1995). An experimental test of "the Mozart Effect": Does listening to his music improve spatial ability? *Perceptual and Motor Skills*, *81*, 1379-1387.

Ozsoy, G. & Ataman, A. (2009). The effects of metacognitive strategy training on mathematical problem solving achievement. *International Electronic Journal of Elementary Education*, *1*(2), 68-83.

Pantev, C., Roberts, L., & Engelien, A. (1998). *Exposure to music is instrumental to the brain*. Munster, Germany: University of Munster.

Pantziara, M., Gagatsis, A., & Elia, I. (2009). Using diagrams as tools for the solution of non-routine mathematical problems. *Educational Studies in Mathematics*, *72*, 39-60.

Parsad, B., & Spiegelman, M. (2011). *A Snapshot of Arts Education in Public Elementary and Secondary Schools: 2009-10* (NCES 2011-078). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.

Pietsch, S. & Jansen, P. (2012). Different mental rotation performance in students of music, sport and education. *Learning and Individual Differences*, *22*, 159-163.

Pietsching, J., Voracek, M., & Formann, A. K. (2010). Mozart effect-shmozart effect: A meta-analysis. *Intelligence, 38*, 314-323.

Polya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method*, 2nd edition. Princeton, NJ: Princeton University Press.

Presmeg, N. C. (1985). *The role of visually mediated processes in high school mathematics: A classroom investigation*. Doctoral Dissertation, University of Cambridge, England.

Presmeg, N. C. (1986a). Visualization in high school mathematics. *For Learning Mathematics*, *63*, 42-46.

Presmeg, N. C. (1986b). Visualization and mathematical giftedness. *Educational Studies in Mathematics*, *17*, 297-311.

Presmeg, N. C. & Balderas-Canas, P. E. (2001). Visualization and affect in nonroutine problem solving. *Mathematical Thinking and Learning*, *3*(4), 289-313.

Prokop, K. L. (2011). *The relationship between studying music and mathematics performance on the New Jersey High School Proficiency Assessment*. Doctoral Dissertation, Montclair State University, New Jersey.

Quinn, D. M. & Spencer, S. J. (2001). The interference of stereotype threat with women's generation of mathematical problem-solving strategies. *Journal of Social Issues*, *57*(1), 55-71.

Raju, P. K. & Clayson, A. (2010). The future of STEM education: An analysis of two national reports. *Journal of STEM Education*, *11*(5-6), 25-28.

Rauscher, F. H. & Shaw, G. L. (1998). Key components of the Mozart effect. *Perceptual and Motor Skills*, *86*, 835-841.

Rauscher, F. H., Shaw, G. L., & Ky, K. N. (1993). Music and spatial task performance. *Nature*, *365*, 611.

Rauscher, F. H., Shaw, G. L., & Ky, K. N. (1995). Listening to Mozart enhances spatial-temporal reasoning: Towards a neurophysiological basis. *Neuroscience Letters*, *185*, 44-47.

Rauscher, F. H., Shaw, G. L., Levine, L. J., Ky, K. N., & Wright, E. L. (1994). Music and spatial task performance: A causal relationship. Presented at the American Psychological Association 102nd Annual Convention, Los Angeles, CA, August 12-16, 1994.

Rauscher, F. H., Shaw, G. L., Levine, L. J., Wright, E. L., Dennis, W. R., & Newcomb, R. L. (1997). Music training causes long-term enhancement of preschool children's spatial-temporal reasoning. *Neurological Research*, *19*, 2-8.

Rauscher, F. H. & Zupan, M. A. (2000). Classroom keyboard instruction improves kindergarten children's spatial-temporal performance: A field experiment. *Early Childhood Research Quarterly*, *15*(2), 215-228.

Raven, J. (1958). *Standard progressive matrices*. London: H. K. Lewis.

Reyes, C. (2010). Success in algebra among community college students. *Community College Journal of Research and Practice*, *34*(3), 256-266.

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2012). Developing procedural flexibility: Are novices prepared to learn from comparing procedures? *British Journal of Educational Psychology*, *82*, 436-455.

Roberts, B. (2004). The social construct of music as a school subject. *Action, Criticism, and Theory for Music Education*, *3*(3). Retrieved from http://act.maydaygroup.org/articles/Roberts3_3.pdf

Ross, D., Reys, R., Chavez, O., McNaught, M. D., & Grouws, D. A. (2011). Student strategy choices on a constructed response algebra problem. *School Science and Mathematics*, *111*(8), 389-398.

Rousseau, D. (2009.) *Improving mathematical problem solving skills: The journey to success* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI 3365587)

Schildknecht, W. R. (1989). *The effect of practice of analytical reasoning skills upon performance on problems in an introductory probability course*. (Doctoral dissertation). University of Maryland.

Schlaug, G., Jancke, L., Huang, Y., Staiger, J. F., & Steinmetz, H. (1995). Increased corpus callosum size in musicians. *Neuropsychologia*, *33*(8), 1047-1055.

Schoenfeld, A. H. (1979, April). *Presenting a model of mathematical problem solving*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Schoenfeld, A. H. & Herrmann, D. J. (1980). *Problem perception and knowledge structure in expert and novice mathematical problem solvers* (Report No. ED200609). Washington, D.C.: National Science Foundation.

