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TRANSMISSION PROBLEM FOR WAVES WITH FRICTIONAL DAMPING

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Abstract. In this paper we consider the transmission problem, in one space dimension, for linear dissipative waves with frictional damping. We study the wave propagation in a medium with a component with attrition and another simply elastic. We show that for this type of material, the dissipation produced by the frictional part is strong enough to produce exponential decay of the solution, no matter how small is its size.

1. Introduction

A number of authors have studied the wave equation with dissipation. We mention for example, the work of Zuazua [5] where it was obtained the uniform rate of decay of the solution for a large class of nonlinear wave equation with frictional damping acting in the whole domain. In this direction, the natural question that arises is about the rate of decay when the dissipation is effective only in a part of the domain. It is the purpose of this investigation, at least in part, to answer this question. We consider the wave propagation over a body consisting of two different type of materials. This is a transmission (or diffraction) problem. It happens frequently in applications where the domain is occupied by several materials, whose elastic properties are different, joined together over the whole of a surface. From the mathematical point of view a transmission problem for wave propagation consists on a hyperbolic equation for which the corresponding elliptic operator has discontinuous coefficients. Even though we consider a case of space dimension one and linear equations with constant coefficients, the problem studied here is interesting by its own.

Existence, regularity, as well as the exact controllability for the transmission problem for the pure wave equation was studied in [2]. The transmission problem for viscoelastic waves was studied by Rivera and Oquendo [4] who proved the exponential decay of solution using regularity results of the Volterra's integral equations and regularizing properties of the viscosity. The asymptotic behavior for a coupled system of equations of waves was studied by Raposo [3] by the same method used in this paper.

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Let k_1, k_2 and α be positive real numbers and $0 < L_0 < L$. The system considered here is

$$u_{tt} - k_1 u_{xx} + \alpha u_t = 0, \quad x \in (0, L_0),\ t > 0, \tag{1.1}$$

$$v_{tt} - k_2 v_{xx} = 0, \quad x \in (L_0, L),\ t > 0, \tag{1.2}$$

satisfying the boundary conditions

$$u(0,t) = v(L,t) = 0, \quad t > 0, \tag{1.3}$$

the transmission conditions

$$u(L_0,t) = v(L_0,t), \quad k_1 u_x(L_0,t) = k_2 v_x(L_0,t), \quad t > 0, \tag{1.4}$$

and initial conditions

$$\begin{gathered} u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x), \quad x \in (0, L_0), \\ v(x,0) = v^0(x), \quad v_t(x,0) = v^1(x), \quad x \in (L_0, L). \end{gathered} \tag{1.5}$$

We are concerned with the asymptotic properties of the system above. The main result of this paper is Theorem 3.6 which shows that the solution of the transmission problem (1.1)–(1.5) decays exponentially to zero as time goes to infinity, no matter how large is the difference $L - L_0$. The approach we use consists of choosing appropriate multipliers to build a functional of Lyapunov for the system.

The notation used throughout this work is the standard one. For instance H^m, $L^2 = H^0$, $W^{m,p}$ and $W^{m,\infty}$ denote the usual Sobolev Spaces (see Adams [1]). By $\mathcal{V}$ we denote the space

$$\mathcal{V} := \{(u,v) \in H^1(0,L_0) \times H^1(L_0,L) : u(0) = v(L) = 0,\ u(L_0) = v(L_0)\}$$

which together with the inner product

$$\langle (u^1, v^1)\,,\, (u^2, v^2) \rangle := \int_0^{L_0} u_x^1 u_x^2\,dx + \int_{L_0}^{L} v_x^1 v_x^2\,dx$$

is a Hilbert space. The energies associated to the equations (1.1) and (1.2) are:

$$\begin{gathered} E_1(t) = \frac{1}{2}\int_0^{L_0} [|u_t|^2 + k_1 |u_x|^2]\,dx, \\ E_2(t) = \frac{1}{2}\int_{L_0}^{L} [|v_t|^2 + k_2 |v_x|^2]\,dx \end{gathered}$$

respectively. We denote $E(t) = E_1(t) + E_2(t)$ the total energy associated to the system (1.1)–(1.5).

The remainder of this paper is organized as follows. In Section 2 we show the existence of weak and strong solutions for the system (1.1)–(1.5), and in Section 3 we show the exponential decay of such solutions.

