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DYNAMICS OF A COMPETITIVE POPULATION SYSTEM WITH IMPULSIVE REDUCTION OF THE INVASIVE POPULATION

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Abstract. Biological invasion refers to the phenomenon that some organisms have been accidentally or artificially introduced into the wild. The invasive populations compete with the local population,and cause damage to the local ecosystem. To protect the local ecosystem, the invasive populations should be artificially reduced. Such processes are seldom studied in dynamical models. In this work, we consider a competitive population system with impulsive reduction of the invasive population. All solutions of the investigated system are proved to be ultimately uniformly bounded. Sufficient conditions are obtained to guarantee the linear stability of the population $x(t)$-extinction periodic solution. This signifies that the alien species invade successfully, and cause the extinction of the native species. The permanency of the conditions is also obtained, which shows that the alien species invade successfully, and they coexist with the native species. Numerical simulations are included to illustrate our results. Through such computation, we find that there exists a threshold of unsuccessful invasion, indicating that the native species cause the extinction of the alien species. These results offer insights that impulsive invasion plays an important role in the dynamics of ecosystem, and provide some reliable tactical analysis for biological resources protection.

1. Introduction

Biological invasion refers to the phenomenon that some organisms have been accidentally or artificially introduced into the wild. The invasive populations compete with the local populations, and cause damage to the local ecosystem. The impacts of biological invasions now rank among the most pervasive threats to native ecosystems and human economies [2, 18]. Invasion by alien organisms is a common worldwide phenomenon. Such invasion by alien species is especially likely to occur on oceanic islands [20]. Biological invasions are rapidly producing planet-wide changes in biodiversity and ecosystem function. Experiments on the competitions are usually employed to investigate biological invasions [4]. Biological invasions are seldom studied in dynamical models [22].

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The theory of differential equations is used in mathematical ecology, and the predator-prey, competitive and cooperative models have been studied by many authors [2, 5, 7, 8, 9, 10, 19, 21]. One of the famous models for population dynamics is the Lotka-Volterra competitive system, as competition is an important biotic process that affects the population dynamics of ecosystems. Many authors [1, 6] have investigated the population dynamics by the theory of impulsive differential equations. Almost all domains of applied science [11, 17] have found the occurrence of impulsive phenomena. Liu and Chen [15] developed the Holling type II Lotka-Volterra predator-prey system model, which may inherently oscillate, by introducing periodic constant impulsive immigration of a predator. Their results showed that the dynamics of such a system is dependent on the impulsive immigration amount of the predator. Meng and Chen [17] formulated a robust impulsive Lotka-Volterra n-species competitive system with both discrete delays and continuous delays. Theirs results indicated that under the appropriate linear bounded impulsive perturbations, the impulsive delay Lotka-Volterra system maintains the original permanence and globally asymptotical stability of the nonimpulsive delay Lotka-Volterra system. Jiao et al. [12] suggested a five-dimensional chemostat model with impulsive diffusion and pulse input environmental toxicant. The results revealed that impulsive diffusion plays an important role on the outcome of the chemostat. Jiao et al. [13] investigated the dynamics of a chemostat model with impulsive input and effect of delayed response in growth. Their results indicated that the discrete time delay has an influence on the dynamical behaviors of the investigated system, and provided a tactical basis for the experimenters to control the outcome of the chemostat. Even though there already is plenty of work using impulsive differential equations to study predator-prey, chemostat and invasive population, few papers can be found to combine impulsive dynamical systems with biological invasions.

The organization of this paper is as follows. In the following section, we introduce the model and background. In Section 3, some important lemmas are presented. In Section 4, we give the linear stability conditions of population $x(t)$-extinction periodic solution of system (2.1), and the permanency condition of system (2.1). In Section 5, a brief discussion is given to conclude this work.

2. The model

In this work, we consider a competitive population system with impulsive reduction of the invasive population

$$
\left.\begin{aligned}
\frac{dx(t)}{dt} &= x(t)(a_1 - b_1 x(t)) - k_1\beta x(t)y(t),\\
\frac{dy(t)}{dt} &= -d_1 y(t) - k_2\beta x(t)y(t),\\
\frac{dz(t)}{dt} &= z(t)(a_2 - b_2 z(t)),
\end{aligned}\right\}\quad t \neq (n+l)\tau,\ t \neq (n+1)\tau,
$$

$$
\left.\begin{aligned}
\Delta x(t) &= 0,\\
\Delta y(t) &= \epsilon d z(t),\\
\Delta z(t) &= -d z(t),
\end{aligned}\right\}\quad t = (n+l)\tau,\ n = 1, 2 \dots, \tag{2.1}
$$

$$\left.\begin{aligned} \Delta x(t) &= -p_1 x(t), \\ \Delta y(t) &= -p_2 y(t), \\ \Delta z(t) &= 0, \end{aligned}\right\} \quad t = (n+1)\tau,\ n = 1, 2 \dots,$$

where it is assumed that the above system is composed of two patches connected by impulsive invasion. Populations $x(t)$ and $y(t)$ inhabit in patch 1, and they have a competitive relation in patch 1. Population $z(t)$ inhabits in patch 2. In patch 1, the intrinsic rate of natural increase and the density dependence rate of population $x(t)$ are denoted by a_1, b_1 respectively, and $\frac{a_1}{b_1}$ denotes the carrying capacity of population $x(t)$. Population $x(t)$ and population $y(t)$ are in competitive relation, where β represents the competitive coefficient. Constants k_1 and k_2 are competitive effects of population $x(t)$ and population $y(t)$, respectively. Constant d_1 represents the death coefficient of population $y(t)$. In patch 2, the intrinsic rate of natural increase and density dependence rate of population $z(t)$ are denoted by a_2, b_2 respectively, and $\frac{a_2}{b_2}$ denotes the carrying capacity of population $z(t)$. Impulsive invasion occurs every τ period (τ is a positive constant). The system evolves from its initial state without being further affected by invasion until the next pulse appears. We define the notation $\Delta y((n+l)\tau) = y((n+l)\tau^+) - y((n+l)\tau)(0 < l < 1)$, where $y((n+l)\tau^+)$ represents the density of population $y(t)$ in the first patch immediately after the $(n+l)$th invasion pulse at time $t = (n+l)\tau$, while $y((n+l)\tau)$ represents the density of population $y(t)$ in the first patch before the $(n+l)$th invasion pulse at time $t = (n+l)\tau, n \in Z_+$. Constant $d(0 < d < 1)$ is the impulsive invasion coefficient, and $1 - \epsilon$ $(0 < \epsilon \leq 1)$ is the loss rate of population $z(t)$ in the invasion process. The constant p_1 $(0 \leq p_1 \leq 1)$ represents the reduction effect of population $x(t)$ accompanying with impulsive reduction of population $y(t)$ at $t = (n+1)\tau, n \in Z_+$. while the constant p_2 $(0 \leq p_2 \leq 1)$ represents the impulsive reduction effect of population $y(t)$ at $t = (n+1)\tau, n \in Z_+$.

