

CHINESE MIDDLE SCHOOL MATHEMATICS TEACHERS' PEDAGOGICAL
CONTENT KNOWLEDGE AND CULTURAL BELIEFS TOWARDS TEACHING

by

Zhaochen Song, M.S.

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Committee Members:

M. Alejandra Sorto, Chair

Hiroko K. Warshauer

Robert Sigley

Melissa A. Martinez

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ABSTRACT

This study investigated the pedagogical content knowledge and cultural beliefs towards teaching of middle school mathematics teachers in Shandong, China. The study illustrates how four middle school mathematics teachers exhibit their knowledge of teaching and knowledge of curriculum in their daily teaching, and how their beliefs about teaching related to such knowledge. Four middle school mathematics teachers participated in this study, and data were collected by means of interviews and classroom observations. The interviews and classroom observations were analyzed using qualitative methods. The significance of this study lies in its contribution to mathematics educators who seek improvement for professional development programs from other cultures and education systems.

The interviews and classroom observations show that the four teachers demonstrated profound pedagogical content knowledge in teaching and curriculum. In particular, their use of questions, the emphasis of prior knowledge and connections between topics, knowledge in curriculum, and teaching coherence was evident in the observations and interview. Though they were using a unified curriculum, their teaching approaches could be vastly different. Two teachers out of four used non-traditional teaching approaches in the observations. Their cultural beliefs towards teaching were established closely to the High School Entrance Exam and how the education system runs in China and hence affected their teaching strategies. Specifically, they valued the use of

practice problems and assessments in their daily teaching, and they all believed that the collective work was helpful to improve their teaching.

This study indicates that teaching approaches of middle school mathematics teachers in China could be driven by their individual beliefs about teaching, their teaching experience, and the pressure from the Entrance Exam. The idea of collective and collaborative work is worth noting in improving the quality of professional development programs.

Keywords: pedagogical content knowledge, cultural belief, middle school mathematics, education in China, professional development

I. INTRODUCTION

Statement of the Problem

To meet the goal of competitiveness in mathematics education globally, international comparison studies (ICS) have received increased attention for sharing, discussing, and debating important issues across countries (Robitaille & Travers, 1992). In recent years, mathematics education in different countries has benefited from the results from ICS, providing an impetus to improve students' performance as well as teachers' effectiveness of teaching. For instance, the report of the Trends in International Mathematics and Science Study (TIMSS) has shown that both fourth and eighth graders' mathematics scores in the U.S. have made a significant increase in their 2007 averages when compared to their 1995 scores over the 12 years (National Center for Education Statistics, 2007). If education systems in different cultures continue examining, comparing, and learning from international mathematics education, it will benefit and help their countries compete and achieve a higher rank globally.

At the same time, international studies indicated that East Asian countries, particularly Chinese students outperform many of their peers in mathematics tests and competitions, and it is reasonable to hypothesize that this learning gap is connected, to some extent, to their teachers, as teaching is a major determinant in students' learning gains (Darling-Hammond, 2000). One of the possible explanations for their good performance is related to teaches and teaching. Teachers, undoubtedly as one of the most significant factors in mathematics education, not only influence students on their content knowledge, but also play a critical role in shaping their misconceptions and confusions (She, Lan, & Wilhlem, 2011). One of the ways to improve students' mathematics

learning is to focus on teachers' knowledge (An, Kulm, & Wu, 2004; An, Kulm, Wu, Ma, & Wang, 2006; Ma, 1999). The National Council of Teachers of Mathematics (2000) pointed out that effective teaching requires "knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17). The international perspectives in mathematics educators developed a deep understanding of various aspects of mathematics teaching and learning and promoted teachers to question their own teaching practices and to develop better strategies in the teaching process (An et al., 2004; National Mathematics Advisory Panel, 2008; J. W. Stigler & Perry, 1988).

In such an era of information and globalization nowadays, the need for teachers to be knowledgeable in other aspects of teaching, such as the knowledge of technology, is increasing. Yet, the pedagogy and the content knowledge of the subject should always be considered as the most important element in teachers' knowledge. It brings my curiosity to see teachers' knowledge in teaching with a cultural context, particularly China, one of the most competitive countries in mathematics education. The well-known study of Ma (1999) showed evidence of quality teaching for mathematics teachers in China. The problem is that we do not know much about teachers and teaching at the middle school level that helps us understand why the students do well.

Thus, I wonder, if a group of teachers in China can be observed and if there is anything I can learn from them in terms of teachers' knowledge of teaching. Although we have learned from the international study of U.S. and Chinese teachers conducted by Ma (1999) about the content knowledge between elementary teachers, we do not know much about how secondary, particularly junior middle school, teachers use their pedagogical knowledge for their effective teaching in China. Studying this group of teachers in detail

about their knowledge and belief could contribute to the international mathematics teacher education literature and many countries, including the U.S., and thus make advances on how to improve student learning from the teachers' perspective.

Researcher's Background

Living in China for the first 17 years of my life, I experienced the education system in China from kindergarten to high school. Spending another year in high school in the U.S., I witnessed the enormous difference in education systems between the two countries at the high school and college levels, especially in the subject of mathematics. Though the overall average scores were about the same, some of my fellow classmates in the U.S. were not able to solve basic algebra problems at senior level, while back in China, I was dealing with calculus, conic sections, and spatial vectors. While in the U.S. I barely had any homework assigned every day for my mathematics classes, I had to do at least one full set of tests besides a tremendous amount of homework back in China. For many years, I have had the feeling that many U.S. students in secondary schools have problems with the fundamentals in mathematics. Having been teaching introductory college-level math courses for over five years, I had even stronger feelings. I have to admit that such a performance gap was formed partly because the students did not expend their utmost effort, but it was not only the students to blame: the focus points, explanation of the concepts, connections between topics, classroom management, and so many other aspects were executed by teachers so differently, that it appeared that the teacher in the classroom was the actual determining factor of the learning process. My experience as a student in both the U.S. and China and as a teaching assistant at a large state university in

the U.S., my knowledge in teaching, as well as my cultural background motivated me to research the area of teachers' pedagogical content knowledge and their cultural beliefs more in-depth, especially as the pedagogical content knowledge is closely related to the actual teaching and directly affects teachers' instructional quality (Krauss et al., 2008).

Middle school education in China

The education system in China has its reputation for being rigid and rigorous in the world, yet it is also considered as one of the most competitive ones in the world. Before 1958, China adopted the education system from the former U.S.S.R. The relationship between the two countries worsened from 1958 and China began to deny the system and gradually create her own education system. Despite the large-scale education reform starting from early 1990's, the school structure has remained until today. Children between ages 12-17 attend secondary schools, which is often divided into junior middle schools (初中 chū zhōng, 7th – 9th grade) and senior middle schools (高中 gāo zhōng, sometimes also referred to as high schools, 10th – 12th grade). For some regions where the students attend elementary schools for five years, the corresponding middle schools take four years (6th grade – 9th grade). In this study, we refer to junior middle schools only as middle schools and refer to senior middle schools as high schools.

For a student in China, the completion of middle school is the end of the nine-year compulsory education. If a student wants to receive education in higher levels, s/he can choose whether to attend a regular high school, a vocational school or a secondary professional school, almost solely based on his/her score in the High School Entrance

Exam (中考, zhōng kǎo). Thus, the most important goal for most middle schools is to prepare students for the best high schools, as better high schools are believed to have higher college enrollment rates. Because of the limited spots in high schools, especially those considered to be the good ones, the High School Entrance Exam is exceptionally competitive. The Exam tests students' knowledge in Chinese, mathematics, foreign language (usually English), and a few other courses, depending on the testing province. For most students, the total score of the Entrance Exam is the ultimate evidence of their three years' academic performance, which they use for admission into high schools.

As almost the only goal in middle school, both the groups of teachers and parents value this one-time opportunity where the students could receive a better quality of education at the next level. For some families, the Entrance Exam is the top priority, and everything that a family does should be supportive of the child's studying. For students, on the other hand, the pressure starts when they enter 7th grade, and it gets tenser as they move up in middle school. In Shandong Province, the region for my study, although it is not recommended by the Bureau of Education, at the end of the 8th grade, most middle school teachers will start rushing to teach material in 9th grade, so that there is more time for rounds of a comprehensive review that last for months.

Teacher education in China

Teacher education in China generally means pre-service education and in-service education. The respective education activities, until the early 1990s, were provided separately by two independent systems of institutions. (Z. L. Yang, Lin, & Su, 1989).

However, along with a rapid shift of the country's economic system from a planned to a socialist market one, and a dramatic transition of social power from bureaucratic control to market forces, the system of higher education (including teacher education) as it existed for at least half a century encountered both internal and external challenges and pressures, and needed to address or overcome them inevitably in the 1990s (Shi & Englert, 2008). As a result, previous institutions that offered teacher education had been amalgamated or upgraded to new ones, and the education programs became more diverse. Non-normal (without teacher education programs) universities started to offer teacher education programs, and normal schools training preschool and elementary school teachers gradually changed to training secondary school teachers or promoted to five-year junior teacher colleges that enroll graduates of junior high schools (Zhu & Han, 2006). The current institutions that offer teacher education include general comprehensive universities, normal colleges and universities, and independent educational (training) institutes.

In order to teach in public schools in China, one must obtain the teaching certificate of the corresponding grade level and subject. All middle school pre-service mathematics teachers in China must have at least a college degree in mathematics or related fields, or a degree in mathematics or related fields from independent educational institutes (Ministry of Education, 2015). Also, all candidates must pass a competitive certification test. The certification test contains various subjects, including education theories, pedagogy, content knowledge, and common knowledge. If a candidate holds a non-education degree, besides an additional test in psychology, s/he will be interviewed to ensure capability of teaching.

Purpose of the Study

The purpose of this study is to contribute to the existing body of knowledge about middle school mathematics teachers' pedagogical content knowledge and beliefs towards teaching from an international perspective by examining Chinese teachers' classroom practices and cultural beliefs about their instructional practices. Results from this study have the potential to advance knowledge in the field of international teacher education and to inform professional developers about alternative teacher training models.

Research Questions

In this study, the following research questions are addressed:

1. What is the pedagogical content knowledge of middle school mathematics teachers from Shandong Province, China? In particular,
 - a) What is their knowledge of teaching mathematics,
 - b) What is their knowledge of mathematical curriculum?
2. What are middle school mathematics teachers from Shandong Province cultural beliefs towards teaching?
3. How are Chinese teachers' cultural beliefs towards mathematics teaching related to their pedagogical content knowledge?

Definition of Terms

The following terms are used throughout this study, and the corresponding definitions are provided here:

Pedagogical Content Knowledge (PCK). Originally, Shulman (1987) identified the term *Pedagogical Content Knowledge* as one of seven knowledge bases for teachers, defining it as “that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding” (p. 8). He mentioned the other six categories as content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational contexts, and knowledge of educational ends. Among those seven categories, PCK was supposed to be of special interest since “it is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue” (p. 8). In this article, Shulman further explained PCK as the ability of the teacher to transform content into forms that are “pedagogically powerful and yet adaptive to the variations in ability and background presented by the students” (p. 15). In his earlier article, Shulman (1986) identified two components that are central to PCK, namely knowledge of instructional strategies and representations and knowledge of students’ conceptions and misconceptions:

for the most regularly taught topics in one’s subject area, the most useful forms of representation of those ideas, the most powerful analogies,

illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others...[PCK] also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates (p. 6).

This study will adopt a refined definition from An et al. (2004), treating the PCK as the knowledge of effective teaching which involves three components: knowledge of content, knowledge of curriculum, and knowledge of teaching.

Cultural Belief. In particular, this study discusses middle school mathematics teachers' cultural beliefs towards teaching. Ernest (1989) originally defined a teacher's belief system as "the teachers' conception of the nature of mathematics and mental models of teaching and learning mathematics." To make this definition fit this study, we define the term *cultural belief* as "the teachers' conception of the nature of mathematics and mental models of teaching and learning mathematics under the influence of culture."

Mathematical Content Knowledge (MKT). MKT is perhaps the most influential reconceptualization of teachers' PCK in mathematics education. The idea of MKT was developed through the overarching constructs (e.g., Ball,

Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008; Hill, Schilling, & Ball, 2004) that covers PCK and content knowledge (CK) together. Shulman's idea of PCK served "as a heuristic, as a tool for helping the field to identify distinctions in teacher knowledge that could matter effective teaching" (Ball et al., 2008, p. 392), and it is more theoretical than practical. On the other hand, the concept of MKT was formed from a series of studies to validate MKT empirically. Moreover, the concept of MKT integrated concepts of PCK and CK, which were considered as two distinct components by Shulman (1986, 1987).

Related studies (e.g., Ball, Bass, & Hill, 2004; Ball, Hill, & Bass, 2005; Ball et al., 2008) indicate that three categories in MKT concern teachers' PCK: knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. To have this knowledge, teachers will need interactions with students with specific mathematics topics and understandings of students' mathematical thinking, an understanding of pedagogical issues that influence students' thinking, and knowledge of the curricular programs. In our study, we follow Ball et al. (2008) and refer to MKT as the mathematical knowledge that teachers need to teach mathematics.

Delimitations

Though there are many factors to effective teaching, this study will focus on teachers' pedagogical content knowledge and their cultural beliefs toward teaching. Also,

it will involve only mathematics teachers who are currently teaching middle school in one region in China.

Summary

In this chapter, I presented my background and motivation for conducting this study. I presented the purpose of this study and my research questions after revealing the current issues of U.S. mathematics education in the global competition. In the next chapter, I will review the existing literature in the field that is related to my study. Chapter II is organized into the following sections: international student achievement studies, studies in mathematics education that involve China, pedagogical content knowledge and theoretical framework, and cultural beliefs towards teaching.

