THE ROLE OF TRANSLANGUAGING IN LATINO MATHEMATICS CLASSROOMS

By

Carlos Alberto Mejía Colindres, B.S., MBA

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Committee Members:

Samuel Obara, Chair

Gilbert Cuevas

Mary Esther Huerta

Alexander White

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DEDICATION

A mi familia: Todd, Daisy y Eva.

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ABSTRACT

Given the concerns about the mathematics achievement gap between Latinos and their majority counterparts, and since English learners (ELs) are an important percentage of Latino students in schools today, it is important to investigate the characteristics of teachers in Latino school districts that have significantly narrowed the mathematics achievement gap in the past. This dissertation is a collective case study of three middle school mathematics teachers participating in a larger project, *CAREER: Mathematics Instruction for English Language Learners* (MIELL). The three participants spoke English and Spanish with different levels of proficiency, and taught in a South Texas school district that has shown important achievements in the mathematics performance of its Latino students. This case study focused on the participants' (a) family background and preparation, (b) support of students' first language, (c) beliefs on the cognitive advantages of bilingualism in mathematics, and (d) use of two languages (or *translanguaging*) in mathematics. Data was collected by means of interviews with the teacher participants and nine classroom videos (three per participant) made during the 2013-2014 academic year. Interviews were analyzed by means of typological analysis and based on topics found in the fields of bilingual education, psycholinguistics, sociolinguistics, and mathematics education. The translanguaging events observed in the participants' videos were coded according to the strategies utilized by the teacher and the teaching goals linked to those strategies. The teacher participants were highly educated

middle school mathematics teachers who had diverse family backgrounds. The aforementioned characteristics influenced the participants' points of view on the cognitive advantages of bilingualism in mathematics and some of their practices in the classroom. The three participants in the study used a variety of linguistic strategies that qualified as translanguaging during their mathematics instruction, including cognate use, collaborative grouping, previewing, revoicing, stimulus of inner speech, and student paraphrasing. These strategies were used to make sense of the content and to elicit students' thinking, but not necessarily to support students' first language. The teaching strategies involving translanguaging identified and described in this case study are relevant to practice, since they can be used by any teacher in any linguistically diverse classroom.

I. INTRODUCTION

Autobiographical Statement

I was born in Tegucigalpa, Honduras, and I am an *elective bilingual* as I chose to develop bilingual abilities (Valdés & Figueroa, 1994). I received my first lessons in English from my mother, who learned English in the late 1940s at a private elementary school in Tegucigalpa that had an English immersion program. All subjects but one (Spanish) were taught in English by American teachers. When my grandparents were no longer able to pay for tuition in that school, my mother had to transfer to a public elementary school, where she had many problems adapting to the new environment and Spanish-only instruction. This transition happened in 5th Grade, and her major challenge was to understand mathematics. She realized that different algorithms for subtraction and division were used, and word problems were incomprehensible for her. Since teachers were not sympathetic to her ordeal, overcoming this obstacle was not easy for her.

Since my mother's education in English was truncated, her command of the English language diminished over the years, but it was good enough to scaffold my learning of that language from my childhood years until my teenage years, when I was able to use Spanish/English dictionaries to read American magazines on my own. Eventually, I used English-only dictionaries and my knowledge of Spanish to write competently in English. It was not until I came to the United States to study that I was able to improve my spoken English, but reaching native-like competency will be a difficult, if not impossible, task.

I obtained a B.S. in Mathematics and an MBA in Honduras. My work experience includes teaching mathematics to middle school students in the Bay Islands, whose native

language was not Spanish, but received instruction in Spanish; and teaching mathematics and statistics for 10 years in Universidad Tecnológica Centroamericana, a private university in Tegucigalpa. I have also been a mathematics teaching assistant at Texas State University. My educational background can be categorized as scholar-academic, with a middle class upbringing.

Background of the Problem

Latino language-minority students. Language-minority students are students from homes where the primary language spoken is not English. For the purposes of this study, language-minority students will be divided in two groups: English learners and bilingual English proficient. English learners (ELs) are students whose native language is not English and are acquiring English in school. These students have also been categorized as: English language learners (ELLs), limited English proficient (LEP), bilingual learners, emerging bilinguals, and English as second language, among others (García, Kleifgen, & Falchi, 2008). For this study, I will use *English learners* to refer to students that, according to state assessments, need to develop the proficiency in English that is necessary to understand academic content. Moreover, I will refer to students whose native language is not English but are not considered ELs as *bilingual English proficient*. The focus of this study is on teachers of Latino ELs and Latino bilingual English proficient students.

Latinos are the fastest-growing segment of the U.S. population. The consequences of this rapid change are being felt particularly in the education system.

Hemphill, Vanneman and the National Center for Education Statistics (NCES, 2011) point out the following:

- In 2010, the U.S. Census Bureau estimated the number of Latinos to be about 50.5 million, or about 16 % of the U.S. population, up 43% from the 2000 census.

- Data collected in 2009 by the U.S. Department of Education indicates that a substantial percentage of Latino students in grades 4 (37%) and 8 (21%) are English learners (ELs).

- In 2009, the National Assessment of Educational Progress (NAEP) mathematics scores for both Latino and White students in grades 4 and 8 nationwide were higher than in 1990, but the achievement gap between Latino and White students did not change significantly at either grade 4 or 8 (21 points and 26 points, respectively).

According to the National Clearinghouse for English Language Acquisition (NCELA, 2011) approximately 5,346,673 ELs were enrolled in public schools (pre-K through Grade 12) for the 2008-2009 school year, and that number represents approximately 10.8% of total public school student enrollment. The NCELA indicates that among the states, California enrolled the largest number of public school EL students, with 1,512,122, followed by Texas (713,218), Florida (257,776), New York (229,260), and Illinois (208,839). In elementary grades, the NCELA reports that 24% of ELs are foreign-born (first generation Americans), while 44% of secondary ELs are foreign-born. Further, the National Education Association (NEA, 2011) indicates that, according to census data, approximately 80% of all ELs in the U.S. are Latino.

Even though ELs are the students that require the most support from the educational system, the population of bilingual English proficient students is larger and has been almost completely ignored by mathematics education researchers. According to the U.S. Census Bureau (see Table 1), in 2011 there were 11.8 million school children that spoke a language other than English (LOTE), but only 2.6 of those children had difficulty speaking English. The latter number is significantly smaller than the number of ELs reported by the states due to the fact that the Census Bureau is based on self-reports and does not ask participants if they can read and write English. Only participants that indicate that they speak English "very well" are considered to be proficient in English.

Table 1

Children (ages 5-17) who speak a LOTE in the United States

	1979[a]	1989[a]	1999[a]	2011[b]	% of growth (1979-2011)
Speakers of a LOTE	3.8	5.2	8.8	11.8	209%
Spanish speakers	2.5	3.6	6.3	8.5	235%
Speakers of a LOTE who have difficulties speaking English	1.3	1.8	2.6	2.6	107%
Spanish speakers who have difficulty speaking English	0.9	1.3	2.3	1.9	111%
Spanish speakers who are proficient in English	1.6	2.3	4.0	6.6	303%

[a] In millions. Source: U.S. Census Bureau, Current Population Surveys.
[b] In millions. Source: U.S. Census Bureau, American Community Survey.

Of the 11.8 children that spoke a LOTE in 2011, 8.5 million (or 72%) spoke Spanish, and 6.6 million of those Spanish-speaking children were proficient in English. Moreover, from 1979 to 2011, the number of Spanish-speaking school children who had difficulty speaking English increased by 111%, and the number of Spanish-speaking school children who were proficient in English increased by 303% (see Table 1). Consequently, Latino bilingual English proficient students are a fast-growing population whose needs regarding mathematics instruction have yet to be determined by research. As García, Kleifgen, and Falchi (2008) point out:

> By focusing only on the elephant's tail, or those students who are not proficient in English, we risk losing sight of the incredible potential of the millions of bilingual and multilingual children in this country who can become national resources in building a peaceful coexistence within a global society and helping the United States remain economically viable in an increasingly multilingual world. (p. 11)

English as a second language (ESL) and bilingual education. The NEA (2011) indicates that approximately 2.5% of teachers who instruct EL students possess a degree in ESL or bilingual education. According to the National Center for Education Statistics (NCES, 2011), during the 2007-2008 academic year, there were 25,000 certified bilingual/ESL teachers working at the elementary level and 21,000 certified bilingual/ESL teachers working at the secondary level. Therefore, the number of teachers with bilingual/ESL certification is insufficient for the large population of mostly Latino ELs enrolled in public schools today, and there is also evidence that these professionals are not prepared to deal with content-specific material like mathematics (Guerrero, 2000). First language instruction is crucial for the academic success of language-minority

students; however, many bilingual/ESL teachers may not have the skills to teach across the curriculum in the students' native language, especially at the upper levels (Guerrero, 2002). Moreover, generic ESL strategies have been criticized because they do not have mathematics content, and teachers implementing them lack the rich knowledge about the role of language in facilitating those strategies or in making mathematics comprehensible and accessible to students (Schleppeegrell, 2010).

Some scholars believe that bilingual education teacher preparation programs are unlikely to help future educators develop academic proficiency in the non-English language of their students, since there are not enough opportunities for these prospective teachers to construct content area language registers before and during coursework in a bilingual education preparation program (Guerrero, 1997). In Texas, academic proficiency in Spanish of prospective bilingual teachers is measured through the Bilingual Target Language Proficiency Test (BTLPT). However, the BTLPT does not measure academic language proficiency in Spanish in specific content areas, such as mathematics.

Many school districts in Texas are implementing the bilingual model called "Gómez and Gómez", which is a "50:50 content model" that assigns different languages for different content areas: Instruction in mathematics is delivered in English, science and social studies in Spanish. The "Gómez and Gómez" model is used in Grades PK through 5th. For Grades 6th through 12th, ESL specialists support teachers that have ELs in their classrooms, and all classes are delivered in English, even in districts where most mathematics teachers speak the minority language of their students.

According to Gómez, Freeman, and Freeman (2005), the "Gómez and Gómez" model selects English as the language of instruction in mathematics for the following reasons:

> (a) Mathematics books have more limited (English-language) text than science or social studies texts; (b) Mathematics is generally a more hands-on subject, with numerous manipulatives available; (c) Mathematics is more universal, and its content cuts across languages; and (d) Generally speaking, Spanish-speaking parents can usually better assist their children in mathematics than in other subject areas due to the strong math education traditionally found in Latin American countries. (p. 154).

The previous statement is probably true for mathematical computational tasks, but cannot be extended to all mathematical activities. Trumbull and Solano-Flores (2011) explain that "mathematics has its own register, a specialized language that includes, among other features, a specific set of terms, notation conventions, and discursive forms…No wonder performance in mathematics is greatly influenced by language skills" (p. 218). Furthermore, Trumbull and Solano-Flores cite studies that demonstrate that language skills become critical when students solve word problems. Given this evidence, it is unlikely that Spanish-speaking parents would be able to assist their EL children if they cannot understand the language in which word problems are written.

Frederickson and Cline (2002) suggest that for ELs, it is "the vocabulary of maths that causes difficulty. The syntax in which mathematical ideas are expressed is often more complex than children are accustomed to in other areas of the curriculum" (p. 347).

In this regard, Cummins (2008) makes a distinction between basic interpersonal communicative skills (BICS) and cognitive/academic language proficiency (CALP). BICS occurs in social, context-embedded situations, such as face-to-face conversations, where ample cues exist to generate meaning. CALP only occurs in schooling and frequently involves context reduced academic situations. For this reason, some language minority students that can hold a conversation in English are unable to understand the academic language used in a mathematics lesson. Consequently, not using students' native language during mathematics instruction may prevent them from fully understanding the content taught.

The "Gómez and Gómez" bilingual model is one of several instructional programs that are classified as *two solitudes* models, since they keep students' native language and target language separate during instruction (Cummins, 2007). This kind of practice counters the *interdependence hypothesis*, which states that instruction in the first language that is effective in promoting proficiency in that language will cause a transfer of this proficiency in the second language. This transfer is called *cross-lingual transfer* by Cummins (2007). Furthermore, students receiving instruction through a two solitudes models are prevented from fully developing metalinguistic awareness, which is the ability to reflect upon the nature and functions of language (Baker, 2011). Metalinguistic awareness is linked to executive control (the ability to selectively attend to relevant knowledge), and it may give bilinguals an advantage in mathematics problem-solving and spatial reasoning (Bialystok, 1999, 2005; Greenberg, Bellana, & Bialystok, 2013).

Depending on how it is implemented, a two solitudes model can be a *subtractive* model (the student speaks a first language and a second one is added while the first is

subtracted) or an *additive* model (the second language is added and the first language is maintained), but none of these models reflect the complex bilingual competence needed in some societies in the 21st century (García, 2009). In addition, if bilingual students are allowed to "spontaneously focus on similarities and differences in their two or three languages, then they are likely to benefit from systematic encouragement by the teacher to focus on language and develop their language awareness" (Cummins, 2007, p. 229). Translanguaging, the ability to use more than one language (Baker, 2011), is the practice of bilingual *dynamic* models, where teachers use both languages to encourage deep understanding of all subjects, including mathematics (Baker, 2011; García, 2009; Esquinca, 2011).

Research on language and mathematics. Research on ELs tends to use monolingual classrooms as the norm, or focus on the obstacles the mathematics register in English presents to these learners, instead of seriously considering any advantages of bilingualism for mathematics learning (Moschkovich, 2010a). In order to conduct research on language and mathematics education that avoids the aforementioned pitfalls, Moschkovich (2010b) recommends the following:

- *Recognize the complexity of language*, by moving away from dichotomies such as everyday/academic, formal/informal, or in-school/out-of-school. In particular, the study of translanguaging in mathematics may benefit from the avoidance of these dichotomies, since multiple registers in English and Spanish can be present.

- *Draw on interdisciplinary approaches and methods*, by using scholarly literature from different relevant fields. Moreover, in order to focus on the mathematical

meanings that participants construct rather than the mistakes they make, researchers need to use frameworks that take into account the mathematical reasoning that participants are constructing "in, through, and with language" (p. 155).

- *Consider language issues in multiple settings,* thus avoiding deficit models that focus only on the challenges participants face and not the resources and competences they bring to the mathematics classroom. For example, bilingual competencies such as translanguaging can be discounted when research is focused on comparisons between monolingual and bilingual speakers.

Statement of the Problem

Given the increasing concerns about the mathematics achievement gap between Latinos and their majority counterparts, and since ELs and bilingual English proficient students are an important percentage of Latino students in schools today, it is important to investigate the characteristics of teachers in Latino districts that have significantly narrowed the mathematics achievement gap in the past, such as their

- family and educational background;
- support of students' first language;
- beliefs on the cognitive advantages of bilingualism in mathematics; and
- use of translanguaging practices in mathematics.

This collective case study focused on three middle school classrooms located in a South Texas school district that has shown important achievements in the mathematics performance of its Latino students. According to the 2012 Census Bureau American Community Survey, 34% of households in the participating school district live in

poverty, which is the second highest percentage of all metro areas in the United States. The district's statistics indicate that 99% of the students for the 2012-2013 school year were Latino, 96% were economically disadvantaged, 59% were at risk, 30% were ELs, and 3% were migrant. However, in reading and mathematics at all grade levels, this district has outperformed other Texas districts serving students with similar income levels. For example, between 2004 and 2007, the participating district experienced a reduction in the gap between Latino students and the state average for White students by 12 percentage points in middle school mathematics (Tupa & McFadden, 2009). A possible explanation for this phenomenon may lie in the mathematics teachers' preparation and understanding of the local culture: The only four year university in the area supplies 75% of the school district's teachers, most of whom graduate with bilingual certification; and what's more, many of those teachers grew up in the area, and therefore share their students' culture (Zehr, 2008).

The participating district is currently implementing an early-exit transitional model for bilingual education at the elementary level for students identified as ELs. PreK ELs receive 80% of instruction in Spanish and 20% is in English; in kindergarten, 75% of instruction is in Spanish and 25% is in English; in first grade, 70% of instruction is in Spanish and 30% is in English; in second grade, 60% of instruction is in Spanish and 40% is in English; and by fifth grade, 20% of instruction is in Spanish and 80% is in English. Middle school and high school ELs are assisted by ESL certified teachers, and content courses are taught through the sheltered instruction methodology that incorporate the Texas English Language Proficiency Standards (ELPS). ELPS are "second language acquisition curriculum standards that support the ability of ELLs to learn the academic

English they need for meaningful engagement in subject-area instruction" (Texas Education Agency, 2011, p. 1).

Newcomers to the participating district are identified as ELs according to the Home Language Survey that is completed for each student during registration. If the parent of the guardian of a newcomer indicates that a LOTE is spoken at home, the student is tested for English proficiency and the results submitted to the Language Proficiency Assessment Committee (LPAC), which determines the placement of the newcomer (see Appendix A for a complete decision chart on newcomers' placement). When EL students enter the ESL/bilingual program, they are assigned an ESL level (Beginning, Intermediate, Advanced, and Advanced High) through the use of Texas English Language Proficiency Assessment (TELPAS). TELPAS is a state-approved language proficiency test that is given to ELs at the end of every academic year, until they exit the ESL program.

Purpose of the Study

In this case study of three middle school mathematics teachers located in a school district in South Texas, I focused on their (a) family background and preparation, (b) support of students' first language, (c) beliefs on the cognitive advantages of bilingualism in mathematics, and (d) use of translanguaging. Furthermore, I used a *pragmatic* interpretative framework for this study, which is associated with the ontological belief that reality is what is useful, practical, and functional; the epistemological belief that reality is known through using many tools of research that generate both objective and subjective evidence; the axiological belief that my values have to be discussed, since

knowledge reflects both my views and the participants' views; and the methodological belief that either quantitative or qualitative approaches can be used in the collection and analysis of data, depending on the phenomenon being studied (Creswell, 2013). Consequently, I collected data by means of interviews and classroom observations, and analyzed the data by means of qualitative methods.

Significance of the Study

The results of this study will make an important contribution to the knowledge base of the following fields:

- *Mathematics education*, by describing how teachers use translanguaging to elicit their students' mathematical thinking.
- *Bilingual education*, by describing (a) the extent to which a dynamic bilingual model is used (in a natural way) by mathematics teachers in the participating district, and (b) how these teachers translanguage in mathematics.
- *Sociolinguistics and psycholinguistics*, by depicting (a) how teachers' family background and life in South Texas have shaped the use of two languages, Spanish and English, and the language register associated with each language, during mathematics instruction; and (b) teachers' awareness of their students' executive control skills as well as their own.
- *Curriculum and instruction*, by providing evidence on how translanguaging could be used in the design of mathematics curriculum materials for bilingual mathematics

classrooms, which could alter the instructional paradigm currently used to teach mathematics to ELs.

Research Questions

My study addressed the following research questions:

1. What are the participants' similarities and differences in their background and preparation?

2. What are the participants' similarities and differences in their support of students' first language?

3. What are the participants' similarities and differences in their perception of the cognitive advantages that bilingual students may have in mathematics?

4. What are the participants' similarities and differences in their use of translanguaging while explaining mathematical concepts?

Definitions of Terms

Bilingual classroom. A classroom that includes one or more student who has varying levels of language proficiency across two languages.

Bilingual English proficient. Students whose native language is not English but are not considered ELs.

Code-switching. A trait of bilingualism and a form of translanguaging in which there is a "juxtaposition within the same speech exchange of passages of speech belonging to two different grammatical systems of subsystems" (Gumperz, 1982, p.59).

Dual language education. In the United States, dual language programs tend to share the following characteristics: (a) a non-English language is used for at least 50% of instruction from PreK through 6th grade, (b) languages are separated during instruction, and (c) both English and non-English speakers are present in near balanced numbers (Baker, 2011).

English learner (EL). A student that, according to state approved assessments, "has sufficient difficulty in the use of English to prevent that individual from learning successfully in classrooms in which the language of instruction is English" (Kindler, 2002, p. 9).

L1 . First language or native language.

L2 . Second language.

Language-minority students. "Students from homes where the primary language spoken is not English" (Menken & Antunez, 2001, p.43).

Latino. Individual whose family was originally from a Spanish-speaking country in North America, Central America, South America, or the Caribbean.

Psycholinguistics. The "study of psychological states and mental activity associated with the use of language" (Psycholinguistics, 2013).

Sociolinguistics. "An orientation to the study of language that stresses the inter-relationship between language and social life, rather than focusing narrowly on language structure" (Sociolinguistics, 2004).

Register. A "set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday, 1977, p. 195).

Translanguaging. "[It] is the process of making meaning, shaping experiences, understandings and knowledge through the use of two languages" (Baker, 2011, p. 288).

Delimitations

The focus of this study was on three middle school mathematics teachers working in public schools in a participant South Texas district, and their Latino students. All participant teachers are bilingual, with different degrees of proficiency in English and Spanish. Moreover, around 30% of middle school students in this district are ELs; the rest are mostly bilingual English proficient.

Summary

In the Unites States, the number of teachers with bilingual/ESL certification is insufficient for the large population of mostly Latino ELs enrolled in public schools today, and these professionals are not prepared to deal with content-specific material like

mathematics (Guerrero, 1997) . When bilingual programs are available, they exclude mathematics, since it is believed that mathematics is a "universal language", when, in fact, it has its own register that includes particular discursive forms not accessible to students when proficiency in both their native language and the second language is developing (Trumbull & Solano-Flores, 2011). Moreover, by keeping languages separated, these bilingual programs get in the way of students developing dynamic bilingual competencies that have been shown to contribute to mathematics problem solving (Cummins, 2007).

Since there is evidence of at least one school district in Texas that succeeded in substantially narrowing the gap between Latino students and the state average for White students during the past decade, it is important to describe the characteristics of teachers working in that district, such as their (a) family background and preparation, (b) support of students' first language, (c) beliefs on the cognitive advantages of bilingualism in mathematics, and (c) translanguaging use. My study attempted to accomplish this goal by using interviews and classroom observations to collect data.

Additionally, the significance of my study lies in the potential contribution of its results to the fields of mathematics education, bilingual education, sociolinguistics, psycholinguistics, and curriculum and instruction. In conclusion, my study will satisfy the need for research in language and mathematics that recognizes the complexity of language, draws on interdisciplinary approaches and methods, and focuses on the bilingual competencies that the participant teachers bring to mathematics classrooms.

II. LITERATURE REVIEW

This chapter contains theoretical framework that guided the study and also the research literature that discusses theoretical and empirical work on topics related to teacher effects and language-minority students. These topics (or typologies) were found in the fields of bilingual education, psycholinguistics, sociolinguistics, and mathematics education, and they contain the knowledge necessary to identify the particular teacher traits that could be associated with the achievement gains in mathematics of language-minority students. The topics discussed in this chapter are: (a) teacher characteristics (e.g., degrees, coursework, credentials, and experience); (b) support of students' first language; (c) bilingualism and mathematics; and (d) translanguaging. The chapter concludes with a summary of the literature and the gaps that were found.

Theoretical Framework

This study on the role of translanguaging in bilingual mathematics classrooms was guided by a socio-cultural framework based on Vygotsky's (1978) theories about (a) the role of teachers as scaffolding tutors, and (b) language as tool of reflection and thinking. This section provides a description of these important theories, which are a critical foundation of all the theoretical frameworks of bilingualism discussed throughout Chapter II.

The zone of proximal development. Vygotsky describes the role of the teacher as that of a scaffolding tutor, who demonstrates and models successful performance while keeping the task at a proper level of difficulty. Teachers and more capable peers assist learners reach their maximum potential by moving them in what Vygotsky calls the *zone*

of proximal development (ZPD). The ZPD is the distance between what students can do independently and what they can do with the support from others. The ZPD is where new understandings are possible by means of collaborative interaction and inquiry. For example, the scaffolding tutor provides meaningful context to academic tasks that may be context-reduced and cognitively complex. Scaffolding can only happen within the ZPD, and consequently it is a temporary device to enable understanding of content. When learning is successful, the scaffold is removed and the student can then attend to the task independently.

Language as a tool for reflection and thinking. The central concept of socio-cultural theory is that the "human mind is *mediated*" (Lantoloff, 2000, p. 1). Vygotsky (1978) argued that humans are not limited to stimulus-response processes, but are able to make indirect connections between incoming stimulation and their responses through mediating links, which can be physical tools (e.g., machines) and psychological or symbolic tools or signs (e.g., language, writing, numbers) which emerge over the course of history and are different from one society to another. According to Vygotsky, language is the tool of all mediating tools. Humans use symbolic artifacts to create an indirect or mediated relationships between ourselves and the world and, as a consequence, we have the power to deliberately regulate our minds from both the outside world and the inside mental world (Lei, 2008).

Teacher Characteristics

When studying teacher effectiveness, the most common factors examined by the research literature are related to teacher background traits like certification status, degrees, coursework, credentials, experience, test scores, grade point average (GPA),

participation in professional development activities, and ratings of the colleges they attended. However, there is no consensus on the relevance of any of the aforementioned characteristics to teacher effectiveness (Nye, Konstantopoulos, & Hedges, 2004; Palardy & Rumberger, 2008; Phillips, 2010; Wayne & Youngs, 2003). Specifically, "there is little scientific evidence that these characteristics have a measurable and consistent direct impact on student achievement" (Palardy & Rumberger, 2008, p. 112).

In recent years, the main motivations to conduct research on teacher effectiveness are the standards movement, the accountability movement and the No Child Left Behind Act (NCLB) of 2001, which demands that a highly qualified teacher be assigned to every classroom in order to achieve educational equity. NCLB defines highly qualified elementary teachers in terms of the background characteristics they bring to the classroom, including a full state certification, a minimum of a bachelor's degree, and demonstrated subject area and teaching skill competence in reading, writing, mathematics and other areas of the basic elementary school curriculum. In the case of secondary teachers, demonstrated subject-area competence is required. The NCLB's intention is to prevent disadvantaged students from being taught at higher rates by unqualified, out of field, or inexperienced teachers (U.S. Congress, 2001).

Despite NCLB's focus on teacher characteristics such as experience and level of academic degree obtained, their effects on student achievement are inconclusive across studies, at least at the elementary level, which is the focus of most studies on teacher effects (see Kukla-Acevedo, 2009; Palardy & Rumberger, 2008; Phillips, 2010). According to Palardy and Rumberger (2008), the problems with most studies focusing on

teacher effectiveness are the breadth of their theoretical frameworks and the appropriateness of their statistical models:

> Many studies employ a theoretical framework narrowly focused on teacher background, neglecting to examine teaching practices and attitudes…. [This may] result in the omitted variable problem, which may bias the teacher effects that are estimated….[Regarding statistical models] most recent studies on teacher effects have used multilevel models to partition the variance in student learning into student and classroom components or into student and school components, [but] most do not correctly partition it into all three important components" (p. 115).

Studies on teacher effects in middle school and high school are more consistent and supportive of the importance of teacher characteristics on student achievement than those for elementary school. For example, a review of 21 studies that controlled for students' prior achievement and socioeconomic status (SES) determined that "high school students clearly learn more from teachers with certification in mathematics, degrees related to mathematics, and coursework related to mathematics" (Wayne & Youngs, 2003, p. 107). A more recent study also showed that the number of mathematics teachers' content courses was indeed a good predictor of student achievement in Grade 8th (Telese, 2012).

Less is known about the relationship between teacher effects and the mathematics achievement of language-minority students. Ethnographic studies (e.g., Cahnmann, 2005) have described effective teachers for language-minority students as more than just

translators, but as revisers of curriculum materials that connect what is known to what is unknown by making these materials accessible, engaging, and challenging.

The skill of using translation as a critical revision of curriculum materials may be linked to *translanguaging*, which "is the process of making meaning, shaping experiences, understandings and knowledge through the use of two languages" (Baker, 2011, p. 288). Translanguaging goes beyond *code-switching* (the process of going back and forth between one language to the other), although it includes it (García, 2009). In the case of bilingual mathematics classrooms, it might be that teachers' skill in translanguaging is playing an important mediating role between their preparation and their bilingual students' achievement in mathematics.

Support of Students' First Language

The activation of students' prior knowledge is necessary when new knowledge is acquired. Effective teachers of language-minority students understand that the best way to activate their previous knowledge is by supporting their students' first language. What follows is a discussion of theories and research about the importance of supporting bilingual learners' native language.