3. Auxiliary lemmas

Before discussing the main results, we will give some definitions, notation and lemmas. Denote $f = (f_1, f_2, f_3)$ the map defined by the right hand of system (2.1). The solution of system (2.1), denoted by $Z(t) = (x(t), y(t), z(t))^T$, is a piecewise continuous function $Z : R_+ \to R^3_+$, where $R_+ = [0, \infty), R^3_+ = \{Z \in R^3 : Z > 0\}$. $Z(t)$ is continuous on $(n\tau, (n+l)\tau] \times R^3_+$ and $((n+l)\tau, (n+1)\tau] \times R^3_+$. According to Reference [1], the global existence and uniqueness of solutions of System (2.1) is guaranteed by the smoothness properties of f, which denotes the mapping defined by the right-side of system (2.1).

Let $V : R_+ \times R^3_+ \to R_+$. Then V is said to belong to class V_0, if

(i) V is continuous on $(n\tau, (n+l)\tau] \times R^3_+$ and on $((n+l)\tau, (n+1)\tau] \times R^3_+$ for each $Z \in R^3_+, n \in Z_+$. $\lim_{(t,Y)\to((n+l)\tau^+, Z)} V(t, Y) = V((n+l)\tau^+, Z)$ and $\lim_{(t,Y)\to((n+1)\tau^+, Z)} V(t, Y) = V((n+1)\tau^+, Z)$ exist.

(ii) V is locally Lipschitzian in Z.

Definition 3.1. Let $V \in V_0$, then for $(t, Z) \in (n\tau, (n+1)\tau] \times R^3_+$, the upper right derivative of $V(t, Z)$ with respect to the impulsive differential system (2.1) is defined as

$$D^+V(t, Z) = \lim_{h\to 0} \sup \frac{1}{h}[V(t+h, Z+hf(t, Z)) - V(t, Z))].$$

Lemma 3.2. *There exists a constant $M > 0$ such that $x(t) \le M$, $y(t) \le M$ and $z(t) \le M$ for each solution $(x(t), y(t), z(t))$ of system (2.1) with t large enough.*

Proof. Define $V(t) = x(t) + y(t) + z(t)$. When $t \ne n\tau$ and $t \ne (n+l)\tau$, we have

$$\begin{aligned}
&D^+V(t) + d_1V(t)\\
&= (a_1 + d_1)x(t) - b_1x^2(t) - k_1\beta x(t)y(t) - k_2\beta x(t)y(t) + (a_2 + d_1)z(t) - b_2z^2(t)\\
&\le (a_1 + d_1)x(t) - b_1x^2(t) + (a_2 + d_1)z(t) - b_2z^2(t)\\
&= -b_1[x(t) - \frac{a_1 + d_1}{2b_1}]^2 + \frac{(a_1 + d_1)^2}{4b_1} - b_2[z(t) - \frac{a_2 + d_1}{2b_2}]^2 + \frac{(a_2 + d_1)^2}{4b_2}\\
&\le \frac{(a_1 + d_1)^2}{4b_1} + \frac{(a_2 + d_1)^2}{4b_2} =: \xi.
\end{aligned}$$

When $t = (n + l)\tau$, we have

$$\begin{aligned}
V((n + l)\tau^+) &= x((n + l)\tau) + y((n + l)\tau) + z((n + l)\tau) - (1 - \epsilon)dz((n + l)\tau)\tau)\\
&= V((n + l)\tau) - (1 - \epsilon)dz((n + l)\tau)\tau)\\
&\le V((n + l)\tau).
\end{aligned}$$

When $t = (n + 1)\tau$, we have

$$\begin{aligned}
V((n + 1)\tau^+) &= x((n + 1)\tau) - p_1x((n + 1)\tau) + y((n + 1)\tau)\\
&\quad - p_2y((n + 1)\tau) + z((n + 1)\tau)\\
&= V(n\tau) - p_1x((n + 1)\tau) - p_2y((n + 1)\tau)\\
&\le V((n + 1)\tau).
\end{aligned}$$

From [14], for $t \in ((n\tau, (n + 1)\tau]$, we have

$$V(t) \le V(0^+)e^{-d_1t} + \frac{\xi}{d_1}(1 - e^{-d_1t}) \to \frac{\xi}{d_1}, \quad \text{as } t \to \infty.$$

So $V(t)$ is ultimately uniformly bounded. Hence, by the definition of $V(t)$, there exists a constant $M > 0$ such that $x(t) \le M, y(t) \le M$ and $z(t) \le M$ for t large enough. The proof is complete. □

If $x(t) = 0$, we have the following subsystem of system (2.1):

$$\begin{gathered}
\left.\begin{aligned}
\frac{dy(t)}{dt} &= -d_1y(t),\\
\frac{dz(t)}{dt} &= z(t)(a_2 - b_2z(t)),
\end{aligned}\right\} \quad t \ne n\tau,\\
\left.\begin{aligned}
\Delta y(t) &= \epsilon dz(t),\\
\Delta z(t) &= -dz(t),
\end{aligned}\right\} \quad t = n\tau,\\
\left.\begin{aligned}
\Delta y(t) &= -p_2y(t),\\
\Delta z(t) &= 0,
\end{aligned}\right\} \quad t = n\tau.
\end{gathered} \tag{3.1}$$

The analytic solution of system (3.1) on $(n\tau, (n + l)\tau]$ is obtained as follows:

$$\begin{gathered}
y(t) = y(n\tau^+)e^{-d_1(t-n\tau)}, \quad t \in (n\tau, (n + l)\tau],\\
z(t) = \frac{a_2e^{a_2(t-n\tau)}z(n\tau^+)}{a_2 + b_2[e^{a_2(t-n\tau)} - 1]z(n\tau^+)}, \quad t \in (n\tau, (n + l)\tau].
\end{gathered} \tag{3.2}$$