2. Existence of solutions

We begin this section defining what is meant by weak solution to our transmission problem.

Definition 2.1. The couple $(u(x,t), v(x,t))$ is a weak solution of the system (1.1)–(1.5) when

$$(u,v) \in\ L^\infty(0,T;\mathcal{V}) \cap W^{1,\infty}(0,T; L^2(0,L_0) \times L^2(L_0,L)),$$

and satisfies

$$\begin{aligned}
&-\int_0^{L_0} u^1\phi(0)\,dx-\int_{L_0}^{L} v^1\psi(0)\,dx-\int_0^T\int_0^{L_0} u_t\phi_t\,dx\,dt-\int_0^T\int_{L_0}^{L} v_t\psi_t\,dx\,dt\\
&+k_1\int_0^T\int_0^{L_0} u_x\phi_x\,dx\,dt+k_2\int_0^T\int_{L_0}^{L} v_x\psi_x\,dx\,dt+\alpha\int_0^T\int_0^{L_0} u_t\phi\,dx\,dt=0
\end{aligned}$$

for any

$$(\phi,\psi)\in\ L^\infty(0,T;)\cap W^{1,\infty}(0,T;L^2(0,L_0)\times L^2(L_0,L)),$$

such that

$$(\phi(T),\psi(T))=(0,0).$$

Theorem 2.2. *Let us take* $(u^0,v^0)\in\ (H^2(0,L_0)\times H^2(L_0,L))\cap\mathcal{V}$ *and* $(u^1,v^1)\in\mathcal{V}$ *verifying the transmission conditions. Under this conditions the solution* (u,v) *of (1.1)–(1.5) satisfies*

$$(u,v)\in\bigcap_{k=0}^{2}W^{k,\infty}(0,T;H^{2-k}(0,L_0))\times H^{2-k}(L_0,L).$$

Proof. The existence is proved using Galerkin method. In order to do so we take a basis $\{(\phi^0,\psi^0),\,(\phi^1,\psi^1),\,(\phi^2,\psi^2),\cdots\}$ of $\mathcal{V}$ and let

$$(u_m^0,v_m^0),(u_m^1,v_m^1)\in\ span\{(\phi^0,\psi^0),(\phi^1,\psi^1)\cdots(\phi^m,\psi^m)\}$$

be a projection of the initial state on a finite dimensional subspace of $\mathcal{V}$. Standard results on ordinary differential equations guarantee that there exists one and only one solution

$$(u^m(t),v^m(t)):=\sum_{j=1}^{m}h_{j,m}(t)(\phi^j,\psi^j)$$

of the approximated system,

$$\int_0^{L_0} u_{tt}\phi^i\,dx+\int_{L_0}^{L} v_{tt}\psi^i\,dx+k_1\int_0^{L_0} u_x\phi^i_x\,dx+k_2\int_{L_0}^{L} v_x\psi^i_x\,dx+\alpha\int_0^{L_0} u_t\phi^i\,dx=0 \tag{2.1}$$

$i=0,1,2,\dots,m$, with initial data

$$(u^m(0),v^m(0))=(u_m^0,v_m^0),\quad (u_t^m(0),v_t^m(0))=(u_m^1,v_m^1).$$

We show next that the above solution remain bounded for any $m\in\mathbf{N}$. In order to do so, we first multiply equation (2.1) by $h'_{j,m}(t)$ and then sum up in i, to obtain

$$\frac{d}{dt}E^m(t)=-\alpha\int_0^{L_0}|u_t^m|^2\,dx.$$

Integrating the identity above from 0 to t, we get

$$E^m(t)\le E^m(0)$$

showing that the first order energy $E^m(t)$ is uniformly bounded for $m\in\mathbf{N}$.

Now we denote the second order energy by

$$\mathcal{E}^m(t)=\frac12\int_0^{L_0}[|u_{tt}^m|^2+k_1|u_{xt}^m|^2]\,dx+\frac12\int_{L_0}^{L}[|v_{tt}^m|^2+k_2|v_{xt}^m|^2]\,dx.$$

Differentiating equation (2.1) with respect to t, we get

$$\begin{aligned} &\int_0^{L_0} u_{ttt}\phi^i\,dx + \int_{L_0}^{L} v_{ttt}\psi^i\,dx + k_1\int_0^{L_0} u_{xt}\phi^i_x\,dx \\ &+ k_2\int_{L_0}^{L} v_{xt}\psi^i_x\,dx + \alpha\int_0^{L_0} u_{tt}\phi^i\,dx = 0. \end{aligned} \tag{2.2}$$