Bilingual acquisition. Evidence supports the assumption that an innate faculty underlies language acquisition. Children all over the world go through similar stages when they acquire the grammatical systems of their specific languages; this would be difficult to explain if humans did not have an innate faculty that figures in language acquisition (Myers-Scotton, 2006). Both children who end up as monolinguals and those who are early bilinguals go through similar stages of acquisition (Genese, 2003, Myers-

Scotton, 2006). The natural outcome for children is to speak whatever parents and other caregivers speak to them, whether it is one language or two or multiple languages (Baker, 2011; García, 2009; Myers-Scotton, 2006).

Language acquisition is largely complete by the age of three or four, depending on the complexity of grammatical elements in the language. This acquisition, which includes pronunciation, grammar and a limited vocabulary, occurs without explicit instruction (Myers-Scotton, 2006). From the socio-cultural standpoint, the process of acquiring two languages may be different from acquiring only one language, since children may resist acquiring some additional languages. How much the family and community support acquiring certain languages matters for very young children (Barron-Hauwaert, 2011; Myers-Scotton, 2006, Tuominen, 1999).

From age six to 12, children continue the oral development of complex grammar rules, vocabulary expansion, and discourse. This development is accomplished through using the language. In contrast, formal instruction introduces reading and writing, and each grade level adds to the cognitive complexity of language development needed to study and learn subject matter. By adolescence, language proficiency developed in and out of school has reached a very complex level; however, there are aspects of language acquisition that continue across an individuals' lifetime (Ovando & Combs, 2012). Children that develop strong academic proficiency in both languages are expected to develop cognitive advantages over monolinguals (Bialystok, 1999, 2005; Greenberg, Bellana, & Bialystok, 2013).

Social language, academic language, and bilingual learners. Cummins (2000, 2008) expressed the distinction between social language skills and academic language skills. He labeled the former *basic interpersonal communication skills* (BICS) and the latter *cognitive academic language proficiency* (CALP). BICS occurs in context-embedded, conversational situations, where there is a wide range of non-verbal cues and face-to-face interaction, and involves the development of literacy necessary to conduct activities such as shopping and using public transportation. BICS not only depends upon external contextual support, but also contextual internal support, which involves the interests, motivations, and life experiences of the individuals involved in a conversation.

Cummins (2000, 2008) also explained that CALP occurs in context-reduced academic situations, and it is what is needed in order to engage in the IRE/IRF (Initiation-Response-Evaluation/Feedback) cycle that commonly occurs in classroom discourse. CALP is associated with higher order thinking, but Cummins indicated that CALP is not superior to BICS and that they develop together through social interaction.

The differentiation between BICS and CALP explains why many bilingual learners encounter difficulties learning mathematics and other subjects:

> In the United States, various programs aim to give language minority students sufficient English language skills to enable them to converse with peers and teachers and to operate in the curriculum. Having achieved surface fluency, they may be transferred to regular classes….Cummins' distinction between BICS and CALP explains why such children tend to fail when mainstreamed" (Baker, 2011, p. 172).

Cummins (2000, 2008) maintained that social interaction and the contextualization of academic language are essential when teaching ELs. A way of contextualizing academic language is by allowing the interaction of students' first language (L1) and second language (L2) in the classroom, since cognitively demanding communication develops through the interdependence of the languages.

The interdependence hypothesis and the Common Underlying Proficiency. Based on the concepts of BICS and CALP, Cummins (2000) found that conversational skills in L2 could be acquired in two years, while the decontextualized academic language skills in L2 could take five to seven (or more) years to develop. Furthermore, Cummins suggested that instruction in L1 that is effective in promoting proficiency in L1 will cause a transfer of this proficiency in L2, provided there is an adequate exposure to L2 and motivation to learn L2. This view is called *the interdependence hypothesis* (Cummins, 1981), and it has been supported by research. For example, Thomas and Collier (2002) found that immigrant students' L1 proficiency at the time of their arrival in the United States is the strongest predictor of academic English development.

Furthermore, Cummins' (1981) *Common Underlying Proficiency* (CUP) model is derived from his interdependence hypothesis, and affirms that the two languages of a bilingual individual operate through the same central processing system. Cummins uses the image of two icebergs that are separated above the surface, but are fused underneath the surface, to represent the two different languages of a bilingual individual, controlled by the same central cognitive engine. What is learned in one language does not have to be learned in the other; in contrast, when one or both languages are not fully developed, cognitive functioning and academic performance may be affected.

Research on language universals (e.g., Hinkel, 2005) supports Cummins' CUP, since it has determined that all languages have common properties at deep structural levels. However, CUP does not explain the findings of research on the brain and bilingualism that has supported the Sapir-Whorf hypothesis instead (Baker, 2011). This hypothesis, also called *the linguistic relativity principle*, states that the structure of a language affects the ways in which its speakers conceptualize their world (Pavlenko, 2005). For example, research has shown that languages like Chinese, Japanese, and Korean allow for more flexibility of mental number manipulation than other languages, like English or Spanish (e.g., Miura & Okamoto, 1989). This is due to the fact that the notion of place value is an inbuilt component of linguistic encoding of numbers in Asian languages, but needs to be understood and internalized by English- and Spanish-speaking children. Moreover, research on narrative discourse has shown that bicultural bilingual children take different stances when asked to tell the same personal experience in their two languages (Koven, 1998), suggesting that languages may indeed create different worlds for their speakers.

The threshold hypothesis. Cummins (1976) described the relationship between cognition and degree of bilingualism through the *threshold hypothesis,* which posits that high bilingual proficiency in two languages is associated with more positive cognitive effects. Psycholinguistics research on executive control has supported this hypothesis (Bialystok, 1999, 2005; Greenberg, Bellana, & Bialystok, 2013). García (2009) explains the relevance of the threshold hypothesis as follows:

> [The threshold hypothesis is useful because it explains why] subtractive bilingualism, with children acquiring a second language at the expense of their

> first language, would most likely impair the possibility of developing the second language appropriately, since learners could not use their first language in making sense of new linguistic and cognitive situations (p. 70).

The problem with the threshold hypothesis is that it does not define the level of bilingual proficiency that would lead to negative cognitive effects and the level of proficiency that would lead to cognitive advantages (Baker, 2011). It is also unclear if the threshold level is for each of the two languages or for both languages jointly (García, 2009).

Bilingual education. Bilingual education is different from traditional language education programs that teach a second or foreign language, since the former uses language as a medium of instruction, and the latter teaches language as a subject. But what truly separates those two kinds of programs is the broader scope of bilingual education, which is "the use of two languages to *educate generally, meaningfully, equitably, and for tolerance and appreciation of diversity"* (García, 2009, p. 6). There are different ways of interpreting what bilingualism is, and these interpretations lead to different models of bilingualism that have been used through the years with varying degrees of success.

During the 20th century, two models were used; the *subtractive* model and the *additive* model. In the former model, the student speaks a first language and a second language is added while the first one is subtracted, and the result is a monolingual speaker of the second language. In the latter model, a second language is added to the first language, but the first language is preserved. Students in a subtractive model may

lack full communicative competence in both the home language and English (Ovando & Combs, 2012). Even though the additive model has benefits, it also requires that bilingual individuals behave like two monolingual individuals, and never mix the languages. This model has been used in countries where the native and the second language have equal prestige, such as in Canada.

In the 21st century, other bilingual models have emerged that better reflect the complexity of bilingual competence, and that respond to the needs of many societies. One of those models is *dynamic bilingualism*, which incorporates the concept of plurilingualism. The language goal is to foster the use several languages to varying degrees of competency and for different proposes (García, 2009). Dynamic bilingualism is a holistic concept of which translanguaging is a process (Lewis, Jones, & Baker, 2013).

Successful Schools for Multilingual Children. Lucas, Henze, and Donato (1990) indicated that language-minority students have often been blamed for their underachievement in U.S. schools, since they are considered to be difficult or culturally and linguistically deprived. Valencia and Solórzano (2004) explained that this is the consequence of the ideological premise that marginalized children, particularly language minority students, suffer from cultural, linguistic and other personal deficits that explain their persistent abysmal educational success rate. According to Valencia and Solórzano, *the deficit ideology* has had different rationales through time, such as genetics, poverty, culture and language, and home environment. Valencia and Solórzano indicated that the deficit ideology is powerful and has pervasively controlled much of U.S. educational thinking and practice.

Opposed to deficit thinking, Lucas et al. (1990) argued that schools are responsible for the quality of education students receive and, by conducting a study of six high schools in California and Arizona that had large populations of Latino students and had been recognized by local, state and/or federal agencies for excellence, they sought to understand the factors that contributed to these schools' success.

Lucas et al. detected eight features they believed to be the most important in promoting the success of language minority students. One of them was *to place an especially high value on the students' languages and cultures*. In these schools, the ability to speak a language in addition to English was treated as an advantage rather than a liability, and a number of White and Latino teachers who were not native Spanish speakers had learned the language in order to better understand and communicate with their students. In order to understand their students' past experiences, a group of teachers from one school observed mathematics teaching in a Mexican school. Lucas et al. explained that this experience helped these teachers understand how Mexican students were taught and that made it easier to teach them long division, for example.

Lucas et al. mentioned that another way in which these schools demonstrated respect for the students' cultures and language was through their staffing, since it is important for parents and students to see Latinos in leadership positions. Hiring minority staff in leadership positions to act as role models is also an example of *making high expectations for language-minority students' achievement concrete.* Lucas et al. pointed out that this goal is also achieved by challenging students with difficult questions and problems, while making complex ideas and materials accessible to language-minority

students through visuals, board work, group work and clear and explicit class expectations.

In this respect, Solórzano and Solórzano (2004) indicated that teacher actions and attitudes directly affect the achievement of all students, and since a large number of teachers were raised in a society that maintains varieties of the deficit model, they teach ELs from stereotypical perspectives that falsely characterize students. Solórzano and Solórzano affirmed that low expectations and lack of teacher professional development focused on teaching ELs allows for societal stereotypes to turn into a self-fulfilling prophecy.

Code-switching. Code-switching is defined as "juxtaposition within the same speech exchange of passages of speech belonging to two different grammatical systems of subsystems" (Gumperz, 1982, p.59). According to García (2009), code-switching can be *intrasentential*, where the switch occurs within the boundaries of a clause or a sentence; and *intersentential*, where the switch occurs at a clause or sentence boundaries.

There are several distinctions among types of code switching. Some sociolinguistics reserve the term *code-mixing* to refer to switching for immediate access to an unknown term, usually a single noun or noun phrase, and use *code-switching* to refer to changing completely from one language to the other within the course of the same conversation (Baker, 2011; García, 2009; Ovando & Combs, 2012). An example of code-mixing is "*Median* es lo mismo que el número del medio", while an example of intersentential code-switching is "This problem is hard. *No sé que hacer*".

Another useful distinction is that of the term *loan* or *language borrowing*, which indicates "the taking of individual lexical items from other languages" (García, 2009). One example is when native English speakers interject the words "salsa" or "delicatessen" in their speech. Nevertheless, it is bilinguals' speech that contains the most loans. García (2009) extends the classification of loans to *phonologically assimilated,*as in the use of *bíldin* referring to certain buildings; and *morphologically assimilated,* as in the use of *lonchar* instead of *almorzar*. García indicates that morphologically assimilated loans are also known as *loan blends* by Haugen (1953). Finally, García explains that when bilingual speakers borrow only the meaning of a word, the loan is called *loan shift* by Haugen. An example of loan shift is the use of in U.S. Spanish of the word *aplicación* with the English meaning of "form" or a document in which an application is made, when *aplicación* actually means "diligence" in Spanish.

The social variables that encourage or inhibit code-switching in bilingual classrooms have been the focus of several studies. For example, Zentella (1978) conducted a study of code-switching among Puerto Rican children attending a New York City public school. Zentella analyzed the correlation between interaction rules, as perceived by the participants, and code-switching. The code-switching strategies of a seven-year-old boy, an eight-year-old girl, and a 10-year-old girl in the bilingual third grade class were investigated in two types of interactions: An individual interview with the researcher, a female New York Puerto Rican, and a domino game in which all three children participated. The interviews were carried out in Spanish and English, and the interviewer began in Spanish and then switched to English without warning.

Zentella (1978) observed that child-adult dyads in question-answer interactions inhibit code-switching, and informal peer interactions stimulate it. This finding implied that the full range of a child's linguistic ability and code-switching patterns may not be noticeable nor used in formal classroom lessons. Zentella explained that if the child and teacher are members of the same speech community, a greater degree and variety of code-switching should be expected, making miscommunication events less frequent. Zentella pointed out that this ideal situation is jeopardized by two critical factor, including that (a) teachers may have learned to be ashamed of their variety of Spanish, and (b) teachers may be under policy or governance disallowing language codes to be mixed.

Another factor that can further affect code-switching patterns in the classroom point to any differences in values between teachers and students due to language and cultural shift; for instance, a teacher born and educated in Puerto Rico teaching U.S. born Puerto Ricans. Yet another factor is the organization of the bilingual program: The bilingual school may be parent-run or controlled by another group; transitional or maintenance in philosophy.

Bilingual learners' code-switching in mathematics classrooms. According to Moschkovich (2007), the differences between code-switching and code-mixing can be subtle, but they might be relevant in mathematics education research when analysis of conversations about mathematics among children of different ages is conducted. For example, Zentella (1997) reported that a study by McClure with Mexican-American children found that older children (ages 9-13) code-switched more than younger children, and younger children (ages 2-9) code-mixed more. According to Moschkovich,

> [the aforementioned] study reminds us that young children and adolescents may participate in different language practices and that the distinction between code-switching and code-mixing may be relevant when comparing children of different ages. This study might also lead us to expect this difference to be evident during mathematical conversation. (p. 130)

Research has yet to determine if bilingual teachers are aware of how code-switching is used by their bilingual students, and if they make use of this knowledge during instruction. One consistent finding in sociolinguistics is that code-switching is a natural adaptation of bilinguals to language contact situations, and not a reflection of a low level of proficiency in a language or the inability to recall a word (Baker, 2011; García, 2009; Ovando & Combs, 2012). Consequently, it would be incorrect to use "someone's code-switching to reach conclusions about their language proficiency, ability to recall a word, or knowledge of a particular technical word" (Moschkovich, 2007, p. 132). Regarding mathematical communication, Moschkovich further explains that code-switching may provide resources as phrases in two languages that are located in the student's mathematical register and allow multiple ways of participating in the mathematical discourse. This use of code-switching is consistent with Cummins' (2007) interdependence hypothesis.

Bilingual teachers' code-switching in mathematics classrooms. The flexible use of languages in bilingual classrooms has seldom been researched (see Jacobson, 1981; Jacobson & Faltis, 1990), even though language is used flexibly in most of these classrooms. Practices like code-switching and translanguaging are widely employed in spite of them being discouraged by policy-makers (Baker, 2011; García, 2009; Ovando &

Combs, 2012). Moreover, some linguistics and educators are of the opinion that switching back and forth between two languages during one lesson "may not control for instructional balance of the two languages, and it may not motivate students to learn the second language because first-language explanations are so immediately available" (Ovando & Combs, 2012, p. 249).

Cummins (2007) explained that flexible language practices in bilingual classrooms clash with what Howatt (1984) calls *monolingual principle*, which "emphasizes instructional use of the target language (TL) to the exclusion of students' L1, with the goal of enabling learners to think in the TL with minimal interference from L1" (Cummins, 2007, p. 223). According to Cummins, there are three major theoretical perspectives that are not easily reconciled with the monolingual principle: (a) engaging prior understandings (Bransford, Brown, & Cocking, 2000), (b) interdependence across languages (Cummins, 1981), and (c) multilingualism as a qualitatively different system from monolingualism (e.g., Herdina and Jessner, 2002). The aforementioned perspectives challenge the assumption that L2 should be used exclusively during instruction while ignoring students' L1.

The perspective of *engaging prior understandings* derives from cognitive psychology research and focuses on building on students' prior knowledge in order to promote optimal learning. According to Cummins (2007):

> Prior knowledge, skills, beliefs and concepts significantly influence what learners notice about their environment and how they organize and interpret their observations. Prior knowledge refers not just to information or skills previously

> acquired in a transmission-oriented instructional sequence but to the totality of the experiences that have shaped the learner's identity and cognitive functioning. This principle implies that when students are being educated through a second language … instruction should explicitly attempt to activate students' prior knowledge and build relevant background knowledge as necessary (p. 232).

Furthermore, the *interdependence hypothesis* states that instruction in L1 that is effective in promoting proficiency in L1, will cause a transfer of this proficiency in L2, provided that there is an adequate exposure to L2 and motivation to learn L2 (Cummins, 1981). According to Cummins (2007) some types of cross-lingual transfer are *transfer of conceptual elements* (i.e., understanding the concept of abscissa), and *transfer of specific linguistic elements* (i.e., knowledge of the meaning of *penta* in *pentagon*).

The perspective of *multilingualism as a qualitatively different system from monolingualism* has been supported by a number of theorists that emphasize that bilinguals (and multilinguals) possess *dynamic* cognitive systems that are qualitatively different from the cognitive systems of monolinguals (Cummins, 2007). Cook (2007) is one such theorist, who introduced the notion of *multi-competence* to refer to multilinguals' particular mental structures, which involve a two-way transfer across languages.

The previously discussed theoretical perspectives support flexible language arrangements in bilingual classrooms, but the effectivity of these arrangements in promoting learning gains in ELs in subjects such as mathematics has yet to be tested empirically. García (2009) classifies these flexible language arrangements as those which

lead to *convergence* or dominance of one language over the other, and those who lead to *multiplicity* of languages.

Flexible convergent arrangements. One pattern of flexible convergent arrangement is *random code-switching* (García, 2009; Ovando & Combs, 2012).When bilinguals mix both language codes in speech, alternating between the two, they code-switch. Linguistics consider this practice to be a creative use of language by bilinguals who know both languages well (Collier, 2004), and it is similar to the style-switch shown by monolinguals when they change from a formal register to an informal register (García, 2009). However, when teachers use the same kind of random code-switching they use in their communities during instruction, they may not be developing the academic language of their students to the fullest extent.

Random code-switching is called *concurrent translation*, and its purpose in the classroom could be to imitate the language students use, to engage emotionally with them, or to take disciplinary action (Baker, 2011; García, 2009). By not having control of why a switch is made, teachers may code-switch towards the more powerful language, to the detriment of fully developing bilingualism in their students. This arrangement is found in programs with a subtractive bilingual framework (García, 2009).

Jacobson (1981) proposed a pedagogy called *New Concurrent Approach*, which consisted of non-random, intersentential switches. The main problem that resulted from this approach is that bilingual teachers would use it to create a pattern of quick translation from one language to the other, preventing the conceptual understanding of the material and overwhelming students (Ovando & Combs, 2013). Consequently, the *New*

Concurrent Approach failed to spread because it was based on an artificial use of language that teachers resisted (García, 2009).

Flexible multiplicity arrangements. Recent research has contradicted the negative views on code-switching by showing that, when properly understood and used *responsibly,* it can enhance the cognitive skills of bilingual learners (Lin, 2013; Then & Ting, 2009, 2011; Tian & Macaro, 2012). The importance of code-switching in bilingual classrooms has reached a point where at least one model of the effectiveness of teachers' code-switching has been developed (e.g., Lee, 2012). García (2009) describes *responsible code-switching* as a pedagogical technique that can be used to provide the definition of a word, to provide a linguistic summary, and to provide a summary of a lesson in one language so that the child can derive more meaning and focus on the language structures.

Other flexible multiplicity arrangements are *preview/view/review* and *translanguaging.* García (2009) describes the *preview/view/review* arrangement as follows:

> The instructor gives the gist, the preview, most often in the home language of the students, then teaches the lesson in a second language, and then reviews in a language understood by the students. But sometimes this process is reversed, especially when students are at the initial stages of the emergent bilingual period (p. 301).

Bilingualism and Mathematics

This section reviews literature from sociolinguistics and psycholinguistics regarding the cognitive costs and benefits of bilingualism in mathematics. Among the

costs of bilingualism is the slower retrieval time for arithmetic facts; and among the benefits of bilingualism are high levels of convergent thinking, metalinguistic awareness, and executive control.

Costs of bilingualism in mathematics. Moschkovich (2007) explains that the cognitive benefits and costs of bilingualism in learning mathematics have been studied by psycholinguistics and sociolinguistics through different theoretical perspectives. Psycholinguistic studies view language as an individual cognitive phenomenon and are conducted in experimental settings, whereas sociolinguistic studies stress the social nature of language and are conducted in naturally occurring settings. An example of the difference between each field how the concept of *bilingualism* is defined:

> A researcher working from a psycholinguistic perspective might define a bilingual person as any individual who is in some way proficient in more than one language….In contrast, a researcher working from a sociolinguistic perspective might define a bilingual person as someone that participates in multiple language communities (Moschkovich, 2010a, p. 5).

For this study, I use the sociolinguistic definition of *bilingualism*. With respect to the costs of bilingualism, psycholinguistic studies have described them as the additional mental effort that causes disadvantages in academic performance (e.g., Grabner, Saalbach, & Eckstein, 2012; Kempert, Saalbach & Hardy, 2011). One of these costs is a slower retrieval time for arithmetic facts when bilingual adults are asked not to use their preferred language (Marsh & Maki, 1976). Therefore, psycholinguistic studies "suggest that classroom instruction should allow bilingual students to choose the language they

prefer for carrying out arithmetic computation, either orally or in writing" (Moschkovich, 2007, p. 139). Since sociolinguistics (e.g., Stroud, 2004) have challenged the notion of *semilingualism,* or the lack of native-like competence in either of the student's two languages, code-switching during arithmetic computation is no longer considered a deficiency and should be encouraged in mathematics classrooms.

Benefits of bilingualism in mathematics. From the early 19th century to approximately the 1960s, the belief among scholars was that bilingualism had a negative effect on intelligence. However, that belief was based on studies methodological flaws, including administering IQ tests to bilinguals in English only (Baker, 2011). Even today, many school systems are unaware of the cognitive benefits that bilingualism may accrue for the children themselves, their states, and the world (García, 2009). As a consequence, subtractive models of language education are imposed on bilingual children, forcing monolingualism.

Divergent and critical thinking. In the 1930s, Vygotsky observed that bilingual children had two ways of describing the world, and as a result they had more flexible perceptions and interpretations (García, 2009). Psychologists call this skill *divergent thinking* or *creative thinking,* and studies conducted by Bialystok (2005) among others (e.g., Kharkhurin, 2008) have indicated that creativity may indeed be an indirect beneficiary of bilingualism. Tasks that engage creativity, such as asking participants to conceptualize unusual uses for common objects, require individuals to ignore the typical use or the appearance of these objects, allowing them to entertain alternatives. Bialystok explains that the specific cognitive processes that appear to be enhanced by bilingualism would most likely have an impact on *fluid intelligence,* which is the aspect of general

intelligence that involves the ability to reason quickly and think abstractly. Bialystok adds that, as an aspect of fluid intelligence, creativity requires *executive control*, which is the ability to select relevant knowledge and ignore misleading information. For example, in order to propose an innovative use for a common object, an individual needs to execute control by ignoring the regular use of the object with the purpose of attending to the object's subtler features.

Metalinguistic awareness and executive control. Vygotsky (1962) noticed that children's knowledge of two languages was conducive to greater linguistic awareness and linguistic flexibility. Vygotsky further explained that a bilingual child's ability to express the same thought in different languages helped him "see his language as one particular system among many, to view its phenomena under more general categories, and this leads to the awareness of his linguistics operations" (p. 110). Vygotsky was referring to what is known today as *metalinguistic awareness*, which is defined as "the ability to reflect upon and manipulate the structural features of spoken language, treating language itself as an object of thought, as opposed to simply using the language system to comprehend and produce sentences" (Tunmer & Herriman, 1984, p. 12).

Baker (2011) explained that recent studies on metalinguistic awareness in bilinguals have shifted from being on the person and on the product to focus only on the person and her process of thinking, rather than the products of thinking. By shifting this focus, bilinguals are studied not necessarily for comparisons to monolinguals, but because bilinguals offer more critically cognitive and language processing (García, 2009).

Additionally, Bialystok's (1999, 2005) research indicated that bilingual children perform better than their monolingual peers in tasks that demand high levels of control, which is "the ability to selectively attend to specific aspects of a representation, particularly in misleading situations" (p. 636). However, Bialystok found that there is no bilingual advantage in tasks for which the solution relies primarily on high levels of analysis of representational structures, or the ability to represent increasingly explicit and abstract structures.

Bialystok illustrated the difference between analysis and control as follows: A sentence like "Why is the dog barking so loudly?" makes few demands on both analysis and control, as children recognize it as an acceptable sentence. However, a sentence like "Why the dog barking so loudly?" increases the demands of analysis, by introducing grammatical errors to the sentence. The demands for control are increased by introducing distracting information, as in the sentence "Why is the cat barking so loudly?" According to Bialystok, "the error in meaning is a compelling magnet for . . . [children's] attention and it is difficult to respond that these sentences are grammatically correct" (p. 636).

Kempert, Saalbach and Hardy (2011) conducted a study in which elementary monolingual (German) and bilingual (German/Turkish) school children were presented with mathematics problems that demanded analysis ("Sabine had 3 euros. Then Philipp gave her 5 euros. How many euros does Sabine have now?") and control ("Maria had 3 marbles. Together these marbles cost 90 cents. Then Hans gave her 5 marbles. These cost 1.50 euros. How many marbles does Maria have now?"). Results were consistent with Bialystok's (1999; 2005) since they:

> showed that immigrant children with a substantially lower proficiency in German than that shown by their monolingual peers performed equally well as their monolingual peers on tasks designed to test executive control while attaining significantly lower scores than their monolingual peers on ordinary word problems (Kempert et al., 2011, p. 557).

A recent study by Greenberg, Bellana, and Bialystok (2013) conducted with bilingual and monolingual 8-year olds showed that there is a bilingual advantage in a task involving complex spatial reasoning. What is more, a modified version of the spatial reasoning task used in Greenberg et al.'s study has proven to be effective in identifying children with high academic potential at early age. Consequently, there is now evidence that the precocious development of executive control abilities by bilingual children also support superior performance in complex reasoning tasks that require a higher-level problem solving ability and, moreover, one that is fundamental to success in an academic environment (Greenberg, Bellana, & Bialystok, 2013).

Limitations of research results on bilingual advantages. Baker (2011) identified the following limitations for the findings on bilingual advantages:

- Researchers who find cognitive advantages mostly focus on balanced bilinguals (bilinguals with equal and strong competence in their two languages).
- It is not clear which types of children share the benefits of bilingualism. Is it only elective bilinguals or middle class children?
- Experimenters' expectations can affect the outcomes and results of studies involving humans. The choice of psychological tests and the choice of a sample may reveal a built-in bias towards finding positive results on bilingualism and cognitive advantages.

- Different languages and cultures have specific cognitive effects. Recent research has suggested that bilingualism could be extremely beneficial not only for enriching the speakers' linguistic repertoires, but in offering them alternative conceptualizations crucial for flexible and critical thinking (Pavlenko, 2005).

Translanguaging

In this section the flexible multiplicity arrangement called *translanguaging* will be defined and linked to both Cummins' bilingual theoretical frameworks and dynamic bilingual models. Moreover, research on translanguaging and its use in bilingual mathematics classrooms will be discussed.

Origin and definition. *Translanguaging* "is the process of making meaning, shaping experiences, understandings and knowledge through the use of two languages" (Baker, 2011, p. 288). Translanguaging is based on the observation that children pragmatically use their two languages inside and outside the classroom in order to construct understandings (Baker, 2011; García, 2009).

Baker (2011) explained that the term *translanguaging* was created by Cen Williams to name a pedagogical practice which switches the language mode in bilingual classrooms. This is done by assigning different languages to the lesson's input (reading/listening) and output (speaking/writing), and this is systematically varied. Therefore, translanguaging goes beyond code-switching and translation, but includes both. Originally, Cen Williams used the Welsh word *trawsiethu* in 1984 to refer to his pedagogical practice, which remains a developing and important concept in Welsh

bilingual education. For García (2009), translanguaging is hybrid language use that is a systematic, strategic, affiliative, and sense-making process.

Connection to Cummins' bilingual theoretical frameworks. Baker (2011) established four potential advantages of translanguaging:

1. Promoting a deeper and fuller understanding of the subject matter. In order to read and discuss a topic in one language, and then to write about it in a different language, the subject matter has to be processed and understood, since students' prior knowledge and interdependence of their two languages is utilized.

2. Helping bilingual learners develop oral communication and literacy in their new, weaker language, since they have to work in their two languages.