Considering the third and fourth equations of system (3.1), we have

$$\begin{aligned} y((n+l)\tau^+) &= y(n\tau^+)e^{-d_1 l\tau} + \epsilon d \frac{a_2 e^{a_2 l\tau} z(n\tau^+)}{a_2 + b_2(e^{a_2 l\tau}-1)z(n\tau^+)}, \\ z((n+l)\tau^+) &= (1-d)\frac{a_2 e^{a_2 l\tau} z(n\tau^+)}{a_2 + b_2(e^{a_2 l\tau}-1)z(n\tau^+)}. \end{aligned} \tag{3.3}$$

The analytic solution of system (3.1) on $((n+l)\tau, (n+1)\tau]$ are

$$\begin{aligned} y(t) &= y((n+l)\tau^+)e^{-d_1(t-(n+l)\tau)}, \quad t \in ((n+l)\tau, (n+1)\tau], \\ z(t) &= \frac{a_2 e^{a_2(t-(n+l)\tau)} z((n+l)\tau^+)}{a_2 + b_2[e^{a_2(t-(n+l)\tau)}-1]z((n+l)\tau^+)}, \quad t \in ((n+l)\tau, (n+1)\tau]. \end{aligned} \tag{3.4}$$

Considering the fifth and sixth equations of system (3.1), we have

$$\begin{aligned} y((n+1)\tau^+) &= (1-p_2)y((n+1)\tau^+), \\ z((n+1)\tau^+) &= z((n+1)\tau^+). \end{aligned} \tag{3.5}$$

Then, we obtain the stroboscopic map of system (3.1),

$$\begin{aligned} y((n+1)\tau^+) &= (1-p_2)e^{-d_1\tau}y(n\tau^+) + \frac{(1-p_2)\epsilon d a_2 e^{[a_2 l - d_1(1-l)]\tau} z(n\tau^+)}{a_2 + b_2(e^{a_2\tau}-1)z(n\tau^+)}, \\ z((n+1)\tau^+) &= \frac{(1-d)a_2 e^{a_2\tau} z(n\tau^+)}{a_2 + b_2(e^{a_2\tau}-1)z(n\tau^+)}. \end{aligned} \tag{3.6}$$

Making notation as $A = (1-p_2)e^{-d_1\tau}$, $B_1 = (1-p_2)\epsilon d a_2 e^{[a_2 l - d_1(1-l)]\tau}$, $C_1 = e^{a_2\tau} - 1$, $B_2 = (1-d)a_2 e^{a_2\tau}$, $C_2 = e^{a_2\tau} - 1$, we can rewrite (3.6) as

$$\begin{aligned} y((n+1)\tau^+) &= Ay(n\tau^+) + \frac{B_1 z(n\tau^+)}{a_2 + b_2 C_1 z(n\tau^+)}, \\ z((n+1)\tau^+) &= \frac{B_2 z(n\tau^+)}{a_2 + b_2 C_2 z(n\tau^+)}. \end{aligned} \tag{3.7}$$

There are two fixed points of (3.7) are obtained as $G_1(0,0)$ and $G_2(y^*, z^*)$, where

$$\begin{aligned} y^* &= \frac{B_1(B_2 - a_2)}{b_2[a_2 C_2 + (B_2 - a_2)C_1]}, \quad B_2 > a_2, \\ z^* &= \frac{B_2 - a_2}{b_2 C_2}, \quad B_2 > a_2. \end{aligned} \tag{3.8}$$

Lemma 3.3. ((i) *If $(1-d)e^{a_2\tau} < 1$, the fixed point $G_1(0,0)$ of (3.7) is globally asymptotically stable;*

(ii) *If $(1-d)e^{a_2\tau} > 1$, the fixed point $G_2(y^*, z^*)$ of (3.7) is globally asymptotically stable.*

Proof. For convenience, we make a notation as $(y^n, z^n) = (y(n\tau^+), z(n\tau^+))$. The linear form of (3.7) can be written as

$$\begin{pmatrix} y^{n+1} \\ z^{n+1} \end{pmatrix} = M \begin{pmatrix} y^n \\ z^n \end{pmatrix}. \tag{3.9}$$

Obviously, the local dynamics of $G_1(0,0)$ and $G_2(y^*, z^*)$ are determined by linear system (3.9). The stabilities of $G_1(0,0)$ and $G_2(y^*, z^*)$ are determined by the eigenvalue of M less than 1. If M satisfies the *Jury criteria* [7], we can know that the eigenvalue of M is less than 1. That is

$$1 - \operatorname{tr} M + \det M > 0. \tag{3.10}$$

(i) If $(1-d)e^{a_2\tau} < 1$, namely, $B_2 < a_2$, $G_1(0,0)$ is the unique fixed point of System (3.7), we have

$$M = \begin{pmatrix} A & \frac{B_1}{a_2} \\ 0 & \frac{B_2}{a_2} \end{pmatrix}. \tag{3.11}$$

Obviously, $A < 1$, calculations give

$$\begin{aligned} 1 - \operatorname{tr} M + \det M &= 1 - (A + \frac{B_2}{a_2}) + A\frac{B_2}{a_2} \\ &= (1-A)(1 - \frac{B_2}{a_2}) > 0. \end{aligned}$$

From Jury criteria, $G_1(0,0)$ is locally stable, then it is globally asymptotically stable.

(ii) If $(1-d)e^{a_2\tau} > 1$, namely, $B_2 > a_2$, $G_1(0,0)$ is unstable, and $G_2(y^*, z^*)$ exists, and

$$M = \begin{pmatrix} A & \frac{a_2 b_2 B_1 C_2}{a_2} \\ 0 & \frac{a_2}{B_2} \end{pmatrix}. \tag{3.12}$$

For

$$1 - \operatorname{tr} M + \det M = 1 - (A + \frac{a_2}{B_2}) + A \times \frac{a_2}{B_2}$$
$$= (1-A)(1 - \frac{a_2}{B_2}) > 0.$$

From Jury criteria, $G_2(y^*, z^*)$ is locally stable, then it is globally asymptotically stable. This completes the proof. □

The following lemma can be proved easily, so we omit its proof.