II. LITERATURE REVIEW

Introduction

This chapter reviews the prior research conducted in the relevant fields in my study. It contains four sections: international students' achievement studies, studies in mathematics education that involves China, pedagogical content knowledge and theoretical framework, and cultural beliefs toward teaching. The section on international students achievement studies reveals the gap in student performance in international studies; the section of studies in mathematics education that involves China examines the studies conducted in, but not necessarily limited to, teachers' pedagogical content knowledge (PCK) in China; the section on pedagogical content knowledge and theoretical framework examines the origin, nature, and development of PCK commonly used today and propose an adopted theoretical framework; lastly, the section of cultural beliefs toward teaching examines the cultural factors that could affect teacher's teaching practice.

International student achievement studies

The internationalization of teacher education has long been an interest of instructors of education programs in the U.S. (Hiebert, Gallimore, & Stigler, 2002; Klassen, 1972). It provided opportunities for sharing, discussing, questioning their own teaching practices and found better choices in constructing the teaching process (J. W. Stigler & Hiebert, 1999). The Trends in International Mathematics and Science Study (TIMSS) 2007 is the fourth time since 1995 that this international comparison of student

achievement has been conducted. Gonzales et al. (2008) reported that the average U.S. mathematics score was higher than those of students in 37 of the 47 other countries at eighth grade, and the average mathematics score for eighth grade, 506 points, was 16 points higher compared to the 1995 average of 492. Although a series of continual large-scale studies have shown U.S. students making significant progress in the international mathematics tests over the last decade (National Center for Education Statistics, 2007), international assessments from TIMSS (1999 & 2007) and PISA (2009 & 2012) revealed that the disparity between U.S. students' mathematics achievement and those from the other countries had not improved. From a more specific perspective, Mills and Holloway (2013) investigated the relationship between student achievement in statistics and factors at the student and teacher/classroom level using the U.S. 8th-grade data from TIMSS in 2007 and indicated that TIMSS students' exposure to and learning of statistics-related concepts appeared to lag behind the expectation set forth in the Data Analysis and Probability Standard created by National Council of Teachers of Mathematics (2000). Mills and Holloway also suggested that the quality and training of available statistics teachers are significant concerns. Current comparative studies mainly focus on identifying the distinctions for students' achievement within various content and competence domains between the top-performing Asian countries and the U.S. (She et al., 2011). All five countries that outperformed U.S. in the eighth-grade mathematics test in TIMSS 2007 are from Asia, and so are the five out of seven countries that had a higher percentage of students performing at or above the advanced benchmark than the U.S. (Gonzales et al., 2008).

Beginning from the year of 2000 and conducted every three years, The OECD Program for International Student Assessment (PISA) is another international study that focuses on 15-year-old students' ability to use their knowledge and skills to meet real-life challenges. Similar to TIMSS, PISA was not designed to provide individual student scores, but rather national and group performance estimates in reading, mathematics, and science literacy (Fleischman, Hopstock, Pelczar, & Shelley, 2010). The term "mathematics literacy" is defined by PISA as:

An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (Organization for Economic Cooperation and Development (OECD), 2009, p. 84).

U.S. 15-year-olds' mathematics literacy in 2009 (487) was higher than the U.S. average in 2006 (474), and not measurably different from the U.S. average in 2003 (483), the earliest time point to which PISA 2009 performance can be compared in mathematics literacy (Fleischman et al., 2010). The authors also pointed out, that participating in the study for the first time as a partner economy, students from Shanghai, China scored the highest on average in the PISA 2009. The report from OECD also showed that Shanghai students' mean score in mathematics was the equivalent of nearly three years of schooling above the OECD average; in fact, seven out of the ten highest-ranking economies in mathematics were in Asia.

Such gaps in testing scores through various international studies indicate the need of change in our education system (Fleischman et al., 2010; Gonzales et al., 2008;

Organization for Economic Cooperation and Development (OECD), 2013). Specifically, She et al. (2011) and Mills and Holloway (2013) suggested that the teacher is the most important factor in the process of learning mathematics. The gaps in those test scores, as well as the suggestions in those articles, motivated me in studying the potential cause of such differences, and hence the knowledge of teachers in China.

Studies in mathematics education that involve China

There has been some research comparing Chinese and U.S. mathematics teachers' content knowledge. For instance, Ma's work (1999) focused on comparing elementary teachers' content knowledge. In her study, a group of Chinese teachers with the equivalent of a 9th-grade mathematics education outperforms college-trained U.S. teachers when asked to respond to four mathematical teaching scenarios. Ma indicated that Chinese elementary mathematics teachers who received more mathematical training seemed to know more mathematics for teaching than the U.S. teachers, yet she focused mainly on teachers' mathematical content knowledge and didn't fully account for cultural contexts, learning objectives, and teachers' beliefs towards teaching. Many of Ma's examples provided indications about how teachers apply their mathematics knowledge in teaching; however, a systematic study that combined the mathematical knowledge with pedagogical content knowledge, and cultural beliefs was not conducted. Though the interest in the connection between teacher knowledge of mathematics and student learning persists, as measured in most of past studies, content knowledge alone does not ensure effective teaching performance and may not be the best investment of teacher development time (Kahan, Cooper, & Bethea, 2003).

In China, a school-based teaching group system was built decades ago; nowadays, such a teaching research group exists almost in every school, town, and city in China. Different from western culture, the classroom teaching of Chinese mathematics teachers is open for colleagues' observations, studies, and discussion (Y. Yang, 2009). The main goal of the teaching-research group is to solve practical teaching problems collaboratively. Yang's study (2009) showed, by a case study of one teacher, showed how a Chinese mathematics teacher improved his teaching quality in such teaching research activities. Besides, Liang, Glaz, and DeFranco (2012) investigated the characteristics of a group of award-winning grades 7-12 mathematics teachers from the Shandong Province in China and illustrated how their expertise had been developed over the years. In addition to investigating these award-winning teachers' characteristics, the authors also explained how the teachers' teaching expertise was developed. This study used a qualitative research method to examine the characteristics of award-winning grades 7-12 mathematics teachers in China, providing an insightful view of mathematical teaching and teacher preparation in a different cultural context. In-depth interviews were used as the major method of data collection, and broad document view was used as a supplemental data collecting method. The analysis of the data identified the characteristics of the award-winning teachers. First, these teachers were all passionate about mathematics and enjoyed sharing their passion through teaching. This group of teachers actively participated in teaching research through the application of teaching research in the classroom, collaboration with peers, and systematic lesson preparation. In addition, they applied technology to teaching. Finally, these teachers engaged in teaching research in order to expand their professional opportunities (Liang et al., 2012). Eight out

of ten participants in this study published numerous teaching research papers or books. In the most extreme cases, one of the teachers published more than 500 articles in newspapers and more than 100 papers in twenty national popular journals.

This study also suggested that the better a teacher is at teaching research, the more professional opportunities are provided to him/her. All the teachers who were interviewed pointed out that they had benefited from taking an active role in teaching research, and the positive effect motivated them to keep going (Liang et al., 2012). One theme emerging from this study is that in-service training played a significant role in building these teachers' teaching expertise. As a part of in-service training, teaching research was conducted throughout the entire professional life of the teachers. This study indicates that the schools in China encourage teachers to engage in teaching research and reward those who perform well in teaching research by giving those honors or early promotion and more opportunities for professional development and advancement. Teaching research, combined with teacher collaboration, plays a powerful role in improving teaching effectiveness, reinforcing each other. Both are fundamental for an in-service teacher training system in China, and the U.S. teacher training system could learn from it to improve the quality of teaching.

In another study, She et al. (2011) examined the U.S. and Chinese middle-level teachers' responses for differences in pedagogical content knowledge using an interview protocol compromised of algebraic questions. The purpose of the research was to determine the differences in pedagogical content knowledge between Chinese and American teachers when observing their problem-solving processes in specific algebraic areas. Four teachers from the West Texas area of the U.S. and four teachers from one

school in a large city of the Sichuan Province in southwestern China were interviewed. A set of eight algebraic word problems was given to participants to solve during the interview session. When solving those problems, teachers were encouraged to “think aloud,” a method that reveals people’s thinking process. The responses from the teachers were collected and coded into four themes within the domains of content knowledge and knowledge of teaching strategies (p. 37).

The author in this article found that teachers in the U.S. were more likely to use concrete models and practical approaches in problem-solving and promoting students’ knowledge skills. Compared to Chinese teachers, however, the U.S. teachers seemed to lack a deep understanding of mathematical concepts as well as connections between concepts. On the other hand, Chinese teachers were inclined to utilize theories and procedures in teaching. They tended to integrate conceptual knowledge as a network that made the knowledge applicable in multiple situations.

The results of this study support the idea that teaching for understanding is the key to successful mathematics education. In the Chinese teachers’ points of view, procedural proficiency is valued as much as the conceptual understanding, because it not only resulted from a genuine mastery of knowledge but also resonates conceptual understanding to some degree. On the other hand, U.S. teachers have more ambivalent attitudes toward procedural learning, such as the attitudes expressed by U.S. participants in the use of cross multiplication (She et al., p. 43). According to the authors, the findings in this study may identify factors that contribute to the discrepancy in mathematics achievement between American and Chinese students that are attributed to teacher impact. The lack of evidence revealed an obvious discrepancy in teachers’ content

knowledge in algebra; nevertheless, there were some significant differences between teachers in the U.S. and China about their problem-solving strategies and teaching methods. These findings provided some insights into the teachers' pedagogical content knowledge through the lens of an international comparative study.

An et al. (2004) compared the PCK of middle school mathematics teachers in the U.S. and China. The authors found that Chinese teachers emphasized developing procedural and conceptual knowledge through reliance on traditional, more rigid practices, which reflected their value for teaching mathematics content, while the U.S teachers emphasized a variety of activities designed to promote creativity and inquiry in attempting to develop students' understanding of mathematical concepts (An et al., p. 145). Focusing on the pedagogical content knowledge in teaching, the authors also pointed out, that in order to have broad and deep pedagogical content knowledge for effective teaching, teachers need to be able to have the following abilities: "to connect prior knowledge and concrete models to new knowledge, focusing on conceptual understanding and procedural development; to identify and correct students' misconceptions by using questions and various activities; to engage students in learning by providing various representations and examples; and to promote students' thinking by focusing activities and questions" (p. 169). Both countries showed benefits and limitations in learning and teaching mathematics.

Cai (2004) reports two studies that examined the impact on early algebra learning and teachers' beliefs on the U.S. and Chinese students' thinking. In this paper, he mainly talked about the latter, as the second study focused on teachers' beliefs and teaching strategies. In the second study, a group of 59 Chinese elementary mathematics teachers

from Guizhou Province and a group of 52 U.S. middle school mathematics teachers from Delaware, North Carolina, Pennsylvania, and Wisconsin participated voluntarily. The data were collected while both groups of teachers participated in professional development activities in their respective regions. In the first part, each of the teachers was asked to evaluate a set of 28 student's responses using a 5-point scoring rubric, while in the second part, a group of distinguished teachers were chosen from each region and interviewed about the reasons for their scoring each of the 28 responses. Later, those distinguished teachers were asked to judge the sophistication of the representations and strategies used in the responses to each problem.

The quantitative analysis of data in Cai (2004) showed that the U.S. teachers gave significantly higher overall mean scores than Chinese teachers did [$t(109)=3.68$, $P < 0.05$]. Of the 28 students' responses, the U.S and Chinese teachers rated significantly differently on 13 responses, with U.S. teachers more lenient than Chinese teachers on 12 of those, and Chinese teachers more lenient on only one. The only response which Chinese teachers scored significantly higher than U.S. teachers involved the number theory problem, which allowed for multiple correct answers. The findings of the second study also showed that U.S. and Chinese teachers view students' responses involving concrete strategies and visual representations differently (Cai, 2004). Although both U.S. and Chinese teachers value responses involving more generalized strategies and symbolic representations equally high, Chinese teachers had a much higher expectation of using the generalized strategies to solve problems for sixth graders, while U.S. teachers did not. This study contributed to our better understanding of the differences between the U.S. and Chinese students' mathematical thinking. It also established the feasibility of using

teachers' scoring of student responses as an alternative and effective way of examining teachers' beliefs (p. 158).

In their comparative study, Zhou, Peverly, and Xin (2006) noticed differences between Chinese and U.S. teachers in their PCK. Specifically, the PCK that the authors studied was the teachers' identification of the concepts to be included in instruction to ensure students' understanding of fractions, and teachers' knowledge of students' prior knowledge to learn fractions. The result in Zhou's study concluded that a "teaching gap" of teachers might parallel the well-known "learning gap" of U.S. students in large-scale international tests, such as PISA and TIMSS, where the words "teaching gap" and "learning gap" were mentioned in Stigler and Hiebert (1997).

Peng (2007) conducted a study to investigate the knowledge growth of mathematics teachers during professional activities based on the task of the lesson explaining. A lesson explaining is an activity that has been developed in China from an evaluative resource to an effective form of teacher professional development with the value of emphasizing teacher reflective practice. In lesson explaining, a teacher explains the teaching process and related issues about the lesson to colleagues and mathematics experts who then comment and discuss what was explained (Y. Chen, 2005). The analysis of Peng's study showed how an individual teacher developed his own subject matter knowledge in probability and how the professional community developed its pedagogical knowledge, PCK collectively during the professional activities based on the task of the lesson explaining.

In the literature, numerous journal articles compared and analyzed the mathematics teachers' PCK as well as their beliefs and teaching strategies between the

U.S. and China. While there is one article that identified the characteristics of award-winning mathematics teachers in Shandong Province of China, all the other articles that conducted comparative studies mentioned other provinces/regions in China. Most of the research conducted in this area is qualitative, with a few quantitative analyses, as a large number of participants is difficult to obtain and resource-consuming. Though these articles investigated the characteristics and teaching strategies of teachers, none discussed the teachers' conception of knowledge about teaching. Many studies provide indications about how teachers apply their mathematics knowledge in teaching but stopped before having a systematic study of how mathematical and pedagogical knowledge were integrated. It was suggested by An et al. (2004) that knowledge of teaching is one of the most important factors that help us understand teacher's thinking of their teaching. For a future study, we should also increase the scope of the number of teachers, having a larger number of teachers from other places to participate in the study.