3. Facilitating home-school cooperation. Children are able to ask their minority language parents for help in solving their assignments.

4. Developing bilingual learners' L2 ability and content learning at the same time. Translanguaging allows the integration of children with different levels of competency in L2, and consequently, their CALP and BICS can develop jointly.

All the aforementioned potential advantages are based on translanguaging's conceptual separation from the *monolingual principle* and *two solitudes* models, which keep L1 and L2 separated during instruction. As García (2009) explained, "Cummins' interdependence hypothesis provides the basis for advancing our understandings of bilingualism in ways that fit the exigencies of twenty-first-century translanguaging" (pp. 70-71).

Translanguaging in bilingual mathematics classrooms. Research studies on translanguaging in mathematics have been limited mostly to student code-switching while problem-solving (e.g., Moschkovich, 2007; Setati, Adler, Reed, & Bapoo, 2002) and teachers' code-switching during instruction (Garegae, 2007; Lim & Presmeg, 2011; Setati, 1998; Setati, Adler, Reed, & Bapoo, 2002). Most of these studies have been conducted in Asia and Africa, where multilingualism is common.

One recent study by Esquinca (2011) described the collaborative writing activity of two bilingual (Spanish/English) college students and their use of translanguaging as a meaning-making resource. Esquinca explains that his study is part of a larger ethnographic case study of the development of mathematical discourse among Mexican-origin college students in a teacher education program. The study was conducted in a university located in a US city where 74% of the population speaks Spanish at home.

Esquinca focused his study on a college course titled *Multiple Views on Conceptual Algebra*, which intends to develop future teachers' mathematical conceptual knowledge. Esquinca selected three students enrolled in that course based on their educational background and observed skill in discussing mathematics in Spanish, but only two of them, Yolanda and Betty, participated in that was reported in the study. Unlike the majority of their classmates, these participants were able to use the Spanish mathematics register because they lived and attended primary and middle school in the same Mexican border city, thus developing their basic mathematical knowledge in Mexican schools. Each participants' English proficiency was developing.

The study required Yolanda and Betty to write their own word problems related to the concept of invariance during a 90-minute session. The units of analysis were *literacy events* established by topic, participants, and contextualization cues. The coding of the session's transcription generated four themes: Mathematics communication, mathematical content, language learning, and education.

After the analysis of data, Esquinca determined that participants' knowledge of the word problem genre was able to transfer from Spanish to English through translanguaging, since these word problems had a similar structure in both languages. Moreover, Esquinca drew the following implications for teachers of bilingual students: (a) academic communicative ability in the first language is an invaluable meaning-making tool; (b) a meaningful writing assignment, where a parallel disciplinary genre structure exists in both languages, can be productive and beneficial for bilingual students; (c) when a writing model is provided, it is beneficial for second-language writers to be aware of the stages in which the genre unfolds; and (d) when given the choice, bilingual students, similar to the participants in the study, are capable of interacting in two languages to create meaning.

Esquinca's study is relevant because it shows how translanguaging can be an important meaning-making tool for language-minority students that have a strong mathematical education in their first language. Although the participants' written work showed that their English proficiency was developing, Esquinca did not analyze it through a deficit perspective. However, Yolanda and Betty were asked to write the solutions (word problems) in English, which is considered to hold more prestige in this academic setting, and only their discussions were conducted in Spanish. A more

interesting approach would have asked Yolanda and Betty to write the word problems in both languages, or only in Spanish.

Moreover, Esquinca's study would have been more balanced if he had included participants whose command of the Spanish mathematics register was not as strong as Yolanda's and Betty's. These participants would have discussed their ideas in English and written the word problems in Spanish, hence testing if their knowledge of the word problem genre in English was able to transfer to Spanish. By having participants with different languages abilities, the following questions (among others) could have been answered:

- How similar are the translanguaging skills of participants with different mathematical registers in English and Spanish?
- How could the results be used in the development of a rubric for the quantification of translanguaging skills?

Summary

This chapter contained a review of relevant literature regarding teacher effects and language-minority students, and it was organized into four typologies that will inform the analysis of the data and the interpretation of the results. In the section *Teacher characteristics* it was determined that, in general, "there is little scientific evidence that [teachers'] characteristics have a measurable and consistent direct impact on student achievement" (Palardy & Rumberger, 2008, p. 112). This is due in part to theoretical frameworks narrowly focused on teacher background, neglecting to examine teaching practices and attitudes, and statistical models that do not partition the variance in student

learning into student, classroom, and school components. In the case of bilingual mathematics teachers, the only evidence of the traits that make them effective can be found in ethnographic studies of limited scope. There is no quantitative evidence of the contributions of teachers' preparation and translanguaging in their language-minority students' learning gains in mathematics.

The section *Support of students' first language* revealed that at least one exploratory study of successful schools for bilingual learners has been conducted in six high schools located in California and Arizona (Lucas, Henze, & Donato, 1990). This study detected eight features that contributed to the success of their students, and all of them demonstrated that teachers' preparation and use of language were important factors in the success of the aforementioned schools. All eight features are consistent with Vygotskian socio-cultural theory of learning; in particular, the feature *value is placed on the students' languages and cultures* included (a) learning students' languages, and (b) encouraging students to develop their primary language skills. When teachers know their students' language, they are able to scaffold them from context-embedded situations to context-reduced academic tasks. Lucas, Henze, and Donato's (1990) results are relevant to the present collective case study, since the student population of the participating school district is similar to that of schools in Lucas et al.'s study.

Moreover, this section contained literature that supports the use of code-switching both on the part of students and teachers. Even though the use of concurrent translation is used to support the more prestigious language, responsible code-switching may lead to a more balanced and effective use of the languages in mathematics classrooms. Recent

studies in Asia and Africa support mathematics instruction that incorporates code-switching, but no similar study has been conducted in the U.S.

Further, results from psycholinguistic and sociolinguistic studies regarding the costs and benefits of bilingualism in mathematics indicate that the benefits far outweigh the costs (Moschkovich, 2007). In the case of the cost of bilingualism related to slower retrieval time for arithmetic facts, the problem can be solved by simply allowing bilingual students to code-switch and conduct arithmetic operations in their native language. Psycholinguistic research on executive control is still providing evidence of the benefits of bilingualism on problem-solving and spatial reasoning (e.g., Greenberg, Bellana, & Bialystok, 2013), but the field of mathematics education has yet to incorporate these results in research that involves bilingual classrooms. This kind of research may improve upon the limitations of psycholinguistic studies.

The final section on translanguaging outlines Cen Williams' pedagogical practice which switches the language mode in bilingual classrooms. Translanguaging is used by bilinguals on a daily basis in order to make sense of their bilingual, bicultural worlds. As a pedagogical practice, translanguaging is hybrid language use that is a systematic, strategic, affiliative, and sense-making process (García, 2009). Research on the effectiveness of translanguaging of is currently being conducted through a five-year study in Wales (Lewis, Jones, & Baker, 2012) and at least one qualitative study in mathematics classrooms has been done recently (Esquinca, 2011).

III. QUALITATIVE METHOD

Extensive reviews of the literature indicate that teacher characteristics such as coursework in mathematics and certification in mathematics do affect student achievement in secondary school (Telese, 2012; Wayne & Youngs, 2003). However, "there is little scientific evidence that [teachers'] characteristics [e.g., years of experience, ethnicity, native language] have a measurable and consistent direct impact on student achievement" (Palardy & Rumberger, 2008, p. 112; see also Nye, Konstantopoulos & Hedges, 2004; Phillips, 2010; Wayne & Youngs, 2003). In particular, the characteristics that make an effective mathematics teacher for language-minority students have been described by qualitative studies (e.g., Cahnmann, 2005), while theoretical models have also been developed (see Lee, 2012, for a model of the effectiveness of teacher code-switching), but these models have yet to be tested empirically.

Additionally, there is evidence that dynamic bilingualism is critical to the academic success of language-minority students in the 21st century (Baker, 2011; Esquinca, 2011; García, 2009; Lewis, Jones, & Baker, 2012), so it is important to understand how translanguaging (dynamic bilingualism's most important feature) occurs during mathematics lessons. Since translanguaging might be mediating between teachers' preparation and ELs' achievement in mathematics, the present collective case study seeks to describe the background, education, beliefs and use of translanguaging of three middle school teachers in a South Texas district that has had significant achievements in the mathematics performance of its Latino students.

Research Questions

The study addressed the following questions:

1. What are the participants' similarities and differences in their background and preparation?

2. What are the participants' similarities and differences in their support of students' first language?

3. What are the participants' similarities and differences in their perception of the cognitive advantages that bilingual students may have in mathematics?

4. What are the participants' similarities and differences in their translanguaging use while explaining mathematical concepts?

Study Setting

The study was conducted in a school district in South Texas where 99% of the students for the 2012-2013 academic year were Latino, 96% were economically disadvantaged, 59% were at risk, 30% were ELs, and 3% were migrant. According to the district's statistics, there were 1,970 middle school students categorized as ELs.

I selected the three participants for my study from a sample of 34 teachers participating in the project CAREER: Mathematics Instruction for English Language Learners (MIELL). These 34 teachers represented all 11 middle schools in the district and each had diverse levels of Spanish and English proficiency, years of experience, and percentage of ELs in their classrooms (see Appendix A).

Research Participants

The research group consisted of three middle school mathematics teachers who spoke English and Spanish with different levels of proficiency. Their bilingualism was the main criterion for their selection, but I also used other criteria such as family background (in an effort to incorporate an aspect of diversity) and their use of translanguaging during classroom observations.

The three participants were two female teachers (Mrs. Aguirre and Mrs. Quiróz) and one male teacher (Mr. Long). They worked in different schools and were all certified, experienced teachers. The classrooms observed were an 8th grade section taught by Mr. Aguirre, a resource mathematics section taught by Mr. Long, and a 6th grade section taught by Mrs. Quiróz.

Qualitative Approach

For this study on three middle school mathematics teachers located in a school district in South Texas, I focused on their (a) family background and preparation, (b) support of students' first language, (c) beliefs on the cognitive advantages of bilingualism in mathematics, and (c) translanguaging use. Consequently, this was a *collective case study*, where three case studies offered different perspectives on the aforementioned issues (Creswell, 2013). I collected data by means of interviews with the participants and nine classroom videos (three per participant) made during the 2013-2014 academic year. These interviews and videos helped in the development of an in-depth understanding of the collective case study by accomplishing a two-fold objective: To describe how the participants' life experience and preparation shaped their beliefs on bilingualism and

mathematics, and to describe how they used two languages in their mathematics classrooms.

Interviews

I conducted interviews with the three participants using an interview protocol that contained questions grouped in four major typologies: Teacher characteristics, support of students' first language, bilingualism and mathematics, and translanguaging (see Appendix C). The dates when the interviews took place are shown in Table 2. Mr. Long was the only participant interviewed before the first classroom observation occurred. I interviewed all participants by telephone.

Mrs. Aguirre and Mr. Long met me in person during professional development sessions given to MIELL participants in July 2013, but I met Mrs. Quiróz the day before her third classroom observation, which I was also able to attend. I analyzed the interviews by means of *typological analysis*, which involves dividing the overall data set into categories or groups based on predetermined typologies generated by theory or common sense (Hatch, 2002). In this case, I used the literature gathered for each one of the four typologies considered during my analysis. Moreover, the analysis of interviews were used to answer research questions 1 – 4.

Classroom Observations

Thirty-one participants of the MIELL project were video recorded three times during different times of the school year, including Mrs. Aguirre, Mr. Long, and Mrs. Quiróz. I was able to attend only one video session (Mrs. Quiróz' third). No post-

observation interviews were conducted. The translanguaging events observed in the participants' videos were coded according to the strategies utilized by the teacher and the teaching goals linked to those strategies. Furthermore, the analysis of classroom observations were used to answer research questions 2 and 4.

Table 2

Schedule of interviews and classroom observations per participant

Participant	Interview	1st classroom observation	2nd classroom observation	3rd classroom observation
Mrs. Aguirre	2/9/2014	10/15/2013	1/28/2014	4/30/2014
Mr. Long	10/14/2013	10/25/2013	1/22/2014	5/12/2014
Mrs. Quiróz	11/5/2013	10/21/2013	1/23/2014	5/13/2014

IV. ANALYSIS OF INTERVIEWS

Introduction

In this chapter, I discuss and analyze the interviews of the three participants in my case study. My analysis focuses on answering the following research questions:

1. What are the participants' similarities and differences in their background and preparation?

2. What are the participants' similarities and differences in their support of students' first language?

3. What are the participants' similarities and differences in their perception of the cognitive advantages that bilingual students may have in mathematics?

4. What are the participants' similarities and differences in their use of translanguaging while explaining mathematical concepts?

My transcriptions of the participants' responses to interview questions reflect the way they speak in English and Spanish very closely. Upon transcribing the data, utterances were not easy to follow, and/or contained grammatical errors. The English translation of Spanish utterances was not verbatim. Rather, the intent was that English-speaking readers were able to understand the essence of what was said. However, the original Spanish utterances are preserved so Spanish-speaking readers can fully appreciate them.

This chapter is divided into four sections: Teacher Characteristics, Support of Students' First Language, Bilingualism and Mathematics, and Translanguaging. Each section has a synthesis subsection were I compare the participants' responses. The "Translanguaging" section contains extensive responses in Spanish that I did not translate in order to preserve their flavor. However, I discuss and analyze the most important aspects of these responses at the end of each subsection.

Teacher Characteristics

Mrs. Aguirre. Mrs. Aguirre was born in a border town in Mexico and grew up in the state of Veracruz. Her parents were Mexican migrant workers in the United States, and growing up she would travel back and forth with them:

> "My siblings were born in the States, but I was the only one that was born in Mexico. I would come with a passport—I never cross—I will come with a passport, but I was not a resident."

Mrs. Aguirre's father was from Yucatán and her mother was from Matamoros. Her maternal grandmother was from Nuevo León, and became an American citizen before she passed away. Her maternal grandfather was from Guadalajara. On her father's side, her grandfather was Cuban and her grandmother was from the state of Yucatán: "She was Mayan, so I am also of Mayan descendants." While growing up, Mrs. Aguirre was exposed to multiple languages:

> "I heard a combination of Spanish and a little bit of dialect. My dad would talk dialect with my grandmother. They would speak Mayan. I grew up in an area

> where they also speak a lot of Zapotecan dialect….I could go to the mercado [market] and be able to chat, and ask the prices, and know the numbers, and different things. Also, the city where I grew up was a very important port where we had people coming from all over the world. I was exposed to a little bit of Greek and English itself but not necessarily at home as a family but in the environment where I grew up."

Mrs. Aguirre completed all of her elementary, middle, and part of her high school education in Veracruz. She settled down in South Texas in her late 20s and Longt to the university when she was 39 years old. Even though Mrs. Aguirre was a high school drop-out in Mexico and knew some English, though not enough to communicate in writing and orally, she was resourceful enough to face the challenges of living in the United States:

> "What I did in the States, I took the GED test, which I passed in English at the first attempt. I also passed the entrance exam at the university, but I never received formal language education. I didn't receive formal English language instruction, but I was able to pass all of the exams."

Mrs. Aguirre, now in her early 50s, obtained a Master's degree in Education with an emphasis on mathematics and science at a South Texas accredited university where 90% of students are Latino, and held certifications to teach special education at all grades and mathematics for grades 4-8. Mrs. Aguirre also held a certification in bilingual education. She took college mathematics courses, including Calculus. Though she mostly obtained excellent grades, she did not have a full conceptual understanding of the material:

> "I did not know what I was doing. I was able to follow all the steps, but I didn't fully understand the concept. Yeah, I knew how to solve an equation. Yeah, I knew how to solve a matrix, but I did not know—I didn't have the connection to real life. It was not until I took education courses for math teachers when…[for the first time] I saw algebra tiles, [for] the first time I saw an equation being solved with manipulatives, that's when everything made sense."

Mrs. Aguirre did not use manipulatives while learning mathematics in Mexico either. Blackboard and chalk was mainly used, with few additional visual aids. Regardless, mathematics was her favorite subject while growing up in Mexico:

> "It was a lot of emphasis on calculations and formulas. It was some problem-solving, and there was some vocabulary but not as much as I see now. To me, it was easy. I liked it, it was fun. To me …it was like if I was playing or solving a puzzle."

Calculus was difficult for her, since she took a course geared toward engineers and was not able to understand the application problems or connect the material to the real world. Moreover, she was the only woman in her class. When describing her experience with calculus, she did it through the lens of an EL student, but also through her knowledge of linguistically diverse classrooms:

> "…that was the first 'C' [grade] I ever received….I could not understand how could I get a 'C' if I was working so hard….there was a particular language barrier for me in that class because my professor, he was from Poland…had a PhD in mathematics, and I guess his expectations were very high. When he

> comes, you should already know all this. I just could not—he could not bring his content to my level so I really struggled in that class. As a matter of fact, I was discouraged from taking that course because it was like a big blur to me…and I changed my major from math to literature."

Mrs. Aguirre believed the most important trait required of a mathematics teacher of EL classrooms was knowledge of the content, and not so much that the teacher was bilingual: "You can have a monolingual teacher, but that teacher can also facilitate…those activities in which the students together as a class can help each other [understand the material]." Moreover, Mrs. Aguirre mentioned effective class management as an important factor, in the sense that it should provide a safe environment for students to discuss the material and help each other:

> "I have friends who have a classroom where you can hear a pin drop, everybody's quiet, absolutely quiet and working. Nobody talks, and they all super well-behaved. It seems like they don't even breathe, but learning is not taking place. I have a friend who is an ultra-disciplinary, but there's no learning. When the scores came, it was a shock cuz the scores were so low, but everybody's working and quiet."

Students are apprentices—Mrs. Aguirre indicated—but they are also social learners, "it's not only till the kids are talking and learning socially that…the learning is perpetuated." Teachers of EL classrooms should also be resourceful and implement different strategies, even though the pressure of standardized tests make it difficult to try a variety of methods: "You really have to dedicate time. What [is it that] I'm gonna teach

next week and how can I make this accessible for the English learners? How can they really grasp it?" Furthermore, Mrs. Aguirre believed that establishing rapport with students also helped them gain interest in the subject and attend tutorials:

> "At the beginning of the school year when I have a new student, I talk to them about my personal experience. I talk to them about my experience with education, like going from one school to another city, and then going to the university as a—not a native speaker of the language, and then going to school as an adult and being able to graduate with honors, and being able to change my life, the salary that I had before an education and my salary after an education. I talk to them about those experiences, and I always tell them, 'if I did it, so can you'. The only reason I am telling you all this is because anybody can, if you try and apply yourself."

Mr. Long. Mr. Long had four years of experience teaching middle school resource math for Grades 6-8, considered his Spanish proficiency as "good", and earned a B.S. in mathematics. He held certifications for grades 6-12 mathematics, all level certification in special education and kinesiology. Mr. Long was in his mid-30s and has lived in the participating school district where the research site is located his entire life. In 2013, he resumed teaching mathematics for the first time since 2003, when he taught Geometry and Algebra courses.

Throughout the interview, it became clear that Mr. Long's personal and academic background influenced what he did in his mathematics classroom. Since he was born, raised, and educated in South Texas, he was able to use his own experiences to better

understand the needs of his students. Furthermore, he was pursuing a Master of Arts degree in Special Education, probably because his father lost his hearing around the time Mr. Long was born. Mr. Long was losing his hearing as well and wore a hearing aid during classroom observations. He learned sign language and claims it was helping him teach ELs:

> "I think it does [help] for the reason because sign language is very expressive, your emotions are part of the language, it's not as dry as what you hear, and it's not monotone…. Sign language is very emotional and very expressive. The person's facial expressions help with understanding."

Mr. Long's father was born in Canton, China, and his mother was born in Jalisco, Mexico:

> "My dad was, well, actually my great-grandfather worked on the railroad (on my dad's side), Pacific Union and then he brought my father over here and he lived here alone in the United States in California and then ended up in South Texas and that's where he met my mom. My mom crossed the Rio Grande, so they met here….My mom spoke Spanish…. she actually did not go to school. Her parents did not have the money to [put her in] school, so she mainly was able to learn by reading the Bible and she learned Spanish and how people were talking for the communication. And on my dad's side, he knew English and Chinese and Spanish and the only problem by the time I was born, he had lost his hearing. And so, he would talk to his friends in Chinese and would talk to my mom sometimes in Spanish and I would talk to him in English."

Due to his father's hearing loss, Mr. Long acquired his mother's language (Spanish) as his first language. I asked him if he considered himself trilingual:

> "Myself, not really, because I do not know the language of Chinese; I just know Spanish and English very well. I learned a little bit of sign language so I know some words in sign language more than my Chinese. And what happens is Cantonese is not a language normally used anymore, it's mainly Mandarin and it's kind of difficult practicing with somebody in Cantonese that I learned in childhood."

He became competent in English when he was in 4th grade, but mathematics was a subject that he could learn in spite of his limited English:

> "When I was learning math, it was a lot easier than learning my English because mainly at the house my mom would talk to us in Spanish, so coming to the school system, it was very difficult with my English, my spelling. I still have a lot of difficulty in it. But my math experience was a lot easier. I just associated patterns, associated relationships and stuff like that and in the middle school I had a very good experience where teachers were more…they wanted to teach you and it was a lot easier for me because I was observing everything at the middle school level. And high school was more of a challenge where I challenged myself taking upper level classes up to Pre-Calc and that's why I decided to get a bachelor's in mathematics."

Mr. Long believed that learning mathematics was easier for EL students than learning other subjects, as it was for him. I asked him to tell me more about his

experience teaching mathematics to ELs. In particular, I wanted to know if recent immigrants' prior knowledge in mathematics helped them learn in their new circumstances:

> "The majority [benefits from that]. They have education from their previous countries. It all depends. For example, if they come from Mexico and they've never been to school, then it's difficult for them to understand that there's a concept or just the basics of how to add, subtract and multiply if they've never had an education. The majority, it's a lot easier for them. But at the middle school, there is still some struggling on the basic background, but it is a lot easier and a lot smoother when they are working on their assignment, doing the assignment and the teacher's not there. I would say it's a lot easier, the same way it was for me."

When I asked him about the traits that a mathematics teacher should have when teaching in a bilingual classroom, Mr. Long mentioned well-known sheltered language instruction techniques, but emphasized that *patience* was the most important trait:

> "Some of the traits [teachers] should have when teaching ELLs…their tone and pronunciation and be able to make connections to the language because of the Latin and also some also some of the traits teachers should have is patience, to be able to figure out the weakness of the student and knowing exactly what the student didn't understand so they can be able to more clearly solve the problem. Another trait is including different, various ways of explaining things. They should be able to explain the problems in various ways. Or maybe [the student]

understood it a certain way and now they've introduced a different way and they might not understand it or what the actual concept was. They need to verify if the students understood things. They need to reinforce that with them and make sure they're checking for understanding"

Mrs. Quiróz. Both English and Spanish were spoken in Mrs. Quiróz's childhood home. Her father was a Minnesotan and his native language was English, and her mother was born in Tlalpan, México and spoke Spanish. Regarding her grandparents, she said:

"My maternal grandparents—my grandmother spoke exclusively Spanish, and my grandfather, who is Mexican, was educated at Yale, so he spoke English and Spanish fluently. My paternal grandparents spoke English only—both of them."

When I asked her what her native language was, she replied: "I would have to choose English as my native language, but I think in both languages, so it's a very close call….I'm definitely stronger in English, but my Spanish is pretty decent." Mrs. Quiróz, in her late 40s, was born in Fort Sam Houston, Texas, but attended kindergarten in Coahuila, Mexico as her father went to Vietnam, necessitating that she live with her maternal grandparents for one year. Mrs. Aguirre attended elementary, middle, and high school in Austin, Texas and obtain her Bachelor's and Master's degrees in San Antonio, Texas. Her experience with mathematics in school was not positive:

"I was usually very frustrated in math. I was always lost because the teacher would go too fast, and until I reached—I wanna say junior high school—I had teachers that really would not entertain my questions. They wouldn't allow me to ask questions, and so I would never get answers. Now when I reached secondary

school, that's when I had teachers that were comfortable listening to questions and explaining."

Mrs. Quiróz would ask her mother for help, but this would end up confusing her more as her mother would use the algorithms that she learned in Mexico: "I would see her division problems, and they were quote unquote backwards, and it was like, 'Okay, never mind'…. As I grew up, and I saw her do the math, I understood what she was doing." Mrs. Quiróz's experience as a first grade teacher in a bilingual school in Mexico also helped her understand "the Mexican approach to teaching math." This was a school for "extremely wealthy people" and Mrs. Quiróz was in her early 20s when she had that experience: "Mexican nationals paid tuition whereas children from the American Consulate attended free…. Only two of those children [her students] were American children from the Consulate. All the other children were Mexican nationals." She would teach mathematics in English, but gave significant support in Spanish. The school was not following an established bilingual education model, but was strictly based on learning English as a second language.

Mrs. Quiróz was certain that teaching in Mexico helped her develop the skills necessary to be an ESL teacher in the United States. In the 80s, she received a life-long certification in bilingual education and ESL for grades K-6, and five years ago she was certified as a mathematics teacher for grades 4-8. "In December [2014]—God willing—I will have my principal certificate." Her experience with EL children was very strong:

"I was an itinerant ESL teacher. That was very interesting, because with Northside [a district in San Antonio] I had from three to four campuses every day,

> and it was truly an ESL environment, because I had children from China….I had children from Turkey, so Spanish wouldn't help them. It was very much an ESL class, and there I taught kinder through fifth grade….what the federal programs would do is they would group the children back then by their English level, so I would have all my beginners even though the ages would be mixed. It worked because…the children in the room…weren't very many…maybe three to six kids at a time. I had my beginners all together, and so you taught at a very different level."

I asked how she managed so many languages in the classroom, and she said that visual aids were very important: "You relied heavily on pictures. On posters, so that you could talk about not just nouns but verbs and interactions between people. Posters of a grocery store or posters of a park. Things that were age suitable that kids could relate to that." I then moved our conversation to mathematics teaching and asked her about the traits she thought a mathematics teacher should have when teaching in a linguistically diverse classroom:

> "Well, believe it or not, you really should have a working knowledge of English sentence structure and grammar, especially with heavy emphasis on knowledge of how prepositions are used in English. It definitely helps that my undergraduate degree is in literature, so I'm a grammarian by trade. Then my masters is math and science, so I have a pretty strong background in most of the subjects I teach. It helps tremendously to put—to look at a word problem with my ELLs and say, "Okay, how would you say this in Spanish? For example, something that classically trips up ELLs is the subtraction sentence structure. Subtract four from

twelve. Well, we don't say it that way in Spanish… It's backwards to them. It's backwards to me. We have to act out what taking something from someone else means, and so they get a working understanding."

In fact, the mathematics academic register in Spanish does have an equivalent for the sentence "subtract four from twelve" (*restar cuatro de nueve*), and Mrs. Quiróz asseveration was partially correct. It is not usual to say something like *restar cuatro de nueve* in Spanish while using a colloquial register.

At the time of the interview, Mrs. Quiróz was teaching mathematics to six sections of 6th grade. This was the first time she taught 6th grade, after ten years of teaching 5th grade. Many of her students were ELs, from children that had just crossed the border, to children that had a good grasp of English:

"I only started with this group of kids less than a month ago, so as far as—I don't know how you say in English…*ganando su confianza* [earning their trust]. It's like getting them to open up to me and being comfortable saying, 'I don't understand this.' They're not there yet, because of their age. They're in a very awkward age. They're pre-teens, and they're terrified of looking not smart in front of their peers, so I'm hoping with time that changes. Because what happens when you get them in cooperative groups is you'll hear the ELLs tell each other *'No, eso no es así. Te están preguntando esto'* [No, this is not right. They are not asking you that]. They start deciphering it, but it takes time, and it takes practice…. so now it's not just language. It's the "cool factor", because *nadie, aunque hables español o inglés, no quieres aparecer como una tonta o un tonto*

en frente de las chicas o los chicos [nobody, even if you speak English or Spanish, wants to look like a fool in front of the boys or the girls]. It's a completely new playing field of what barriers you have to overcome as an educator to get the kids to what I call buy-in."