Lemma 3.4. (i) *If* $(1-d)e^{a_2\tau} < 1$, *the trivial periodic solution* $(0,0)$ *of System* (3.1) *is globally asymptotically stable;*

(ii) *If* $(1-d)e^{a_2\tau} > 1$, *the periodic solution* $(\tilde{y}(t), \tilde{z}(t))$ *of System* (3.1) *is globally asymptotically stable, where* $(\widetilde{y_1(t)}, \widetilde{y_2(t)})$ *can be expressed as*

$$\begin{gathered} \tilde{y}(t) = \begin{cases} y^* e^{-d_1(t-n\tau)}, & t \in (n\tau, (n+l)\tau], \\ y^{**} e^{-d_1(t-(n+l)\tau)}, & t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\ \tilde{z}(t) = \begin{cases} \frac{a_2 z^* e^{a_2(t-(n+l)\tau)}}{a_2 + b_2 z^*(e^{a_2(t-(n+l)\tau)} - 1)}, & t \in (n\tau, (n+l)\tau], \\ \frac{a_2 z^{**} e^{a_2(t-(n+l)\tau)}}{a_2 + b_2 z^{**}(e^{a_2(t-(n+l)\tau)} - 1)}, & t \in ((n+l)\tau, (n+1)\tau], \end{cases} \end{gathered} \tag{3.13}$$

where y^* *and* z^* *is determined in* (3.8), *and* y^{**} *and* z^{**} *are determined by*

$$\begin{gathered} y^{**} = y^* e^{-d_1 l\tau} + \epsilon d \frac{a_2 e^{a_2 l\tau} z^{**}}{a_2 + b_2(e^{a_2 l\tau} - 1)z^*}, \\ z^{**} = (1-d) \times \frac{a_2 e^{a_2 l\tau} z^*}{a_2 + b_2(e^{a_2 l\tau} - 1)z^*}. \end{gathered} \tag{3.14}$$

4. The dynamics

In this section, we easily find that there exist trivial periodic solution $(0,0,0)$ and population $x(t)$-extinction boundary periodic solution $(0,\tilde{y}(t),\widetilde{z(t)})$ of system (2.1). We will prove that the trivial periodic solution $(0,0,0)$ of system (2.1) is linear unstable, and prove the population $x(t)$-extinction boundary periodic solution $(0,\tilde{y}(t),\widetilde{z(t)})$ of system (2.1) is linearly stable/unstable. Then, we will prove that system (2.1) is permanent.

Theorem 4.1. (i) *If $(1-d)e^{a_2\tau}<1$, and $(1-p_1)e^{a_1\tau}<1$, then the trivial periodic solution $(0,0,0)$ of (2.1) is linear stable.*

(ii) *If $(1-d)e^{a_2\tau}>1$, or $(1-p_1)e^{a_1\tau}>1$, the trivial periodic solution $(0,0,0)$ of system (2.1) is linear unstable.*

(iii) *If*
$$\ln\frac{1}{1-p_1}>a_1\tau-\frac{k_1\beta(1-e^{-d_1\tau})}{d_1}y^*-\frac{k_1\beta(1-e^{-d_1(1-l)\tau})}{d_1}y^{**},\ \text{and}$$
$$\ln\frac{1}{1-d}>a_2\tau-2\ln[1+\frac{b_2(e^{a_2\tau}-1)}{a_2}z^*]-2\ln[1+\frac{b_2(e^{a_2\tau}-1)}{a_2}z^{**}],$$
hold, then the population $x(t)$-extinction boundary periodic solution $(0,\tilde{y}(t),\widetilde{z(t)})$ of system (2.1) is linearly stable. Where y^* is defined in (3.8).

(iv) *If*
$$ln\frac{1}{1-p_1}<a_1\tau-\frac{k_1\beta(1-e^{-d_1\tau})}{d_1}y^*-\frac{k_1\beta(1-e^{-d_1(1-l)\tau})}{d_1}y^{**},$$
or
$$\ln\frac{1}{1-d}<a_2\tau-2\ln[1+\frac{b_2(e^{a_2\tau}-1)}{a_2}z^*]-2\ln[1+\frac{b_2(e^{a_2\tau}-1)}{a_2}z^{**}],$$
hold, then the population $x(t)$-extinction boundary periodic solution $(0,\tilde{y}(t),\widetilde{z(t)})$ of system (2.1) is linearly unstable. Where y^* and z^* are defined in (3.8), and y^{**} and z^{**} are defined in (3.14).

Proof. Define $x_1(t)=x(t), y_1(t)=y(t)-\tilde{y}(t), z_1(t)=z(t)-\tilde{z}(t)$, we have the following linearly similar system of system (2.1)
$$\begin{pmatrix}\frac{dx_1(t)}{dt}\\ \frac{dy_1(t)}{dt}\\ \frac{dz_1(t)}{dt}\end{pmatrix}=\begin{pmatrix}a_1-k_1\beta\tilde{y}(t) & 0 & 0\\ k_2\beta\tilde{y}(t) & -d_1 & 0\\ 0 & 0 & a_2-2b_2\tilde{z}(t)\end{pmatrix}\begin{pmatrix}x_1(t)\\ y_1(t)\\ z_1(t)\end{pmatrix}.$$
It is easy to to obtain the fundamental solution matrix
$$\Phi(t)=\begin{pmatrix}\exp(\int_0^t(a_1-k_1\beta\tilde{y}(s))ds) & 0 & 0\\ * & \exp[-d_1t] & 0\\ 0 & 0 & \exp(\int_0^t(a_2-2b_2\widetilde{z(s)})ds)\end{pmatrix}.$$
There is no need to calculate the exact form of $*$ as it is not required in the analysis that follows. The linearization of the fourth, fifth and sixth equations of system (2.1) is
$$\begin{pmatrix}x_1((n+l)\tau^+)\\ y_1((n+l)\tau^+)\\ z_1((n+l)\tau^+)\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & \epsilon d\\ 0 & 0 & 1-d\end{pmatrix}\begin{pmatrix}x_1((n+l)\tau)\\ y_1((n+l)\tau)\\ z_1((n+l)\tau)\end{pmatrix}.$$

The linearization of the seventh, eighth and ninth equations of system (2.1) is

$$\begin{pmatrix} x_1((n+1)\tau^+) \\ y_1((n+1)\tau^+) \\ z_1((n+1)\tau^+) \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1((n+l)\tau) \\ y_1((n+l)\tau) \\ z_1((n+l)\tau) \end{pmatrix}.$$

The stability of the population $x(t)$-extinction periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon d \\ 0 & 0 & 1-d \end{pmatrix} \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(\tau),$$

which are

$$\begin{gathered} \lambda_1 = (1-p_1)e^{\int_0^\tau (a_1 - k_1\beta\tilde{y}(s))ds}, \\ \lambda_2 = (1-p_2)e^{-d_1\tau} < 1, \\ \lambda_3 = (1-d)e^{\int_0^\tau (a_2 - 2b_2\widetilde{z(s)})ds}. \end{gathered}$$

(i) For the trivial periodic solution $(0,0,0)$ of system (2.1). According the first condition of this theorem, we easily know that $\lambda_1 = (1-p_1)e^{\int_0^\tau (a_1 - k_1\beta\tilde{y}(s))ds} = (1-p_1)e^{a_1\tau} < 1$, and $\lambda_3 = (1-d)e^{\int_0^\tau (a_2 - 2b_2\widetilde{z(s)})ds} = (1-d)e^{-a_2\tau} < 1$. From the Floquet theory [19], the trivial periodic solution $(0,0,0)$ is linearly stable.