These studies that compared various aspects between the mathematics education systems in the U.S. and China indicated several differences in teachers' teaching practices. For instance, the U.S. teachers valued concrete examples with specific strategies more, while Chinese teachers tended to favor generalized strategies more traditionally. It is reasonable to think that such a difference was caused by different cultural values and beliefs, implying the consideration of cultural values when we conduct the following stages of the study.

Pedagogical Content Knowledge (PCK) and Theoretical Framework

According to a Chinese saying, if one wants to give the students one cup of water, the teacher should have one bucket of water of his own. Shulman (1986) introduced the concept of PCK as an answer to what he called a “missing paradigm” in the field of research on teaching. He pointed out that “to be a teacher requires extensive and highly organized bodies of knowledge” and stated further in one of his later articles that PCK includes knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes, and values, and their philosophical and historical bases (Shulman, 1987). In this article, Shulman defined the term *Pedagogical Content Knowledge* as the ability of the teacher to transform content into a form that is “pedagogically powerful and yet adaptive to the variations in ability and background presented by the students” (p. 15). Although educators from different subject seem not to have an agreement on the definition of PCK, the concept of PCK is very influential in research on teaching and teacher education.

As a systematic review of the way PCK was conceptualized and studied in mathematics education, Depaepe, Verschaffel, and Kelchtermans (2013) examined 60 journal articles that studied PCK and identified different conceptualizations of PCK that in turn had a differential influence on the methods used in the study of PCK. The study revealed four common characteristics that are also in line with Shulman’s conceptualization. First, most scholars agree that PCK connects at least two forms of knowledge: content knowledge and pedagogical knowledge. Second, PCK deals with teachers’ knowledge necessary to achieve the aims of teaching, and it is a form of practical knowledge that teachers need to apply in the act of teaching. Third, it is agreed that PCK is specific to a particular subject matter. As the fourth common characteristic,

content knowledge is assumed to be an important and necessary prerequisite for teachers' PCK (Ball et al., 2005; Capraro, Capraro, Parker, Kulm, & Raulerson, 2005; Lim-Teo, Chua, & Cheang, 2007; Seymour & Lehrer, 2006; Vale, McAndrew, & Krishnan, 2011). Many researchers referred to Ma's (1999) concept of a profound understanding of fundamental mathematics (PUFM) when they try to clarify the content knowledge that teachers need in order to teach effectively (An et al., 2004; Baumert et al., 2010; Capraro et al., 2005; Seymour & Lehrer, 2006; Vale et al., 2011). "A profound understanding, in Ma's description, has three related meanings: deep, vast and thorough. A deep understanding is one that connects mathematics with ideas of greater conceptual power. Vast refers to connecting topics of similar conceptual power. Thoroughness is the capacity to weave all parts of the subject into a coherent whole" (Capraro et al., 2005, p. 109). It refers to teachers' ability to connect different mathematical ideas and the flexibility to think in multiple ways about particular mathematical concepts (Ball et al., 2008).

From an international perspective, numerous PCK studies have been conducted from countries around the world, even though rarely do we see comparative studies among them. For instance, Dalgarno and Colgan (2007) and Sibbald (2009) conducted PCK studies in Canada, focusing more on PCK in general mathematics and conditions that support the development of PCK during teachers' training. Lim-Teo et al. (2007) examined the way in which pre-service teachers' PCK evolves during their teacher training in Singapore, and found that student teachers at the beginning of their programs were generally weak in their mathematical pedagogical content knowledge, yet significantly improved in some aspects of their PCK upon completion of their

mathematics pedagogy courses. Sorto, Marshall, Luschei, and Carnoy (2009) conducted a larger and cognitively oriented study in Panama and Costa Rica that measured third- to seventh-grade teachers' mathematical knowledge of teaching by means of a test and compared to the quality of instruction. Addressing the gaps in pre- and in-service teachers' PCK regarding mathematics in general and using a coding scheme to lesson videos, the authors did not find a significant difference in PCK between teachers in these two countries. Blanco (2004) conducted a study in Spain that took a more situated perspective on PCK, trying to capture teachers' PCK in action through interviews and classroom observations. He explained programs in Spain and several different types of activities which helped to create learning environments referred to as "learning to teach mathematics." Finally, Blömeke, Suhl, and Kaiser (2011) considered the component of knowledge of curriculum and media in PCK and included Botswana in its comparison of the effectiveness of pre-service teachers' PCK in their training in fifteen different countries from Europe, Asia, Americas, and Africa. Blömeke's study revealed significant cultural differences in the effectiveness of teacher education and presented a combination of differential choices of teacher education programs according to background and differential achievement of teachers from these programs.

Teachers' mathematics knowledge is essential to effective teaching and student learning (Ball & Bass, 2001; Shulman, 1987). To teach effectively, teachers must possess the knowledge and skills that consists of (a) general ways to present content to students; (b) understanding of students' common conceptions, misconceptions, and difficulties when encountering particular situations; and (c) specific teaching strategies that can be used to meet students' diverse learning needs, which derives from Shulman's original

notion of PCK (Shulman, 1987). Numerous studies elaborated on the definition of PCK, and this study adopts one of the frameworks from An et al. (2004), defining PCK as the knowledge of effective teaching which involves three components: knowledge of content, knowledge of curriculum, and knowledge of teaching. Such a definition is broader than Shulman's original designation. According to An, Kulm, and Wu (2004), knowledge of content consists of broad mathematics knowledge as well as specific mathematics content knowledge at the grade level being taught. Knowledge of curriculum includes selecting and using suitable curriculum materials, fully understanding the goals and key ideas of textbooks and curricula (National Council of Teachers of Mathematics, 2000). Knowledge of teaching consists of knowing students' thinking, preparing instruction, and mastery of modes of delivering instruction. In the figure below, these three components interconnect and interact intensely in a complex way. The core of pedagogical content knowledge in this study is the knowledge of teaching, and it can be enhanced by content and curriculum knowledge.

Mathematics teaching can be seen either as a divergent or a convergent process. In a divergent process of teaching, teachers tend to focus on the content itself with curriculum knowledge, but such a way of teaching is usually lack of focus, and it ignores students' mathematical thinking. On the other hand, a convergent teaching process focuses on students' mathematical ideas. According to Carpenter and Lehrer (1999), a convergent teaching process consists of four aspects: building on students' mathematical idea, addressing students' misconceptions, engaging students in mathematics learning, and promotes students' thinking mathematically, and these four aspects of convergent teaching compromise the notion of teaching with understanding, which is an essential to

effective teaching. During a convergent process of teaching, students, instead of textbooks, curricula, or teachers, are the center of teaching. Throughout the convergent teaching process, an effective teacher attends to students' mathematical thinking: preparing instruction according to students' needs, delivering instruction consistent with student's levels of understanding, addressing students' misconceptions with specific strategies, engaging students in activities and problems that focus on important mathematical ideas, and providing opportunities for students to revise and extend their mathematical ideas (Kulm, Capraro, Capraro, Burghardt, & Ford, 2001).

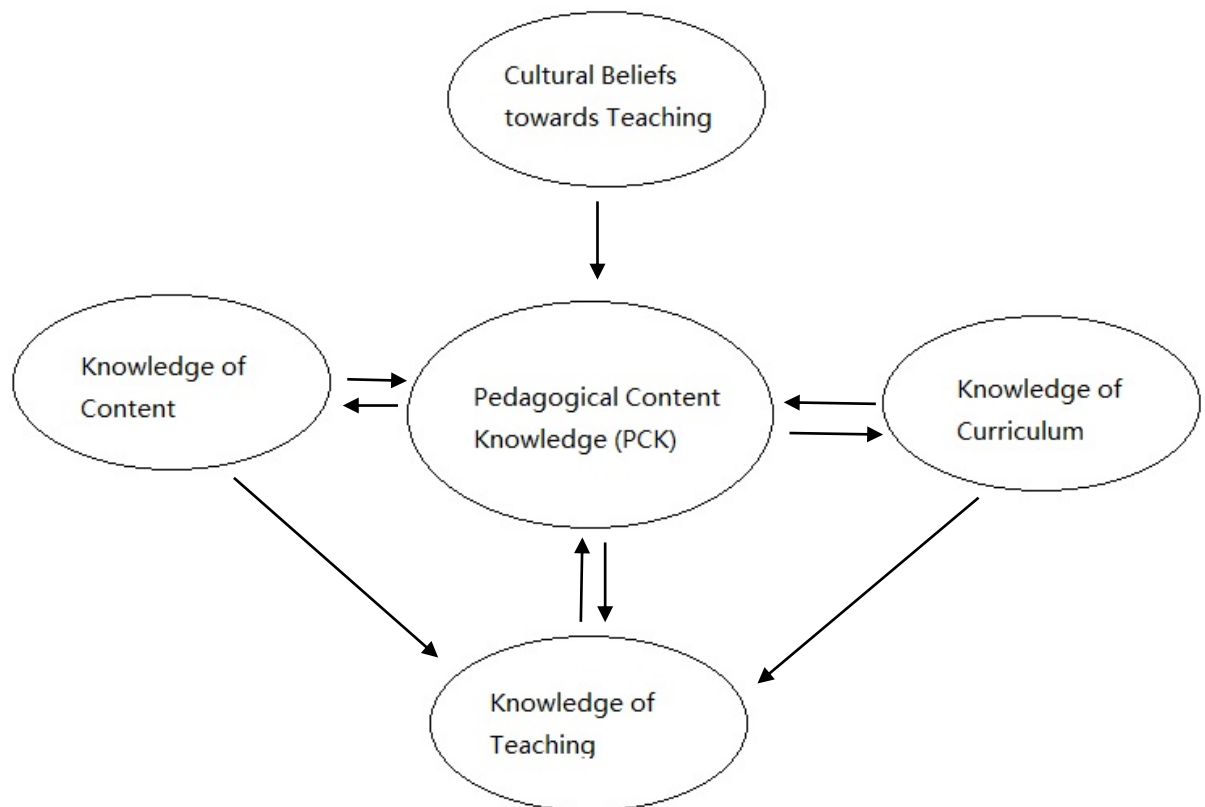


Figure 1. The network of pedagogical content knowledge (An et al., 2004).

In this theoretical framework, we define four aspects regarding a teacher's pedagogical content knowledge: Knowledge of content, Knowledge of curriculum, Knowledge of teaching, and Cultural beliefs towards teaching. The first and last research questions are expected to be answered via examining, analyzing, and comparing the first three aspects of PCK, while the cultural belief aspect is to clarify and enhance the understanding of a teacher's idea about his/her teaching in practice, and is expected to answer the second research question.

Besides the three research questions, hopefully, implication can be derived from the answers from the three questions above:

What, if any, is a feasible reform that we can adopt from Chinese teachers' conceptions in teaching to improve teacher's professional development programs internationally?

Cultural Beliefs towards Teaching

Cultural beliefs towards teaching do not determine what teachers teach, but teachers do draw upon their cultural beliefs as a standard scheme of values to guide their teaching. Through accumulation and participation in intellectual activities, perspective penetrates the holders' minds, and beliefs are gradually formed by constant exposure and osmosis over a period of time (Ball, 1990). Influenced by the tradition of Confucian heritage culture, mathematics teachers in China are often evaluated according to various

aspects, including their performance in institutional exams, academic production, achievements of their students and so on (Peng, 2007).

What a teacher considers to be the desired goals of the mathematics program, his or her own role in teaching, the students' role, appropriate classroom activities, desirable instructional approaches and emphases, legitimate mathematical procedures, and acceptable outcomes of instruction are all part of the teacher's conception of mathematics teaching (Thompson, 1992). According to Thompson, there is four distinctive views of how mathematics should be taught: learner-focused, content-focused, content-focused with an emphasis on performance, and classroom-focused. These four models are used to describe main differences in beliefs in mathematics teaching; however, it is likely for a teacher's beliefs in teaching mathematics to include several aspects in several models than to describe that teacher's belief precisely using only one model (Yu, 2013).

Although there is no overall difference in intelligence, the differences in mathematical achievement of American children and their Asian counterparts are staggering (J. W. Stigler & Stevenson, 1991). Specifically, Chinese students display superiority over the U.S. peers on base-ten counting (Miller & Stigler, 1987), computation and mental mathematics (Cai, 1997; Geary, Bow-Thomas, Fan, & Siegler, 1993), simple problem solving (Cai, 1995), and representational competence (Brenner, Herman, Ho, & Zimmer, 1999). The argument that Stevenson and Stigler (1994) later presented hinged on the dichotomy between effort and ability, trying to explain the reason behind such difference in student mathematics achievement between two cultures. By collecting data from parents from the U.S. and those in Japan and China for over a decade, they demonstrated the existence of a marked difference in the emphases given to

education and concluded that parents in the U.S. attribute their children's success and/or failure to innate ability. On the other hand, however, Chinese and Japanese parents paid greater attention to the effort of children and the environment in which they learn:

We and others have found that American children, teachers, and parents emphasize innate abilities as a component of success more strongly than their Chinese and Japanese counterparts do. ...Chinese and Japanese societies allow no excuses for lack of progress in school; regardless of one's current level of performance, opportunities for advancement are always believed to be available through more effort. High scores on a test are interpreted as a sign of diligence.
(p. 95)

Effort versus ability was only one of the many similar findings that Stevenson and Stigler had. Such belief in hard work is not an abstract conception, but a concrete and practical guideline in people's daily lives in East Asian countries such as China and Japan. The recommendations about learning from the culture of Japanese and Chinese education found a lukewarm reception among American readers. Such discomfort is perhaps understandable because identification of cultural differences often leads to the subject of values and value judgments (Cheng, 1998).