Multiplying equation (2.2) by $h''_{j,m}(t)$ and summing up in i, we obtain

$$\frac{d}{dt}\mathcal{E}^m(t) = -\alpha\int_0^{L_0} |u^m_{tt}|^2\,dx$$

which integrated from 0 to t furnishes

$$\mathcal{E}^m(t) \le \mathcal{E}^m(0).$$

The next step is to estimate the second order energy. Letting $t \to 0^+$ in equation (2.1), multiplying the limit result by $h''_{j,m}(t)$ we get

$$\begin{aligned} &\int_0^{L_0} |u^m_{tt}(0)|^2\,dx + \int_{L_0}^{L} |v^m_{tt}(0)|^2\,dx \\ &= -k_1\int_0^{L_0} u^m_x(0)u^m_{xtt}(0)\,dx - k_2\int_{L_0}^{L} v^m_x(0)v^m_{xtt}(0)\,dx - \alpha\int_0^{L_0} u^m_t(0)u^m_{tt}(0)\,dx. \end{aligned}$$

Integrating by parts the equation above, we get

$$\begin{aligned} &\int_0^{L_0} |u^m_{tt}(0)|^2\,dx + \int_{L_0}^{L} |v^m_{tt}(0)|^2\,dx \\ &= k_1\int_0^{L_0} u^m_{xx}(0)u^m_{tt}(0)\,dx + k_2\int_{L_0}^{L} v^m_{xx}(0)v^m_{tt}(0)\,dx - \alpha\int_0^{L_0} u^m_t(0)u^m_{tt}(0)\,dx. \end{aligned} \tag{2.3}$$

After application of Young's inequality in equation (2.3) we find

$$\begin{aligned} &\int_0^{L_0} |u^m_{tt}(0)|^2\,dx + \int_{L_0}^{L} |v^m_{tt}(0)|^2\,dx \\ &\le c\Big\{\int_0^{L_0} |u^m_{xx}(0)|^2\,dx + \int_{L_0}^{L} |v^m_{xx}(0)|^2\,dx\Big\} + c\int_0^{L_0} |u^m_t(0)|^2\,dx. \end{aligned}$$

which implies that the initial data

$$(u^m_{tt}(0), v^m_{tt}(0)) \quad \text{is bounded in } L^2(0,L_0)\times L^2(L_0,L)),$$

and so is $\mathcal{E}^m(0)$. Whence we have

$$\mathcal{E}^m(t) \quad \text{is bounded for every } m \in \mathbf{N}.$$

The first and second order energy boundedness implies that there exists a subsequence of (u^m, v^m), which we still denote in the same way, such that

$$\begin{gathered} (u^m, v^m) \overset{*}{\rightharpoonup} (u, v) \quad \text{in } L^\infty(0.T;\mathcal{V}), \\ (u^m_t, v^m_t) \overset{*}{\rightharpoonup} (u_t, v_t) \quad \text{in } L^\infty(0.T;\mathcal{V}), \\ (u^m_{tt}, v^m_{tt}) \overset{*}{\rightharpoonup} (u_{tt}, v_{tt}) \quad \text{in } L^\infty(0.T;L^2(0,L_0)\times L^2(L_0,L))). \end{gathered}$$

Therefore the couple (u, v) satisfies

$$\begin{gathered}u_{tt} - k_1 u_{xx} + \alpha u_t = 0\\ v_{tt} - k_2 v_{xx} = 0.\end{gathered}$$

The rest of the proof is a matter of routine. □

3. Exponential stability

With a view toward proving the main result of this paper we formulate and prove a series of five lemmas. They will provide some technical inequalities which play fundamental role in the proof of Theorem 3.6.