(ii) For the trivial periodic solution $(0,0,0)$ of system (2.1). According the second condition of this theorem, we easily know that $\lambda_1 = (1-p_1)e^{\int_0^\tau (a_1 - k_1\beta\tilde{y}(s))ds} = (1-p_1)e^{a_1\tau} > 1$, or $\lambda_3 = (1-d)e^{\int_0^\tau (a_2 - 2b_2\widetilde{z(s)})ds} = (1-d)e^{a_2\tau} > 1$. From the Floquet theory [19], the trivial periodic solution $(0,0,0)$ is linearly unstable.

(iii) For the boundary periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ of system (2.1). According the third conditions of this theorem, we easily know that $\lambda_1 = e^{\int_0^\tau (a_1 - k_1\beta\tilde{y}(s))ds} < 1$, and $(1-d)e^{\int_0^\tau (a_2 - 2b_2\widetilde{z(s)})ds} < 1$, then $\lambda_1 < 1$, and $\lambda_3 < 1$. From the Floquet theory [19], the population $x(t)$-extinction boundary periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ is linearly stable.

(iv) Its proof is similar to (iii). The proof is complete. $\square$

The next task is to investigate the permanence of system (2.1). Before starting this work, we should give the following definition.

Definition 4.2. System (2.1) is said to be permanent if there are constants $m, M > 0$ (independent of the initial value) and a finite time T_0 such that for all solutions $(x(t), y(t), z(t))$ with all initial values $x(0^+) > 0$, $y(0^+) > 0$, $z(0^+) > 0$, $m \le x(t) \le M, m \le y(t) \le M$, $m \le z(t) \le M$ holds for all $t \ge T_0$. Here T_0 may depend on the initial values $(x(0^+), y(0^+), z(0^+))$.

Theorem 4.3. *If*

$$\begin{gathered} \ln\frac{1}{1-p_1} < a_1\tau - \frac{k_1\beta(1-e^{-d_1\tau})}{d_1}y^* - \frac{k_1\beta(1-e^{-d_1(1-l)\tau})}{d_1}y^{**} \quad \textit{and} \\ \ln\frac{1}{1-d} < a_2\tau - 2\ln[1 + \frac{b_2(e^{a_2\tau}-1)}{a_2}z^*] - 2\ln[1 + \frac{b_2(e^{a_2\tau}-1)}{a_2}z^{**}], \end{gathered}$$

hold, system (2.1) *is permanent, where y^* and z^* are defined in* (3.8)*, and y^{**} and z^{**} are defined in* (3.14).

Proof. Let $(x(t), y(t), z(t))$ be a solution of (2.1) with $x(0) > 0$, $y(0) > 0$, $z(0) > 0$. By Lemma 3.2, we have proved there exists a constant $M > 0$ such that $x(t) \leq M, y_1(t) \leq M, y_2(t) \leq M$ for t large enough. We may assume $x(t) \leq M, y(t) \leq M, z(t) \leq M$, for $t > 0$.

Firstly, we need to find a $m_1 > 0$ such that $x(t) \geq m_1$ for t large enough. Otherwise, there must exist a $m_2 > 0$ small enough such that $x(t) < m_2$. By the condition $a_1\tau > \frac{k_1\beta(1-e^{-d_1\tau})}{d}y^*$, we can also select an $\varepsilon > 0$ small enough such that

$$a_1\tau - b_1m_2\tau - k_1\beta\tau\varepsilon - \frac{k_1\beta y^*(1-e^{-d_1l\tau})}{d} - \frac{k_1\beta y^{**}(1-e^{-d_1(1-l)\tau})}{d} > 0,$$

and use the notation

$$\begin{aligned} \sigma := a_1\tau - b_1m_2\tau - k_1\beta\tau\varepsilon - \frac{k_1\beta y^*(1-e^{-d_1l\tau})}{d} \\ - \frac{k_1\beta y^{**}(1-e^{-d_1(1-l)\tau})}{d} > 0. \end{aligned} \tag{4.1}$$

Considering the second equation of system (2.1), we obtain

$$\frac{dy(t)}{dt} \leq -d_1y(t).$$

Then, we have the following comparative differential equation

$$\begin{aligned} &\left.\begin{aligned} \frac{dy_2(t)}{dt} &= -d_1y_2(t), \\ \frac{dz_2(t)}{dt} &= z_2(t)(a_2 - b_2z_2(t)), \end{aligned}\right\} \quad t \neq (n+l)\tau,\ t \neq (n+1)\tau, \\ &\left.\begin{aligned} \Delta y_2(t) &= \epsilon dz_2(t), \\ \Delta z_2(t) &= -dz_2(t), \end{aligned}\right\} \quad t = (n+l)\tau, \\ &\left.\begin{aligned} \Delta y_2(t) &= -p_2y_2(t), \\ \Delta z_2(t) &= 0, \end{aligned}\right\} \quad t = (n+1)\tau. \end{aligned} \tag{4.2}$$

From Lemma 3.4, we know that $y_2(t) \leq \tilde{y}(t) + \varepsilon, z_2(t) \leq \tilde{z}(t) + \varepsilon$ for all t large enough, and $\varepsilon > 0$ is small enough. From the comparative theorem of impulsive differential equation [19], there exists a T, such that for $t > T$,

$$y(t) \leq y_2(t) \leq \tilde{y}(t) + \varepsilon, \tag{4.3}$$
$$z(t) \leq z_2(t) \leq \tilde{z}(t) + \varepsilon. \tag{4.4}$$
$$\tag{4.5}$$

Substituting (4.3) in the first equation of system (2.1), we obtain

$$x(t) \geq x(t)(a_1 - b_1x(t)) - k_1\beta(\tilde{y}(t) + \varepsilon)x(t). \tag{4.6}$$

which can be written as

$$x(t) \geq x(t)[(a_1 - k_1\beta(\tilde{y}(t) + \varepsilon) - b_1x(t)]. \tag{4.7}$$

Substituting $x(t) < m_2$ into (4.7), we have

$$x(t) \geq x(t)[(a_1 - b_1m_2) - k_1\beta(\tilde{y}(t) + \varepsilon)], \tag{4.8}$$

for $t > T$, for some $T > 0$. Let $N_1 \in N$ and $N_1\tau > T$, integrating on $(n\tau, (n+1)\tau]$, and $n > N_1$, we have

$$x((n+1)\tau) \geq (1-p_1)x(n\tau^+)e^{\int_{n\tau}^{(n+1)\tau}[(a_1-b_1m_2)-k_1\beta(\tilde{y}(t)+\varepsilon)]dt}. \tag{4.9}$$

So we obtain

$$x((N_1+k)\tau) \geq (1-p_1)^k x(N_1\tau^+)e^{k\sigma}. \tag{4.10}$$

Then, $x((N_1+k)\tau) \to +\infty$ as $k \to +\infty$, which is a contradiction to the boundedness of $x(t)$. Therefore, there exists a $t_1 > 0$ such that $x(t) \geq m_1$.