Stevenson and Stigler (1992) observed that among Japanese and Chinese parents, "much more importance is given to establishing interdependent relationships between the child and other members of the family and society" (p. 89). There have also been in-depth studies of how cultural norms carry over into educational settings. Tobin, Wu, and Davidson (1989), for instance, used ethnographic approaches to explore differences in preschool education as practiced in Japan, China, and the U.S. They found that Japanese

and Chinese preschool education emphasized the class. Immersion in the child's class taught how to lead a life within one's community. Indeed, in most East Asian systems of education, the class is an essential element of school education (Cheng, 1998).

Discipline, according to Cheng, is another example that shows the cultural difference between the two education systems. In many Western systems of education, discipline is viewed as a necessary evil that fosters orderly learning, and teachers tend to learn classroom management skills to prevent and solve behavior problems. School faculties learn to implement systems of schoolwide discipline only when there is a perspective that school discipline is weak. On the other hand, in East Asian education systems, discipline does not only receive focus for the pragmatic purpose of effective teaching and learning—it is itself a primary objective of education.

There are some scholars who have made comparisons of mathematical beliefs between ancient China and ancient Greece culturally and found that the ancient Chinese beliefs were based on empiricism and pragmatism, in which mathematics was regarded as a tool used to solve practical problems, a kind of skill (Xie & Cai, 2018). Hong Kong scholar Leung (2001) compared and analyzed the content of *Nine Chapters on Mathematical Procedures* and Euclid's *Elements of Geometry*. Leung pointed out that the algorithms and the emphasis on applications are the two major features of traditional Chinese mathematics, which affected the field of mathematical beliefs greatly in China. Such beliefs have endowed great importance to the “double-base” education in China—basic knowledge and basic skills.

China has been using a common curriculum for more than half a century. In the early 1950s, China adopted the Soviet mathematics curriculum that paid more attention to

deductive reasoning using formal and rigorous mathematical language, just like the mathematics curriculum in the former Soviet Union (Wang & Cai, 2007). In the 1970s, according to the Chinese National Ministry of Education (1978), mathematics syllabus for elementary and secondary schools started to require students to solve real-life problems by applying mathematical knowledge. In the late 1980s, Chinese State Education Commission (1988) clearly stated that students need to be able to understand the principles of mathematical operations and use appropriate strategies to solve problems besides being able to calculate correctly. There was no major change to the mathematics curriculum in China until decades later. The wave of curriculum reform when the Basic Education Curriculum Material Development Center (2001) issued Curriculum Standards for nine-year compulsory education.

Cultural beliefs about teaching do not directly dictate what teachers do, but teachers do draw upon their cultural beliefs as a normative framework of values and goals to guide their teaching (Bruner, 1996; Cai, 2004; Cai & Wang, 2010; Wilson & Cooney, 2002). Cai and Wang (2010) conducted a study investigating Chinese and U.S. teachers' cultural beliefs towards effective teaching from the teachers' perspective. Even with some common beliefs, the two groups of teachers thought differently about students' mathematics understanding as well as effective teaching. In Cai and Wang's study, the sample of U.S. teachers valued and emphasized students' understanding of concrete examples, while Chinese teachers emphasized the abstract reasoning after the examples. Cai and Wang also pointed out that the U.S. teachers valued the abilities to manage classrooms, facilitate student participation, and even have a sense of humor, while

Chinese teachers focused more on the solid mathematical knowledge and through careful study of textbooks.

Gap in Literature

Numerous prior researches have been carried out to compare Chinese and U.S. teachers' knowledge of content (e.g., Ma, 1999) and knowledge of effective teaching (e.g., An et al., 2004); however, little research examines teacher' pedagogical content knowledge by integrating the components from it and cultural beliefs. Overall, the proposed study is insightful and could contribute to our teacher development program in the U.S. Because of the difficulties that we have in conducting large-scale studies, having small studies is particularly important and such studies better assist researchers to understand teachers' knowledge of content and teaching strategies. This is helpful when we are conducting cross-cultural studies, as indicated in International Comparative Studies in Education:

There is a great need for small, in-depth studies of local situation that permit cross-cultural comparisons capable of identifying the myriad of causal variables that are not recognized in large-scale surveys...much survey data would remain difficult to interpret and explain without the deep understanding of society that other kinds of studies provide...research in cross-national contexts benefits from increased documentation of related contextual information, it would be useful to

combine large-scale surveys and qualitative methods. (Gilford, 1993, p.
23)

III. METHODOLOGY

Introduction

This chapter describes the methods that were used for this study. In this study, a qualitative research approach was used given the nature of the research questions. The research questions included: 1. What is the pedagogical content knowledge of middle school mathematics teachers from Shandong Province, China? In particular, a) What is their knowledge of teaching mathematics, b) What is their knowledge of mathematical curriculum? 2. What are middle school mathematics teachers from Shandong Province cultural beliefs towards teaching? 3. How are Chinese teachers' cultural beliefs towards mathematics teaching related to their pedagogical content knowledge? Qualitative research is defined by Denzin and Lincoln (2005) as:

...a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them (p. 3).

A qualitative research design is appropriate because of several reasons. For one, quantitative data cannot perfectly answer the research questions. Teachers' PCK and their beliefs cannot simply be converted into numbers and compared. Therefore, qualitative research is needed because I need a deep, complex, and detailed understanding of current

issues. Such detail, according to Creswell (2012), can only be established by talking directly with people, going to their homes or places of work, and allowing them to tell the stories unencumbered by what I expect to find or what I have read in the literature. Moreover, the cultural beliefs towards teaching cannot be easily measured by quantitative data; instead, teachers' own explanations are more straightforward.

Among many qualitative research methods, I chose the case study as my approach. The *case* is defined by Miles and Huberman (1994) as “a phenomenon of some sort occurring in a bounded context.” The case is, “in effect, your unit of analysis” (p. 25). The urge to analyzing the pedagogical content knowledge and the beliefs of individual teachers makes the case study the best choice. In this study, I considered each teacher as a case. Within each case, their PCK and cultural beliefs were considered and studied. Though cultural factors will be considered in this study, I will not choose ethnography because it studies how the culture works rather than an understanding of a single case in-depth and exploring the issue using specific cases. According to Yin (2009), case study research involves the study of a case within a real-life, contemporary context or setting. Even though Stake (2013) stated that case study research is not a methodology but a choice of what is to be studied, others present it as a strategy of inquiry, a methodology, or a comprehensive research strategy (Creswell, 2012; Denzin & Lincoln, 2005; Merriam, 1998; Yin, 2009).

Participants

In this study, I selected five teachers from three middle schools in my hometown Qingdao, the largest city in Shandong Province. According to the report from Shandong Provincial Education Department (2016), in the year of 2015, the city of Qingdao has 231 middle schools with 238,715 students enrolled. China is one of the countries where, to a great extent, the examination scores can determine a student's opportunity for additional education and even future careers (Cai & Nie, 2007), and Shandong is considered to be one of the most competitive provinces in China in terms of education because of its large population.

In May 2016, I spent about seven weeks collecting data in my hometown, Qingdao, Shandong. To obtain access to the schools, I contacted the principals of several middle schools in the city, explained the study to them, and asked whether they would like to introduce some teachers to participate in this study. A consent form containing the IRB number (see Appendix A) was signed by each participant at the beginning of the survey/interview, and pseudonyms were used to protect the participants' identities. As a result, four mathematics teachers from three different middle schools participated in this study, namely:

Ms. Wang, School C, two years of teaching experience;

Ms. Liu, School C, six years of teaching experience;

Ms. Xu, School Y, 22 years of teaching experience;

Ms. Zhang, School N, 25 years of teaching experience.

The three middle schools that participated in this study covered the two most common types of middle school in China: public schools (School Y and School N) and

private schools (School C). In addition, the four teachers also assured a wide range of teaching experience (from two years to 25 years). It was very helpful when I compared the PCK among the Chinese teachers themselves to see if there is any significant difference caused by the experience. See Table 1 for the overall data collection in summer 2016.

During my data collection, Ms. Wang, and Ms. Liu were observed twice in this study, and Ms. Zhang and Ms. Xu were observed three times. All teachers besides Ms. Liu spent more than one class covering a certain topic. In Ms. Liu's case, instead of teaching new content, she was going over practice tests and answered questions, as the High School Entrance Exam was approaching in less than 30 days from when the data was collected. The following table summarizes the overall data collection progress during summer 2016:

Table 1.
The overall data collection in summer 2016

Name	School	Teaching Experience (years)	Grade Teaching	Number of Observations	Interview
Ms. Wang	C	2	7	2	Yes
Ms. Liu	C	6	9	2	Yes
Ms. Xu	Y	22	7	3	Yes
Ms. Zhang	N	25	8	3	Yes

The four teachers' data were used to analyze their pedagogical content knowledge and cultural beliefs towards teaching. A fifth participant, Mr. Qu from school C was sought, but because insufficient data was collected from him, his data would not be used.

Analytic Tools and Procedure

In this study, we focus on two constructs: pedagogical content knowledge (PCK) and cultural beliefs towards teaching. To operationalize our two construct teachers' PCK, I examined and elements or components from the PCK network (see Figure 1), namely, teachers' knowledge in teaching mathematics, teachers' knowledge in mathematical curricula, in addition to their cultural beliefs towards teaching. Classroom observations and interviews were used to examine teachers' PCK. To investigate the teacher's cultural beliefs towards teaching, I mainly focused on the interview responses and possible connections between their interviews and actual classroom teaching. To better design and utilize the interview and observation, I used guidelines in Creswell (2012). This guideline includes a series of steps and tips that a qualitative researcher needs to know and be aware of in every phase of an interview.

Interview Protocol. Interviews are an integral part of doing qualitative research and considered the best data collection technique for a case study of several participants (Merriam & Tisdell, 2015). Even though classroom observations are powerful in finding teachers' PCK in teaching, yet the understanding of the teachers' cultural beliefs towards teaching is only a supposition. Such beliefs, plus the PCK in the curriculum, can be best

obtained through in-depth interviews. The responses in the interviews provide the information for answering the research questions in this study.

The complete PCK interview protocol can be found in Appendix B. To develop the interview protocol that examined teachers' PCK, several interview questions were inspired and adapted from the survey that measures teachers' *Mathematics Knowledge for Teaching* (MKT), which focuses attention on the considerable *mathematical* demands that are placed on classroom teachers (Ball & Bass, 2001; Ball et al., 2004). The concept of MKT was built upon Shulman's notion (1986) of PCK that packed mathematical, pedagogical, and developmental knowledge together and helped teachers to address issues during the process of learning mathematics. On this basis, Ball and Bass (2001) posited a complementary mathematical knowledge that teachers must call upon as needed in their teaching practice to engage students in learning.

Although pedagogical content knowledge provides a certain anticipatory resource for teachers, it sometimes falls short in the dynamic interplay of content with pedagogy in teacher's real-time problem-solving...It is what it takes mathematically to manage these routine and non-routine problems that have preoccupied our interest...It is to this kind of pedagogically useful mathematical understanding that we attend to in our work. (Ball & Bass, 2001, pp. 88-89)

In this study, I examined the survey the authors used to measure teachers' MKT to inform my interview questions. I did not directly use the survey itself but derived interview questions from it. By asking teachers questions such as "how would you teach your students to solve a quadratic equation with the leading

coefficient other than one?” and “how would you respond to your students if they ask why we can’t divide by zero?”, I examined their responses to find possible traces of their PCK in order to answer our first research question, in particular, about their knowledge in teaching mathematics as well as knowledge in mathematical curricula. In particular, to identify teachers’ knowledge in curricula, I presented the current version of the textbook published by Beijing Normal University Press (BNUP), as well as the previous textbook widely used in Shandong Province before 2001, published by People’s Education Press (PEP). Teachers who didn’t teach using PEP textbooks had these textbooks as students in middle schools, and I assumed that they all had an impression about the older version of the curriculum. Some questions in examining teacher’s knowledge in the curriculum include curricula comparison, such as “How would you compare the current and the previous versions of our textbooks, and what do you think about the advantages and disadvantages of the two versions?”. I also asked teachers about practice problems that are aligned with the curriculum, for instance, “What problems would you choose from practice problems book to ask students, and what are the criteria for you to choose them?”

To describe their cultural teaching beliefs, I posted a series of questions in the interview to identify teachers’ beliefs towards teaching under the influence of cultural and societal backgrounds. The questions cover various aspects, asking not only teachers’ beliefs about what happened in the classroom, such as the attitude to tests, practicing, and posing questions, but also their beliefs outside of the class that is related to teaching, such as support from parents, and talent versus practice. These questions were asked during interview, and the responses were analyzed to answer

our second research questions, for example, “How do you think about the number of exams that students have today, and how do these exams affect students’ learning?” and “How do you think the support from students’ parents influence middle school students’ learning?”

Pedagogical Content Knowledge Observation Rubric. The observation rubric can be found in Appendix E. The observation rubric was informed by the Mathematical Quality of Instruction (MQI) instrument design. This instrument was designed to provide scores for teachers on important dimensions of classroom mathematics instruction, and it is consistent with the PCK network framework used for this study. These dimensions include the richness of the mathematics, student participation in mathematical reasoning and meaning-making, and the clarity and correctness of the mathematics covered in class (Hill, Blunk, et al., 2008). The MQI was developed to provide a multidimensional and balanced view of mathematics instruction, and studies suggest that it both returns reliable teacher scores and that scores correlate with student outcomes (Hill, Umland, & Kapitula, 2011; Hill, Umland, Litke, & Kapitula, 2012). Not all dimension of MQI were used, since some segments, such as “Common Core Aligned Student Practices,” do not fit in this study. I adjusted the dimensions in the MQI rubric and used three categories, namely mathematics coherence, curriculum-aligned practice, and working with students and mathematics. The rubric was used in classroom observations, and the responses were analyzed to answer the first research question and provided some insight for the research question three. The observational rubric can be found in Appendix E.

PCK Interview Protocol. The complete interview protocol can be found in Appendix B. The interview protocol was partially adopted from Bower (2016) and

Kennedy, Ball, and McDiarmid (1993). The purpose of the interview is to investigate teachers' teaching strategies and their actions they apply in the classrooms and to certain scenarios, to learn how teachers prepare and reflect for instruction, how they determine student's thinking, how they see the education in general under the current system, and what they believe culturally about teaching the subject of mathematics. The interview questions are expected to further explore teachers' PCK as well as its importance in their teaching.