The previous translanguaging episode is characterized by the differentiation of the genders (*tonta-tonto, chicas-chicos*), a goal that is not easy to achieve in English (this is evident in the English translation above). Furthermore, Mrs. Quiróz relayed that she was happy to be teaching 6th grade, even though it was a very different experience than teaching at the elementary level, where "you have the kids all day, and so you can use the other subjects beautifully to help develop their English…. but here [in middle school] you have the kids 45 minutes." I asked Mrs. Quiróz what traits she thinks a mathematics teacher should have when teaching in a linguistically diverse classroom and her response was similar to Mr. Long's:

"More than anything a teacher has to be patient. A teacher has…to have faith that even if a kid looks like they don't wanna learn or they don't care if they learn, that's not true. It's a show, and so you have to know that, yes, this kid internally—they really do wanna understand…. It's patience, and it's persistence of not lowering your standards but trying to find ways with every child—how can you support that child so that they elevate themselves to the level necessary that they can meet that rigor successfully."

Synthesis. Mrs. Aguirre was the only participant not born in the United States, since her native country was Mexico; however all three participants had at least one

Mexican parent. During the interviews, it was clear that Mrs. Quiróz had the better command of both English and Spanish, followed by Mrs. Aguirre and Mr. Long. Additionally, Mrs. Quiróz grew up in a higher socio-economic status. Nevertheless, Mrs. Aguirre and Mr. Long both had contact with multiple cultures and languages growing up, even though they are considered ethnic minority cultures and languages in the United States. Mr. Long and Mrs. Aguirre considered Spanish to be their first language, and Mrs. Quiróz considered hers to be English.

Mrs. Aguirre received all elementary and almost all of her secondary education in Mexico which made her the participant most likely to understand the mathematics background of recent immigrants. Mrs. Quiróz was the only participant that had teaching experience in Mexico, even though she taught mostly in English. This teaching experience gave her some understanding of mathematics education in Mexico at the elementary level and the algorithms used.

Mrs. Aguirre and Mrs. Quiróz both held master's degrees in Education with emphasis in mathematics and science, while Mr. Long was enrolled in a master's program in special education. However, Mr. Long was the only participant with a strong background in mathematics, since he held a bachelor's degree in Mathematics. Both Mrs. Aguirre and Mr. Long attended the same university in South Texas, whereas Mrs. Quiróz attended two universities located in San Antonio, Texas. Mr. Long was the participant with the closest ties to the region, having been born and educated there. Mrs. Aguirre also had ties in the region.

Regarding teaching credentials, all the participants were certified to teach mathematics at the middle school level. Additionally, both Mrs. Aguirre and Mrs. Quiróz were certified bilingual teachers, and both Mrs. Aguirre and Mr. Long had certifications in special education at all levels. Mr. Long also had a certification in kinesiology. Of the three, only Mrs. Quiróz found mathematics to be a difficult subject growing up. She attended school in Austin, Texas, a larger city with diverse student population.

Both Mr. Long and Mrs. Quiróz indicated that the most important trait a mathematics teacher needed when teaching ELs was patience. This quality was vital to better understand ELs' thinking and to provide the necessary scaffolding so they could comprehend the content taught. Mrs. Aguirre was of the opinion that knowledge of the content was the most important trait a mathematics teacher of ELs required, but she mentioned additional traits, such as class management skills, resourcefulness, and establishing rapport with students.

Support of Students' First Language

Mrs. Aguirre. Mr. Aguirre started her teaching career as a teacher of special education. At that time, children with learning disabilities would be separately instructed in a *resource classroom*, where the teacher of special education would use a separate curriculum. Currently, schools in South Texas follow an inclusion model, where the teacher of special education assists children with learning disabilities in a regular classroom setting. Mr. Aguirre was a teacher in a mathematics resource classroom for five years, teaching students that were at least three years behind their grade level. Since

those students are being assessed at grade level, teachers of special education are charged with closing the achievement gap.

For the last four years, Mrs. Aguirre had been working as an 8th grade teacher. She described the classroom that was videotaped for the MIELL project as containing mostly students considered at risk of academic failure and a few students that were considered gifted and talented. Mrs. Aguirre indicated that only three students were English-monolingual, ten were formally classified as ELs, and the rest were bilingual students at varying stages of language proficiency in English and Spanish. Mrs. Aguirre's 8th grade classroom had a total of 26 students where a few students were recent immigrants, that is, students that arrived in the United States within one year. According to Mrs. Aguirre, some of her students should have been classified as ELs, but their parents refused to have them placed in a bilingual education program:

> "It comes to a survey that the parents come to the school the first day you are enrolling your child, and there's a language survey. In that language survey, the parent immediately opts out of bilingual because they believe that that's not helpful, or they want their kids to really learn English and they opt out of bilingual….They call it parental denial. The parent really doesn't want anything to do with the bilingual education."

Mrs. Aguirre pointed out that ELs received tutorials and some accommodations. For example, ELs were allowed to use the dictionary in class and when they took the English version of the STAAR (Texas' mathematics standardized test); moreover, they were given extensions to complete their assignment. "We're allowed to clarify and in

some instances, you have to translate. Sometimes you can also read out loud for them to push them, like if they have a test and they're not understanding the vocabulary." Mrs. Aguirre also used ELs' life experiences in order to clarify the meaning of words in tests without using direct translation: "You're allowed to clarify. For example, if, let's say, it says convenience store. I can go to that student and say—instead of translate it directly to Spanish, I can say, this is a store where you can buy—like, for example, Wal-Mart."

However, Mrs. Aguirre reported that accommodations are not favored by all, since many teachers and school district authorities believe that some students take advantage of the accommodations they receive as ELs to pass the STAAR, and try to remain in the bilingual program as long as possible: "Now they're saying [many teachers and school district authorities] 'you are passing but you are receiving accommodation, we're not gonna validate your scores like if you're really passing. I'm gonna validate that you pass the day that you didn't receive any accommodation.'"

Mrs. Aguirre used both Spanish and English in her mathematics classroom, along with other strategies in order to make the material more comprehensible to all students:

> "There are some key words that I make emphasis on, that are some few words that I wanna help the students to really understand. When it comes to problem-solving, they really need to understand what is needed from them. Those are the instances in which I will use the Spanish word. For example, if we are approximating, or if you are rounding, or if the problem said something like, what is the closest?"

Another strategy Mrs. Aguirre reported using in her classroom was to begin the lesson with a concept map focused on a keyword. Students were free to use drawings or words associated with the keyword in question:

> "I feel that a lesson needs to be presented, *like they say in the United States*, from scratch. The meaning that even the most basic vocabulary needs to be understood in order for the student to grasp the concept. It's a strategy that you use to help English learners and students with special needs. If I begin from scratch and I help them make a mental picture, a mental image of this word, then they will be able to make the connection.... This is something that has a special meaning and it has a special application in real life."

In the previous statement, Mrs. Aguirre positioned herself as a foreigner in the country where she has been residing legally for many years. The digital data showed that her self-identification was also evident in her classroom, where her accent and grammatical errors contrasted with the native-like English spoken by some of her students. Additionally, her use of thinking maps was a strategy to stimulate inner speech, since students could use their preferred language during this process.

When I asked her about the translanguaging techniques that she used in her classroom, Mrs. Aguirre explained that she used these as a way to promote a fruitful discussion:

> "I think that the most effective activity that you can do is what we call meta-cognition, when they have to reflect about and talk about what is it that's being learned. I want the students to have a conversation. If I just ask them, 'Okay, you

and you, sit down together and talk about percent', it might not happen that way. When it's a class-guided discussion, you know we are all talking about this…. We say it and we say it again. We say it in Spanish, and we show it in—because drawings are more than welcome. I tell 'em, sometimes if you can't write it, sketch it, and we'll talk about the picture….I want to see what it is that they think. I'm looking for their frame of reference."

In order to prepare her students for the STAAR, Mrs. Aguirre reported focusing on key vocabulary and reading comprehension. First, she would drew a sketch to help students see what she was trying to say, and if she achieved this goal she considered herself "fifty percent successful." Mrs. Aguirre considered the mathematics and the calculations involved in solving application problems were not usually what held students back, but the lack of reading comprehension. "I guess I am a math teacher, but I am also a language teacher."

Once students grasped the concepts, Mrs. Aguirre proceeded to do practice and review, "putting a lot of emphasis on my spiral reviewing. We're spiraling already. Everything's coming back, keeping it fresh in their mind….close to the end, [we] do some drilling. That's also important. That doesn't happen unless the learning has taken place." Regarding the use of both English and Spanish in her classroom, Mrs. Aguirre explained that she created an environment where it was perfectly fine to have two words for the same object: "We can say *compass* and we can say *compás*…. that will enrich [the students'] experience." By using two words for the same object, Mrs. Aguirre not only tried to clarify the meaning of words in English, but "to create [an] environment in which it's okay to be bilingual."

When I asked Mrs. Aguirre to estimate the percentage of her class in which she used Spanish, she indicated that she it was 80% of the time when it came to discipline, "because there are some powerful words in Spanish." In terms of teaching content, she used Spanish 5% of the time, since the tests were in English and students needed to be familiar with the content: "If there is clarification needed or the students are not getting it, then I will add a little bit more Spanish." Moreover, Mrs. Aguirre reported that she allowed students to use Spanish, but then would have another student paraphrase in English what was stated in Spanish, or she would paraphrase it herself: "I just say, 'Oh so you're saying, this, this and that'. They can hear their own thinking, but also in English."

Mr. Long. According to Mr. Long, a significant accomplishment when teaching mathematics to ELs occurred when they completed assignments when they worked independently:

> "One of the challenges I see for these students is actually assistance at home. For example, they don't have the reinforcement at home where they could be taught….I don't want to say all of the students, but some of them have economic disadvantages because their parents don't have money, there are problems at home, etc. So, I think that affects the student and doesn't help with all of those disadvantages to make any progress."

Mr. Long also noticed that for some of the ELs that he taught, the academic language associated with mathematics was a third language that they had to learn. By acknowledging this challenge, Mr. Long demonstrated awareness of the linguistic differences that Cummins (2000, 2008) outlined, including the distinction between the

linguistic basic interpersonal communicative skills (BICS) and cognitive/academic language proficiency (CALP). In situations when students could not understand academic language, Mr. Long would scaffold by translating mathematical terms:

> "Sometimes I know some of the terminology they use in Spanish so it's a lot easier for me to explain it in English and the translation is a lot easier for me. I have some colleagues and it's more difficult for them to actually explain what needs to be done."

In the participating school district, mathematics school teachers were advised not to use Spanish during instruction. In spite of this, Mr. Long would support using both English and Spanish in mathematics classrooms. In fact, he claimed having done so in the past:

> "I would strongly agree to teach in both languages. When I was at the high school level, mainly I would say the whole thing in English and some of them knew what to do and they understood and then those people who did not understand what was going on, I did repeat it in Spanish and they were able to understand. I like the idea of being able to teach in both English and Spanish in the classroom."

Nevertheless, when asked directly if he currently used Spanish in his classroom, Mr. Long hesitated and ended up saying he did not. Since he had previously admitted translating mathematical terms for his students, his answer appeared to be contradictory. In contrast, Mr. Long claimed not to have a problem with his students using Spanish while working in groups, which is a typical practice of bilingual students while solving mathematical problems (Moschkovich, 2007).

Mrs. Quiróz. According to Mrs. Quiróz, the key to the success for EL students in standardized tests was that they had a conceptual understanding of the material:

> "There are tremendous gaps, because the elephant in the room that nobody wants to talk about when you go to these math professional development sessions is there are teachers who are teaching tricks and shortcuts. My favorite one that I really get upset about is cross multiplication….when you cross your denominator over, and you only multiply it with the other numerator, and you write the little number on top. That works for third, fourth and fifth grade, because when you're comparing fractions, that's the only number you need. The bigger number wins, so you can tell if it's a greater than or a less than. The problem is when you get to equivalent fractions the students do not understand why you are multiplying with a unit of one. They have difficulty understanding that two halves is one or five-fifths is one."

Mrs. Quiróz told me that it was a big challenge to try to fill in gaps such as the aforementioned one or the gaps in reading decimal numbers, particularly for ELs:

> "One point two four. No! One and twenty four hundredths. If a child is asked to read that in English correctly from the beginning, when you make the transition to fractions—well, think about it. You can hear it—twenty-four hundredths. Oh, okay, I can see the 24 numerator. I can see the hundreds denominator. Correct usage of the mathematic lexicon is essential, especially for ELLs."

Regarding standardized tests, Mrs. Quiróz complained about the strict timelines for testing that did not correspond to the reality of how children learn, and that this prevents teachers from teaching with the depth demanded of them:

> "You have to allow children enough time to feel like they've mastered a substantial amount—70, 80 percent of the material before you assess them. Because what happens is [that] the psychological factor is kicking in. I mean, how would I feel if I get a paper back and I have a 20 on it? Why should I try the next time? It becomes a self-fulfilling prophecy. If you're not in the curriculum driver's seat—if you're the teacher, and you have to follow the directions of administration, and you have to test on a certain timeline, that is affecting your ELLs because they're frustrated. They need more time to assimilate. They need more time to process what you're teaching them. It can be done, but having these artificial testing timelines really undermines that entire factor of *confianza* [trust]."

According to Mrs. Quiróz, another elephant in the room that no one wanted to talk about was how girls were raised in the Latino culture, since some of her female students that were not making academic progress had the "*¿Y qué vamos a cocinar de cena hoy?*" ["What are we cooking for dinner today?"] mentality:

> "If you bring up girls with makeup and hairstyles…they are not gonna have the same spatial reasoning as the guys who get to build stuff with Legos and connect blocks and almost every toy that the boys have—and I know this because I have

two sons—has to do with spatial reasoning. The girls don't get that. They don't get that benefit growing up."

Mrs. Quiroz readily admitted that she used Spanish in her mathematics classroom: "If I have the kids and especially Anglo kids that [say] 'We're not supposed to be learning in Spanish', and it's like, 'You know what, kid? You're gonna have to take two years of Spanish in high school anyway, so get used to it.'" I asked her to provide examples of situations when she switched languages, and she described what had transpired that day in her class:

> "We were doing ratios. The question was, 'You have a two to seven ratio of men to women at a dance recital, and there are 36 people at the recital. How many men are at the recital?' Well, the kids—especially the Spanish speaking kids—wanted to say, 'Well, there are two men. It says so right here.' You have to explain that it's a relationship, so if you have two men and seven women—as I'm explaining all this in Spanish—does that give you your total of 36 people? We go over logical approaches in Spanish."

However, Mrs. Quiróz reported that her classes were mostly taught in English in order to not single out EL students. Instead, her strategy was to seat ELs with fluently bilingual children, so they could discuss the material quietly during a lesson.

Synthesis. All three teachers used Spanish in their mathematics classrooms, but only Mrs. Aguirre and Mrs. Quiróz admitted it directly. Mr. Long denied using Spanish in class, but contradicted this claim during our conversation on several occasions. Months after the interview, Mr. Long's classroom videos revealed that he did speak Spanish

during instruction, so it was clear that he made the decision to use Spanish during the observations as he probably did when he was not observed.

Mrs. Aguirre and Mrs. Quiróz reported that English was the primary language of instruction, but they described different reasons for doing so. In Mrs. Aguirre's case, she was aware that standardized tests were in English and that her students needed to learn the language in order to pass them. In Mrs. Quiróz's case, she reported that she tried not to single out her ELs by constantly speaking Spanish. The digital data indicated that Mrs. Quiróz included ELs during instruction by frequently calling them by name and asking them questions in Spanish. The digital data also showed that Mrs. Aguirre also included her EL students in classroom discussions, but she did it less frequently than Mrs. Quiróz. Moreover, both Mrs. Aguirre and Mrs. Quiróz considered themselves language teachers in addition to being mathematics teachers.

Mr. Long reported that being a Spanish-speaker was an asset when teaching ELs, since he could provide scaffolding by explaining the meaning of mathematical terms in Spanish. In contrast, Mrs. Aguirre expressed that being bilingual was not necessary to teach ELs, but that being resourceful was. Additionally, Mrs. Aguirre mentioned the most uses of translanguaging in her classroom (translation, paraphrasing, disciplining, explaining the meaning of words, and encouraging students to use their favored language in inner speech), followed by Mrs. Quiróz (paraphrasing, collaborative grouping) and Mr. Long (translation).

Bilingualism and Mathematics.

Mrs. Aguirre. Mrs. Aguirre believed there was no difference in how monolingual and bilingual children learned mathematics, and that the strategies teachers use for ELs benefitted all students: "It's not that they [English-monolingual students] learn different. It's that we take for granted that the monolingual student, because he knows the words, he also understands the concept…. The difference is in the way the teachers present it [the material], not in the way the kids learn it."

I asked Mrs. Aguirre if she believed bilingualism provided any learning advantage in mathematics, and her reply was cautious, as if entertaining this possibility for the first time:

> "I think that if a student can—let me see, when they're learning a language, they're learning a new thing, right? Sometimes they're connecting to previous knowledge. They have previous experience. They solved perimeter before, but now they are learning it in the other language…I believe that if they have this schema in the brain for perimeter and now there's another schema that is enhanced because now you know it also in English. There are other examples that I'm adding to your previous knowledge, but not only that, but now calculations come along and problem-solving comes along, so I think it's a richer experience."

Mrs. Aguirre added that it was not easy for teachers to provide this richer experience to bilingual students: "It requires more work, yes. You have to prepare and you have to provide those real-life examples because they don't always have them."

Mr. Long. Research has shown that bilinguals use executive control (the ability to ignore misleading information) in arithmetic problem solving (Bialystok, 2005; Kempert et al., 2011). When asked if he had noticed any difference in how monolingual and bilingual children learn mathematics, Mr. Long indirectly referred to the use of executive control as the main difference between these children:

> "I think the bilinguals learn a lot faster or different ways of solving compared to monolingual. I think…mainly monolingual they tend to do things one way, where the bilinguals tend to do or see different ways to solving problems….For example, improper fractions, mainly the numeric fractions where they have a whole number and a fraction and they want to convert it into a decimal and the bilingual [students] do less steps where the monolingual tend to just follow what they've learned. The bilingual actually analyze where they could shorten the steps that they've already been taught or they already know a different way of solving the problem and they make the connection where that basic was already taught, so it's a lot easier for them to connect one way of doing it to different way of doing it. This is at the middle school level. At the high school level, the algebra equations where they have a variable [an unknown], they were able to connect and use common sense to find the variable plus they also could actually solve the equation the whole algebraic way."

According to Mr. Long, the source of this bilingual advantage in mathematics was them being able to make connections from "their primary language to a secondary language" and the fact that they have the "knowledge of knowing a different language". Both statements clearly refer to metalinguistic awareness, which is the ability to treat

language as an object of thought (García, 2009), and has been shown to be associated with executive control (Bialystok, 2005).

Mrs. Quiróz. I asked Mrs. Quiróz if she thought there was a difference in how monolingual and bilingual children learned mathematics, and she said that bilingual children seemed to grasp mathematical concepts much more quickly than monolingual children, but warned me that her views might have been biased. She also mentioned reading studies about the cognitive advantages of bilingualism, and she also indirectly mentioned executive control as the possible source of that advantage:

> "Now, I don't know if that's due to a development. These studies of neuron development—that the more you challenge the child the more neuron connections there are, and therefore they're able to progress much more quickly and to a much higher level than children who don't have those neurons stimulated. I guess I would hypothesize that because kids who are in a bilingual environment—you have to be thinking of different lexicons. You speak one way when you're with your thirteen-year-old peers. You speak another way when you're with your teacher. You speak another way when you're with your parent, and so I think kids that have to use their brain in more situations like that—yeah, I think that does assist them to understand concepts—conceptual lessons—more easily than the monolingual kids."

Moreover, Mrs. Quiróz mentioned an example in which she had observed that advantage:

> "Anyway the Spanish speaking kids—once you get them to understand what a word problem is saying—not telling them how to find the solution to the problem—the scenario that they're describing [in the problem] and then the question that is issued at the end. The bilingual learners tend to be able to process the scenario much more quickly and with much more facility. It's easier, and it's like, 'Okay, this is going on in this scenario, and this is what they're asking. Oh, okay, so this is the approach I need to take.' I would say word problems, definitely. I know that sounds contradictory, because they're ELLs, but once they understand what the word problem is telling them and then asking them, they—I really do see a deeper conceptual understanding with the bilingual kids—with the ELLs."

In the previous statement, Mr. Quiróz chose to describe ELs as bilingual children, focusing on what they will become, and not on what they lack. Moreover, Mrs. Quiróz indicated a bilingual advantage in solving word problems that has been supported by research (e.g., Kempert et al., 2011).

Synthesis. Of the three participants, only Mrs. Aguirre seemed not to be fully aware of the cognitive advantages of bilingualism in mathematics, but she indicated that learning mathematical terms in another language should provide some cognitive benefits, given that it was "a richer experience". In contrast, Mr. Long and Mrs. Quiróz both cited concrete examples of the cognitive advantages that bilingualism provides their students in mathematics. Additionally, while Mr. Long indicated how bilingual students were creative when problem solving, and even referred indirectly to metalinguistic awareness as the source of this creativity, Mrs. Quiróz indicated that once these children understood

what a word problem was asking, they were able to decide what approach to take on their own. Mrs. Quiróz also explained that having to switch languages according to the situation (executive control) was behind bilingual students' advantage in mathematics.

Translanguaging

Mrs. Aguirre. I asked Mrs. Aguirre to respond mathematics questions in a different language than the language in which I asked the question. For example, if I asked a question in English, her task was to respond in Spanish, and vice versa. The mathematical questions in this exercise involved the concepts of mean and median. I chose these concepts because research has shown that teachers have misconceptions about them (see Jacobbe, 2012), and because the term "median" has a cognate that some bilingual teachers are not aware of ("*mediana*"). The transcription of the translanguaging exercise follows:

Interviewer: How would you clarify the difference between the concepts of mean and median to your students?

Mrs. Aguirre: *Para explicarles la diferencia entre el medio y el promedio, probablemente les haría una línea de números donde ellos pueden ver…físicamente pueden visualizar la diferencia. Los números están en orden, este número que está en medio, es el medio, es el número de en medio. Y luego para explicarles el promedio les enseñaría que físicamente esa misma línea de números se sumó completamente y se dividió entre el número de…ay, se me está*

olvidando el español…el número de ítemes que estamos sumando. ¿Contesté su pregunta?

Interviewer: *Sí, pero quisiera que me explicara un poco más esta diferencia. ¿Me la podría explicar un poco más, pero en inglés?*

Mrs. Aguirre: I like to explain that the median number is not so much the numerical value, but it has to do with the position in which this number is standing in the number line. Where is this number located? Where can you find it? It should be exactly in the middle, not necessarily the numerical value or the weight it has. If you're talking about the mean, it has to do with the weight, what weight this number has. This example that I can always give my students is their grade. What are your grades? You had a very good test then you have a bad test grade. Then you calculate it in this particular manner, that's where you get your mean, which is your average. Did I answer your question?

Interviewer: Yes, but I would like to know when would you use the mean and when would you use the median? Are there situations when one is better than the other?

Mrs. Aguirre: The decision which one to use is gonna depend on what do you need it for, okay?

Interviewer: Wait a second. When I ask you the question in English, you are supposed to answer it in Spanish.

Mrs. Aguirre: *Depende qué es lo que quieres enseñar, vas a decidir cuál medida vas a ocupar. Si estás…si le quieres enseñar a tus papás que vas pasando bien en la clase y resulta que el examen que tomaste en el medio del semestre tenía un buen grado o era el grado más alto, tú quieres enseñarles a tus padres el número de en medio. Pero si tuviste un grado muy malo pero uno bueno nada más, entonces les quieres enseñar a tus papas que todavía estás pasando la clase, entonces les quieres enseñar el promedio, porque te van a decir "sí pasaste, pero fíjate que también reprobaste". Entonces les vas a decir, "sí pero este es mi promedio y todavía estoy pasando la clase". Ahí podrás hacer la decisión cuál quieres usar. ¿Quieres usar el número que está en medio, o quieres usar el promedio?*

Interviewer: *Cuando calculas la mediana, ¿cómo lo haces?*

Mrs. Aguirre: It's very important that you order your numbers in a particular order. We begin with the one with the smallest value. It's the least number, which means the number—the smaller in numerical value, and then from least to greatest. This is where I use a lot of—I can't think of the word right now either in English or Spanish, but it's something I do with my hands when I make my students repeat. The word is least and I make this sign with my hands that I'm referring to something small, and it looks like this.

When I say the word is the greatest, I open my arms and I show like something really big, and it looks like this. Not only I'm

saying it and repeating it, but I'm also making this motion with my hands that this is also a mnemonic tool where they can clearly identify the least and the greatest, and then find the number in the middle. Did I answer the question?

Interviewer: Yes. Is there only one case when you are calculating the median, or are there other cases?

Mrs. Aguirre: *Es que puedes usar el medio… lo necesitas a veces, por ejemplo, si quieres graficar diferentes medidas que hay…entonces, casi para todo hay ciertas medidas que necesitas encontrar el medio, por ejemplo si vas a calcular los cuartos…aquí me va a fall*ar un *poquito, pero es el* interquartile, *y* when I'm trying to do a box and whiskers plot. *Es muy interesante, un buen ejemplo en el cual los niños leen cuál es el medio, porque voy a encontrar el medio primero de todas, esa información de toda la data que estoy usando y luego el medio otra vez.*

Interviewer: *Y cuando tienes un número par de datos o un número impar de datos, ¿hay alguna diferencia en cómo calculas la mediana?*

Mrs. Aguirre: *Sí, porque*…because not always you have sets of numbers in which you gonna have just one number in the middle. You gonna have sets of data in which you end up with two numbers at the center. That's when you have to find the mean of those median numbers. How you do that? Well, you add those two numbers, then divide by two, and that's your new median.

Interviewer: How do you translate the word mean in Spanish?

Mrs. Aguirre: *Promedio. La palabra es "el promedio".*

The translanguaging exercise above revealed both strengths and weaknesses in Mrs. Aguirre's knowledge of (a) the mathematical concepts involved, (b) the use of academic language in both English and Spanish, and (c) her instructional techniques. The mean and the median are statistical concepts that are introduced in the middle school, and Mrs. Aguirre demonstrated familiarity with both concepts, even though she struggled with their translation in Spanish – she used *promedio* (average) for *mean*, *medio* (middle) for *median,* and *cuartos* (fourths) for *quartiles.* The correct translation in Spanish for those terms are *media*, *mediana,* and *cuartiles,* respectively. Moreover, she used the English word *data* instead of the Spanish word *datos.*

Furthermore, Mrs. Aguirre was able to recognize the word *mediana* when I asked her a question in Spanish about that concept, probably because *median* is a cognate, but she kept using the word *medio* nevertheless, which is a morphologically assimilated loan from English to Spanish. During our translanguaging exercise, she used the loan shift *grado,* borrowing the meaning of the English word *grade* (García, 2009). In Spanish, *grado* means either *grade* (as in a level in school) or *degree,* whereas *calificación* means *grade* in the context that Mrs. Aguirre used it.

There was an instance of code-mixing and intersentential code-switching (García, 2009) when Mrs. Aguirre was trying to explain in Spanish how to construct boxplots: "*…por ejemplo, si vas a calcular los cuartos…aquí me va a fallar un poquito, pero es el* interquartile, *y* when I'm trying to do a box and whiskers plot." Another instance of intersentential code-switching occurred when I asked a question in Spanish and she was

supposed to respond in English: "*Sí, porque*…because not always you have sets of numbers in which you gonna have just one number in the middle." The extension of the first episode of intersentential code-switching demonstrates that the dominance of the English language in South Texas' classrooms is strong.

Mrs. Aguirre's concern for making the material accessible to all students was evident during the translanguaging exercise. For example, she indicated that she would use the number line (a visual aid) to clarify the concepts of mean and median, and would use gestures to convey meaning. Furthermore, she brings interesting insights about the measures in question, such as the location of items in a data arrangement being more important that than their individual values when finding the median. In parts of the conversation, sociological aspects are mixed with mathematical concepts in order to promote behaviors that Mrs. Aguirre perceives as necessary for her students to adapt to the life in the United States. For example, she indicated that the main factor when choosing between the mean and the median is to achieve a favorable or convenient outcome, and that was the point of her example about the students' grades. By doing this, she also revealed a statistical misconception: The median is the result that occurs in the middle of a process.

It was not clear if Mrs. Aguirre knew that the mean can be interpreted as a balance point. She did refer to data points as weights, but chose to talk about the mean in terms of its algorithm. Additionally, she mentioned the two different cases for the median only when pressed, but even then it was not clear if she remembered that the simplest case occurs when the total number of data items is odd.