Lemma 3.1. *The total energy $E(t)$ satisfies*

$$\frac{d}{dt}E(t) = -\alpha \int_0^{L_0} |u_t|^2 \, dx.$$

Proof. Multiplying equation (1.1) by u_t and integrating in $(0, L_0)$ we have

$$\int_0^{L_0} u_t u_{tt} \, dx - k_1 \int_0^{L_0} u_t u_{xx} \, dx = -\alpha \int_0^{L_0} |u_t|^2 \, dx$$

which integrated by parts leads to

$$\frac{d}{dt}\frac{1}{2}\int_0^{L_0} [|u_t|^2 + k_1 |u_x|^2] \, dx = -\alpha \int_0^{L_0} |u_t|^2 \, dx + k_1 u_x(L_0) u_t(L_0). \tag{3.1}$$

Multiplying equation (1.2) by v_t and performing an integration in (L_0, L) we get

$$\int_{L_0}^{L} v_t v_{tt} \, dx - k_2 \int_{L_0}^{L} v_t v_{xx} \, dx = 0.$$

After integrating by parts we arrive at

$$\frac{d}{dt}\frac{1}{2}\int_{L_0}^{L} [|v_t|^2 + k_2 |v_x|^2] \, dx = -k_2 v_x(L_0) v_t(L_0). \tag{3.2}$$

Adding (3.1) with (3.2) and using the transmission conditions (1.4) we conclude

$$\frac{d}{dt}E(t) = -\alpha \int_0^{L_0} |u_t|^2 \, dx. \tag{3.3}$$

□

Lemma 3.2. *There exist positive constants C_0 and C_1, independent of initial data, such that the functional defined by*

$$J_1(t) = \int_0^{L_0} (x - L_0) u_t u_x \, dx$$

satisfies

$$\frac{d}{dt}J_1(t) \le -C_1 E_1(t) + C_0 \int_0^{L_O} |u_t|^2 \, dx + \frac{k_1 L_0}{2} |u_x(0)|^2.$$

Proof. Multiplying equation (1.1) by $(x-L_0)u_x$ and performing an integration in $(0,L_0)$ we get

$$\int_0^{L_0}(x-L_0)u_xu_{tt}\,dx-k_1\int_0^{L_0}(x-L_0)u_xu_{xx}\,dx=-\alpha\int_0^{L_0}(x-L_0)u_xu_t\,dx. \quad (3.4)$$

Note that

$$\frac{d}{dt}(x-L_0)u_xu_t=(x-L_0)u_xu_{tt}+(x-L_0)u_{xt}u_t. \quad (3.5)$$

Now using (3.5) in (3.4) we get

$$\begin{aligned}\frac{d}{dt}\int_0^{L_0}(x-L_0)u_xu_t\,dx&=\int_0^{L_0}(x-L_0)\frac{1}{2}[\frac{d}{dx}|u_t|^2]\,dx\\&\quad+k_1\int_0^{L_0}(x-L_0)\frac{1}{2}[\frac{d}{dx}|u_x|^2]\,dx-\alpha\int_0^{L_0}(x-L_0)u_xu_t\,dx\end{aligned}$$

and performing integration by parts we get

$$\begin{aligned}\frac{d}{dt}\int_0^{L_0}(x-L_0)u_xu_t\,dx.&=-\frac{1}{2}\int_0^{L_0}|u_t|^2\,dx-\frac{k_1}{2}\int_0^{L_0}|u_x|^2\,dx\\&\quad-\alpha\int_0^{L_0}(x-L_0)u_xu_t\,dx+\frac{k_1L_0}{2}|u_x(0)|^2\end{aligned}$$

from which it follows that

$$\frac{d}{dt}J_1(t)\le -C_1E_1(t)+C_0\int_0^{L_0}|u_t|^2\,dx+\frac{k_1L_0}{2}|u_x(0)|^2.$$

□

Lemma 3.3. *There exists a positive constant C_2, independent of initial data, such that the functional defined by*

$$J_2(t)=\int_{L_0}^{L}(x-L_0)v_tv_x\,dx$$

satisfies

$$\frac{d}{dt}J_2(t)\le -C_2E_2(t)+\frac{k_2(L-L_0)}{2}|v_x(L)|^2.$$

Proof. Multiplying equation (1.2) by $(x-L_0)v_x$ and performing an integration in (L_0,L) we get

$$\int_{L_0}^{L}(x-L_0)v_xv_{tt}\,dx-k_2\int_{L_0}^{L}(x-L_0)v_xv_{xx}\,dx=0. \quad (3.6)$$