Data in this study were collected in three phases. In the first phase, teachers answered the modified and translated version of LMT survey (Learning Mathematics for Teaching, 2005) that is designed to test middle school mathematics teachers' MKT. All teachers were given one week to complete the survey individually. The modified LMT survey (adopted from Kennedy et al., 1993) was translated into Chinese so that the teachers could work on. The LMT scores could later be used to compare with a U.S. national sample. It is only used to posit teachers as a reference, and the score itself is not for answering any of the research questions in this study.

In the second phase, the data was collected by observing and videotaping the teachers' actual teaching in the classrooms. Each teacher was observed at least twice in May 2016, about a month before the end of the spring semester, and each lesson took 45 minutes. It was close to the High School Entrance Examination, and all 9th-grade classes were doing practices and reviews for the Exam. The observations were conducted at a pre-arranged date and time, and field notes and audiotape recordings were taken during the observations. The researcher then transcribed the video recordings and translated the transcripts into English.

In the third phase, the teachers were interviewed concerning their pedagogical content knowledge in teaching and curriculum, their views about beliefs towards teaching, issues in current mathematics education in China. The interviews were video- and audio-recorded to prevent potential technical issues. The data were then transcribed and translated to English, and the responses were analyzed to answer our research questions 1 b), 2, and 3.

Data Analysis

For the analysis of data, coding procedures described in Gibbs (2008) was used. Coding, according to Gibbs, is how you define what the data you are analyzing is about, and it involves “identifying and recording one or more passages of text or other data items such as the parts of pictures that, in some sense, exemplify the same theoretical or descriptive idea” (Gibbs, 2008). The protocol itself was not translated into Chinese; instead, when I interviewed the teachers, I just asked the questions in Chinese.

To investigate teachers’ pedagogical content knowledge and answer research question 1 a)., 15 different coding categories were adopted from An et al. (2004) (see Table 2 for the list of categories and definitions that the authors used to examine the knowledge for teaching mathematics). I had gone through the literature and found that the study An et al. (2004) conducted is the most relevant, as their study also investigated and compared middle school mathematics teachers’ PCK between the U.S. and China. After going over several different classroom observation protocols, such as the Reform-Oriented Teaching Observation Protocol (RTOP) and Instructional Quality Assessment

(IQA), it is found that MQI fits this study the best. Pertinent dimensions were hence used with their corresponding descriptions, from the MQI observational protocol to examine the mathematics curricula component, for the quality teaching usually involves the deep understanding and thorough knowledge of the curriculum. Table 4 describes the categories that I used in this study. Specifically, the use of the curriculum-aligned practice, the knowledge of curriculum, and curriculum coherence could be observed during teaching, yet the knowledge of the overall curriculum was likely to be found in the interviews.

Table 2.

Categories for describing teachers' pedagogical content knowledge in teaching

Category	Brief definition
1.	Prior knowledge: Know students' prior knowledge and connect it to new knowledge.
2.	Concept or definition: Use concepts or definitions to promote understanding.
3.	Rule and procedure: Focus on rule and procedure to reinforce knowledge.
4.	Draw a picture or table: Use a picture or table to show a mathematical idea.
5.	Give an example: Address a mathematical idea through examples.
6.	Connect to concrete model: Use concrete model to demonstrate mathematical idea.
7.	Students who do not understand prior knowledge: Students lack in understanding of prior knowledge.
8.	Provide students the opportunity to think and respond: Promote students to think problems and give them chances to answer questions.
9.	Manipulative activity: Provide hands-on activities for students to learn mathematics.
10.	Attempts to address students' misconceptions: Identify students' misconceptions.
11.	Use questions or tasks to correct misconceptions: Pose questions or provide activities to correct misconceptions.
12.	Use questions or tasks to help students' progress in their ideas: Pose questions or provide activities to increase the level of understanding for students.
13.	Provide activities and examples that focus on student thinking: Create activities and examples that encourage students to ponder questions.

Table 2. Continued

14.	Unintelligible response: Provide a response that is not relevant to the question.
15.	Incorrect: Provide a wrong answer.

Note. This table is adapted from An et al. (2004).

Two categories from An et. Al (2004), “use one representation to illustrate concepts: apply repeated addition to address the meaning of fraction multiplication” and “use both representations to illustrate fraction multiplication,” were removed in this study as Chinese middle school mathematics teachers would very rarely mention the additions or multiplications of fractions in class. They almost always expect students to compute such operations fluently, and I would expect to have zero instances for all four teachers. To answer question 1 a), “What is the knowledge of teaching mathematics of middle school mathematics teachers from Shandong Province, China?” the codes listed in Table 2 were grouped to aligned with the different dimensions of teaching mathematics PCK component adapted from An et al., 2004. See Table 3 for a summary of the dimensions and essential elements.

Table 3.

Categories for describing four aspects of PCK in teaching mathematics

Dimensions of Knowledge of Teaching Mathematics Component	Essential elements	Category number
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Table 3. Continued.

Building on students' math ideas	1. Connect to prior knowledge	1, 7
	2. Use the Concept or definition	2
	3. Connect to concrete model	
	4. Use rule and procedure	6, 3
Addressing students' misconceptions	1. Address student's misconceptions	10
	2. Use questions or tasks to correct misconceptions	11
	3. Use rule and procedure	3
	4. Draw picture or table	4
	5. Connect to concrete model	7
Engaging students in math learning	1. Manipulative activity	9
	2. Connect to concrete model	6
	3. Give examples	5
	4. Connection to prior knowledge	1
Promoting students' thinking about mathematics	1. Provide activities to focus on students' thinking	13
	2. Use questions or tasks to help students' thinking	12
	3. Provide opportunity to thinking and respond	8

Note. This table is adopted and modified from An et al. (2004).

To find out teachers' pedagogical content knowledge in mathematical content, self-reported responses of questions about the mathematical background in the interviews were analyzed to identify teachers' knowledge of mathematical content indirectly. Instead of taking a math exam, teachers were asked about their experience in the way of

being a middle school mathematics teacher, as well as experiences as a student who learned mathematics.

To answer question 1 c), “What is the knowledge of math curriculum of middle school mathematics teachers from Shandong Province, China?” I adapted categories that relate to the PCK mathematical curriculum component, and they are summarized in Table 4.

Table 4.
Categories for describing teachers’ responses to pedagogical content knowledge questions in curriculum

Dimensions of knowledge of mathematical curriculum	Brief description
1. Curriculum aligned practice	Teachers provide curriculum-aligned practices in class, provide explanations, and guide students to work/explain the practice problems.
2. Curriculum coherence	Teachers provide linking and connections between topics, guide students to observe patterns and generalization, and provide multiple procedures or solution methods from other places in the curriculum.
3. Knowledge of curricula	Teachers have knowledge of the math curricula in general.
4. Knowledge of curriculum material	Teachers have knowledge of the curriculum-related material, such as

Table 4. Continued

textbooks, practice books, lesson plan, etc.
--

Coherence has long been a construct in research related to discourse, and recently, it has become a construct in educational research (Cai, Ding, & Wang, 2014). Schmidt, Wang, and McKnight (2005) defined coherence as “a sequence of topics and performances consistent with the logical and if appropriate, hierarchical nature of the disciplinary content from which the subject matter is derived.” Though at the micro level, coherence refers to “connections between propositions in composite sentences and successive sentences” (Van Dijk, 1997, p. 4), coherence in this study focused on teachers’ global knowledge of the curriculum.

To answer research question 2, “What are middle school mathematics teachers from Shandong Province cultural beliefs towards teaching?”, I designed five aspects, and their corresponding descriptions are listed in Table 5.

Table 5.

Categories for describing teachers’ responses to cultural beliefs towards teaching.

Aspects of Cultural beliefs towards teaching	Brief description
1. Beliefs about practice	Teachers provide opinion in giving practice-based lessons vs. instruction-based lessons

Table 5. Continued.

2. Beliefs about exams	Teachers provide opinion in frequent exams and the preparation of the High School Entrance Exam
3. Beliefs about parental support	Teachers provide opinion in support from support
4. Beliefs about using questions	Teachers provide insights on how to use questions to promote students' learning.
5. Beliefs about lesson preparation	Teachers provide insights about how to prepare classes

Transcriptions were made for all interviews and classroom observations, and all transcripts were also translated into English prior to the analysis of data. The responses to the interview questions as well as field notes from the observations were analyzed with the same categories to clarify teachers' idea and beliefs. These data are expected to confirm the consistency among their responses to questionnaires, interview questions, and their actual teaching practices.

I noticed that all components in Table 2, 3 and first two components in Table 4 are observable through classroom observations, while the last two components in Table 4 and components in Table 5 can mainly be obtained from teachers' responses in the interviews. It is worth noting that many responses from the interviews are interwoven and connected with their actual teaching in the classroom. When the data was being analyzed, with the focus of the tables that correspond to a certain research question, responses from other sources were also considered. It is important to notice that a teacher's pedagogical content knowledge is not only embodied in an observation or an interview; it is the

teacher's teaching philosophy that permeates through her life. Overall, there were several cycles of data analysis, each of which identified a category in PCK or cultural beliefs. The connection between the PCK and cultural beliefs towards teaching was later considered when all components were identified.

Ethical Issues

Before conducting this study, formal approval from the Institutional Review Board for conducting this study was obtained. All participants were informed about their voluntary participation and their right to withdraw at any time. A consent form (see Appendix A.) that explain this study in detail was reviewed and signed by each participant before his/her data collection. All data that were collected are kept confidential, and their privacy of identities will be protected. All names in the videos of observations and interviews, including teachers' and students that are called, are covered and changed to pseudonyms that cover identities during transcription.

When I was recording the videos, I tried out my best not to have students' faces recorded. Since all students faced one direction in the classroom observations, most students' faces were not included. However, inevitably, a few students' faces were recorded in the video since they were either doing board work or presentation. These faces will be covered in the videos are publicly used. In addition, all students' names that are called by the teachers will be altered in transcription.

Validity and Dependability

According to Creswell (2012), validation in qualitative research is an attempt to assess the “accuracy” of the findings, as well as the distinct strength of qualitative research in that the account made through extensive time spent in the field, the detailed thick description, and the closeness of the researcher to participants in the study all add to the value or accuracy of a study. The strategies of qualitative validations I used in this study are triangulation and peer review. By collecting and analyzing multiple forms of data from multiple participants, it provided for a means of triangulation of findings; specifically, Creswell suggested that qualitative researchers locate evidence to document code or theme in a different source of data. This is exactly what I did: coding both the interviews and actual teaching with another coder and try to build up a connection by themes that I have designed. The peer coder is also a doctoral student who studies mathematics education and speaks Chinese as a native speaker. During the triangulation, the coder was given transcript excerpts and the coding scheme. She then independently coded the excerpts. The results were then compared and discussed if there were significant agreement (i.e., different categories coded for the same conversation). The discussions continued until 90% of the code agreement in an excerpt was reached. The process of peer coding was another way to validate my results. Having an external check of my data analysis, the peer reviewer played the role of “devil’s advocate”; the arguments from another perspective could help me fortify the coding results. It made me rethink the meanings, interpretations, and even the coding categories of the results for my research.

In this study, I positioned myself as an outsider as well as an insider. According to positivist tradition, the outsider perspective was considered optimal for its “objective”

and “accurate” account of the field, while insiders were believed to hold biases in observations and interpretations (Chavez, 2008). As suggested by Chavez, I categorized myself as an external-insider, who was socialized outside the community but endorses the cultural perspective and values of the indigenous community (p. 475). Not living in the country for more than ten years and never teaching in public school system in China makes me study teachers’ pedagogical content knowledge and beliefs completely like an outsider; on the other hand, for teachers in China, I will be staying with them, interviewing them and observing their classes myself, and studying this education system in which I spent over ten years. I have experienced all stages of basic education in China, with knowledge of how the system looks like and what to expect. From this perspective, I am conducting this study as an insider.

Summary

In this chapter, I introduced the constructs, analytic tools, and the procedure of data collection in this study. The dimensions in each component of pedagogical content knowledge and cultural beliefs towards teaching were considered and shown in several tables. Following the procedure of data analysis, in the next Chapter, I will show the results that I’ve collected and analyzed. The following three research questions are to be answered in the next two Chapters:

1. What is the pedagogical content knowledge of middle school mathematics teachers from Shandong Province, China? In particular,
 - a) What is their knowledge of teaching mathematics, and

b) What is their knowledge of mathematical curriculum?

2. What are middle school mathematics teachers from Shandong

Province cultural beliefs towards teaching?

3. How are Chinese teachers' cultural beliefs towards mathematics

teaching related to their pedagogical content knowledge?

By the end of the data analysis, one of the expected outcomes of this study is to gain insight into teachers' different pedagogical content knowledge and their beliefs towards teaching in a region in China. Another expected outcome is to see how Chinese teachers, as their teaching experience grows, have different pedagogical content knowledge in a rather unified and centralized education system. The main goal of this study is to contribute to the existing body of knowledge about middle school mathematics teachers' pedagogical content knowledge from an international perspective, and hence possibly point out a direction for the improvement of professional development training for middle school mathematics teachers in other countries, such as the U.S. Another goal of this study is to help teachers—starting from the four teachers who participated the study to, hopefully, middle school mathematics teachers in general—to identify the areas in their pedagogical content knowledge that can be improved and hence raise the quality of their instructions.