Mr. Long. I asked Mr. Long to explain the difference between the concepts of mean and median to me as if I were an EL student. The transcription of the translanguaging exercise follows:

Interviewer: Let's imagine that you are in the classroom and you have to explain the concepts of 'mean' and 'median' to your students. In particular, you have to explain this concept to an ELL. How would you explain the difference between those two concepts to an ELL?

Mr. Long: Mainly relate it to something that they already understand. For example, for the learner who already knows English and that's the only language they speak, making the relationship between something, for example, the 'median' is the one in the middle and that's what they're looking for. And understanding the vocabulary and adding that visual, the difference between bilingual. And if they didn't understand the vocabulary, I'd try to find a problem to demonstrate the difference between 'median' and 'mean so they could make that relationship.

Interviewer: Imagine I am your student, I am an ELL and you're trying to explain the difference between 'mean' and 'median' but you know I won't be able to understand this explanation in English. Can you try explaining it to me in Spanish?

Mr. Long: You want to say it in Spanish, right, for an ELL student, to teach the difference between 'mean' and 'median' in Spanish?

Interviewer: Imagine he's a 6th grader and he doesn't know English at all and you're trying to explain it in Spanish. How would you explain it in Spanish?

Mr. Long: Okay (Pause). Median *es lo mismo que el número del medio. Tienes un…diferentes números y los pones en orden. Al ponerlos en orden, buscas el número que está en medio y la orden que los pones es chiquito a grande o de grande a chiquito. El número de en medio es el* median. *Ahora el* mean… *es el promedio, el promedio es de que sumas los números y luego te divides por las cantidad de números que tienes.*

Mr. Long translated *mean* as *promedio* (average), and *median* as *el [número] del medio* (the number in the middle). This shows that two common misconceptions with respect to the terms "mean" and "median" emerged in Mr. Long's Spanish translations of the terms: (a) the view of the term "mean" as synonymous with "average", and (b) the view of the median as "something in the middle" (see Jacobbe & Carvalho, 2011). The fact that Mr. Long chose to define the mean algorithmically and not conceptually may indicate either a lack of conceptual understanding on his part, or his desire to be understood by his students. That being said, his algorithmic definition of the median had good elements as well as missing elements, since he mentioned that the median would be the same regardless of the chosen order (from least to greatest or vice versa), but did not consider the case when there is an even number of elements in the data set.

Mrs. Quiróz. For this translanguaging exercise, I asked Mrs. Quiróz to teach the concepts of mean and median as if I were one of her EL students. Once we discussed

what my level of English understanding was (intermediate), she proceeded to explain the concepts as follows:

Mrs. Quiróz: Okay, so we have these five numbers, and they're listed with commas, so we're going to list them again vertically. That means up and down. We have—pretend that these are five grades on five assignments from class. These are your five grades, and you wanna find out what your grade is for the class. Well, the way I do that, as your teacher, is I'm going to average them using mean. It's spelled M-E-A-N, but it doesn't mean like I'm gonna be mean to you. This is a math chart, and I'm gonna show you what it means. We're going to line up the numbers so that all the ones place are lined up, and we're gonna use graph paper. We're going to add them. We have a sum. Okay, in English when you add all the numbers together, the answer you get with addition is called a sum. I write that out. Okay. Now that we have our sum we're gonna count how many grades there were. One, two, three, four, five. Okay, I have five. Now we're gonna divide the sum by five, and we'll do the division problem, and so this quotient—this would be your grade. This is called finding the mean of this group of numbers. Then—not at the same time—later in the day I'm going to teach—what was it? Median or mode?

Interviewer: The difference between median and mean.

Mrs. Quiróz: Okay, so we did mean just now, right? Okay, now we're gonna take the same five numbers, and we're going to list them least to greatest.…We're gonna put the number that's the smallest first, and then the next smallest—all the way until we get to the biggest number. That's called least to greatest. Now we're going to put our fingers—let's put our right finger at the top and our left index finger on the bottom, and we're going to keep moving our fingers from the bottom to the top and the top to the bottom until we get to the middle number. That middle number is your median. This is two different ways of finding an average because these are both ways of finding an average. We have the word—oh, you said only in English, so it means I can't translate.

Interviewer: Go ahead. Do what you do. You can switch if you want.

Mrs. Quiróz: Okay, so the word average means *promedio*. Do you hear the word *medio?* It's sort of like what's the middle number? What's the number that would represent all of these numbers? In math we have different ways of doing that. We have mode that we haven't looked at. We have median, which is when you line them up, and you find the number in the middle. Look at the word median. Do you see how it looks very similar to the word *medio*? It's the number *en medio*. It's a number in the middle. That's why it's called median, and mean—I have no idea why it's called mean! What we do is we add the numbers, and then we divide by the

number of—for example grades that we have. When you use division, that's mean, but when you look for the number in the middle, that's median. Those are two different ways of finding an average.

Interviewer: Now, can you do the same thing—but in Spanish?

Mrs. Quiróz: Okay, I use the word promedio for mean, so I don't know if I'm translating that correctly….Okay. *Ahora vamos a ver dos modos distintos de sacar un promedio de un grupo de esos cinco números, y aquí en el libro te los ponen así como letras con comas pero eso es un poquito confuso, así que vamos a poner los números verticalmente en la hoja, y vamos a ver los cinco números. Y para que lo entiendas un poquito mejor, una situación real, en vida real, que maestros usan esto es para sacar tu promedio de tu calificación para mi clase. Entonces, vamos a darnos cuenta o hacernos cuenta que estas son calificaciones, entonces el primer modo que yo como maestra puedo usar para sacar un promedio es…vamos a sumar los números y luego vamos a dividir por cinco. ¿Y por qué cinco? Porque hay cinco calificaciones. Si hubiese siete, hubiésemos dividido por siete. Pero aquí tenemos cinco números, así que vamos a dividir por cinco. Una vez que sacamos el resultado, ese número es un promedio de todos los cinco números, y si ves el número, vas a ver que es un poquito más grande que algunos de los números y un poquito más chiquito que*

otros, entonces es un promedio. Y puedes escuchar la palabra medio, aunque es un número en medio, un número que representa todos los números. Okay, *hay otro modo que te voy a enseñar para también sacar un promedio. Vamos a poner los números, pero esta vez sí importa en qué orden los ponemos. Así que vamos a empezar con el número más chiquito y vamos a seguir hasta que llegamos al número más grande. Tienen que estar en orden. Y esta vez lo que vamos a hacer es que vamos a encontrar el número que queda en medio, así que tienes dos números arriba, dos abajo, ¿cuál es el que está en medio?* Okay. *Ese también es un promedio, y si lo comparas con el otro que sacamos puede ser que sí son muy similares. Entonces esos son dos modos distintos en matemáticas que tú puedes sacar un promedio.*

Interviewer: *Mrs. Quiróz, tu explicaste sólo uno de los dos casos para encontrar the median. Explicaste que tenías cinco números, ¿verdad? ¿Qué pasaría si tuvieras seis?*

Mrs. Quiróz: *Vamos a tomar otro ejemplo. Vamos a tener seis números. Entonces, obviamente no va haber un número de en medio, porque como tenemos seis y ese es un número par, vamos a tener tres arriba, tres abajo, y un hueco en medio, ¿verdad? Pero lo que podemos hacer es que podemos tomar el tercer número y el cuarto número donde esta ese hueco y podemos investigar si el número existe en medio de esos dos números. Entonces lo vamos a escribir*

así y vamos a obtener todos los números que existen entre esos dos números, y vamos a seguir buscando hasta que encontramos un número en medio. Si encontramos que sigue siendo par, entonces puedes decir que va a tener una mitad.

Interviewer: *Entonces en el caso cuando tienes seis números, ¿cuál es la mediana?*

Mrs. Quiróz: *El número que existe entre el segundo número y el tercer número. O sea que si tienes una lista. Vamos a suponer que tienes 2, 4, 6, y 8. Entonces tú dices, pues el número de en medio debe de estar entre el 4 y el 6, pero no hay, no hay nada escrito. Si es cierto, pero tú sabes que existe entre 4 y 6. ¿El 5? Muy bien, el número de en medio.*

Interviewer: *¿Tú explicas así en clase? ¿Das explicaciones completas en español como esta?*

Mrs. Quiróz: *Sí. En el caso real de estar en una clase, yo sí trato…sí hago eso, o voy al texto, porque ya muchos de los textos tienen la traducción en español. Y buscamos la palabra correcta antes de empezar la lección, y yo les digo…nunca trato de ocultar cuando no se algo. Yo quiero que los niños vean cuando no sé algo, porque lo que voy a hacer es voy a ir a buscar cuál es la palabra correcta para que durante la lección sí yo estoy usando el vocabulario apropiado. Entonces, eso es lo que haría en una situación real, porque sí me gusta usar el vocabulario preciso.*

Interviewer: *Me podrías explicar más sobre los ELLs que tienes ahora. ¿Tú dirías que acaban de venir o ya tienen tiempo de estar aquí en los Estados Unidos?*

Mrs. Quiróz: *Tengo dos niños, un niño y una niña, es muy difícil para ellos ahorita. Acaban de llegar. Para seguir la lección en inglés sí es un reto para ellos. Nada más ahorita después de que yo…tres semanas y media ya están empezando a agarrar la confianza… "yo no entiendo eso", "me lo puede traducir". Pero son muy tímidos. Un niño…sé que hay alguna otra cosa sucediendo, no es nada más el lenguaje. Pero la niña sí está avanzando como uno pensaría, está ganando más inglés con cada día, con la interacción con otros niños y con las otras clases de lectura y ciencia, sí está avanzando en su inglés, pero hace 4 semanas cuando yo llegué, casi nada de inglés.*

Interviewer: *¿Y son de México?*

Mrs. Quiróz: *Sí, los dos son de Matamoros.*

During this exercise, Mrs. Quiróz gave detailed explanations in Spanish, and claimed to do the same with her students (I confirmed this when I observed one of her classes directly and when I watched videos of two more of her classes). Among the strengths of Mrs. Quiróz explanations of the concepts of mean and median are that she (a) correctly called them both *averages* in English and *promedios* in Spanish; (b) came up with concrete ways of finding the median when the number of data items was odd (using fingers to cover the numbers); and (c) described a completion method in Spanish to find

the median in the case when the number of data items is even (if the two middle numbers are 4 and 6, one must complete the sequence and the median of those numbers is the median of the whole set). However, this strategy is useful only when the two middle numbers are close to each other and they are integers; moreover, it is not a good strategy if, after completing the sequence, there are still an even number of values.

The translanguaging exercise revealed that Mrs. Quiróz did not know the Spanish word for *median*, and she also translated mean as *promedio* (average) but acknowledged that she might be wrong. Furthermore, she only defined the mean by using its algorithm, and did not define it in other ways, such as a balance point. When the exercise was compleated, we continued our conversation in Spanish. Mrs. Quiróz told me be about the two ELs she had in the classroom that was being observed by the MIELL project, a boy and a girl that had recently crossed the border. They were both very shy, but the girl was adapting faster than the boy, who, according to Mrs. Quiróz, may have had more problems than just the language.

Synthesis. In the three translanguaging exercises I described here, I used a different approach with Mrs. Aguirre than the one I used with the other two participants. On one hand, my approach with Mrs. Aguirre was closer to what translanguaging is as a pedagogy, where the input and output languages were switched in a structured way. Even though I tried to establish the tone of a peer-to-peer conversation, I am aware that Mrs. Aguirre might have perceived me as being in a position of power. It was difficult for her to use the academic mathematics register in Spanish and would switch to English in order to express herself in a clearer way. On the other hand, my approach with Mr. Long and

Mrs. Quiróz was less structured, and they talked to me as if I were an EL student. This might have helped them be more comfortable with the exercise.

Furthermore, the translanguaging exercises became more complex as I gained more experience as an interviewer. Consequently, Mr. Long's translanguaging exercise was considerably shorter than Mrs. Aguirre's and Mrs. Quiróz's since he was the first participant I interviewed.

All three participants did not know the Spanish words for *mean* and *median*, and this revealed that their concerns about precision are mostly reserved for English mathematical terms. Likewise, their conceptual understanding of these terms was not deep, regardless of the language they used. They all defined the mean in terms of its algorithm; and none of them mentioned that the median was not susceptible to extreme values. When explaining how to find the median, they all referred to the simplest case (when there is an odd number of items in the data set), and only mentioned the other case when pressed. This might have to do with these teachers' tendency to make the material as accessible as possible for all students, or with their knowledge that the simplest case is the one that is assessed in standardized tests.

Of all three participants, Mrs. Quiróz was the one that spoke Spanish the most, and did not code-switch to English. I also observed that, when explaining the median in Spanish, both Mr. Long and Mrs. Quiróz used the word *chiquito* (smallish) to refer to the smallest value of the data set. Since they were talking to me as if I was an EL student, they used a Spanish diminutive to establish rapport with me.

V. ANALYSIS OF CLASSROOM OBSERVATIONS

Introduction

In this chapter, I discuss and analyze the digital data collected during classroom observations with the three participants in my case study. My analysis focuses on answering the following research questions:

2. What are the participants' similarities and differences in their support of students' first language?

4. What are the participants' similarities and differences in their translanguaging use while explaining mathematical concepts?

The transcriptions of the participants' classroom observations closely reflect the way they and their students spoke in English and Spanish. Upon transcribing the data, utterances were not easy to follow, and/or contained grammatical errors. The English translation of Spanish utterances was not verbatim. Rather, the intent was that English-speaking readers were able to understand the essence of what was said. However, the original Spanish utterances are preserved so Spanish-speaking readers can fully appreciate them. The chapter concludes with a synthesis of the classroom observations, where the use of translanguaging by the three participants is compared.

The classroom observations were conducted in October 2013, January 2014, and April-May 2014. Mr. Long and Mrs. Quiróz were interviewed before any analysis of their classroom videos. Therefore any classroom dynamics were unknown. The interview with

Mrs. Aguirre was conducted days after her second classroom observation. In this case, the digital data for her first classroom observation had been partially analyzed.

First Classroom Observation

Mrs. Aguirre. Mrs. Aguirre started the lesson by determining her 8th grade students' pre-conceptions of the word *percent*. She gave each student a sheet of paper with the word *percent* written in the middle of the page and asked them to make a concept map by writing or drawing something related to the word *percent* that was based on their own experiences. Since *percent* has a cognate in Spanish (*por ciento*), this task could have been engaging students' metalinguistic awareness, requiring each to draw connections with their conceptual knowledge. Mrs. Aguirre gave her students three minutes to work on their maps, and in the meantime supervised their work. She noticed that Raquel, one of her students that sat in the first row, was struggling and had not worked much on the conceptual mapping task:

Mrs. Aguirre:	Anything else? You can put words. Where have you seen it [the word *percent*] before?
Raquel:	*En las tiendas* [in stores].
Mrs. Aguirre:	*En las tiendas.* Okay. How about a store? At the mall? [Raquel nodded]. Okay, you can put the mall. [Raquel wrote *mall* on her worksheet]. And where do you see the percent at the mall?
Raquel:	*Como en…cualquier cosa.* [Like in …anything].
Mrs. Aguirre:	*¿Cualquier cosa?*

Raquel: *Zapatos.* [Shoes. She pointed at the words "shoe sales" in her work].

Mrs. Aguirre: Shoe sales? Very good! What else? When you go to the register and you pay the bill, let's say that the price was $10.00 and you end up paying $11.00. Why is that?

Raquel: *Hay más* [There is more].

Mrs. Aguirre: *¿Qué es más?* [What is more?]. What are they adding to your bill?

Raquel: *Más dinero* [More money].

Mrs. Aguirre: *Más dinero. Pero, ¿qué es ese dinero?* [More money. But, what is that money?]

Raquel: Taxes. [She wrote down the word *taxes* on her worksheet. See Figure 1].

Mrs. Aguirre: Taxes! Very good!

During the previous conversation, Mrs. Aguirre used translanguaging to help Raquel unpack her prior knowledge about percent. Shopping was an activity that Raquel was able to connect to the concept of percent (shoe sales and taxes in this case) and Mrs. Aguirre accepted Rachel's responses in either English or Spanish. Furthermore, Mrs. Aguirre made frequent use of revoicing during this episode as a way to encourage Raquel to express her ideas or explain herself. *Revoicing* occurs when a teacher restates or incorporates student comments or answers into questions or statements to build a discourse based on the contributions of students (Inan, 2014).

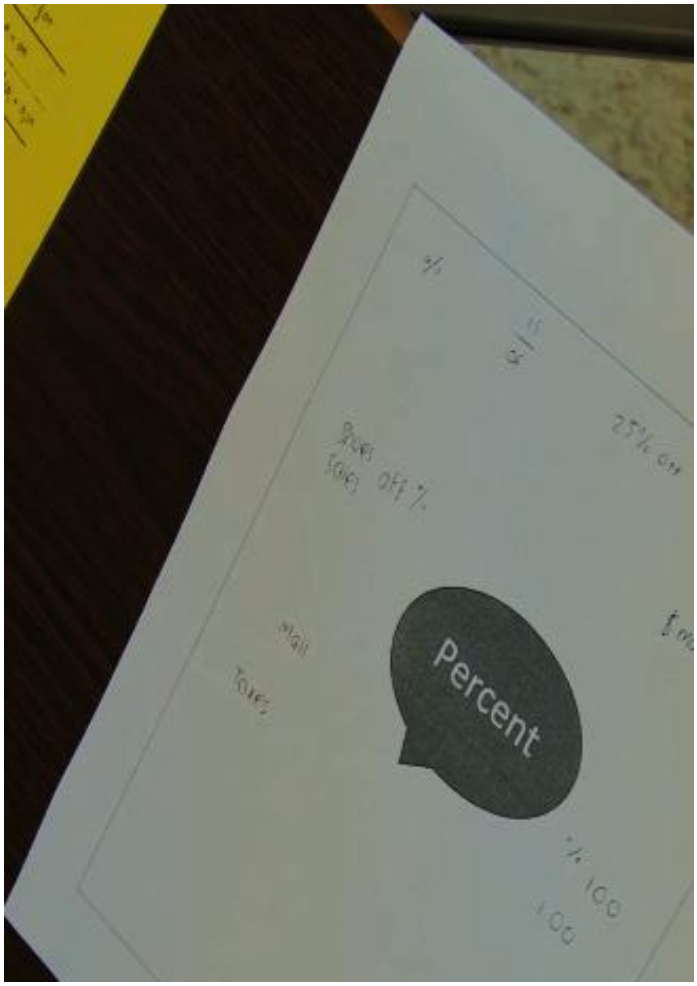


Figure 1. Rachel's concept map of the word "Percent". Rachel's work showed her knowledge of the percent symbol. It also showed the connections she established between percentages and shopping, such as discounts and taxes.

Next, Mrs. Aguirre started a class discussion by sharing individual students' work[1]. The work was shown to the entire class and students were asked to explain their responses. When it was time for Arturo (a recent immigrant) to share his work, he described one of the ideas in his concept map in Spanish:

[1] This vignette has been previously reported (see Sorto, Mejía Colindres, & Wilson, 2014).

Mrs. Aguirre: Arturo? *No sea penoso. A ver, platíqueme ¿de qué se trata esto?* [Don't be shy. Let's see, talk to me. What is this about?]

Arturo: *Pues esto se trata de una playera de que está al cincuenta por ciento y cuesta cien dólar.* [This is about a T-shirt that is 50 percent off and it costs 100 dollars].

Mrs. Aguirre: Okay [She pointed at Arturo's work. See Figure 2].

Arturo: *Entonces nada más la tuvieron así durante dos días y la bajan a cincuenta dólar.* [It had that price only for two days and they lowered the price to 50 dollars].

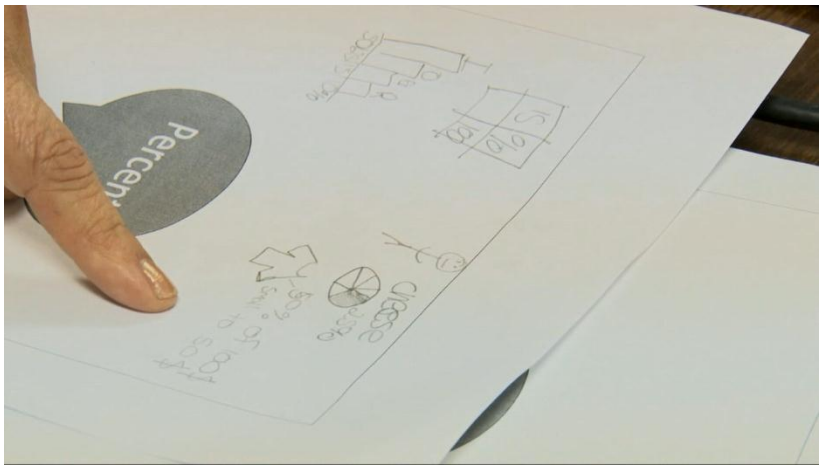


Figure 2. Arturo's concept map. His work contained his discounted T-shirt example and words in English.

At this point, Mrs. Aguirre had many pedagogical choices, including translating the explanation herself, since she understood what Arturo was saying. However, she chose to utilize the paraphrasing skills of other bilingual students in the class to make Arturo's explanation part of the entire class discussion. Thus, she solicited Sara's help:

Mrs. Aguirre: Okay. *A ver ¿Qué dijo?* Sara, what did he say? Help me.

Sara: *De que si el 50 por ciento lo resta a 100…*[If 50 percent is subtracted from 100…]

Mrs. Aguirre gently re-routed Sara's efforts, since she was interested in Sara's English interpretation of what Arturo said in Spanish.

Mrs. Aguirre: *En inglés*. [In English] What did he say?

Sara: From $100.00 he took out 50 percent off it, kinda like that, right?

Mrs. Aguirre: Uh-huh, yes.

Sara: You took out 50 percent, and yeah, I don't know. Like from $100.00 you took out 50 percent, $50.00 off. I think, yeah.

The previous translanguaging episode illustrated Mrs. Aguirre's awareness of the paraphrasing skills of her students. Bilingual English proficient students use their knowledge of the English language and U.S. cultural traditions to speak for their parents, other relatives and members of their communities (Orellana, Dorner, & Pulido, 2003). Consequently, it was natural for Sara to paraphrase what Arturo said.

Sara's paraphrasing in the previous vignette had potential benefits for all students. By acting as a Spanish interpreter, Sara may have been deepening her understanding of the mathematical concept by interpreting what was said and expressing it in her own words. When Sara paraphrased Arturo's example, she made use of her decontextualizing skills, which in turn were an important part of reasoning abstractly and quantitatively. On the other hand, Arturo was able to use his preferred language to explain his reasoning of a task given in English, the language in which he was developing proficiency. Allowing students to express their solution to mathematical tasks in their

preferred language helped them connect to their prior mathematical knowledge and everyday experiences.

Additionally, Sara's interpretation of Arturo's words may have helped him learn the academic language from one of his peers, complementing the formal academic instruction given by Mrs. Aguirre. At the same time, other students in a similar situation as Arturo's and Sara's were also likely to increase their mathematical understanding of the material.

A third relevant translanguaging episode transpired when Mrs. Aguirre was discussing percentage of change with her students. In this case, translanguaging was utilized to deepen understanding and sociopolitical engagement through a discussion involving discounts:

Mrs. Aguirre:	If they give you a 100% discount, how much are you paying?
Students:	Zero! It'll be free!
Mrs. Aguirre:	Do you see a lots of sales like that?
Students:	No!
Mrs. Aguirre:	Not really, right? But let me tell you. There was a one sale last year at *Pizza Patrón*, and all you needed to do was get in line and say: *Pizza por favor!* [She pronounced it as a native English speaker would]. When you said *Pizza por favor!* [she used the same English accent] they would give you free pizza. And you saw lines of people. They brought grandma and grandpa and *tío* [uncle] and everybody. Well…they didn't do it this year again, because there

> was always somebody to complain: “I can’t speak Spanish and say *Pizza por favor!* [English accent] and you are discriminating against me!” So no more *Pizza por favor!* [Students laughed].

In this vignette, Mrs. Aguirre used a discussion about discounts to address issues of linguistic inequality occurring in the community. Mrs. Aguirre’s use of Spanish (with an English accent) was strategic, and aimed to the development of critical consciousness in their students.

Mr. Long. There were only six students in Mr. Long’s resource mathematics classroom, and the environment was relaxed. Students participated frequently and used translanguaging freely among themselves and to answer Mr. Long’s questions. Mr. Long also used translanguaging during instruction, although most of the lesson was given in English. Mr. Long frequently encouraged his students to translanguage in inner speech, and would use Spanish strategically to engage students.

The lesson focused on ratios and proportions. Mr. Long solved problems involving proportions by cross multiplication, a method that tends to obscure the mathematics behind it. However, this was the solution that the curriculum materials required. He also used an interactive board and switched colors often when using the pen to highlight actions to be taken, such as cross multiplication arrows. Mr. Long started the lesson by discussing the meaning of the words *product* and *proportion*.

Mr. Long: Johnny, what do you think *products* mean?

Johnny: I have no idea.

Mr. Long: Think of a word….Johnny, what do you think product mean? Think in Spanish if you think.

Johnny: *Como un objeto* [It's like an object].

Mr. Long: *Un objeto.* It could be an object, but we are thinking about product now. In mathematics we have operations. What operation you think we are talking about when we say products? Crissy?

Crissy: *¿Producto?*

Mr. Long: *Producto.* And then when you have *producto*, is what?

Johnny: *¿Un porcentaje?* [A percentage?]

Mr. Long: Okay, we are getting close. So now, the product is something that every time you see that word, I want you to associate with this [he wrote × on the board]. Multiplication…Now, the word *proportion*. Think of a connection. Sellina, when you hear it, pro-PORTION, what comes to your mind first? [Long pause]. *¿Qué palabra se te viene a la mente cuando oyes proporción?*

Sellina: Ratios?

Mr. Long: Ratios? Very good. We've been working on ratios.… I am going to give you an example. Suppose you are going to cook something for your family. You know how much you are gonna cook, for example…you're gonna make a…*un caldo de res* [beef broth]. But normally you do it for yourself. You cook it for yourself but now you have to make a bigger pot because your *tía* [aunt], your *tío* [uncle] and your *abuelo* [grandfather] are gonna come.

Sellina: The whole family.

Mr. Long: Your whole family. So you have to set up a proportion.

Mr. Long wrote the problem $\frac{6}{12} = \frac{25}{50}$ on the board and told students that the goal was to determine if the two ratios represented a proportion. Then he proceeded to solve the problem by cross multiplication (see Figure 3).

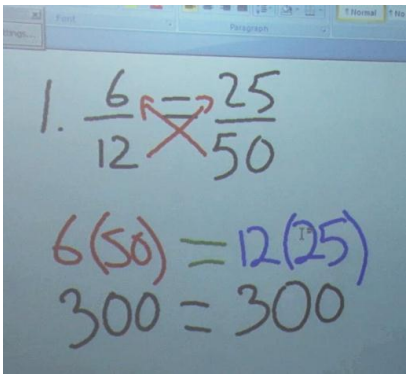


Figure 3. Mr. Long demonstrating that $\frac{6}{12} = \frac{25}{50}$ by cross multiplication. Different colors were strategically used to highlight operations.

Mr. Long: Johnny, do you understand the cross product? You see a little cross right here? *¿La cruz? Multiplicas cincuenta por seis y esta parte es trescientos* [You multiply 50 by 6 and this part is 300]. Okay. And then the other side…you get the cross product twelve twenty-five, equals to three hundred.

In the explanation above, Mr. Long used Spanish to explain one part of the cross product, and English to explain the other part. Therefore, Mr. Long was not translating what he previously said. Since the two products in the cross multiplication turned out to be the same, his use of two languages in this case suggested linguistic equality.

Later, Mr. Long proposed a proportion involving an unknown, $\frac{2}{3} = \frac{N}{27}$. By using cross multiplication, he obtained $3(N) = 27(2)$.

Crissy: *¡No está la en!* [There is no en!]

Mr. Long: I know! It's not there. What do you think? *N* is a variable. We don't know what it is.

Crissy: *No le entiendo.* [I don't understand].

Mr. Long: Right now you are going to see it. We just do the first step, like you did on the others part. It's the first step. Are you with me, Johnny?

Johnny: *La ene puede ser cualquier número.* [The en can be any number].

Mr. Long: *Eso es lo que vamos a buscar,* okay? *Pero los pasos son iguales* [That´s what we are going to find out, okay? The steps are the same though].

Diego: So it cannot be just any number?