In the next step, we intend to prove the boundedness of $y(t)$ and $z(t)$. From Lemma 3.2, we know $x(t) < M$ for $t > 0$. Substituting $x(t) < M$ into the second equation of system (2.1), we have

$$\begin{gathered}
\left.\begin{aligned}
\frac{dy(t)}{dt} &> -(d_1 + k_1\beta M)y(t),\\
\frac{dz(t)}{dt} &= z(t)(a_2 - b_2 z(t)),
\end{aligned}\right\} \quad t \neq (n+l)\tau, t \neq (n+1)\tau,\\
\left.\begin{aligned}
\Delta y(t) &= \epsilon d z(t),\\
\Delta z(t) &= -dz(t),
\end{aligned}\right\} \quad t = (n+l)\tau,\\
\left.\begin{aligned}
\Delta y(t) &= -p_2 y(t),\\
\Delta z(t) &= 0,
\end{aligned}\right\} \quad t = (n+1)\tau.
\end{gathered} \tag{4.11}$$

The comparative differential equation of (4.11) can be written as

$$\begin{gathered}
\left.\begin{aligned}
\frac{dy_3(t)}{dt} &= -(d_1 + k_1\beta M)y_3(t),\\
\frac{dz_3(t)}{dt} &= z_3(t)(a_2 - b_2 z_3(t)),
\end{aligned}\right\} \quad t \neq n\tau,\\
\left.\begin{aligned}
\Delta y_3(t) &= \epsilon d z_3(t),\\
\Delta z_3(t) &= -dz_3(t),
\end{aligned}\right\} \quad t = n\tau,\\
\left.\begin{aligned}
\Delta y_3(t) &= -p_2 z_3(t),\\
\Delta z_3(t) &= 0,
\end{aligned}\right\} \quad t = n\tau.
\end{gathered} \tag{4.12}$$

As in Lemma 3.4., we can obtain a globally asymptotically stable periodic solution

$$\begin{gathered}
\widetilde{y_3(t)} = \begin{cases} y_3^* e^{-(d_1+k_1\beta M)(t-n\tau)}, & t \in (n\tau, (n+l)\tau],\\ y_3^{**} e^{-(d_1+k_1\beta M)(t-(n+l)\tau)}, & t \in ((n+l)\tau, (n+1)\tau], \end{cases}\\
\tilde{z_3}(t) = \begin{cases} \frac{a_2 z_3^* e^{a_2(t-(n+l)\tau)}}{a_2 + b_2 z_3^*(e^{a_2(t-(n+l)\tau)}-1)}, & t \in (n\tau, (n+l)\tau],\\ \frac{a_2 z_3^{**} e^{a_2(t-(n+l)\tau)}}{a_2 + b_2 z_3^{**}(e^{a_2(t-(n+l)\tau)}-1)}, & t \in ((n+l)\tau, (n+1)\tau], \end{cases}
\end{gathered} \tag{4.13}$$

where y_3^* and z_3^* are determined as

$$\begin{gathered}
y_3^* = \frac{B_3(B_4 - a_2)}{b_2[a_2 C_4 + (B_4 - a_2)C_3]}, \quad B_4 > a_2,\\
z_3^* = \frac{B_4 - a_2}{b_2 C_4}, \quad B_4 > a_2,
\end{gathered} \tag{4.14}$$

here $B_3 = (1-p_2)\epsilon d a_2 e^{[a_2 l-(d_1+k_1\beta M)(1-l)]\tau}$, $C_3 = e^{a_2\tau}-1$, $B_4 = (1-d)a_2 e^{a_2\tau}$, $C_4 = e^{a_2\tau}-1$, and y_3^{**} and z_3^{**} are determined as follows:

$$\begin{gathered}
y_3^{**} = y_3^* e^{-(d_1+k_1\beta M)l\tau} + \epsilon d \frac{a_2 e^{a_2 l\tau} z_3^{**}}{a_2 + b_2(e^{a_2 l\tau}-1)z^*}, \\
z_3^{**} = (1-d)\frac{a_2 e^{a_2 l\tau} z_3^*}{a_2 + b_2(e^{a_2 l\tau}-1)z_3^*}.
\end{gathered} \tag{4.15}$$

There exists a $t_1 > 0$, for $t > t_1$, and exists a $\varepsilon_1 > 0$ small enough such that

$$\begin{aligned}
y(t) &> y_3(t) \geq \widetilde{y_3(t)} - \varepsilon_1 \\
&\geq [y_3^* e^{-(d_1+k_1\beta M)l\tau} + y_3^{**} e^{-(d_1+k_1\beta M)(1-l)\tau}] - \varepsilon_1 =: m_3,
\end{aligned}$$

and

$$z(t) > z_3(t) \geq \tilde{z}_3(t) - \varepsilon_1 \geq [z_3^* + z_3^{**}] - \varepsilon_1 =: m_4.$$

That is to say, $y(t) > m_3$ and $z(t) > m_4$ for $t > t_1$. This completes the proof. $\square$