IV. RESULTS

The primary goal of this study is to understand the pedagogical content knowledge and cultural beliefs towards the teaching of a selected group of middle school mathematics teachers in China. This study uses primarily case studies and qualitative methods to analyze data collected from classroom observations as well as teacher interviews. This chapter describes the findings of the study through four case reports. Each report is divided into five sections: lesson description, evidence of pedagogical content knowledge in teaching, pedagogical content knowledge in content, pedagogical content knowledge in curriculum, and cultural beliefs towards teaching. These four sections addressed the research questions outlined in Chapter I and stated again below:

1. What is the pedagogical content knowledge of middle school mathematics teachers from Shandong, China? In particular,
 - a) What is their knowledge of teaching mathematics,
 - b) What is their knowledge of mathematical curriculum?
2. What are selected middle school mathematics teachers' cultural beliefs towards teaching?
3. How are selected teachers' cultural beliefs towards mathematics teaching related to their pedagogical content knowledge?

The first four sections in each case report are used to answer the first research question. The second research question is answered in the fifth section of each case, in addition to the evidence from the first section. For the last research question, all sections

of each case are considered for the relationship between teachers' PCK and their cultural beliefs towards teaching.

Videotaped lessons over various topics taught by Ms. Wang, Ms. Xu, Ms. Zhang, and Ms. Liu were analyzed to capture important features of their pedagogical content knowledge, in particular, their knowledge in teaching and knowledge in the curriculum. Short pre- and post- interviews were conducted with the teachers to provide more information about how they planned to teach, how they thought about the classes they just taught, and what they considered as the key points in the classes. Each classroom observation lasted for 45 minutes, and all four teachers used the traditional classroom setting in China, where teachers teach in front of the class, and all students face the same direction. Although observing two or three lessons may not be considered enough exposure to classroom data to draw claims about teachers' pedagogical knowledge, researchers have empirical evidence that supports that three observations are enough (Hill, Ball, et al., 2008). Also, it was observed, and confirmed by interviews, that teachers show their best efforts to demonstrate their knowledge of curricula, the expectations of students, and their own teaching experiences.

To strengthen the interpretation of coded classroom observations, in addition, to identify other factors that possibly affect the four teachers' teaching and their learning to teach, the main interviews were coded to triangulate the findings from the analysis of the three teachers' observations and the materials that they used in classes. In the following sections in each report, I analyzed four teachers' lessons, their beliefs, and teaching methods that they used. I then discussed how their teaching, their understanding of the

curriculum, and their cultural beliefs towards teaching reflected their pedagogical content knowledge.

Case 1: Ms. Wang

Ms. Wang is a mathematics teacher in Qingdao, Shandong, China. She graduated from a normal university in Shandong in 2014 with the degree of mathematics and got her middle school mathematics teaching certificate in 2015. She had taught mathematics in an after-class tutorial school, a school that offers out-of-classroom tutoring and more practice, in the year of 2014-2015, before she started teaching mathematics in the current school where my data was collected. From here on, the school will be referred as School C. At the time of this study, she had been teaching middle school mathematics for almost two years, and she was assigned to teach two sections of 7th grade in the school. She considered herself as one who is very good at mathematics and admitted that it was one of the reasons that motivated her to become a teacher.

Lesson Description

The class observations for Ms. Wang took place in the same classroom on May 12 and May 13, 2016. In these two classes, Ms. Wang covered the introductory sections of axial symmetry and axial-symmetric figures. Ms. Wang started the first class by briefly going over the material from the previous class—the idea of the congruent shapes, followed by showing students a PowerPoint slide of axial symmetric/non-axial symmetric figures in real life. She then guided the students to investigate the characteristics of axial

symmetric shapes by continually asking questions about previous knowledge (congruent shapes) and by gesturing with her hands the concept:

For instance, my hands. Clap, they match together. Along with the straight line in the middle, assuming that it exists, will they completely match if I “fold” them along it? Do they match together? Are they congruent? My hands have thickness and temperature. You are supposed to learn the idea of temperature in your biology class, and in our math class, we do not want to have such thickness and temperature. What should we do?

It seemed at first that Ms. Wang’s was trying to illustrate her hands like a pair of axial-symmetric figures, yet she mentioned that they could not consider other physical properties of the objects other than shapes and sizes in a middle school mathematics class. She then took out a pair of hands cut from a folded paper and introduced the definition of axial-symmetric figures, followed by the axis of symmetry.

For the next eight minutes, Ms. Wang led students going over the definitions several times using different sayings, with the identifications of possible axial-symmetric figures shown on the PowerPoint slides. She spent five out of eight minutes distinguishing the terms “axial-symmetric figures” and two “figures being axial symmetric,” comparing their similarities and differences on board in two columns. After the first class, Ms. Wang explained in the post-observation interview about why she emphasized the definitions again and again, despite the fact that they may seem simple definitions to understand:

It is all about the wordings in the tests...there are all kinds of ‘traps’ hidden in the test and one careless reading would cause huge problem...that’s why I emphasized again and again in class about the different saying of those definitions, for instance, the difference between axial-symmetric figures and axial symmetry...one mistake could cause a series of problems.

Ms. Wang then asked the students to draw and count the axes of symmetry for some figures in the textbook. While students were working on the problems, she walked around looking carefully at the students’ work. Five minutes later, she went back to the podium and started to show some students work without revealing their names and with the purpose of addressing a common mistake: “When I was walking around, how did some of us do? ...If you do this vertically (drew the figure on the board with a wrong axis) one said that it is an axis without even looking at it... Why can’t we say that it is one?” she asked students for comments.

In the next practice of her class, Ms. Wang asked the students to recall the shapes they had learned and identified whether those shapes were axial-symmetric: “From the figures that we learned from 7th grade or even elementary, what are some of the common axial-symmetric figures in mathematics? ...some of us said triangle. Is an arbitrary triangle axial-symmetric? OK, isosceles and equilateral...circles...” she wrote and drew the figures on the board while speaking to students, with an explanation for each figure. “Squares and rectangles. Does parallelogram work? ...if you fold one (folding a parallelogram paper) you will see that it doesn’t work.” In addition to those figures, Ms. Wang went further to more fundamental geometric elements: “Besides these, what else?

The simplest, what about the line segments? [Students: Yes.] What about an angle? [Students: Yes.] Both line segments and angles are (axial-symmetric figures).”

After going over the reasoning of basic shapes/geometric figures, the class started doing exercises on an exercise book. The exercises focused on the identification and number of axial symmetries over several figures—from a letter to a snowflake, from the logo of a bank to a Chinese character. Ms. Wang walked around while the students were working on the exercises, pointing out their mistakes individually. The in-class exercise lasted for about 20 minutes, which was the majority of the class time. For the last five minutes of the class, Ms. Wang checked the answers together as a whole class. She ended the lesson by summarizing the procedure of how to find the axes of symmetry.

The second class observation of Ms. Wang took place the next day, and it was a continuation of her first class, in which she led the class to explore more properties of axial-symmetric figures. By showing the class a drawing of an axial-symmetric plane with some points labeled, Ms. Wang emphasized, that the connection of a pair of corresponding points is perpendicularly bisected by the axis of symmetry. More exercises were assigned from exercise books, and the problems were raised to a new level: how to find the axes of symmetry for figures in the Cartesian plane? The topics and ideas learned from last class were emphasized again, and the level of difficulty of problems was immediately raised. The exercise problems from the exercise books were not only intended to test whether students had understood the idea of axial symmetry, but also their prior knowledge in geometry. The following figure illustrates one of the problems the students were working on.

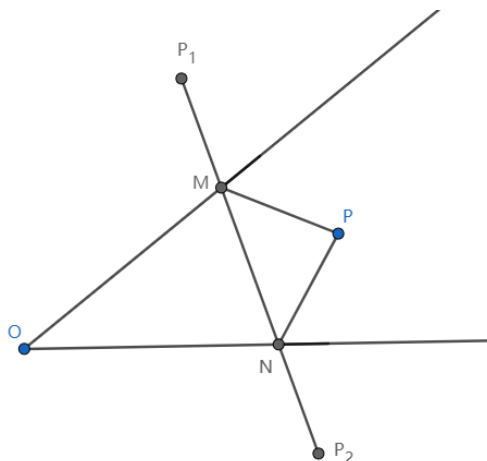


Figure 2. Geometry problem 1 in Ms. Wang's class

In the figure above, P_1 and P_2 are axial symmetric to P , with the symmetry OM and ON , respectively. If $P_1P_2=5$, find the perimeter of $\triangle PMN$.

Ms. Wang called a student to stand up and explain his idea about this question. Later in the second post-observation interview, Ms. Wang admitted that she considered that student a “good student.” The student, however, answered the question without too much confidence.

Student1: Emm... (paused for a few seconds) I think the answer should be 5. The perimeter of PMN should be equal to the length of P_1P_2 .

Instead of confirming and praising the student for having the correct answers, Ms. Wang asked the student for reasoning:

Ms. Wang: Why? How do you know?

Student1: Because...you can reflect P_1M to PM ...they look the same. So if you bend them over like that (“bend” the line segments in the air) ... they become PMN and five happens to be its perimeter.

Ms. Wang: Look the same... “look the same” is not accurate enough...when you do problems either as practice or in the test, you need to be very careful, use what is taught, and show every step. Now, all we know is that, since OM is the axial symmetry and P, P_1 is a pair of corresponding points of that symmetry, OM is the perpendicular bisector of PP_1 . That’s all we know so far...how do we use this for our next step? (called another student’s name) Student2?

Student2: We can connect P and P_1 , P and P_2 ...

Ms. Wang: Yes (connected the points on the board) ...and then?

Student2: Since OM is the perpendicular bisector...

Ms. Wang: Wait a second. Why is it a perpendicular bisector? (to class)
Class?

Class (together): Since OM is the axis of symmetry of P and P_1 , therefore it perpendicularly bisects the connection of P and P_1 .

Ms. Wang: Very good... (to student2) and?

Student2: Then we got two right triangles ($\triangle PAM$ and $\triangle P_1AM$), and they are congruent.

Ms. Wang: The reason is...?

Student2: SAS.

Ms. Wang: Wonderful. Please sit down. Now once we've shown that these two triangles are congruent, we can say P_1M and PM are of the same length indeed, and now we can do the bending like [Student1] said. Same reasoning for P_2N and PN . Therefore the perimeter of $\triangle PMN$ is 5.

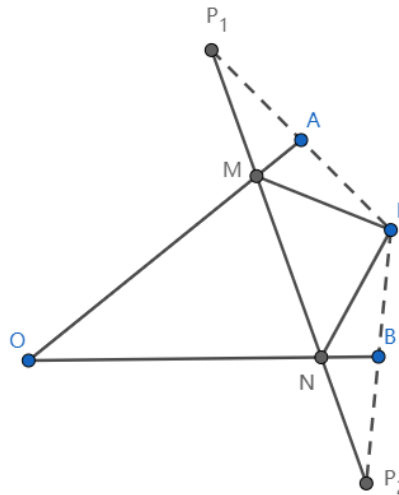


Figure 3. Solution to Problem 1

Ms. Wang then led the class to write down the detailed steps on the board and introduced another property of a pair of axial-symmetric figures, that the corresponding line segments and angles are congruent. Though it seemed that solving that problem went smoothly, Ms. Wang admitted, during the post-observation interview, that this example was a bit too hard for students at the current stage: “It would be very easy for students to tackle this problem (the problem above) a few lessons later; I hesitated for a while before I put this in my lesson plan. However, I was thinking about psychology (chuckles), that students tend to use what they just learned to solve a problem. If I put this example a few classes later, they would come up with the congruent method.”

Another important technique required in China's middle school mathematics curriculum is to fluently use compass and straightedge to construct basic geometric drawings and solve application problems (Ministry of Education, 2012). Right after the example above, she assigned the problem illustrated in Figure 4:

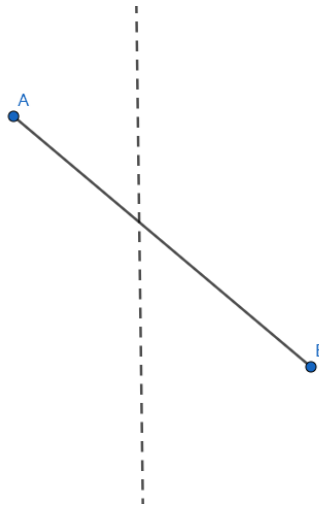


Figure 4. A construction problem regarding axial symmetry

The picture given above is half of an axial-symmetric figure.

Given the line segment AB and the axis of symmetry, use compass and straightedge to construct the other half of this axial symmetric figure, $A'B'$.

Ms. Wang was lenient on her students for this problem, as the students had not yet learned how to draw a line that passes through a point and is perpendicular to a given line. Nevertheless, she emphasized again, about the properties of a pair of axial symmetric figures. She explained after class, in the post-observation interview: “I need to

constantly remind the students about what they have already learned and what to use; otherwise they will be off-track and go everywhere.”

For the last 15 minutes of Ms. Wang’s second observation, she assigned more practice problems from the practice book and walked around to check if there was any mistake. While she was walking around, she noticed that a student used an alternative method to solve the problem. She immediately showed that student’s work on board. Using that problem, she started a discussion about multiple methods to approach the problem.

Ms. Wang: Do we see the alternative method done by [Student3]? As we learned before, how many points do we need to determine a straight line?

Class: Two points.

Ms. Wang: so now we have A’ determined, we just need another point to determine the line. But where is that point?

Class: [paused a few seconds] It’s on the axis of symmetry.

Ms. Wang: If one point on one half of the figure is on the axis, should the second half of the figure have one point on the axis? And where should that point be on?

Students: ...It should be on the axis. The second half would have the same point on the axis.

Ms. Wang: Does it determine a part of the line segment?

Class: Yes!

Ms. Wang: And now what we need to do (use a compass to measure the length of AB) is to measure the length of AB using our compass and extend the line segment until we hit that length. Excellent.

At the end of the class, Ms. Wang summarized the properties of axial symmetric figures on the board again and assigned homework for the day.

Lesson summary. In her two classes, Ms. Wang went over the introductory sections of axial-symmetric figures. She was active in her instructions with questioning and sharing reasoning. In her classes, she focused on the definitions and properties, in addition to a large amount of practice. Moreover, she frequently used manipulatives and visual representations to make students better understand the concept and properties of axial-symmetric figures.