Mr. Long: Right now you're gonna see which number it is. But we need to solve it. *Vamos a encontrar la solución aquí, para ver qué número va ahí.* Because it has to be a proportions. We don't know what that letter is, but we're going to find it.

Students seemed to confuse the variables they had used with linear functions, and the *N* in this problem, which was best described as an unknown. In this case, Mr. Long should have emphasized that if the two ratios represented a proportion, then *N* can only take one value. In any case, the use of translanguaging by both teacher and students in

this situation reflected the need for both parties to make sense of a complex issue through the use of two linguistic repertoires.

Mr. Long wrote the next step on the board, $3N = 54$, being careful that the equal signs in all steps were aligned (see Figure 4). Once more, Mr. Long asked their students to translanguage in inner speech; in particular, when making mental calculations.

Mr. Long: Now, there's different ways you can do this. Mentally…*Mentalmente. Lo puedes pensar. Tres por un número es igual que 54.* [You can think about it. Three times a number equals 54]. You can look at your multiplication chart [he showed the multiplication chart in the interactive board, but it was a 12 by 12 chart]. I don't have it here, but you can also think it.

Laura: Can we divide?

Mr. Long: We can divide.

Laura: Three!

Mr. Long: Hold on, hold on.

Laura: Eighteen!

Mr. Long: Hold on, Laura.

Crissy: She goes ahead. *¡Bien rápido que va!*

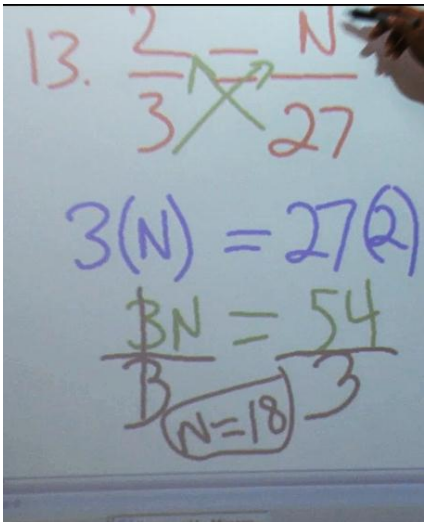


Figure 4. Mr. Long and his students finding *N* in the proportion $\frac{2}{3} = \frac{N}{27}$. As in the solution to the problem in Figure 3, he used different colors to highlight operations on the interactive board, and carefully aligned the equal signs for every equivalent equation.

Mrs. Quiróz. The class started with a review of the previous week's lesson on equivalent fractions. Mr. Quiróz distributed a worksheet with review problems to her 6th graders, and made sure that Beatriz, a recent immigrant, got one of the worksheets by asking her: "Did you get one, Beatriz? *¿Sí?*" For much of the class Beatriz was the focus of Mrs. Quiróz's attention. Mrs. Quiróz would ask Beatriz questions in Spanish frequently to check for understanding. Beatriz's classmates did not seem to mind Mrs. Quiróz's focused attention on Beatriz.

Mrs. Quiróz: *Beatriz, voy a revisar el primero, así que voy a poner las fracciones en el pizarrón* [Beatriz, I will go over the first one, so I will write the fractions on the board].

Mrs. Quiróz asked one student to read the directions for Part I on the worksheet, and then asked another student to read them again. After this, she asked students to find the keywords in those directions:

Mrs. Quiróz: Please circle two words, "all" and "equivalent." Beatriz, "all" *es "todo", "todas las fracciones"*.

Sally: I need a pencil.

Mrs. Quiróz: I don't have any more pencils, they all walked away last week. If someone could lend Sally a pencil I would really appreciate it. *Beatriz ¿tienes un lápiz extra que le puedas prestar?* [Do you have an extra pencil that you could lend her?]. Patty?

As it is evidenced in the paragraph above, Mrs. Quiróz tried to make Beatriz, a shy and quiet student, participate in every single event that occurred in class. Mrs. Aguirre chose to involve Beatriz in classroom activities by means of code-switching that was responsible as opposed to random, since there was a pedagogical goal behind Mrs. Quiróz's decision of switching languages (García, 2009).

Next, Mrs. Quiróz wrote the fractions $\frac{1}{4}, \frac{2}{8}, \frac{3}{5}, \frac{4}{24}$ on the board, and proceeded to find out if they were all equivalent or not.

Mrs. Quiróz: This is the part I am really interested in, because many of you struggled last week. *Beatriz, esta es la destreza que muchos batallaron la semana pasada. Entonces quiero…*make sure you are writing what I'm writing. *Escribe lo que yo escribo*. And if there is

any question at all you raise your hand [Mrs. Quiróz raises her hand] so we can go over it. Alright?

At this point, Mrs. Quiróz drew a rectangle to the left of $\frac{1}{4}$ where she would write the fraction that, when multiplied by $\frac{1}{4}$, gives $\frac{2}{8}$ (see figure 5).

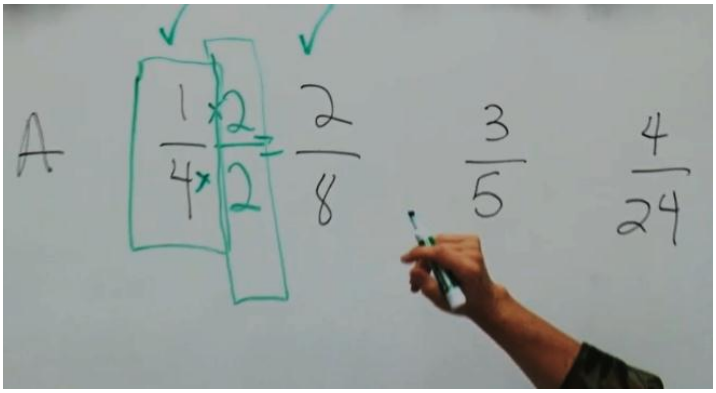


Figure 5. Mrs. Quiróz testing if $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent. This was the first step to test if the fractions $\frac{1}{4}, \frac{2}{8}, \frac{3}{5}, \frac{4}{24}$ were all equivalent or not.

Mrs. Quiróz: We are going to begin with the one fourth. *Un cuarto. Y te preguntas ¿qué hice con este numerador para llegar a dos? ¿Cómo lo puedo convertir?* What am I going to do to the one? Ted?

Ted: Multiply times two.

Mrs. Quiróz: *¿Está bien, Ted? ¿Lo voy a multiplicar por dos? ¿Uno por dos es?*

Ted: *Dos.*

Mrs. Quiróz: *¡Dos!* Yes, perfect! Now, everybody…What would your denominator have to be if this is going to equal one?

Students: Two! The same as the numerator!

Mrs. Quiróz: The same as the numerator. Does anyone remember the property that we are using?

Pepe: Division.

Mrs. Quiróz: No. Let me ask you this, let me guide you a little bit. If I were the number four and I were to multiply myself with a one, would I still keep my *identity?* [She emphasized the word].

Sally: Yes! They're not changing.

Mrs. Quiróz: The property is called the identity property…*La propiedad, Beatriz, la propiedad de identidad. Cuando multiplicas con un uno, no se cambia el valor* [When you multiply by one, the value does not change].

Once Mrs. Quiróz demonstrated that $\frac{1}{4}$ and $\frac{2}{8}$ were equivalent, she told her students that, when finding out if $\frac{3}{5}$ was equivalent to the other fractions, it was better to use $\frac{1}{4}$ again. Her work is shown in Figure 6.

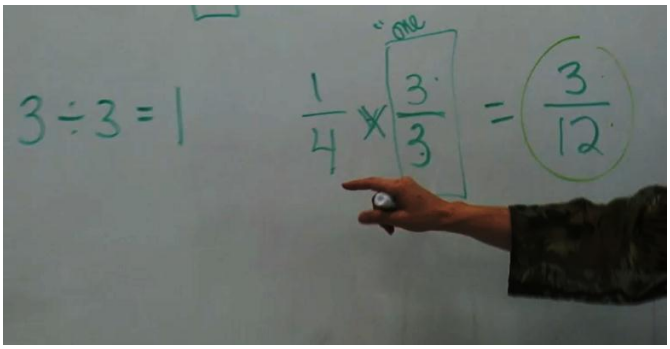


Figure 6. Mrs. Quiróz showing that $\frac{1}{4}$ is not equivalent to $\frac{3}{5}$. She multiplied $\frac{1}{4}$ by $\frac{3}{3}$ and emphasized that $\frac{1}{4}$ was being multiplied by one in two different ways: By explicitly noting that 3 ÷ 3 = 1 and by writing "one" on top of $\frac{3}{3}$.

Mrs. Quiróz: What would I have to do the one to get to three?

Lucy: Multiply it times three.

Mrs. Quiróz: Multiply it times three. Let's do it down here so it's a little more clear. Beatriz, *si tienes un numerador de tres aquí, que tiene que ser el denominador para que esto se queda valor de uno* [Beatriz, if you have a numerator of three here, what should the denominator be to get one? (When Mrs. Quiróz said *uno*, she wrote "one" on the board)].

Beatriz: [Inaudible].

Mrs. Quiróz: *Si voy a multiplicar, ¿con qué número? ¿Qué tiene que ser este número?* [If I am multiplying, by what number? What should that number be?]

Beatriz: [Inaudible].

Mrs. Quiróz: Okay. *Y esto es lo que quiero batallar…lo que quiero revisar para que no batalles. ¿Qué puedo dividir…? ¿Tres dividido por qué me da uno?* [This is what I want to review so you don't struggle. Three divided by what gives you one?]

Beatriz: *Tres* [Three].

Mrs. Quiróz: *¡Exactamente! Tres dividido por*…[That's right! Three divided by…]

Beatriz: *Tres* [Three].

Mrs. Quiróz: *Tiene que ser tres* [It must be three].

Since $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12} \neq \frac{3}{5}$, Mrs. Quiróz concluded that the four fractions given in exercise A were not equivalent. She emphasized that it was not necessary to investigate if $\frac{4}{24}$ was equivalent to $\frac{1}{4}$, since the directions referred to *all fractions* being equivalent, and they had already determined that one of them was not. Next, Mrs. Quiróz asked students to solve problems B and C independently, while she supervised their work.

Problem B asked students to determine if the fractions $\frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{5}{10}$ were equivalent, and Mrs. Quiróz approached Beatriz and helped her compare the first two fractions:

Mrs. Quiróz: *Ahora vas a analizar letra B igual. Vas a poner uno y medio y vas a preguntarte, ¿con qué puedo multiplicar para que me salga dos cuartos? ¿Qué tiene que ser esta fracción aquí?* [Now you are going to do the same to analyze B. You are going to write one half and ask yourself, what can I multiply it by to get two fourths.]

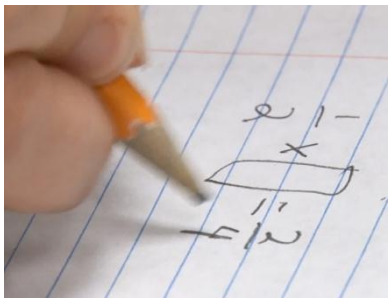


Figure 7. Mrs. Quiróz acting as Beatriz's scaffolding tutor. She did this by first setting up the problem about equivalent fractions in Beatriz's journal.

Mrs. Quiróz wrote the problem for Beatriz, drawing a rectangle in the place where $\frac{2}{2}$ should go (see Figure 7). Next, she asked Beatriz about the numerator and denominator of the unknown fraction:

Mrs. Quiróz: *¿Uno por qué número te da dos?* [One times what number gives you two?]

Beatriz: *Uno por dos* [One times two].

Mrs. Quiróz: Okay. *Entonces vas a poner el dos aquí. Entonces para que esto sea un valor de uno, ¿qué tiene que ser el denominador?* [Then you are going to write the two here. Now, for this to have the value of one, what should the denominator be?]

Beatriz: *Dos* [two].

Mrs. Quiróz: *Exactamente, pon el dos. ¿Esto es cierto? ¿Dos por dos es cuatro?* [That's right. Write the two down. Is it true that two times two is four?]

Beatriz: *Sí* [Yes].

Mrs. Quiróz: Okay, *en este instante ya sabes que mitad y dos cuartos sí son equivalentes. Ahora vas a utilizar un medio por algo para ver si es igual a seis octavos.* Okay, *ahora tú enséñame a mí. ¿Qué es el primer, la primera pregunta que haces para ver esto?* [Okay. Right now you know that one-half and two fourths are equivalent. Now you will use one-half times something to check if it is six eights. Okay, now you teach me. What is the first question you need to ask to check this?]

Beatriz: *Uno por qué número* [One times what number].

Mrs. Quiróz: *Exactamente. Entonces ¿qué tiene que ser este número?* [That is right. Then, what is that number?]

Beatriz: *Seis* [Six].

Mrs. Quiróz: *Seis. Y mira lo que pasa. ¿Qué es el producto de dos por seis?* [Six. And look what happens. What is the product of two times six?]

Beatriz: *Dos por seis, doce* [Two times six, twelve].

Mrs. Quiróz: *¿Y ese es doce?* [And that is twelve? (She points at eight)].

Beatriz: No.

Mrs. Quiróz: Okay, *aquí vas a tachar. Entonces B… ¡Adios!* [Okay, you will cross out this. Goodbye B!]

Mrs. Quiróz then paired Beatriz with a bilingual girl and encouraged them to speak in Spanish to solve problem C. After students worked on the problems and their solutions were discussed in class, Mrs. Quiróz explained the meaning of the percent symbol (%) by stimulating her students' metalinguistic awareness.

Mrs. Quiroz: *Beatriz, ¿cuántos años hay en un siglo, en un* century? [Beatriz, how many years are there in a century?]

Beatriz: *Cien* [One hundred].

Mrs. Quiroz: *Exactamente, dilo otra vez* [That is right. Say it again].

Beatriz: *Cien.*

Mrs. Quiroz: Beatriz has already told you how many years are in a CENT-ury.

Students: A hundred!

Mrs. Quiroz: So what do you think this word [cent] means?

Students: [Crosstalk]

Mrs. Quiroz: How many years in a century?

Students: A hundred!

Mrs. Quiroz: How many cents in a dollar?

Students: A hundred!

Mrs. Quiroz: What do you think *cent* could mean?

Students: *¡Cien!*

Mrs. Quiroz: *Cien.* It means a hundred.

Once the meaning of the word *cent* was clear by connecting it to the Spanish word *cien*, Mrs. Quiróz was able to move the discussion to the word *percent.* In this vignette, translanguaging was used as a tool for cross-linguistic transfer and identity investment. The former use of translanguaging was achieved through the utilization of cognates (*cien* and *cent*), while the latter was achieved by establishing Beatriz, a recent immigrant, as the source of valuable knowledge.

Second Classroom Observation

Mrs. Aguirre. Mr. Aguirre distributed the worksheet *TEKS/STAAR Spiraled Practice* among her students, to review the topics covered in class. The worksheet was stylistically close to the STAAR test and Mrs. Aguirre used it as a test preparation tool. First, she asked students to solve the problems on their own, while she supervised their work. During this time, some students asked questions in Spanish, even though explanations were given in English. For example, after Mr. Aguirre told a student that he needed to use his formula sheet in order to solve an area problem involving a trapezoid, he wrote the pyramid surface area formula $S = \frac{1}{2}Pl + B$ by mistake on the worksheet and asked "*Qué significa* pee ell?"

Next, Mrs. Aguirre explained a problem about linear equations in two variables to the whole class, since she observed that some students were struggling with it (see Figure 8). She proceeded to solve part A of problem on the whiteboard as shown in Figure 9. By using arrows connecting the *x* and *y* coordinates of the point (2, -3) to the *x* and *y* in the equation $4x - y = 11$, Mrs. Aguirre was helping all students understand the substitution that followed.

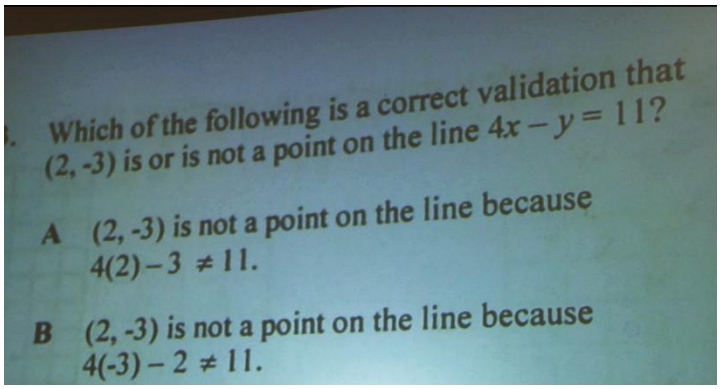


Figure 8. Problem in the TEKS/STAAR Spiraled Practice. Students were asked to critique arguments regarding the point (2, -3) being a solution to $4x - y = 11$ or not.

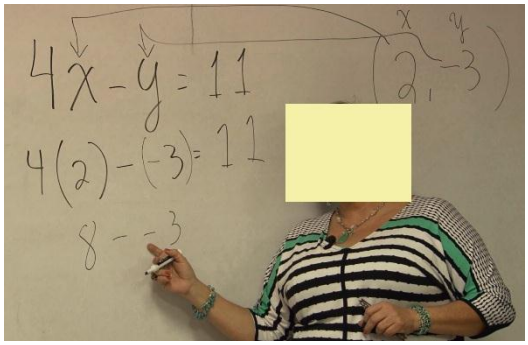


Figure 9. Mrs. Aguirre substituting (2, -3) into $4x - y = 11$. She used arrows to indicate how the substitution was done.

When Mrs. Aguirre reached the point where students had to decide what to do with the expression $-(-3)$, she initiated a discussion that involved translanguaging as a meaning-making tool.

Mrs. Aguirre: *¿Por qué, Bobby? ¿Por qué si tengo dos negativos se va a hacer positivo?* [Why Bobby? How come if I have two negatives, it becomes positive?]

Bobby: *Porque es como el "chinito"*. [Because it's like the "Chinese boy"]

Mrs. Aguirre: *Es como el "chinito"*. Okay. *¿Cómo se mira el "chinito"?* Like this? [It's like the "Chinese boy". Okay. How does the 'Chinese boy" look like? Like this?]

Mrs. Aguirre drew a face on the whiteboard, while some students laughed and said "negative, negative, positive" as she drew negative signs as the eyes and a positive sign as the mouth (see Figure 10). She had previously used this mnemonic device, and translanguaging acted in this case as the mediating tool between students' prior knowledge and the mathematics at hand.

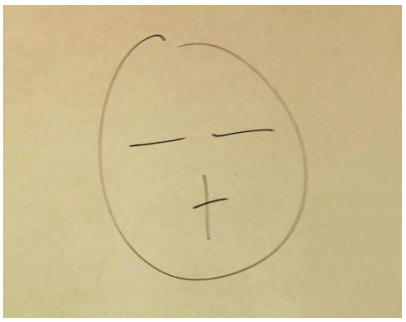


Figure 10. Drawing used as a mnemonic device. Mrs. Aguirre linked the drawing to the Spanish word *chinito*.

The point of this mnemonic device was that students remembered what the sign of the product of two numbers was by using the leftover sign in the face of the *chinito*. Is this case, for example, the result of multiplying the negative numbers -1 and -3 (the eyes of the *chinito*) was positive 3 (the mouth of the *chinito*).

Later, Mrs. Aguirre distributed another worksheet containing problems involving the Pythagorean Theorem. Before students started on the problems, Mrs. Aguirre addressed Arturo, who had been quiet during most of this class period: *"A veces no te dan todas las medidas que necesitas y tu tienes que interpretar o hacer una calculación con el Teorema de Pitágoras"* [Sometimes you are not given all the dimensions that you need and you have to interpret or make a calculation with the Pythagorean Theorem.] Mrs. Aguirre's attentiveness to meaning-making was evidenced in this instance and the previously mentioned use of translanguaging, since she was adapting instruction to students from diverse linguistic backgrounds (García & Wei, 2014).

Mr. Long. Students present in Mr. Long's resource classroom on the second observation included Crissy, Johnny, Laura, and Sellina. As in the first observation, translanguaging played an important role in Mr. Long's instruction, since it was clear that these students were still learning English. The lesson of the students focused on angles, and Mr. Long used his students' prior knowledge to connect the English words *acute* and *obtuse* to the Spanish words *agudo* and *obtuso*, respectively. However, he was not as successful with the translation of *right angle* (*ángulo recto*), since he used *derecho* as the translation of *right*. This is an example of how words that are used in more than one register in both in English and Spanish may end up being used in the wrong register when translated. *Derecho* or *derecha* do mean *right* in Spanish, but only when used as the opposite of *left*. Nevertheless, Mr. Long was able to use his own experiences as a young mathematics student to talk about mistakes that, according to him, are common for students whose native language is Spanish.

Mr. Long: I want you to highlight this word, "Angles". When I first learned it, I spelled it "angels", and it's very difficult. You gotta remember that the L, and then the E and the S. That is the only way I can spell it correctly. So gotta think "angles". Sometimes you misspell it by saying "angels" or you see it "angels" because it looks the same, but you gotta think "angles", and it ends with L-E-S. It's just something you have to remember. English is very difficult. You just gotta remember. When I see it, I see, I would think "angel", but that's not correct, it's "angle", it's spelled differently….The first angle that we are going to identify is the acute angle. What is an acute angle? Have you ever heard of this name in Spanish or no? Johnny?

Johnny: No.

Mr. Long: You, Laura? *¿Sabes cómo se llama el ángulo en español?* Acute?

Laura: *No lo recuerdo* [I don't remember].

Mr. Long: *Yo lo he oído "agudo"* [I've heard it's "agudo"].

Laura: *Sí.*

Mr. Long: *Agudo.* When I learned Spanish they taught me that *agudo* is "small" and that's all I remember. And another thing that I think about when I think about this acute angle is the "A". "A" starts at the beginning, that is small and then you get bigger to get to the O. "Obtuse". *¿Como se dice obtuse en Español? ¿Te acuerdas?*

¿Abtuso? [How do you say *obtuse* in Spanish? Do you remember? *Abtuso*?].

Crissy: *Obtuso.*

Mr. Long: *Obtuso.* That how you said. Right angle is *ángulo derecho*.

Laura: *Si usted me dice que busque un* right angle, *puede ser como el pizarrón?* [If you ask me to find a right angle, could it be like the board?]

Mr. Long: *El pizarrón. Porque aquí el ángulo es un 90. Una pantalla de una tele…* [The board. Because here the angle is 90. A TV screen…]

Laura: *¿De 90 pulgadas?* [A 90 inches one?]

Mr. Long: *De 90 grados, no pulgadas. Las teles todas se miden diagonal, por eso dicen 32 pulgadas, te van a decir diagonal. Ahorita estamos hablando de los grados, que viene siendo ángulo de la esquina, o los ángulos de aquí de un rayo a otro rayo. Vamos a identificarlos acute, agudos, chiquitos. ¿Qué tamaño? De cero a menos de 90 grados. Ya saben más o menos los que es un* acute. [90 degrees, not inches. All TVs are measured in diagonal. That is why they say 32 inches, they will tell you the diagonal. Right now we are talking about degrees, which is the corner angle, or the angle from one ray to the other. We will identify them (as) acute, smallish. What size? From zero to less than 90 degrees. Now you have an idea of what an acute is].

Mr. Long asked his students to draw examples of acute, right, and obtuse angles on a foldable (a graphic organizer), and suggested drawing clocks with hands in different positions in order to illustrate the aforementioned angles. He pointed at the classroom clock behind them so they used it while coming up with examples.

Mr. Long: How would you draw two o'clock? Crissy? Where does the big hand go? Two o'clock. *Las dos pe eme o dos a eme. ¿Cómo se pone las manos?* [2:00 PM or 2:00 AM. Where would you place the clock hands?]

Crissy: *La pequeña en el dos. ¿Dos a eme, dijo? ¿Pe eme?* [The small one at the two. You said 2:00 AM? PM?].

Mr. Long: Okay.

Crissy: *Y la grandota en…en el doce* [And the big one at the twelve].

Mr. Long: *En el doce. ¡Muy bien! Good job* [he drew the clock on the board. See Figure 11]

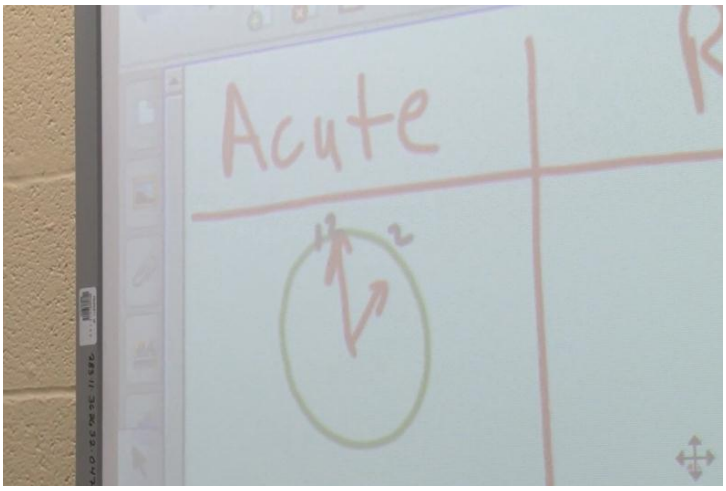


Figure 11. Mr. Long's example of an acute angle. Clocks are common objects and most students are familiar with them.

In the preceding vignette, Mr. Long used translanguaging to successfully unpack Crissy's prior knowledge of time reading. This happened in spite of him using the word *manos* instead of *manecillas,* the word that is used in Spanish for watch hands. The following vignette also showcases Mr. Long's skill in using students' life experiences:

Mr. Long:	When I was younger, I used to play a game that…it was called Pac-Man, because he liked…
Crissy:	*¡Ah sí! ¿Verdad?* [Oh yes! Right?]
Mr. Long:	What do you see in Pac-Man?
Crissy:	*La boca, la boca.*
Mr. Long:	The mouth. Was that acute or obtuse?
Crissy:	*Así* [Like this (she used one hand to imitate Pac-Man's mouth)]. Obtuse.
Mr. Long:	Uh?
Crissy:	Obtuse.
Mr. Long:	Okay. You can draw an obtuse. You make sure you draw it enough to be obtuse [Crissy then drew a Pac-Man on her foldable].
Crissy:	*¡No! ¡Es* acute!
Mr. Long:	Ah! Very good!

Mrs. Quiróz. Students were going to finish up work with foldables involving quadrilaterals that they had started the previous day. Desks were arranged in such a way that students were grouped in teams of four. First, Mrs. Quiróz reviewed the material (areas and perimeters) and checked for understanding. She focused on an area table for squares, where the first column contained the length *s* of the side of the square, and the

other column contained the corresponding area s^2. Students had filled out the table for $s = 1, 2, 3,\ldots, 10$. As in the first classroom observation, she would use Spanish to ask Beatriz questions:

Mrs. Quiroz: *¿Por qué estoy multiplicando este número con el mismo número? ¿El uno aquí o el tres allá? ¿Por qué estoy multiplicando con el mismo número? Es un cuadrado, ¿verdad? ¿Y qué sabemos de un cuadrado? Que cada lado es…* [Why am I multiplying this number by itself? One here and three there? Why am I using the same number? It´s a square, right? And what do we know about squares? That each side is…]

Beatriz: *Que todo…Que cada lado es igual.* [That all…That each side is equal].

Mrs. Quiroz: *Es igual. Si este es tres, sé que voy a multiplicar con tres.* Okay, *vamos a suponer que tenemos un cuadrado que es cinco por cinco.* [Is equal. If this (number) is three, I know I will multiply by three. Okay, we are going to assume that we have a five by five square].

Beatriz: *Se multiplica por cinco* [You multiply by five].

Mrs. Quiroz: *¿Y sabes que es el producto? ¿Cinco por cinco?* [And do you know what the product is? Five times five?]

Beatriz: *Veiticinco.* [TLongty-five].

Mrs. Quiróz: *Así es.* That's the area. [That is right. That is the area].

In this translanguaging episode, there were some inaccuracies in the use of the academic Spanish register in mathematics (e.g., squared units are not mentioned; use of

multiplicando con instead of *multiplicando por*). In spite of the aforementioned inaccuracies, translanguaging was beneficial in this case because it promoted the participation of all students.

Once students began working in teams, Mrs. Quiróz would supervise their work and answer their questions. On one occasion, Mrs. Quiróz explained that the word *variable* was used in Spanish also.

Mrs. Quiróz: [In the formula $A = s \times s$] Every time your side changes, what's going to happen to your area?