Notice that

$$\frac{d}{dt}(x-L_0)v_xv_t=(x-L_0)v_xv_{tt}+(x-L_0)v_{xt}v_t. \quad (3.7)$$

Now using (3.7) in (3.6) we get

$$\frac{d}{dt}\int_{L_0}^{L}(x-L_0)v_xv_t\,dx=\int_{L_0}^{L}(x-L_0)\frac{1}{2}[\frac{d}{dx}|v_t|^2]\,dx+k_2\int_{L_0}^{L}(x-L_0)\frac{1}{2}[\frac{d}{dx}|v_x|^2]\,dx$$

and performing integration by parts we get

$$\frac{d}{dt}\int_{L_0}^{L}(x-L_0)v_xv_t\,dx.=-\frac{1}{2}\int_{L_0}^{L}|v_t|^2\,dx-\frac{k_2}{2}\int_{L_0}^{L}|v_x|^2\,dx+\frac{k_2(L-L_0)}{2}|v_x(L)|^2$$

from which it follows that
$$\frac{d}{dt}J_2(t) \le -C_2E_2(t) + \frac{k_2(L-L_0)}{2}|v_x(L)|^2.$$
□

Now we must control the punctual terms $|u_x(0)|^2$ and $|v_x(L)|^2$ present in the inequalities given by the lemmas 3.2 and 3.3 respectively. In order to do so we introduce the two following lemmas.

Lemma 3.4. *Let us take $p \in C^1(0, L_0)$ with $p(0) > 0$ and $p(L_0) = 0$. Then, there exist positive constants C_0, C_4, N_0 independent of initial data, such that the functional defined by*
$$J_3(t) = N_0J_1(t) + \int_0^{L_0} pu_tu_x\,dx$$
satisfies
$$\frac{d}{dt}J_3(t) \le -C_4E_1(t) + N_0C_0\int_0^{L_0}|u_t|^2\,dx.$$

Proof. Multiplying equation (1.1) by $p\,u_x$ and performing an integration in $(0, L_0)$ we get
$$\int_0^{L_0} p\,u_xu_{tt}\,dx - k_1\int_0^{L_0} p\,u_xu_{xx}\,dx = -\alpha\int_0^{L_0} p\,u_xu_t\,dx. \tag{3.8}$$
Notice that
$$\frac{d}{dt}p\,u_xu_t = p\,u_xu_{tt} + p\,u_{xt}u_t. \tag{3.9}$$
Now using (3.9) in (3.8) we get
$$\begin{aligned}&\frac{d}{dt}\int_0^{L_0} p\,u_xu_t\,dx\\ &= \int_0^{L_0} p\,\frac{1}{2}[\frac{d}{dx}|u_t|^2]\,dx + k_1\int_0^{L_0} p\,\frac{1}{2}[\frac{d}{dx}|u_x|^2]\,dx - \alpha\int_0^{L_0} p\,u_xu_t\,dx\end{aligned}$$
and performing integration by parts we get
$$\begin{aligned}&\frac{d}{dt}\int_0^{L_0} p\,u_xu_t\,dx\\ &= -\frac{1}{2}\int_0^{L_0} p'\,|u_t|^2\,dx - \frac{k_1}{2}p(0)\,|u_x(0)|^2 - \frac{k_1}{2}\int_0^{L_0} p'\,|u_x|^2\,dx - \alpha\int_0^{L_0} p\,u_xu_t\,dx,\end{aligned}$$
from which it follows that
$$\frac{d}{dt}\int_0^{L_0} p\,u_xu_t\,dx \le -\frac{k_1}{2}p(0)\,|u_x(0)|^2 + C_3E_1(t).$$
Denoting
$$J_3(t) = N_0J_1(t) + \int_0^{L_0} pu_tu_x\,dx,$$
we have
$$\begin{aligned}\frac{d}{dt}J_3(t) &\le -N_0C_1\,E_1(t) + C_3\,E_1(t) + \frac{N_0k_1L_0}{2}\,|u_x(0)|^2 - \frac{k_1}{2}p(0)\,|u_x(0)|^2\\ &\quad + N_0\,C_0\int_0^{L_0}|u_t|^2\,dx.\end{aligned}$$

Now taking N_0 such that $N_0\, C_1 > C_3$ and choosing $p(0) = N_0 L_0$ we conclude that

$$\frac{d}{dt} J_3(t) \le -C_4 E_1(t) + N_0 C_0 \int_0^{L_0} |u_t|^2\, dx.$$

□

Lemma 3.5. *Let us take $q \in C^1(L_0, L)$ with $q(L_0) = 0$ and $q(L) < 0$. Then, there exist positive constants C_5 and N_1 independent of initial data such that the functional defined by*

$$J_4(t) = N_1 J_2(t) + \int_{L_0}^{L} q v_t v_x\, dx$$

satisfies $\frac{d}{dt} J_4(t) \le -C_5 E_2(t)$.