Shaw, G. L. (2000). *Keeping Mozart in Mind*. San Diego, CA: Academic Press.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 20-26.

Small, D. (2002). An urgent call to improve traditional college algebra programs. Summary of the Conference to Improve College Algebra (U.S. Military Academy, West Point, NY, February 7-10, 2002). Retrieved from http://www.contemporarycollegealgebra.org/national_movement/an_urgent_call.html

Steele, K. M., Ball, T. N., & Runk, R. (1997). Listening to Mozart does not enhance backward digit span performance. *Perceptual and Motor Skills*, *84*, 1179-1184.

Steele, K. M., Bass, K. E., & Crook, M. D. (1999). The mystery of the Mozart Effect: Failure to replicate. *Psychological Science, 10*(4), 366-369.

Stough, C., Kerkin, B., Bates, T., & Magnan, G. (1994). Music and spatial IQ. *Personality and Individual Differences*, *17*, 695.

Stylainou, D. A. (2002). On the interaction of visualization and analysis: The negotiation of visual representation in expert problem solving. *Journal of Mathematical Behavior*, *21*, 303-317.

Stylainou, D. A. (2011). An examination of middle school students' representation practices in mathematical problem solving through the lens of expert work: Towards an organizing scheme. *Educational Studies in Mathematics, 76*, 265-280.

Swanson, H. L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, *82*(2), 306-314.

Taylor, J. M. & Rowe, B. J. (2012). The "Mozart Effect" and the mathematical connection. *Journal of College Reading and Learning*, *42*(2), 51-66.

Taylor, K. L. & Dionne, J. P. (2000). Accessing problem-solving strategy knowledge: The complementary use of concurrent verbal protocols and retrospective debriefing. *Journal of Educational Psychology*, *92*(3), 413-425.

Texas Higher Education Coordinating Board & Texas Education Agency. (2009). Texas College and Career readiness standards. The University of Texas at Austin: Austin, TX.

Thurstone, L. L. & Thurstone, T. G. (1947). *Primary mental abilities*. Chicago, IL: Science Research Associates.

Tillman, L. C. (2002). Culturally sensitive research approaches: An African-American perspective. *Educational Researcher*, *31*(9), 3-12.

Tucker, A. & Bushman, B. J. (1991). Effects of rock and roll music on mathematical, verbal, and reading comprehension performance. *Perceptual and Motor Skills*, *72*(3), 942.

van der Ven, S. H. G., Boom, J. Kroesbergen, E. H., & Leseman, P. P. M. (2012). Microgenetig patterns of children's multiplication learning: Confirming the overlapping waves model by latent growth modeling. *Journal of Experimental Child Psychology*, *113*, 1-19.

Vaughn, K. (2000). Music and mathematics: Modest support for the oft-claimed relationship. *Journal of Aesthetic Education*, *34*(3/4), 149-166.

Walkington, C., Petrosino, A., & Sherman, M. (2013). Supporting algebraic reasoning through personalized story scenarios: How situational understanding mediates performance. *Mathematical Thinking and Learning*, *15*(2), 89-120.

Webb, N. L. (1977). Mathematical problem solving project technical report I: Documents related to a problem-solving model. Part A: A review of the literature related to problem-solving tasks and problem-solving strategies used by students in grades 4, 5, and 6. Final report.

Wechsler, D. (1976). *Wechsler Intelligence Scale for Children—Revised*. San Antonio, TX: Psychological Corporation.

Weinberger, N. M. (1998). The music in our minds. *Educational Leadership*, *56*(3), 36-40.

Wenger, W. & Wenger, S. H. (1990). Training music sight-reading and perfect pitch in young children as a way to enhance their intelligence. *Journal of the Society for Accelerative Learning and Teaching*, *15*, 77-89.

Whitehead, B. (2001). *The effect of music-intensive intervention on mathematics scores of middle and high school students*. (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI 3023878)

Wilburne, J. M., Marinak, B. A., & Strickland, M. J. (2011). Addressing cultural bias. *Mathematics Teaching in the Middle School*, *16*(8), 460-465.

Winsler, A. & Naglieri, J. (2003). Overt and Covert verbal problem-solving strategies: Developmental trends in use, awareness, and relations with task performance in children aged 5 to 17. *Child Development*, *74*(3), 659-678.

Wolf, R. H. & Weiner, F. F. (1972). Effects of four noise conditions on arithmetic. *Perceptual and Motor Skills*, *35*, 928-930.

Wolfe, D. (1983). Effects of music loudness on task performance and self-report of college-age students. *Journal of Research in Music Education*, *31*, 193-201.

Yackel, E. B. (1984). *Mathematical beliefs survey*. West Lafayette, IN: Purdue University.

Zazkis, R., Dubinsky, E., & Dautermann, J. (1996). Coordinating visual and analytic strategies: A study of students' understanding of the group D4. *Journal of Research in Mathematics Education*, *27*(4), 435-457.

Zhu, Z. (2007). Gender differences in mathematical problem solving patterns: A review of literature. *International Education Journal*, *8*(2), 187-203.

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