Pedagogical content knowledge in teaching

The table below shows the number of instances observed during Ms. Wang's classes.

Table 6.
Ms. Wang's PCK in teaching in classroom observations

Categories	Ms. Wang	
	Instances	Percent
Prior Knowledge	4	6.3
Concept of definition	10	15.6
Rule and procedure	4	6.3
Draw Picture or table	8	12.5
Give example	8	12.5
Connect to concrete model	2	3.1

Table 6. Continued.

Students who do not understand prior knowledge	2	3.1
Provide student opportunity to think and respond	12	18.8
Manipulative activity	4	6.3
Attempts to address students' misconceptions	5	7.8
Use questions or tasks to correct misconceptions	3	4.7
Use questions or tasks to help students' progress in their ideas	0	0.0
Provide activities and examples that focus on student thinking	2	3.1
Unintelligible response	0	0.0
Incorrect	0	0.0
	64	100.0

The most frequently observed instances about Ms. Wang's PCK in teaching during the two observations of her were the emphasis on concepts of definitions and opportunities for students to think and respond. She frequently mentioned the concepts of the definition during her classes to guide students through the practice problems. Because of the nature of the topic (axial symmetric figures), she used several activities to clarify the differences and similarities between two concepts and some concrete models to connect the mathematical ideas with real life. She made no unintelligible or incorrect responses during the observations.

Connections between models and abstract thinking. It was observed that Ms. Wang used a few activities to introduce the ideas of axial symmetrical figures,

emphasizing the connections between the mathematical models and abstract thinking. She created and utilized several manipulatives to illustrate students the axial-symmetric figures in addition to PowerPoint slides before using the concept in a problem that required coordinates. At the beginning of the first observation, she used her own hands as a model to demonstrate a real-life example of axial-symmetric figures; at the same time, she also mathematized such an example to a model—a pair of “hands” cut from a folded piece of paper, so that the student can deduct the mathematical properties considering only the figures themselves.

When she used her hands as an example, she mentioned imagining the axis of symmetry, even though it did not exist: “...my hands...they match together, along with the straight line in the middle, *assuming* that it exists. If I ‘fold’ my hands along such line, would they completely match (shows the blue hands)? What happens if they are not ‘attached’ (shows the orange hands)?”

Emphasis on the concept/definition. The node “concept and definition” was identified and coded ten times in the two observations of Ms. Wang’s classes. Ms. Wang used words such as “think from the definitions!”, “go back to your notes for those properties,” “how did we define it minutes ago?” to constantly remind students to look back at the definition to solve the practice problems.

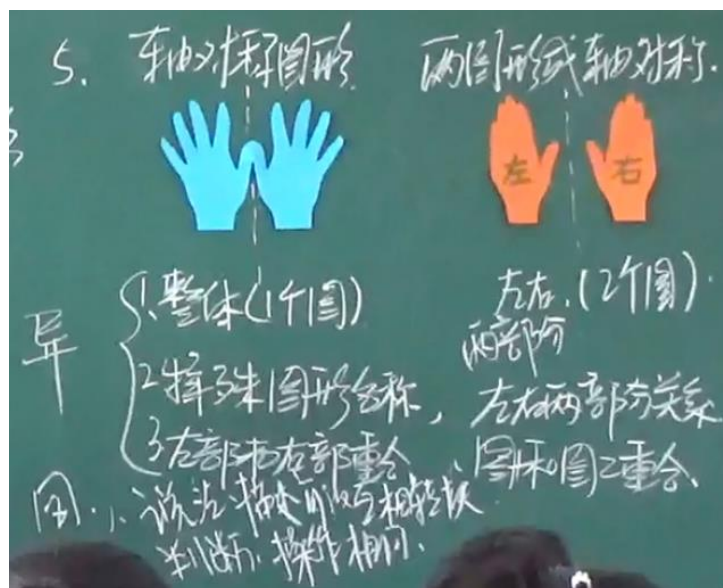


Figure 5. An axial-symmetric figure versus two axially symmetric figures

Table 7.

Differences and similarities between axial-symmetric figures and two figures being axially symmetric (translation of board work)

	Axial-symmetric figures	Two figures being axially symmetric
Differences	One whole figure	Two figures
	Name of a special figure	The relationship between two parts
	Left-hand part matches right-hand part	Left-hand figure matches right-hand figure
Similarities	The descriptions are interchangeable; same procedure to identify	

Not only did Ms. Wang frequently mention definitions to help students solving practice problems, but she was also very rigorous about the details of the concepts. The figure and table above showed how Ms. Wang addressed the slight differences between

an axial-symmetric figure and two figures that are axially symmetric, with which students are often confused. She used “hands” as models and listed the differences and similarity between the two concepts.

Opportunities for students to think and respond. During the post-observation interview, Ms. Wang was asked about the methods that she used to prompt student thinking in her classes, and she explained her idea:

Teachers should have the capability to teach students in accordance with their aptitude. We can't only teach those who perform and leave others behind. However, if we only cover easy problems, it is also unfair for good students. I have students who get the idea even before the first example, and students who can't get it after 45 minutes. That's how wide the gap is. That's why we need to prepare problems in different levels of difficulty and guide children to do harder and harder problems.”

However, the opportunities that Ms. Wang provided to students to think and respond in her classes seem to be too short. For most of the questions she prompted during the observations, she only waited for one to two seconds before the class or herself revealed the answers.

In addition, Ms. Wang provided different approaches during the interview to deal with students' misconception. When she was asked about to “teach” the Pythagorean Theorem, the following conversation occurred.

Researcher: What would you do if students couldn't understand this idea?

Ms. Wang: You first need to know why. What happened to that student? Was it because he didn't memorize the formula, or that he didn't know how to apply the formula? There are many things we can do if he didn't memorize the formula. For the second situation, we can sit down and discuss what situation can we use this formula. Kids will take time to get used to it. There is a process for kids to accept a certain concept. As s/he sees more, s/he will gradually form a structure in his head: how do I tackle this problem when I see this and that? This is part of mathematical thinking.

She added a few seconds later, reluctantly: "Well, if all these don't work...we'll have to go back to a more traditional way. Rote memorization and a huge amount of practice will probably work as well.

Use of prior knowledge. Ms. Wang explicitly mentioned prior knowledge in her two classes. For instance, she mentioned prior knowledge in geometry when explaining practice problems about identifying axial-symmetric figures:

Ms. Wang: Alright, we just did some figures that are a little bit complicated. From the figures that we learned from 7th grade or even elementary, what are some of the common axisymmetric figure in mathematics [writes on the board: common axial-symmetric figures]?

Students: [naming all kinds of shapes] Triangles, squares, circles, trapezoids...

Ms. Wang: OK, some of us said triangles. Is an arbitrary triangle axial symmetric?

Students: No.

Ms. Wang: It has to be a special triangle, right? Which one?

Some students: Isosceles, equilateral.

Ms. Wang: Isosceles and equilateral triangles, right? And circles... We just did that in the last problem. How many axes of symmetry?

Some students: Infinitely many.

Ms. Wang: Infinitely many. Remember, each diameter in a circle is one of its axes of symmetry...

In this vignette, without specifically reviewing the properties of geometric shapes, Ms. Wang prompted students to recall their prior knowledge about basic geometry shapes and connected it to the topic of the day. A minute later in class, she extended to content even back to elementary school. "What more-basic geometric elements are axial-symmetric and why?"

Use of questions. In her observed lessons, Ms. Wang did not call an excessive number of students to stand up and answer questions. Most of her questions were for the entire class and usually about prior knowledge and definitions. During her interview, she mentioned that she would call individual students to answer questions when she thought they lost concentration. She, however, provided another perspective in the use of questions other than checking students' concentration:

Besides keeping them focused...some students in my class are shy and don't know how to express themselves. Sometimes we know that they have an idea,

but they couldn't express it or write the proper steps on test papers...so what I do is to call a whole row to go through the steps of a problem, one by one and step by step. I will involve that student on purpose, and I hope that I can establish that student's confidence by just letting him/her have that one correct step.

Pedagogical content knowledge in curriculum

In this study, teachers' pedagogical content knowledge in the curriculum is analyzed from four aspects: curriculum-aligned practices, curriculum coherence, knowledge of mathematical curricula in general, and knowledge in curriculum-related material. Their knowledge about curriculum-aligned practices and knowledge in curriculum-related material are mainly observed from the interviews, while the curriculum coherence and knowledge of mathematical curricula, in general, were observable mainly in the classroom observations. Teachers' knowledge about curriculum in different elements is operationalized mainly based on answers to an interview.

Knowledge in the mathematical curriculum in general. Ms. Wang could quickly recall the flow and sequencing of mathematical topics in the middle school curriculum, even for the grade that she was not teaching. "...In the three sections that introduce Pythagorean theorem, only the first section is about the formula itself. The second and third sections, how to determine if it's a right triangle and real-life applications, are the repetitive practices of the theorem. Students go back and forth to practice how to apply this theorem, and eventually gain mastery in it." She mentioned this when asked to explain her teaching strategies for the topic of Pythagorean Theorem.

When Ms. Wang talked about the curriculum, she was able to elaborate on the middle school curriculum and to connect the middle school curriculum to elementary and high school curricula. “In the subject of mathematics education in China, I think the elementary and middle school curricula are somehow disjointed. We need to either make elementary math more rigorous or to make middle-school math easier. If students could not adjust themselves, they would have a tough time in 7th grade.” She further explained: “I am not saying the difficulty of the problems. What I mean by ‘not jointed’ is regarding students’ reading and comprehension skills. Elementary teachers in China tend to focus on kids’ arithmetic skills. Kids can compute so fast mentally. But when they go to middle school, such skill is not that useful—not exactly useless, I mean, it is indeed important, but it (elementary curriculum) magnifies the importance of arithmetic too much. In my opinion, what students need to exercise is their comprehension. Once you learned ‘how to learn’ and have sharp learning ability, they will perform much better.”

She then critiqued the uneven difficulties throughout the middle school curriculum:

From my perspective, the content in 7th grade is a bit hard, yet the material in 9th grade is a bit too easy. But we have to do this in China, because of the High School Entrance Exam. The whole 9th year is to review for the Entrance Exam, and we need to move more new content to 7th and 8th grade. Such unbalance leads the disjoint between 9th grade and 10th grade (first year in high school). However, when I look back from high school, the six textbooks in middle school is equivalent to one in high school, in terms of the depth of material covered. One chapter in my college math book is

equivalent to one textbook for high school. Similarly, if you look at elementary from middle school, the curriculum is composed of fragments of knowledge. It's...natural. That's why we could improve student learning if we can improve the connection between school levels.

Curriculum Coherence. Since Ms. Wang only went over introductory sections in her two observations, the evidence in curriculum coherence during her observations was not very strong. She, however, showed her knowledge in curriculum coherence during the interview.

Though she had only been teaching for two years, Ms. Wang was aware of the prerequisites for her 7th-grade classes as well as follow-ups for mathematical topics in middle school. For instance, in this vignette where she was asked to “prepare” the quadratic equation $2x^2 - 5x = 3$, she mentioned the topics in 8th and 9th grades that, in her opinion, connected to the topic of quadratic equations.

Researcher: If you were to teach students to solve this equation, $2x^2 - 5x = 3$, what would you do? Have you taught 9th grade before?

Ms. Wang: No, not yet.

Researcher: Well then, let's change it to a 7th-grade problem. I believe your classes have already gone over linear equations on one variable. They had learned something similar in elementary grades, hadn't they?

Ms. Wang: No, no, the linear equations they learned before was too simple. They would use the property of an equation, to add or subtract a number on both sides of the equation and it remains the same. Now in 7th grade, they

will learn to move a term around, which is based on the property of the equations. They will also learn to clear the fraction coefficients. Clearing fractions also lay a foundation for solving rational equations, which is a lot more challenging to students since they need to deal with the (variable in) denominators; they have to be clear about those rules. Then finally they reach that quadratic equation [pointing on the paper], and you will show them certain methods to solve it.

In addition, Ms. Wang was able to explain the teaching strategy in general for a topic that she had never taught before. In the following vignette, Ms. Wang was asked about the teaching strategy for Pythagorean Theorem in 8th grade. She explained the routine of teaching and talked briefly about her teaching strategy in general.

Researcher: Is there a strategy for you to teach so that students could understand the idea (of Pythagorean Theorem) very quickly? Things like using manipulatives, software, et cetera?

Ms. Wang: Well, no matter which tool you use, the main idea is the moving and additivity principles of areas. Students do some cutting and moving of shape and see equivalent areas, and we guide them to come up with some formulas. Pythagorean Theorem is interesting, and the activities are not hard to find, as there are hundreds of proofs from all places over time. However, it is more straightforward for students to understand this concept via areas the first time. It could even help them in some other topics if they are very familiar with this process.

Curriculum aligned practice. In class, Ms. Wang used practice problems in the textbook to fortify students' understanding. She walked around while students were doing problems and gave comments when she found a mistake. The whole class yelled out the answers for each problem and checked for the answers. There was one problem in the textbook that asks students to identify if capital letters and some common Chinese characters are axial symmetric (the letters and characters are printed in bold fonts so that some could look symmetric). Ms. Wang spent five minutes on that particular one and wrote down examples on the board. Later in the post-observation interview, Ms. Wang explained:

Most of the problems in our textbooks are trivial and not suitable for the tests; so mostly I assign textbook problems in class to fortify students' understanding. On the other hand, some textbook problems are quite intriguing. The textbook problems often use real-life scenarios, so I would usually take them to motivate students. For example, the problem that identifies the symmetry of Chinese characters and English letters. That's what our students use every day, and it is motivating. Some students don't like pure mathematical problems, but most of my students are interested in this one.

Curriculum-related material. Besides practice problems from the textbook, Ms. Wang also used problems from a practice book called *New Classroom: Synchronous Study and Exploration* (New Classroom). According to her, the problems in this book are "much more rigorous, more mathematical, and fits the exams better." "When we prepare

lessons, we need to not only refer to this single practice books,” she said, “there are hundreds of kinds of practice books in the bookstore if you go there, each of which we can learn from. Right now there are six practice books in my office. Even though I usually assign problems from New Classroom in class, if I found a great problem from other books, I will put it in my slides.”