Pedro: It's gonna make it…

Mrs. Quiróz: It's gonna change. That's right, and that's why they are called *variables*. I am trying to think of a word in Spanish—cuz Spanish is Latin-based—that means change…*Cambiar*…

Pedro: *Cambiar* [To change].

Mrs. Quiróz: Do you'all ever use the word *variar* [to vary]? *Ah! Sabes dónde…*You know where you have read it? *Variedad.* A variety. Yes? Have you heard that word? [Three students said no, but Luis nodded].

Luis: I sure have, because my cousins are…

Mrs. Quiróz: As you get older, and you increase your vocabulary, you'll hear the word variety, or—in Spanish—*variedad*, and that means you'll have a whole bunch of different choices. It can change. So it's called the variable because it can change.

As indicated by the data for the first and second classroom observations, a major goal of Mrs. Quiróz's was the development of metalinguistic awareness in her students. In this case, however, this awareness was not created by eliciting students' thinking, but by means of unnecessary scaffolding.

Later, Mrs. Quiróz approached Beatriz and went over her foldable. Mrs. Quiróz noticed that Beatriz had problems finding perimeters of squares, and had also completed the perimeter table on the wrong side of the foldable (the "area" side). Mrs. Quiróz's strategy to help Beatriz understand the formula $P = 4s$ was to show that it was an equivalency of two expressions, and to provide a concrete example using grid paper.

Mrs. Quiroz: *Beatriz, déjame ver tu…*Okay. *Sabes lo que significa* equivalency? [Beatriz, let me see your…Okay. Do you know what equivalence means?]

Beatriz: *Equivalencia* [low tone of voice].

Mrs. Quiroz: *Exactamente. Es la misma palabra en español—equivalencia. Y esta palabra es también idéntica. ¿Qué es? Expresión. Y lo que quieren que—ustedes, todos los niños—entiendan es que cuando tienes una expresión es igual a otra expresión, todo junto, las dos cosas que son iguales…* [That´s right. It is the same Word in Spanish—*equivalencia*. And this other word is also identical. What is it? *Expresión.* And what they want to—you, all children—understand is that when you have an expression that is equal to another expression, everything together, the two things that are equal…]

Mrs. Quiróz wrote down "expression = expression" (see Figure 12) in Beatriz's foldable (*foldable* being a word she was not able to translate in Spanish) but pondered the value of that choice.

Mrs. Quiróz: *Es que no quería…no quiero enseñarles esto con palabras porque se ve muy confundido. Pero si lo ves así, tiene sentido. El perímetro es igual a…cuando tomas el lado…¿Este es área o perímetro?* [It´s just that I didn't…I don't want to teach this with words to the class because it looks confusing. But if you see it that way, it makes sense. The perimeter is equal to…when you take the side…Is this area or perimeter?]

Beatriz: *Esta es área* [This is area].

Mrs. Quiróz: *Bueno, tienes perímetro de este lado. Pero bueno, vamos a terminar. Cuando tú tienes un cuadrado que es…un*, two, *uno, dos, tres, cuatro…uno, dos, tres, cuatro…uno, dos, tres, cuatro.* [Well…you have the perimeter on the other side (of the foldable). But let's finish. When you have a square that is (she counted) *un*, two, *uno, dos, tres, cuatro…uno, dos, tres, cuatro…uno, dos, tres, cuatro*].

Mrs. Quiróz tried to draw a four by four square using the grid, but since she utilized a mix of Spanish and English during the measurement of the first side of the square, she ended up drawing a five by four rectangle (see Figure 12). Mrs. Aguirre found out about her mistake when she tried to find the perimeter of the quadrilateral.

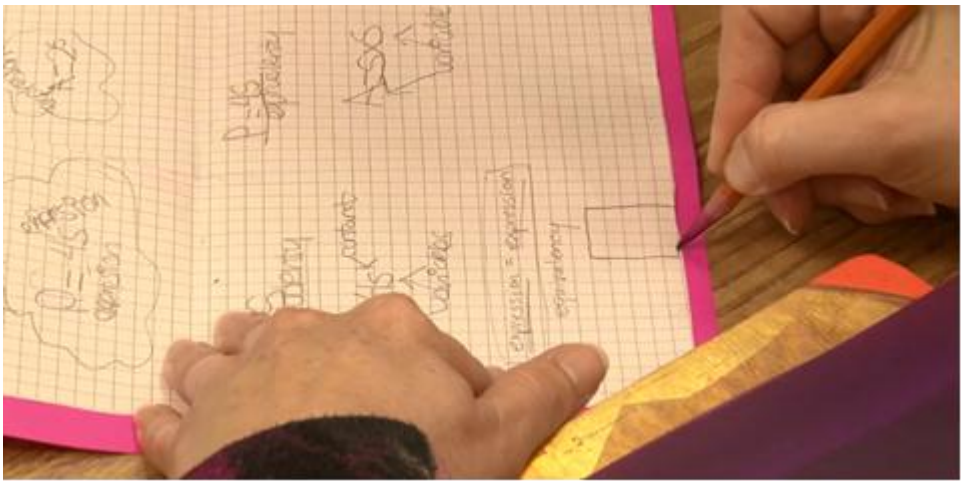


Figure 12. Mrs. Quiróz drawing a 5 × 4 rectangle in Beatriz's foldable. Her intention was to draw a 4 × 4 square.

Mrs. Quiróz: *Si tú cuentas todo…Perímetro, perdón. Entonces, de aquí a aquí…uno, dos, tres, cuatro, cinco. Uno, dos…No. Estoy equivocada. ¿Qué tengo que hacer? No es un cuadrado, ¿verdad? Porque es cuatro por cinco.* [If you count everything…Perimeter, sorry, Then, from here to here…one, two, three, four, five. One, two…No. I'm wrong. What do I have to do? It's not a square, right? Because it's four by five].

Mrs. Quiróz erased the rectangle, and drew a four by four square. She measured the lengths of the sides of the larger rectangle by counting the sides of the smaller (one by one) squares that formed each side. Since it was not clear what was it that she was counting (the smaller squares or the sides of these squares whose union formed the side of the larger square), this introduced a misconception in Beatriz's concept of perimeter, as shown in the transcript below.

Mrs. Quiróz: *Si cuento todas las unidades, todo alrededor, ¿qué tanto van a ser?* [If I count all the units, everything around it, how many are they?

Beatriz: *Doce* [Twelve].

Mrs. Quiróz ignored this answer and proceeded to explain to Beatriz why the perimeter was sixteen, by adding the lengths of two adjacent sides and inferring that the sum of the lengths of the two other adjacent sides was the same.

Mrs. Quiróz: *¿Cuatro más cuatro?* [Four plus four?].

Beatriz: *Ocho* [Eight].

Mrs. Quiróz: *¿Ocho más ocho?* [Eight plus eight].

Beatriz: *Diesciseis* [Sixteen].

Mrs. Quiróz: *Entonces, por eso estas multiplicando por cuatro. El lado es cuatro. Cuatro por cuatro, dieciséis. Te dice qué tanto es el rededor* [That is why you are multiplying by four (in the formula). The side is four. Four times four, sixteen. It tells you how much the border is].

The digital data indicates that Beatriz counted the twelve squares shown in Figure 13, and thought that twelve was the perimeter of the larger square. This misconception was motivated by a lack of precision in the mathematical language used by Mrs. Quiróz and her gesturing while counting, since it was not clear if she was counting segments or squares. ELs pay close attention to teachers' gestures, and this may introduce unintended misconceptions (Sorto et al., 2014).

The vignette above illustrates that the effectiveness of translanguaging as a meaning-making tool requires teacher's attention to precision in the use of the mathematical register of the student's L1. Moreover, the vignette also illustrates how one-to-one correspondence may be affected if languages are switched while counting.

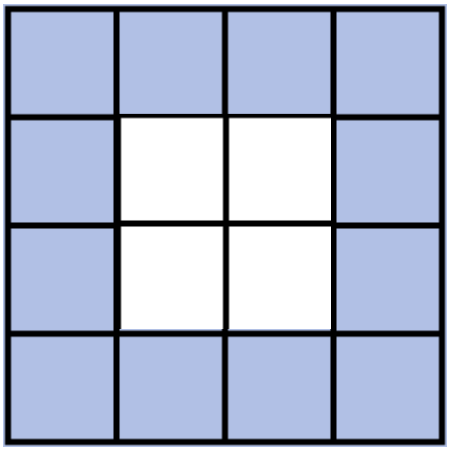


Figure 13. Beatriz's misconception. Mrs. Quiróz pointed at the shaded squares when finding the perimeter of the larger square. Beatriz counted them and indicated that 12 was the perimeter of the square.

Third Classroom Observation

Mrs. Aguirre. This class took place when state-mandated testing had concluded. The classroom dynamics were different in the sense that students were not being prepared for the standardized test. Mrs. Aguirre indicated that she was preparing her students for high school. The lesson was focused on the translation of verbal phrases into algebraic expressions, and at one point Mrs. Aguirre asked her students to come up with words that meant addition, and she would write them down on the board (see Figure 14).

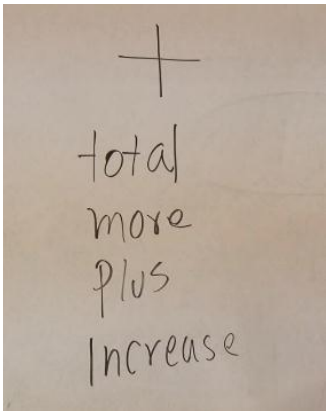


Figure 14. Student-generated list of words that are related to addition. Mrs. Aguirre would write down the words below the addition sign.

Mrs. Aguirre involved Arturo in the discussion: "Arturito, any words that you think…*Palabras que quieren decir que vas a hacer más, que vas a sumar.* Words that indicate that you need to add." Arturo, who was now sitting in the third and final row (he sat in the first row during the first class observation, and in the second row during the second observation) said something that Mrs. Aguirre interpreted as "it is already here [on the board]".

Translanguaging was used sparsely throughout this lesson, and only to encourage Arturo and Rocío to participate. Rocío was also a recent immigrant. The following was another significant yet brief translanguaging episode that transpired during the lesson:

Mrs. Aguirre: Who can tell me what a variable is?

Rocío: *Es como una letra* [it is like a letter].

Mrs. Aguirre: *Es como una letra.* It can be a letter.

In this episode, Mrs. Aguirre used revoicing in Spanish when she repeated what Rocío said, and revoicing in English when Mrs. Aguirre subtly rephrased what Rocío

said. Consequently, translanguaging provided an opportunity for the teacher to reformulate Rocío's contribution and increase Rocío's metalinguistic awareness, by connecting the cognates *letra* and *letter*.

Mr. Long. The focus of this lesson was to give an overview of some of the mathematics topics that are covered in 8th grade. The four students in Mr. Long's resource classroom included 7th graders Laura, Johnny, Sellina, and Frank (a new student). Mr. Long wanted to prepare them for what was ahead, so he used the *8th Grade STAAR Ready Assessment* and solved a few problems on finding algebraic rules and geometric dilations.

In this lesson, Mr. Long's dual role as a mathematics and language teacher was more apparent than in previous observations. Mr. Long showed concern for his students' learning correct pronunciation and emphasized the use of new terms that he knew were difficult for ELs. For example, when Laura referred to the vertex Z in ΔXYZ, Mr. Long corrected her pronunciation: "*Zee*, not *cee*. Z*ee*". Another example was the introduction of the prime symbol ($'$) in dilations; Mr. Long made his students repeat the word *prime* several times and even found a Spanish word that had a similar sound: "*Es como premio, pero* prime".

Moreover, Mr. Long used translanguaging often in this lesson, mostly because of Laura's frequent questions posited in Spanish. Laura and, to a lesser degree, Johnny were invested in the class. Sellina and Frank showed very little interest, in spite of Mr. Long's attempts to involve them. Sellina's behavior during the second observation also indicated disengagement.

Mr. Long first solved a problem about algebraic sequences (see Figure 15) and wrote the word "Table" right beside the table of values, clarifying that this was not a table such as a wood table in the classroom (he tapped it). Since they had to find out which of the proposed expressions fit the pattern, Mr. Long discussed the meaning of the word "substitute" as a verb. Laura indicated that it meant *cambiar* in Spanish, and Johnny said it meant "change". Mr. Long then mentioned that he preferred to say *plug in,* and pointed at the computer cords that were plugged into the outlets.

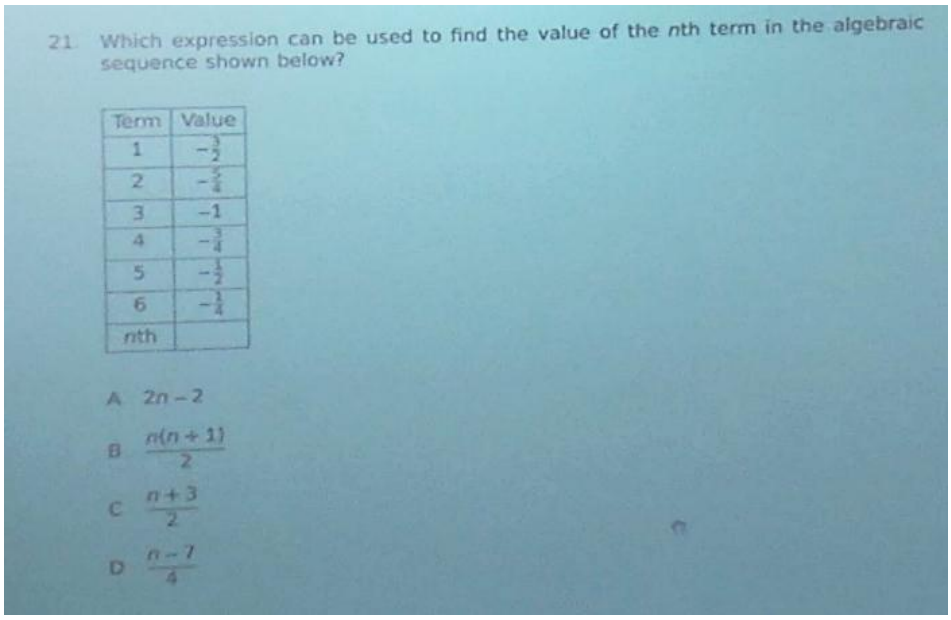


Figure 15. Multiple choice problem about algebraic sequences. Mr. Long chose this problem from the *8th Grade STAAR Ready Assessment*.

Laura: *¿La ene tiene un valor?* [The *n* has a value?]

Mr. Long: *El valor está aquí. Estos son los valores. Puedes escoger cualquiera* [The value is here. These are the values. You can choose any].

Laura: Oh! *ene es como cualquiera de esos.* [Oh! *n* is like any of those].

Mr. Long: *Pero tiene que estar en esta columna* [But it has to be in this column].

Laura: *Ya entendí. Estamos descubriendo cual número de esa tabla es*…[I understand, We are descovering which number in the table is…]

Mr. Long: *Cual función va funcionar para que te de esta respuesta.* Okay? *Es muy simple, ¿verdad? Pones este número aquí y si funciona, esta es tu respuesta* [Which function would work so you can get this answer. Okay? It´s very simple, right? You plug in the number and if it works, this is your answer].

Mr. Long's method was naive, since using only one value of *n* to test each expression may work to discard expressions, but not to find the likely *n*th term of the sequence. However, Mr. Long's method would have been easier to use on a test. Next, Mr. Long chose a problem about geometric dilations in which students had to find the scale factor used. In this problem, ΔABC was transformed into ΔXYZ. Mr. Long explained that they had to decide if ΔXYZ was an enlargement or a reduction of ΔABC, and then compare each option in the multiple choice item to the number one. Mr. Long and his students decided that ΔXYZ was an enlargement of ΔABC and that the scale factor had to be greater than one. Laura did not remember what symbol was used for *greater than*, and which one was used for *less than.* In particular, she wanted to know if the opening of the symbol (*el espacio* or the space) had to be located next to the smallest number. Mr. Long suggested that it was better to focus on the tip of the symbols $<$ and $>$. He drew three points on the board, and connected them to form the symbol *less than* (see Figure 16). He pointed out that, since the vertex of the symbol had only one point, he

was going to write the number 1 next to it; and since there were two points to the right, he was going to write the number 2 next to them.

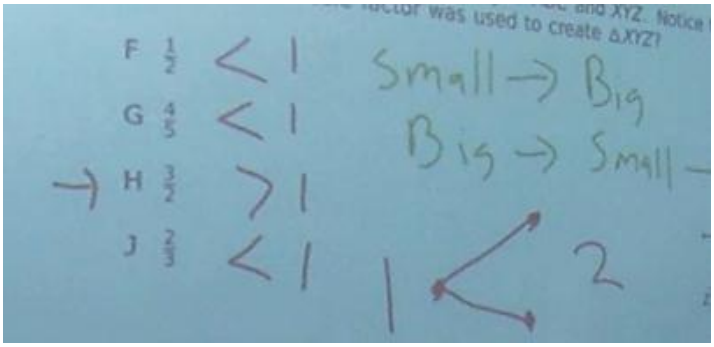


Figure 16. Mr. Long's mnemonic device. The purpose of the device is that students remember how to use the symbols > and < .

Mr. Long: One is less than two. In Spanish, I've heard it *"el piquito para el chiquito"* ["the little beak for the little one"].

Laura: *La flecha para el uno y el espacio para el dos* [The arrow for one and the space for two].

In this vignette, Mr. Long used the three points and translanguaging as mnemonic devices that complemented each other. Moreover, Figure 16 shows that Mr. Long compared each option in the multiple choice problem to one, and that he had marked $\frac{3}{2}$ (option H) as the clear answer. Nevertheless, he tested $\frac{3}{2}$ and determined that it was indeed the desired scale factor.

Mrs. Quiróz. In this lesson, Mrs. Quiróz first reviewed addition and subtraction of integers, which had been covered the previous day, and then explained multiplication and division of integers. As in the previous two classroom observations, her class was

conducted in English with brief, but significant, translanguaging episodes. For example, Mrs. Quiróz used translanguaging as a way of previewing the rules that would be used for multiplication (and division) of integers.

As usual, Mrs. Quiróz would ask questions in Spanish to Beatriz, but less frequently than in prior observations. Beatriz's responses were given in an audible tone of voice and she demonstrated more confidence around her peers. Furthermore, Mrs. Quiróz did not give Beatriz individual assistance this time, since she was now able to work more independently.

Mrs. Quiróz's first mathematics question to Beatriz had to do with the addition of integers that had the same sign. Mrs. Quiróz used the Spanish word *señal* as a translation of the English word *sign.* In this case, she translated a word of the mathematics English register into a word of the Spanish conversational register. The correct translation of *sign* in this context is *signo,* whereas *señal* is used for signs such as traffic signs, or gestures.

Mrs. Quiróz:	*Beatriz, cuando tenemos señales iguales, positivo y positivo, ¿vamos a sumar o restar?* [Beatriz, when we have equal signs, positive and positive, are we going to add or subtract?]
Beatriz:	*Vamos a sumar* [We are going to add].
Mrs. Quiróz:	*Sumar, muy bien* [Add, very good].

When explaining the problems $11 - (-3)$ and $-10 - (-5)$, Mrs. Quiróz used translanguaging to make a rule accessible to all: *"Los dos negativos se convierten a positivos.* Yes, so…two negatives equal positive, and that's only with subtraction, okay?" In the previous examples, Mrs. Quiróz demonstrated imprecision in the use of

mathematical language, in both English and in Spanish. As was evidenced in the second classroom observation, her imprecisions may introduce misconceptions; nevertheless, on this occasion that did not happen, as it was demonstrated by her students' correct responses. The following is another example of Mrs. Quiróz's use of translanguaging to preview the rules that would be used during the lesson.

Mrs. Quiróz: *Si tu contestas "sí" a la pregunta ¿las señales son iguales?, [entonces] el producto va a ser positivo. ¿Las señales son iguales? No. [Entonces] el producto va a ser negativo.* [If you answer "yes" to the question *are the signs the same?* (Then) the product is positive. *Are the signs the same?* No. (Then) the product will be negative].

Next, Mrs. Quiróz asked her students to work on several problems on multiplication of integers, and she used translanguaging to give directions and encouragement to her students.

Mrs. Quiróz: *Vamos a la misma hoja que ayer, el lado izquierdo era para sumas y restas.* The left side [of the sheet] was for addition and subtraction *y a mano derecha vamos a practicar*…what are we going to practice on the left side?

Students: Multiplication and division!

Mrs. Quiróz: The worst thing you can do is just to copy and not think and not try. What do we say in Spanish? *La peor lucha es la que no se hace,* right? You gotta try it to be successful at it.

Students solved the problems individually, and then Mrs. Quiróz discussed them as a class. She asked Beatriz about the sign of sixteen in $2 \times 8 = 16$.

Mrs. Quiróz: *Beatriz, ¿por qué me están diciendo que el producto aquí debe ser un dieciséis positivo?* [Beatriz, why are they telling me that this product must be positive sixteen?]

Beatriz: *Porque…es diez.* [Because it's ten].

Mrs. Quiróz: *Porque dos por ocho son*…[Because two times eight is…]

Beatriz: *Ehhh…dieciséis* [Mmm…sixteen].

Mrs. Quiróz: *Dieciséis. ¿Y por qué es positivo?* [Sixteen. And why is it positive?]

Beatriz: *Porque el dos es positivo.* [Because two is positive].

Mrs. Quiróz: *Y el ocho también es…[*And eight is also…]

Beatriz: *Positivo.* [Positive]

Mrs. Quiróz: *Los dos son iguales…es positivo* [Both are the same…It's positive].

Later, Mrs. Quiróz asked Beatriz to explain the solution to the problem -5×-5. Beatriz was not asked to give mathematical explanations in the first and second classroom observations.

Mrs. Quiróz: *Beatriz, ¿quieres intentar el [problema] número nueve?* [Beatriz, would you like to try (problem) nine?]

Beatriz: *Se multiplica el cinco por el cinco, igual a veinte. Y como los dos… todos iguales, es igual a veinte positivo* [You multiply five

times five, equals tLongty. And since both…all are the same, it equals positive twenty].

Mrs. Quiróz: *Muy bien, VEINTICINCO positivo* [Very good, positive TWENTY-FIVE]

Beatriz: *Veinticinco positivo* [Positive twenty-five].

In the previous vignettes, Mrs. Quiróz used translanguaging as a way to corroborate Beatriz's procedural understanding and scaffolded accordingly. Beatriz's willingness to explain problems indicated that this approach had succeeded in involving her in the learning activities of Mrs. Quiroz's classroom.

Synthesis of the Classroom Observations

Support of students' first language. All three participants allowed their students to use Spanish in class, but instruction was conducted mostly in English. In Mrs. Aguirre's case, the data showed that she used translanguaging less and less with each observation and, at the same time, Arturo, a recent immigrant, would participate less and less in class. I interpreted Mrs. Aguirre's actions as a "sink or swim" strategy that other teachers in the school district told me they use in order to take EL students out of their comfort zone. "If you always talk to them in Spanish, then they never learn English" one of the teachers told me, especially since all of them speak Spanish at home.

As Mrs. Aguirre reported during our interview, she was both a mathematics and an English language teacher, and the same can be said of Mr. Long and Mrs. Quiróz, who used translanguaging more often during all observations. Moreover, the digital data did

not reveal that the teacher participants used curriculum materials or other resources in Spanish, or that they used written translanguaging.

It was clear that the participants' greatest concern was to accelerate the acculturation of ELs, as much as teaching them mathematics, and believed that an important part of this acculturation was that ELs learned English, the dominant language. Mrs. Aguirre, Mr. Long, and Mrs. Quiróz demonstrated this priority in subtle ways as well, by not being able to manage the mathematics register in Spanish fluently. Teachers usually focus on what is a priority, and learning academic Spanish is not one of them, since standardized tests are in English. The classroom observations revealed that Spanish is mainly used as a tool to support the understanding of the curriculum in English.

Translanguaging. Table 3 shows the different teaching strategies that involved translanguaging that I was able to detect during classroom observations, and the teachers that used them. These strategies were adapted from a list created by García and Wei (2014). The participating teachers all utilized strategies such as using cognates, collaborative dialogue with students, revoicing, translating, and translanguaging in speaking. Cognates were used implicitly by Mrs. Aguirre (e.g., the word "percent" in one worksheet) or explicitly by Mr. Long and Mrs. Quiróz (cent and *cien*; obtuse and *obtuso*). Collaborative dialogue was used for problem solving in class or during individual work. Revoicing involved repeating what students said in Spanish, paraphrasing it in English or both. Translating was done implicitly (through revoicing) or explicitly (more often by Mr. Long and Mrs. Quiróz).

Table 3

Teaching strategies involving translanguaging used by the participants

Teaching strategy	Teacher
Cognate use	Mrs. Aguirre, Mr. Long, Mrs. Quiroz
Collaborative dialogue	Mrs. Aguirre, Mr. Long, Mrs. Quiróz
Collaborative grouping	Mrs. Quiroz
Previewing	Mrs. Quiróz
Revoicing	Mrs. Aguirre, Mr. Long, Mr. Quiróz
Stimulus of inner speech	Mrs. Quiróz, Mr. Long
Student paraphrasing	Mrs. Aguirre
Translanguaging in speaking	Mrs. Aguirre, Mr. Long, Mrs. Quiróz
Translating	Mrs. Aguirre, Mr. Long, Mrs. Quiróz
Use of mnemonic devices	Mrs. Aguirre, Mr. Long

Translanguaging in speaking was done by the teacher during instruction or when assisting students, by some students when participating in class (especially when questioning what was said), and I also observed it in collaborative groups, formal or informal. Some strategies were not used by all teachers, and a few were used by only one teacher, revealing personal choices that could be linked to their background and education. For example, Mrs. Quiróz's translanguaging choices were oriented towards the development of her students' metalinguistic awareness, and her instruction in general to be focused on metacognition. Furthermore, she did not seem comfortable using mnemonic devices, which is something both Mrs. Aguirre and Mr. Long used often. Additionally, Mrs. Aguirre and Mrs. Quiróz used strategies that no other teacher used

during classroom observations. Mrs. Aguirre encouraged bilingual English proficient students to act as paraphrasers for EL students, while Mrs. Quiróz used translanguaging to preview the lesson and encouraged students to use translanguaging in collaborative groups.

Moreover, Mrs. Aguirre and Mr. Long both encouraged translanguaging during inner speech, but in different ways; by means of concept maps in the case of the former, and by asking students to think in Spanish in the case of the latter. All teachers used translanguaging as an affiliative tool to establish rapport with students. Nevertheless, Mrs. Aguirre was the only teacher that used translanguaging to interrogate linguistic inequality.

Additionally, Mrs. Aguirre and Mr. Quiróz intentionally included recent immigrants during instruction and involve them in the classroom dynamics. In particular, Mrs. Quiróz did this by making Beatriz, her recently immigrated student, a source of valuable information that was used to guide instruction.

VI. DISCUSSION

Introduction

In this collective case study examining the instructional approaches of three middle school mathematics teachers located in a school district in South Texas, it was vital to examine each participants' (a) family background and preparation, (b) support of students' first language, (c) beliefs on the cognitive advantages of bilingualism in mathematics, and (d) the varying modes of translanguaging that they used. More specifically, my study addressed the following research questions:

1. What are the participants' similarities and differences in their background and preparation?

2. What are the participants' similarities and differences in their support of students' first language?

3. What are the participants' similarities and differences in their perception of the cognitive advantages that bilingual students may have in mathematics?

4. What are the participants' similarities and differences in their use of translanguaging while explaining mathematical concepts?

This chapter discusses the findings derived from the analysis of the interviews and classroom observations with the three participants of the case study. I used the analysis of interviews to answer research questions 1 – 4, and the analysis of the classroom observations to answer research questions 2 and 4. I discuss the results in four sections

that are associated to my research questions: Teacher characteristics, Support of students' first language, Bilingualism and mathematics, and Translanguaging. The chapter concludes with a discussion on the study's implications for practice, limitations, and directions for future research.

Teacher Characteristics

Mrs. Aguirre settled in South Texas in her late 20s and attended the university when she was 39 years old. Even though she was a high school dropout in Mexico and her knowledge of English was modest, she was able to pass the GED test in English on her first attempt, pass the entrance exam at the university, and was able to succeed academically. This is consistent with the interdependence hypothesis, which posits that instruction in L1 that is effective in promoting proficiency in L1 will facilitate a transfer of this proficiency in L2, provided that there is adequate exposure to L2 and motivation to learn L2 (Cummins, 1981). Moreover, Mrs. Aguirre's academic success was consistent with Thomas and Collier's (2002) results that indicated that students' L1 proficiency at the time of their arrival in the United States is the strongest predictor of their academic success.