5. Discussion

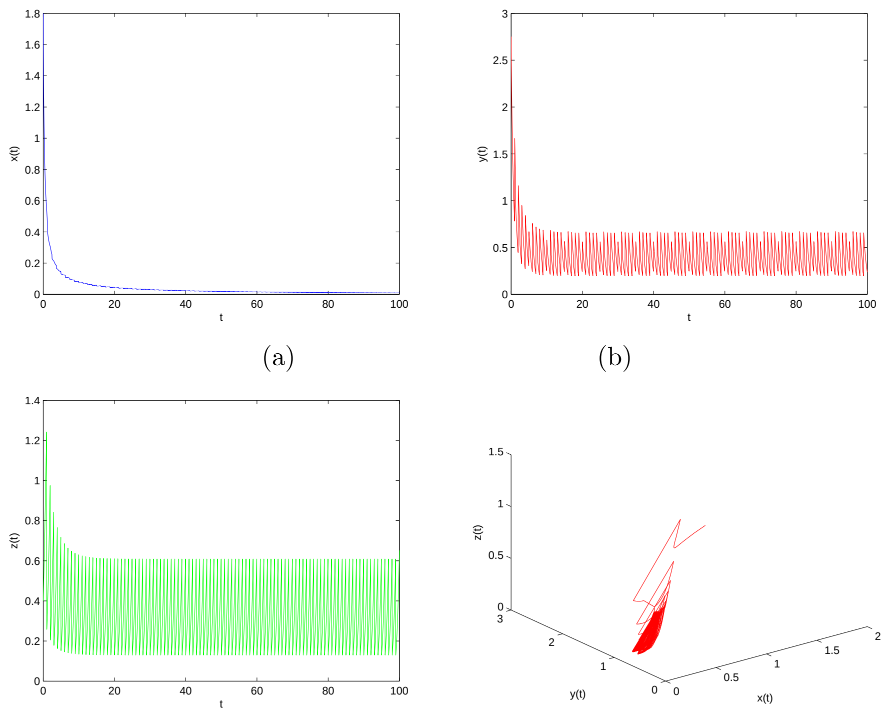
In this work, we considered a competitive predator-prey system with impulsive reduction of the invasive population. We have proved that all solutions of system (2.1) are uniformly ultimately bounded. The stability of the conditions of population $x(t)$-extinction boundary periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ of system (2.1) is obtained. The permanent conditions of system (2.1) are also obtained. From Theorem 4.1 and Theorem 4.3, we know that there exists a threshold of impulsive invasion parameter, which can make notation as d_0. If $d > d_0$, the population $x(t)$-extinction solution $(0, \tilde{y}(t), \tilde{z}(t))$ of System (2.1) is stable. If $d < d_0$, System (2.1) is permanent. That is, if $d > d_0$, the population $y(t)$ invades from patch 1 to patch 2 successfully, and causes the native species $x(t)$ to extinct. The invasive behaviors do harm to the native biodiversity. If the $d < d_0$, the population $y(t)$ invades from patch 1 to patch 2 successfully, then the alien species $y(t)$ coexist with the native species$x(t)$. We can reach the optimal invasion effect by controlling the threshold d_0.

If it is assumed that $x(0) = 2$, $y(0) = 2$, $z(0) = 2$, $a_1 = 0.2$, $b_1 = 1$, $k_1 = 0.5$, $k_2 = 0.5$, $\beta = 0.6$, $d_1 = 1$, $a_2 = 2$, $b_2 = 1$, $d = 0.8$, $\tau = 1$, $\epsilon = 0.9$, $l = 0.5$, then the population $x(t)$-extinction periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ of system (2.1) is stable, see Figure 1.

If it is also assumed that $x(0) = 2$, $y(0) = 2$, $z(0) = 2$, $a_1 = 1$, $b_1 = 0.5$, $k_1 = 0.5$, $k_2 = 0.5$, $\beta = 0.6$, $d_1 = 1$, $a_2 = 2$, $b_2 = 1$, $d = 0.5$, $\tau = 1$, $\epsilon = 0.7$, $l = 0.5$, then system (2.1) is permanent, see Figure 2.

If it is also assumed that $x(0) = 2$, $y(0) = 2$, $z(0) = 2$, $a_1 = 1$, $b_1 = 0.5$, $k_1 = 0.5$, $k_2 = 0.5$, $\beta = 0.6$, $d_1 = 1$, $a_2 = 2$, $b_2 = 1$, $d = 0.01$, $\tau = 1$, $\epsilon = 0.7$, $l = 0.5$, then the population $y(t)$-extinction solution of system (2.1) is stable, see Figure 3.

From the numerical analysis, we can further guess that there are two thresholds on parameter d, which can be written as d^* and d^{**} with assumption $d^* > d^{**}$. When $1 > d > d^*$, the population $x(t)$-extinction solution $(0, \tilde{y}(t), \tilde{z}(t))$ of system (2.1) is stable. That is, if $1 > d > d^*$, the population $y(t)$ invades from patch 1 to patch 2 successfully, and exclude the native species $x(t)$ to extinct, The invasional behaviors do harm to the native biodiversity. When $d^{**} < d < d^*$, system (2.1)



(a) (b)

(c) (d)

FIGURE 1. The population $x(t)$-extinction periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ of system (2.1) is stable with $x(0) = 2$, $y(0) = 2$, $z(0) = 2$, $a_1 = 0.2$, $b_1 = 1$, $k_1 = 0.5$, $k_2 = 0.5$, $\beta = 0.6$, $d_1 = 1$, $a_2 = 2$, $b_2 = 1$, $d = 0.8$, $\tau = 1$, $\epsilon = 0.9$, $l = 0.5$. (a) Time-series of $x(t)$; (b) time-series of $y(t)$; (c) time-series of $z(t)$; (d) the phase portrait of the stable population $x(t)$-extinction periodic solution $(0, \tilde{y}(t), \tilde{z}(t))$ of system (2.1).

is permanent. That is to say, if $d^{**} < d < d^*$, the population $y(t)$ invades from patch 1 to patch 2 successfully, and the alien species $y(t)$ coexist with the native species $x(t)$, and the invasional behaviors will do no harm to the native biodiversity. When $0 < d < d^{**}$, the population $y(t)$-extinction solution of system (2.1) is stable. That is to say, if $0 < d < d^{**}$, the population $y(t)$ invades from patch 1 to patch 2 unsuccessfully, and the alien species $y(t)$ will be excluded to extinction by the native species $x(t)$.