Proof. Multiplying equation (1.2) by $q\, v_x$ and performing an integration in (L_0, L) we get

$$\int_{L_0}^{L} q\, v_x v_{tt}\, dx - k_2 \int_{L_0}^{L} q\, v_x v_{xx}\, dx = 0. \tag{3.10}$$

Notice that

$$\frac{d}{dt} q\, v_x v_t = q\, v_x v_{tt} + q\, v_{xt} v_t. \tag{3.11}$$

Now using (3.11) in (3.10) we get

$$\frac{d}{dt} \int_{L_0}^{L} q\, v_x v_t\, dx = \int_{L_0}^{L} q\, \frac{1}{2} [\frac{d}{dx} |v_t|^2]\, dx + k_2 \int_{L_0}^{L} q\, \frac{1}{2} [\frac{d}{dx} |v_x|^2]\, dx$$

and performing integration by parts we arrive at

$$\frac{d}{dt} \int_{L_0}^{L} q\, v_x v_t\, dx = -\frac{1}{2} \int_{L_0}^{L} q'\, |v_t|^2\, dx + \frac{k_2}{2} q(L)\, |v_x(L)|^2 - \frac{k_2}{2} \int_{L_0}^{L} q'\, |v_x|^2\, dx,$$

from which it follows that

$$\frac{d}{dt} \int_{L_0}^{L} q\, v_x v_t\, dx \le \frac{k_2}{2} q(L)\, |v_x(L)|^2 + C_4 E_2(t).$$

Denoting

$$J_4(t) = N_1 J_2(t) + \int_{L_0}^{L} q v_t v_x\, dx,$$

we have

$$\frac{d}{dt} J_4(t) \le -N_1 C_2\, E_2(t) + C_4\, E_2(t) + \frac{N_1 k_2 (L - L_0)}{2}\, |v_x(L)|^2 + \frac{k_2}{2} q(L)\, |v_x(L)|^2.$$

Now taking N_1 such that $N_1\, C_2 > C_4$ and choosing $q(L) = -N_1(L - L_0)$ we conclude that

$$\frac{d}{dt} J_4(t) \le -C_5 E_2(t).$$

□

Now we are in position to show the main result of this paper.

Theorem 3.6. *Let us denote by (u, v) a strong solution of system (1.1)–(1.5), as in Theorem 2.2. Then there exist positive constants C and ω, such that*

$$E(t) \le C\, E(0) e^{-\omega\, t}.$$

Proof. Let us define
$$\mathcal{L}(t) = N_2\,E(t) + J_3(t) + J_4(t).$$
From Lemma 3.1 we have
$$\frac{d}{dt}E(t) = -\alpha\int_0^{L_0} |u_t|^2\,dx.$$
From Lemma 3.4 we have
$$\frac{d}{dt}J_3(t) \le -C_4E_1(t) + N_0C_0\int_0^{L_0} |u_t|^2\,dx.$$
From Lemma 3.5 we have
$$\frac{d}{dt}J_4(t) \le -C_5E_2(t).$$
In fact we have
$$\frac{d}{dt}\mathcal{L}(t) \le -C_4E_1(t) - C_5E_2(t) + (N_0C_0 - N_2\alpha)\int_0^{L_0} |u_t|^2\,dx.$$
Taking N_2 large enough it follows
$$\frac{d}{dt}\mathcal{L}(t) \le -C_6\,E(t)$$
Since $\mathcal{L}(t)$ is equivalent to $E(t)$, we conclude that there exist positive constants C and ω, such that
$$E(t) \le C\,E(0)e^{-\omega\,t}.$$
□

Theorem 3.6 can be extended easily to weak solutions by using density arguments and the lower semicontinuity of the energy functional $E(t)$. This is the content of the following corollary whose proof is omitted.

Corollary 3.7. *Under the same hypotheses of Theorem 3.6, there exists positive constants $\bar{C}$ and $\bar{\omega}$, such that*
$$E(t) \le \bar{C}\,E(0)e^{-\bar{\omega}t}.$$

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