Mr. Long and Mrs. Quiróz—the two other participants—were raised bilingually, but Mrs. Quiróz's parents were more educated and had better socio-economic status. This was reflected in Mrs. Quiróz fluency in both English and Spanish. Furthermore, Mrs. Aguirre and Mr. Long both had contact with multiple cultures and languages growing up, even though they are considered minority cultures and languages in the United States. Mr. Long and Mrs. Aguirre considered Spanish their first language, and Mrs. Quiróz considered hers to be English.

Since Mrs. Aguirre received all elementary and almost all of her secondary education in Mexico, this made her the participant most likely to understand the mathematics background of recent immigrants. Mrs. Quiróz was the only participant who had teaching experience in Mexico, even though she taught mostly in English. This teaching experience provided her some understanding of how mathematics is taught in Mexico at the elementary level and the algorithms used for instruction.

Additionally, the teacher participants were highly educated middle school mathematics teachers, and hence, this made their students more likely to be successful (Wayne & Youngs, 2003). Mrs. Aguirre and Mrs. Quiróz both held master's degrees in education with emphasis in mathematics and science, while Mr. Long was enrolled in a master's program in special education. However, Mr. Long was the only participant with a strong background in mathematics, holding a bachelor's degree in mathematics. Both Mrs. Aguirre and Mr. Long attended the same university in South Texas whereas Mrs. Quiróz attended two universities located in San Antonio, Texas. Mr. Long was the participant with the closest ties to the region, having been born and educated there. Mrs. Aguirre also had close ties to the region.

Regarding their teaching credentials, all the participants were certified to teach mathematics at the middle school level. Additionally, both Mrs. Aguirre and Mrs. Quiróz were certified bilingual teachers, and both Mrs. Aguirre and Mr. Long had certifications in special education at all levels. Mr. Long also had a certification in kinesiology. Of the three, only Mrs. Quiróz found mathematics to be a difficult subject growing up.

Both Mr. Long and Mrs. Quiróz indicated that the most important trait a mathematics teacher needed when teaching ELs was patience to uncover these students' mathematical thinking and to provide the necessary scaffolding so they could succeed. Mrs. Aguirre reported that knowledge of the content is the most important trait a mathematics teacher of ELs should have, but she also included class management, resourcefulness, and establishing rapport with students as important traits.

Support of Students' First Language

I used both interview responses and classroom observations to examine how the three participant teachers supported their students' first language during instruction. All three teachers used Spanish in their mathematics classrooms, but only Mrs. Aguirre and Mrs. Quiróz reported this during our interview. Mr. Long reported that he did not use Spanish to teach; however, the data indicated that he did speak Spanish during instruction. Moreover, Mr. Long's use of varying types of translanguaging in his class increased with each observation. As a consequence, Laura, an EL student, would increasingly participate during instruction, motivated to ask clarifying questions.

In contrast, Mrs. Aguirre used translanguaging less and less with each observation. At the same time, Arturo, a recent immigrant of hers, would participate less and less in class. Mrs. Aguirre told me during our interview that she taught mostly in English because students were tested in that language, and therefore the linguistic choices she made during classroom observations were consistent with that statement.

Furthermore, Mrs. Quiróz reported that she used mostly English during instruction because she did not want to single out her EL students by constantly speaking

Spanish. The data revealed that she included EL students during instruction by frequently calling them by name and asking them questions in Spanish. The data also revealed that Mrs. Aguirre similarly included EL students during instruction, but not as frequently as Mrs. Aguirre. Moreover, both Mrs. Aguirre and Mrs. Quiróz identified themselves as language and mathematics teachers.

Mr. Long perceived that being a Spanish speaking teacher was an asset when teaching ELs, since he could provide scaffolding by explaining the meaning of mathematical terms in Spanish. In contrast, Mrs. Aguirre expressed that being bilingual was not necessary to teach ELs, but that being resourceful was a critical trait. Additionally, interview and classroom observation data indicated that Mrs. Aguirre was the participant that used the greatest variety of translanguaging modes in her classroom.

All three participants allowed their students to use Spanish in class, but instruction was conducted mostly in English. Moreover, there was not evidenced that these teachers used curriculum materials or other resources in Spanish, or that they translanguaged in writing.

The data indicated that these three teachers' greatest concern was to accelerate the adaptation process of ELs, as much as teaching them mathematics. And an important part of this acculturation was that ELs learned English, the dominant language. Mrs. Aguirre, Mr. Long, and Mrs. Quiróz demonstrated this priority by not being able to use the mathematics register in Spanish fluently. Teachers usually focus on what is the priority, and learning academic Spanish is not one of them, since standardized tests are in English.

In summary, the evidence revealed that participants mainly used Spanish as a tool to support the understanding of the curriculum in English, but this was also done in a classroom environment where high importance was given to students' language and culture. Even though these teachers' focus on developing their students' academic language in English may limit students' access to positive cognitive effects (Cummins, 1976), research has indicated that successful schools for multilingual children place an especially high value on their students' languages and cultures (Lucas, Henze, & Donato, 1990).

Bilingualism and Mathematics

Studies have pointed out that bilingualism is linked to cognitive advantages such divergent or creative thinking, executive control, and spatial reasoning (e.g., Bialystok, 2005; Greenberg, Bellana, & Bialystok, 2013). Of the three participants, only Mrs. Aguirre was not fully aware of the aforementioned cognitive advantages of bilingualism in mathematics, but she indicated that learning mathematical terms in another language should provide some cognitive benefits, given that it was "a richer experience". In contrast, Mr. Long and Mrs. Quiróz both cited concrete examples of the cognitive advantages that bilingualism provided their students in mathematics.

While Mr. Long indicated how bilingual students were creative when problem solving, and even referred indirectly to metalinguistic awareness as the source of this creativity, Mrs. Quiróz indicated that, once these children comprehended what a word problem was asking, they were able to decide what approach to take on their own, and

she also explained that having to switch languages according to the situation (executive control) was behind bilingual students' advantage in mathematics.

Mrs. Aguirre learned English later in life, while Mr. Long and Mrs. Quiróz grew up learning both English and Spanish and lived in bilingual communities. This difference influenced each participant's points of view on the cognitive advantages of bilingualism and some of their actions in the classroom. Further, although all the participants taught mostly in English, the data indicated that Mrs. Aguirre's use of Spanish diminished with each subsequent observation.

Translanguaging

The three participants in the study used a variety of linguistic strategies that qualify as translanguaging during their mathematics instruction. These strategies were used to make sense of the content and to elicit students' thinking, but not necessarily to support students' first language. These teachers' main concern was to help their EL students adapt to their new environment as quickly as possible, and in this sense mathematics lessons were also an instrument of acculturation. Nevertheless, these teachers also used translanguaging as a way to establish rapport with ELs, to affirm knowledge of a common identity, and to interrogate linguistic inequality.

The translanguaging exercises conducted during interviews revealed that the participants could use translanguaging as a pedagogy where the input language (listening) and the output language (speaking) were different. The interview data also revealed that they all had a limited knowledge of a Spanish-language register in academic mathematics for the grade level they taught. Additionally, misconceptions or partial conceptions in

mathematics permeated from one language to the other. The most problematic words for the participants were those in the mathematics register that are also used in multiple registers in both English and Spanish. For example, Mr. Long translated *right angle* as *ángulo derecho*, which literally means *angle on the right*.

The participants' use of the academic language of mathematics in English was mostly accurate, and this was a direct consequence of both the curriculum materials and standardized tests being in English. Since Spanish was used as a meaning-making tool, the participant teachers did not invest time to improve their knowledge of academic language of mathematics in Spanish. However, Mr. Long and especially Mrs. Quiróz were able to use cognates to develop their students' metalinguistic awareness on several occasions.

The translanguaging exercise I conducted during interviews consisted in asking participants to explain the difference between the concepts of mean and median in both Spanish and English. I used a different approach with Mrs. Aguirre than the one used with the other two participants. On one hand, my approach with Mrs. Aguirre was closer to what translanguaging is as a pedagogy: The input (listening) and output (speaking) languages were switched in a pre-planned, structured, and intentional way. The data showed that it was difficult for Mrs. Aguirre to use the academic language of mathematics in Spanish and would switch to English in order to express herself more clearly. On the other hand, my approach with Mr. Long and Mrs. Quiróz was less structured, asking them to explain concepts as if I were an EL student.

The data revealed that all three participants did not know the Spanish words for *mean* and *median* and that their conceptual understanding of these terms was not deep, regardless of the language they used to explain them. They all defined the mean in terms of its algorithm and none of them mentioned that the median was not susceptible to extreme values. When explaining how to find the median, they all referred to the simplest case (when there is an odd number of items in the data set), and only mentioned the other case when pressed.

Of the three participants, Mrs. Quiróz spoke Spanish with the most proficiency, and did not code-switch to English. When explaining the median in Spanish, both Mr. Long and Mrs. Quiróz used the word *chiquito* ("smallish") to refer to the smallest value of the data set. Since I had asked them to respond as if I were an EL student, this may have influenced their use of the Spanish diminutive.

The classroom observations revealed that participants used the following teaching strategies that involved translanguaging:

- *Cognate use* to develop students' metalinguistic awareness (e.g., Mrs.Aguirre's worksheet with the word *percent*; Mr. Long angles lesson involving the cognates *agudo* and *acute*; Mrs. Quiróz finding the relationship between the cognates *cien* and *cent*).
- *Collaborative dialogue* between teacher and students to find the solution to a problem (e.g., Mrs. Aguirre and Raquel discussing the word *percent*; Mr. Long and Larissa exchanging ideas on inequalities; Mrs. Quiróz and Beatriz talking about perimeters of squares).

- *Collaborative grouping* of ELs and English proficient bilinguals to create the social interaction necessary for the simultaneous development of the basic interpersonal communication skills and the cognitive academic language proficiency (e.g., Mrs. Quiróz pairing Beatriz with a bilingual classmate).
- *Previewing* in Spanish the rules or properties to be used in a lesson to build background knowledge (e.g., Mrs. Quiróz previewing the sign rules when multiplying and dividing integers).
- *Revoicing* to reflect on what students said by repeating or rephrasing it (e.g., Mrs. Aguirre rephrasing in Spanish and English what Rocío said about variables; Mr. Long and Mrs. Quiroz repeating what students said).
- *Stimulus of inner speech* to unpack students' prior knowledge (e.g., Mrs. Aguirre using mental maps; Mr. Long asking Johnny to think in Spanish).
- *Student paraphrasing* in English what other students said in Spanish to promote deeper understanding (e.g., Mrs. Aguirre asking Sara to paraphrase what Arturo said).
- *Translanguaging in speaking* to make sense of the content, to establish rapport with students and to interrogate linguistic inequality (e.g., Mrs. Aguirre telling the *Pizza por favor!* story; Mr. Long explaining inequalities to Larissa; Mrs. Quiróz explaining the meaning of *cent* as a prefix or suffix).
- *Translating* to make students gain immediate access to an unknown word (e.g., Mrs. Aguirre revoicing *letra* as *letter*; Mr. Long revoicing *boca* as *mouth*; Mrs. Quiróz explaining Beatriz that *all* is *todo*).

- *Use of mnemonic devices* to unpack prior knowledge (e.g., Mrs. Aguirre utilizing the "chinito" device so students remembered the sign rules of multiplication; Mr. Long using the phrase *el piquito para el chiquito* referring to inequalities).

Teacher participants used translanguaging in a pragmatic way to accelerate the acculturation for their EL students. They used translanguaging without hesitation because the learning environment enabled them to use either language. Therefore, English proficient bilingual students might have benefitted indirectly from these teachers' translanguaging by processing the information in two different languages.

Furthermore, I did not observe instances of translanguaging where the input was in reading and the output was in writing. All curriculum materials and resources were in English, and both teachers' and students' written work was also in English. Limiting the use of translanguaging to listening/speaking also limits the bilingual proficiency of students and—according to the threshold hypothesis (Cummins, 1976)—students' access to positive cognitive effects.

Implications for Practice

The analysis of interviews and classroom videos for this collective case study revealed the important role of translanguaging as a sense-making, affiliative tool that is used in Latino classrooms to unpack students' prior knowledge, elicit students' mathematical thinking, and scaffold student learning. Even though the participants of this study were bilingual, the teaching strategies involving translanguaging that they used in their mathematics classrooms can be used by any teacher in any linguistically diverse classroom.

For example, it is not necessary for a teacher to speak fluent Spanish to revoice a mathematical term in Spanish used by a student. A teacher that revoices in foreign languages would seek to establish rapport with ELs through the affiliative nature of translanguaging. By briefly using a minority language, a teacher would be expressing her intentions to learn from their students and join their community.

Furthermore, the internet is a rich resource of multilingual media that could be used to implement other teaching strategies involving translanguaging, such as previewing mathematical content in linguistically diverse classrooms. If the access to technology is problematic, other strategies can be used, such as asking bilingual English proficient students to act as paraphrasers for EL students. However, teachers should use additional support (such as written work) to better understand ELs' mathematical thinking when asking other students to act as paraphrasers.

Limitations of the Study

This collective case study was conducted in a school district located in a bilingual (English/Spanish) community in South Texas. The student population of the district was ethnically homogenous (Latino) and the participants were three Latino bilingual teachers certified to teach mathematics at the middle school level. Moreover, the participants held other certifications, obtained (or were pursuing) graduate degrees, and translanguaged in their mathematics classrooms.

The most important limitation of the study is that I did not conduct any post-observation interviews with teachers or students. Consequently, I could not determine the reason for some teachers' actions or students' misconceptions with certainty. In these

cases, I proposed explanations based on the available evidence on video and what teachers told me about their practice during our interviews.

Directions for Future Research

I conducted this collective case study in order to fill an identified gap in the literature on the influence of translanguaging in a South Texas school district that has had important achievements in the mathematics performance of Latino middle school students. The description of the participants' educational and family background, support of students' first language, their views of the cognitive advantages of bilingualism, and the variety of translanguaging modes used while teaching mathematics offer relevant information to mathematics education, sociolinguistics, psycholinguistics, and bilingual education for future explorations.

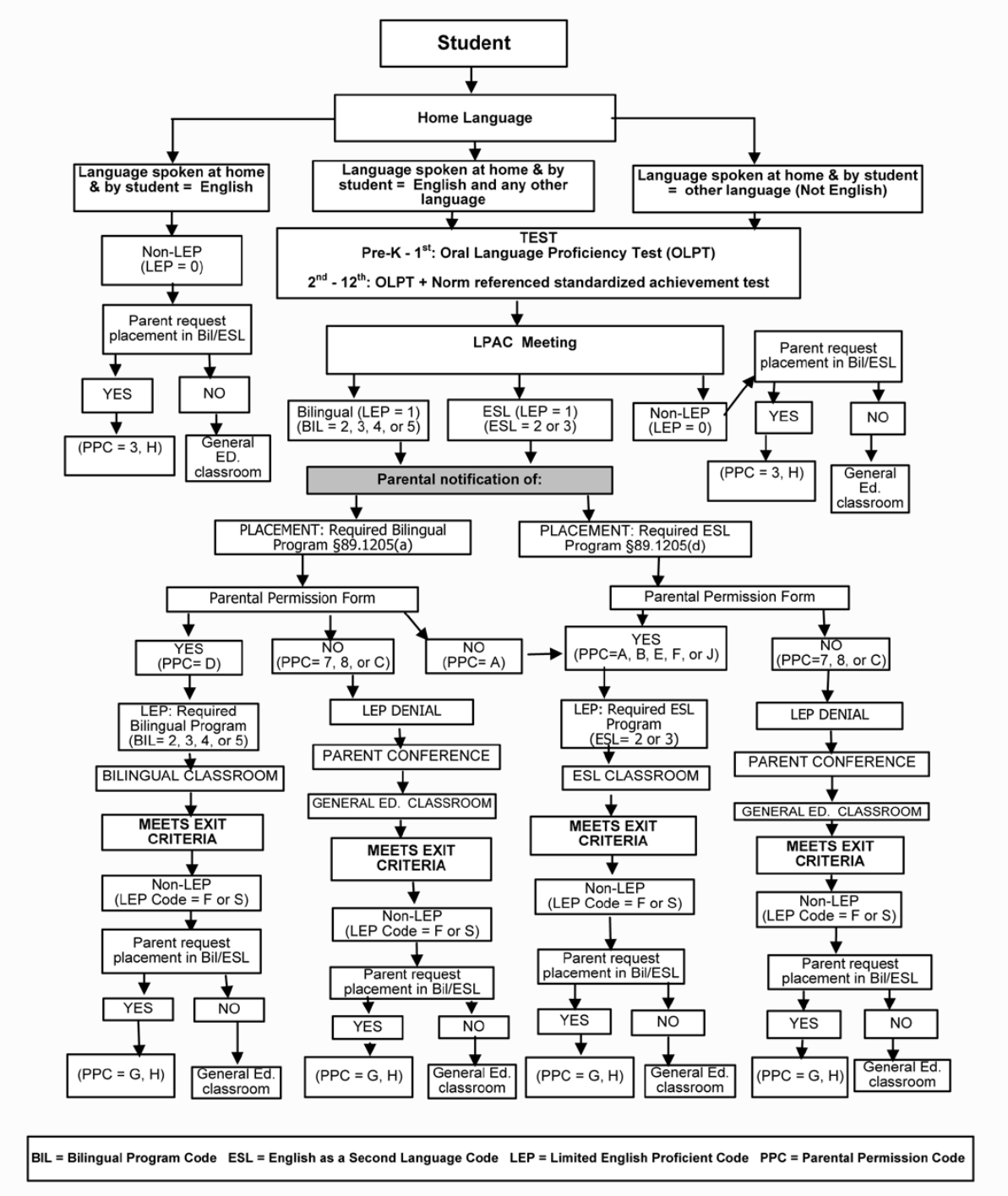
More evidence on the benefits of translanguaging in mathematics education needs to be gathered, ideally through interventions in which the effectiveness of translanguaging as a pedagogy is tested in linguistically diverse mathematics classrooms. Furthermore, research on translanguaging as a tool for assessing students' mathematical knowledge is necessary to clarify its benefits when used in formative and summative assessments.

Psycholinguistics research has shown that bilingualism is associated to cognitive advantages in tasks involving complex spatial reasoning and problem solving skills (Greenberg et al., 2013; Kempert et al., 2011). Therefore, research on translanguaging in linguistically diverse mathematics classrooms has the potential of transforming

mathematics education with the purpose of extending the cognitive advantages of bilingualism to all ELs.

APPENDIX SECTION

APPENDIX A: LIMITED ENGLISH PROFICIENT DECISION CHART



APPENDIX B: DESCRIPTION OF THE MIELL SAMPLE

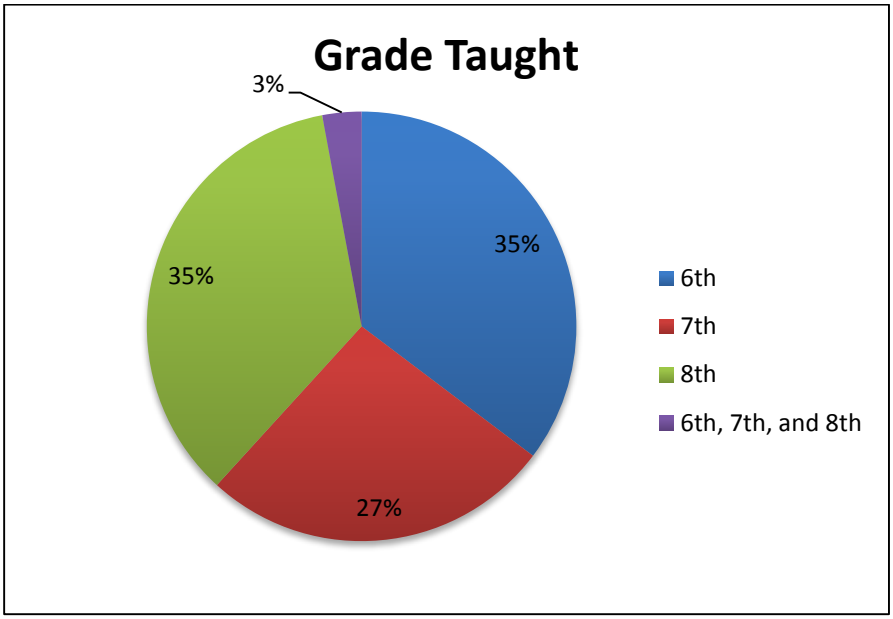


Figure 1B. Chart of the distribution of the participants by grade taught.

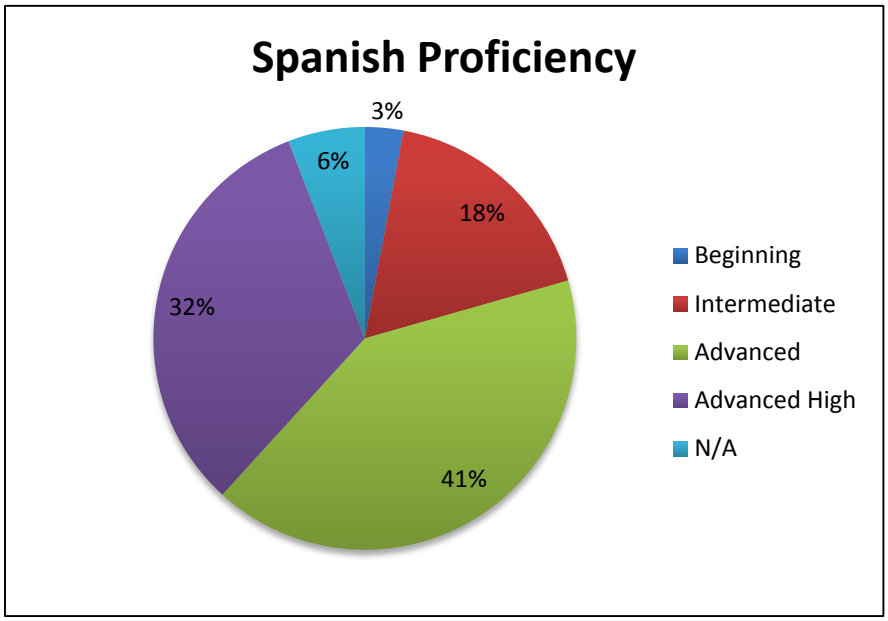


Figure 2B. Chart of the distribution of participants according to their Spanish proficiency.

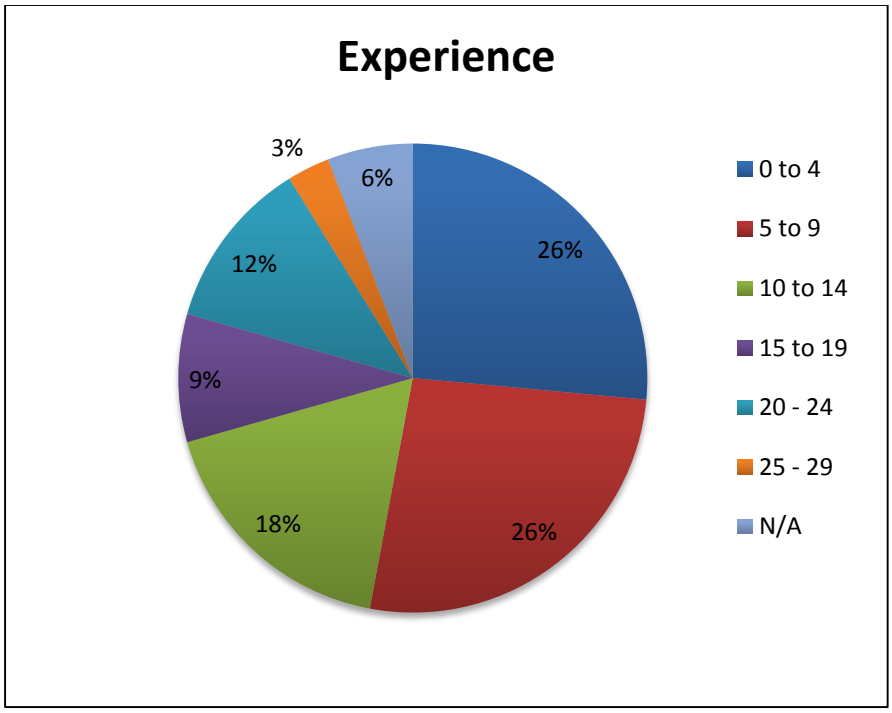


Figure 3B. Chart of the distribution of the participants according to their experience (in years)

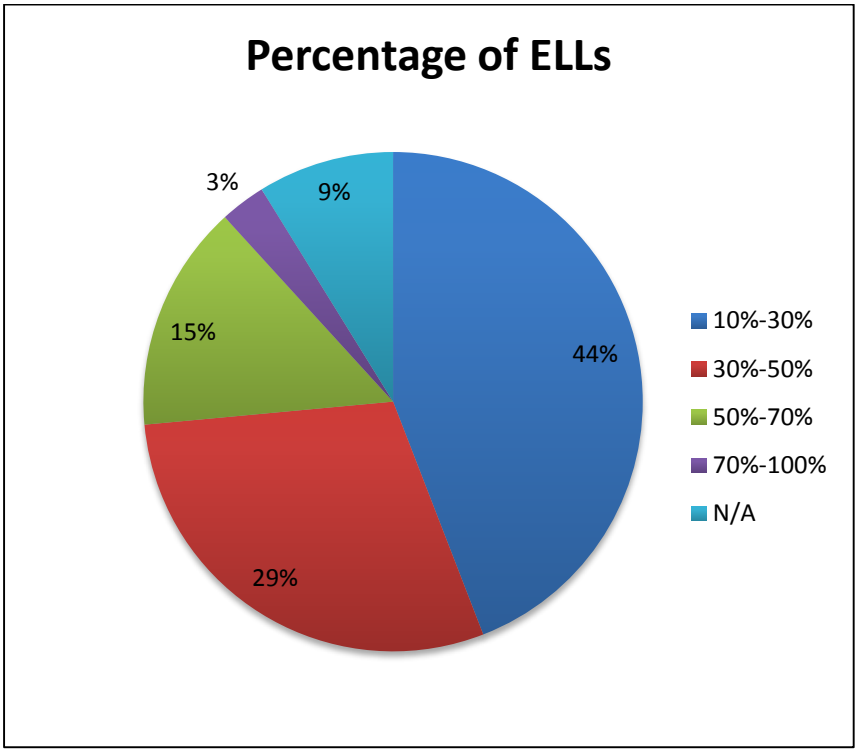


Figure 4B. Chart of the distribution of the participants according to their perception of the percentage of ELs in their classrooms.

APPENDIX C: INTERVIEW PROTOCOL

My name is Carlos Mejía. I am a doctoral student at Texas State University in the Mathematics Education program. The purpose of my study is to determine which bilingual (Spanish/English) teachers' traits are conducive to English language learners' achievement gains in mathematics. Your participation in this study is voluntary. The interview will be recorded, and the recording will be destroyed after analysis. Your name will never appear in any research instrument, including the transcript of this interview.

Teacher Characteristics

1. Where were you born? Where were your parents and grandparents born? Which languages were spoken in your childhood home?
2. Where did you attend school? What was your experience learning mathematics?
3. What are your teaching credentials? What is your teaching experience in mathematics? What is your experience teaching ELLs? Have you ever taught mathematics abroad?
4. What are the traits that a mathematics teacher should have when teaching in a bilingual classroom? Of those traits that you mentioned, is there one that is more important than the others?

Support of Students' First Language

5. What are some of your successes teaching mathematics to ELLs? What are the challenges of teaching mathematics to ELLs? How do you overcome those challenges?
6. How do you prepare your ELL students for the standardized mathematics test? What would prepare your students for the standardized mathematics test if they were all monolingual?
7. What is your opinion on switching from English to Spanish while teaching mathematics? What is your opinion on allowing students to speak/write in Spanish while solving mathematics problems?
8. If you use Spanish during mathematics instruction, when do you do it? Please provide examples.

Bilingualism and Mathematics

9. Have you noticed any difference in how monolingual and bilingual children learn mathematics? Can you provide examples?
10. Does bilingualism provide any learning advantage in mathematics?

Translanguaging

11. How would you clarify the difference between the concepts of *mean* and *median* to an English language learner? How would you explain this difference in Spanish?
12. Is there anything else you would like to say about teaching mathematics in bilingual classrooms?

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