Combining the above numerical computation with Theorems 4.1 and 4.3, we can also guess that there are two thresholds about parameter τ, which can be written as τ^* and τ^{**} where $\tau^* > \tau^{**} > 0$. When $\tau > \tau^*$, the population $y(t)$-extinction solution of system (2.1) is stable. That is, if $\tau > \tau^*$, the population $y(t)$ invades from patch 1 to patch 2 unsuccessfully, and the alien species $y(t)$ is excluded to extinction by the native species $x(t)$. The invasional behaviors do no harm to the native biodiversity. When $\tau^* > \tau > \tau^{**}$, system (2.1) is permanent. That is to say, if $\tau^* > \tau > \tau^{**}$, the population $y(t)$ invades from patch 1 to patch 2 successfully, and

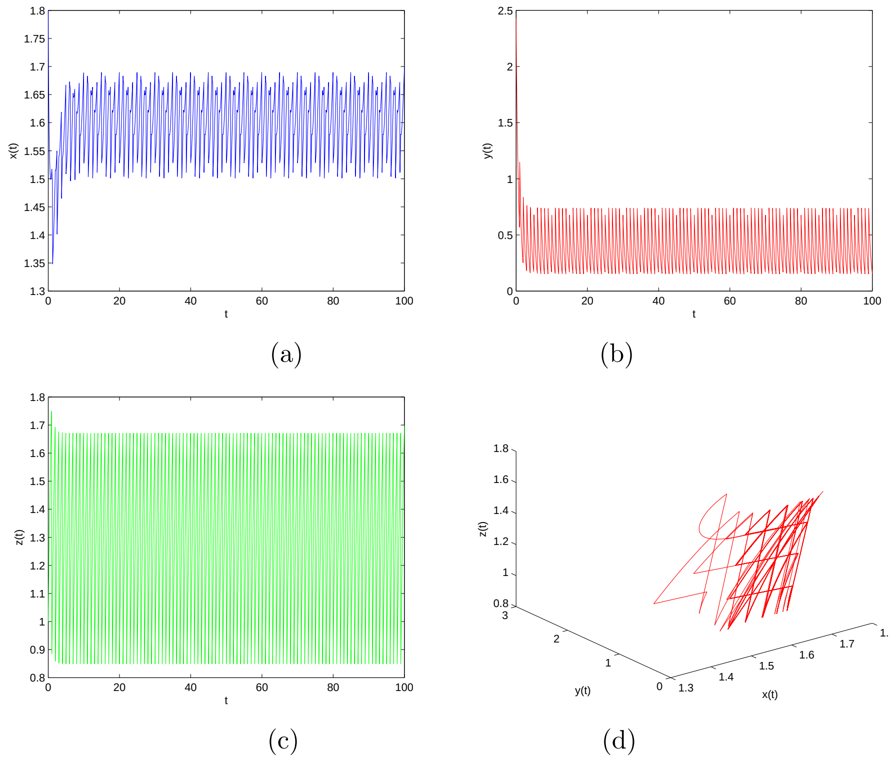


FIGURE 2. Permanence of system (2.1) with $x(0) = 2$, $y(0) = 2$, $z(0) = 2$, $a_1 = 1$, $b_1 = 0.5$, $k_1 = 0.5$, $k_2 = 0.5$, $\beta = 0.6$, $d_1 = 1$, $a_2 = 2$, $b_2 = 1$, $d = 0.5$, $\tau = 1$, $\epsilon = 0.7$, $l = 0.5$. (a) Time-series of $x(t)$; (b) time-series of $y(t)$; (c) time-series of $z(t)$; (d) the phase portrait of the permanence of system (2.1).

the alien species $y(t)$ coexist with the native species $x(t)$. The invasional behaviors also do no harm to the native biodiversity. When $0 < \tau < \tau^{**}$, the population $y(t)$-extinction solution of system (2.1) is stable. That is, if $0 < \tau < \tau^{**}$, the population $y(t)$ invades from patch 1 to patch 2 successfully, and exclude the native species $x(t)$ to extinction. The invasional behaviors will do harm to the native biodiversity. Our results show that the impulsive invasion amount and invasion period play important roles for the the dynamics of system (2.1). Our results also provide reliable tactic basis for the practical biodiversity management.

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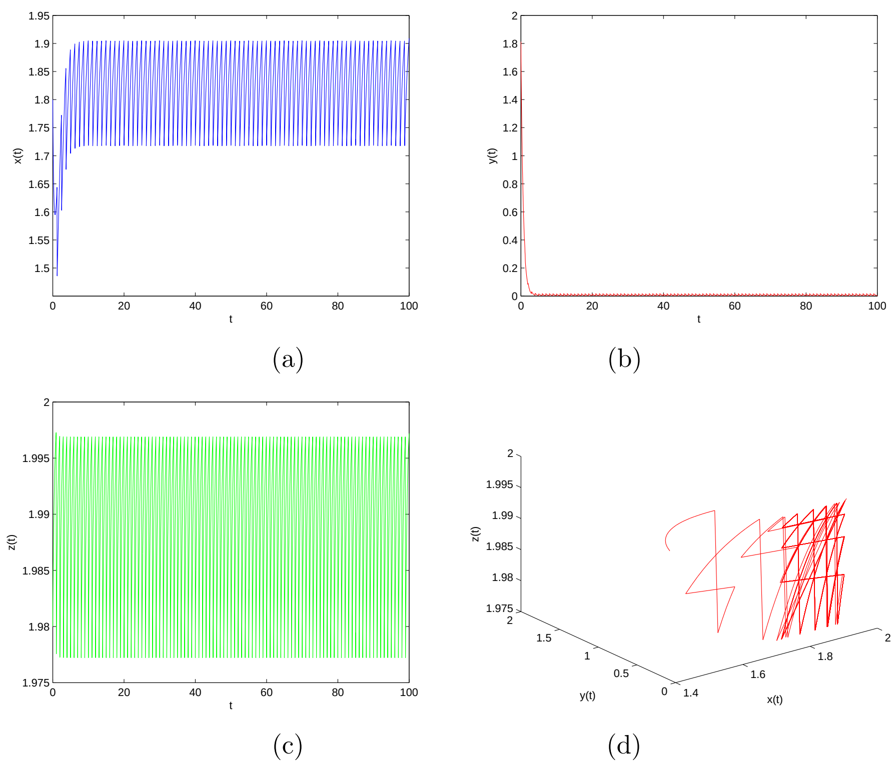

FIGURE 3. The population $y(t)$-extinction periodic solution of system (2.1) is stable with $x(0) = 2$, $y(0) = 2$, $z(0) = 2$, $a_1 = 1$, $b_1 = 0.5$, $k_1 = 0.5$, $k_2 = 0.5$, $\beta = 0.6$, $d_1 = 1$, $a_2 = 2$, $b_2 = 1$, $d = 0.01$, $\tau = 1$, $\epsilon = 0.7$, $l = 0.5$. (a) Time-series of $x(t)$; (b) time-series of $y(t)$; (c) time-series of $z(t)$; (d) the phase portrait of the population $y(t)$-extinction periodic solution of System (2.1).

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