Cultural Beliefs towards teaching

Teachers’ cultural beliefs towards teaching were analyzed from five aspects mainly throughout the interviews, namely, beliefs about practices, beliefs about examinations, beliefs about parental support, beliefs about using questions, and beliefs about collaborative lesson preparation. The responses were coded and analyzed to answer the second and third research questions.

Belief in examinations. During the interview, Ms. Wang showed her belief about examinations when she was explaining her teaching in general. She stated that she didn’t require her 7th-graders to write down all the detailed steps and started to talk about exams:

I sometimes tell my students, since you are facing the education system that solely uses test scores to evaluate your performance, for problems such as multiple choices and fill-in-the-blanks, you don’t need to write down specific steps and have a clear idea. Let me get it straight. I don’t recommend students always to guess the answers, but if that student wants to get a better score, s/he needs to know some tricks to guess. For instance, a very complicated fill-in-

the-blank problem could end up with the answer to one. Another thing is that if a student knows the answer but couldn't write down steps, can we really say that s/he doesn't know the idea? Students in middle schools are still learning about logical thinking; most of them cannot write down perfect steps in one try. Even though they have to write down steps to get points in the exams, we teachers must guide them in class to develop their logical thinking.

Belief in practices. In her two classroom observations, Ms. Wang spent at least half of the class time doing practice problems. During the interview, however, when asked about the view of giving practice problems to students, Ms. Wang said, "Students are still in transition from elementary to middle school in 7th grade. I tried hard not to burden them too much with practices. As long as they understand the material, I'm OK with it." She added a few seconds later, "but there are just so many types of problems that are potential test problems. You don't go over them, students lose points, and it is your fault. They will need to practice even more as the level goes up. I can already see that some of them will have a tough time in 9th grade. That transition is painful, but everyone has to pass through it."

When students needed help for practice problems, guided instructions seemed to be Ms. Wang's preferred teaching method—if it didn't work, she would switch to a more traditional "memorize, apply, and practice" method. "We can guide them and derive the formula with him, using moving and additivity principles and let them explore the formula. The last thing we can do is to use rote memorization and try to understand it while solving practice problems," she explained her belief when she was asked to explain the teaching strategy about Pythagorean Theorem.

Ms. Wang also believes that, by doing practice problems, students benefit not only for their exams but also for their logical thinking in the future. She mentioned this point of view in the second post-observation interview while going over the practice problem (this problem can be found on page 58).

We have to be rigorous in those geometry problems, and logic must flow.

You can't use a corollary that seems to be correct in your proof. For instance, [draws on the paper, see figure below] it is equidistant for a point on the perpendicular bisector to the two ends of the line segment. We then obtain two congruent triangles and hence two congruent angles. Can we say that these two angles are congruent because the point is on the perpendicular bisector? No. Even without proving the congruence one needs to show the isosceles triangles. When you write the proofs, theses statement must be included to get credit. Well, it's helpful even for their future. In high school, they will learn syllogism, which is another practice for students' logical reasoning. They will use logic in many aspects of their lives in the future.

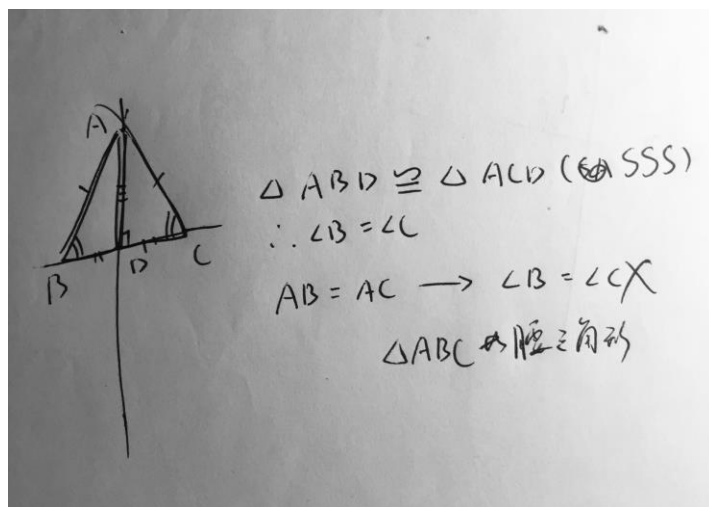


Figure 6. Ms. Wang's work shows the rigor of logical thinking

Belief in collaborative work. Ms. Wang believes that collaborative work helped her as a less-experienced teacher. “What we do in School C is to turn in a plan at the beginning of the semester, which is like a syllabus with a lot more details. Once approved, we prepare for classes accordingly. But I was benefited more is not from this plan; it’s from my colleagues. As you saw before, all of our math teachers share an office. My seniors gave me great tips and suggestions that helped me get better. Yes, I think it is very helpful, especially for new teachers like me.”

Ms. Wang also mentioned that the preparation of detailed lesson plans was of great help as a rehearsal of teaching. “I write detailed notes for each of my class, not just an outline. As a novice teacher, this is particularly helpful. Some items I usually include in my lesson plans are objectives, materials, my teaching approaches, examples—sometimes with different methods to solve, practice problems and their solutions, and summaries,” she explained. “It is very time-consuming but worth it. When I read the notes before classes, I get clearer ideas of what is going to happen today. Even though the

classes won't always follow the notes, I feel more confident by 'rehearsing' it before class."

Belief in parental support. During the interview, Ms. Wang pointed out that teaching not only occurs in the classroom when she was about the question of "how do you think the support from the parents affects your teaching?" She even stated that parents play a more prominent role that most people would think:

I must admit that a teacher is a very respected job in China...however, sometimes we are taking too many responsibilities. Parents in recent years tended to rely more and more on teachers and started to blame teachers if their kids did not behave or achieve in school, and they think that they can hand the job to teachers to have a well-educated kid. That is almost impossible—we need understanding and support from parents. Most people think that school is the place where children get an education, but I would say that parents play a much more significant role in education. I mean, I will teach your kid for at most three years, but you will accompany your children for a much longer time.

Belief in student engagement. This node was not an element in the design, but Ms. Ms. Wang believes that student engagement is one of the keys for students to succeed in middle school; she also pointed out that teachers must play an active role in teaching to keep such engagement. Though it was not observed in her actual teaching, Ms. Wang mentioned the different situations in her two classes:

One of the apparent differences, now between my two classes, is that one of the classes is much more interactive than the other. It is the class of which I am

the head teacher, and I trained my students to be responsive in my class. I ask them to head up when I speak and nod if they understand; if they don't (understand), they will frown or sit still. By observing their movement, I will decide what to do next. The other class is much less interactive—probably because I am not their head teacher [chuckles]. I have to repeat myself several times to make them head up...and very few respond to me when I ask “is there any question.” “Does everyone understand?” “Yes!” But when I call a student and ask him “what did I just say,” usually I get answers like “I don't know.” Some students pretend to focus in class. Overall, my own class performs much better than the other one.

Summary

Ms. Wang is a mathematics teacher who values the concepts and definitions in mathematical topics. She emphasized the concepts in various ways to ensure students' understanding, including using manipulatives, tables, and different examples. She was explicit in using prior knowledge in her classes when explaining problems. When asked to compare textbooks, she showed rich knowledge about the mathematical curriculum that she was teaching and was well aware of the prerequisite and follow-up topics in middle school. As a teacher that had only taught for five years, she appreciated the collaborative work done among her and colleagues. However, she didn't call many students to stand up and answer questions—the main goal of calling students seemed to be keeping students focused. Moreover, the opportunities that she provided to students to think and respond in classes seem to be too short.

Case 2: Ms. Xu

Ms. Xu started teaching in 1989 and had been teaching for 27 years by the time of this study. Before she started teaching mathematics in School Y in 2000, she had been teaching in various other schools, including vocational schools and high schools. She had been teaching all three middle school grade levels, and she was teaching two sections of 7th grade when the data was collected. She is currently the leader of the mathematics teachers' group and responsible for the management of collective lesson preparations. Having both parents as teachers, Ms. Xu admitted that the imperceptible effect from her parents is probably the most important reason why she is a teacher today.

Lesson Description

Throughout the three 8th-grade lessons that Ms. Xu taught, she covered two different topics in three days: on May 24, 2016, Ms. Xu gave a practice class about the proof problems in geometry. She concluded the chapter of axial symmetry and started a new chapter, Introductory Probability. The following two classes on May 25 and May 26 were the continuations of the new chapter, in which Ms. Xu led the class to explore the concepts and idea of probability. The first two classes of Ms. Xu took place in the multimedia classroom in where the SMART Board is accessible, and the class moved back to their regular classroom for the third observation.

Without saying too much, Ms. Xu started her lesson by showing the following geometry proof problem on the SMART Board:

Point P is a point on the angle bisector of $\angle AOB$. PC is perpendicular to OA , and PD is perpendicular to OB , with perpendicular feet C , D , respectively.

1) Does $\angle PCD \cong \angle PDC$? Why?

2) Is OP the perpendicular bisector of CD ? Why?

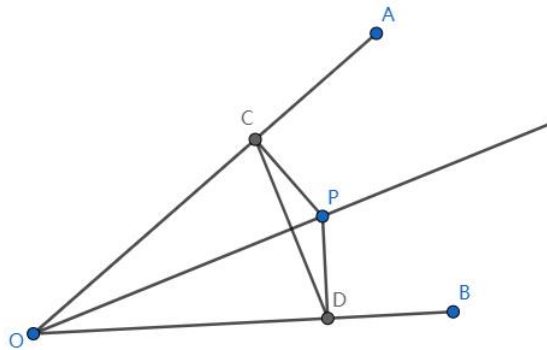


Figure 7. Ms. Xu's in-class geometry problem, observation 1

This is a problem in which the conclusion must be obtained by proving two sets of congruent triangles. Ms. Xu called several students' names, asking them to provide ideas about the steps of the proof. For the last two steps, she even invited a student to come to the board and draw the necessary conditions of congruence on the SMART Board. She kept asking questions to the class about the prerequisite knowledge and properties while going through steps with students, and emphasized a key property of isosceles triangles, that “the three lines (angle bisector, perpendicular bisector, the altitude of the base) of an isosceles triangle coincide.”

Ms. Xu then asked if there is another approach other than using the congruence twice. One student went to the front of the classroom and suggested using the properties of the perpendicular bisector of a line segment. Ms. Xu immediately took over the suggested idea and guided the class through the problem. She clearly stated her expectations: “I only expect that a third or a half of us understand this advanced methods...for the rest of us, make sure that we know how to prove it using the method of congruence...put however many marks that you want to remind you of its importance, and make sure you practice after class”.

In the second and third observations, Ms. Xu started a new chapter called “Introductory Probabilities.” Instead of simply introducing definitions and jumping to the practice problems, Ms. Xu spent almost 20 minutes in the second observation for the following task:

There are two situations when you throw a thumbtack: “point up” and “point down.” Do you think the probability of getting the “point up” is the same as getting a “point down”?

In the following vignette, Ms. Xu went over the first two steps in the general procedure of conducting a statistic study. Instead of showing the entire procedure, she prompted the steps naturally along what students suggested.

Ms. Xu: So...do you think that we have the same probabilities (for the two situations)?

Class: [all talking at once; some said “yes,” and some said “no”]

Ms. Xu: [Student1], what do you think?

Student1: I just feel that they won't be the same.

Ms. Xu: You *feel* that they are not the same. Did all of us make some hypotheses? We hypothesize (writes “hypothesis” on the board)—which situation is more likely to happen?

Class: [all talking at once; some said “point up” and some said “point down”]

Ms. Xu: You all have different answers. Can anyone convince others at this moment? No? Then what do we need to do?

Class: Experiment!

Ms. Xu: [writes “experiment” on the board] Let's conduct the experiment.

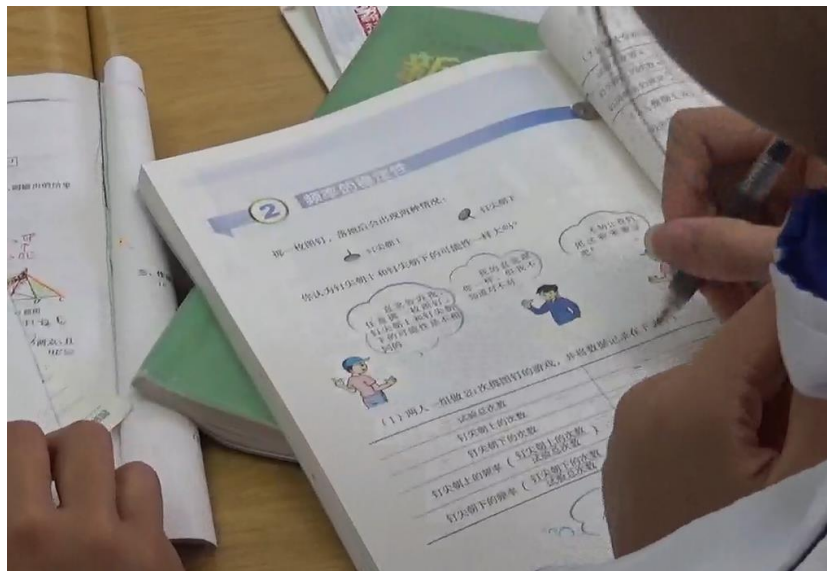


Figure 8. Two students tossing a thumbtack and recording their result

Ms. Xu asked students to toss a thumbtack and record how it landed 20 times in pairs. By observing the significantly different results among groups, students realized that they needed to run the experiment more times to make the result convincing:

Ms. Xu: So, after we have results from all groups, how do you see the difference in different groups?

Some students: The difference is rather big.

Ms. Xu: The difference is rather big. Then how do we improve our result, to make it more convincing?

Class: We can run the experiment multiple times.

Ms. Xu: OK, we'll do nothing else but let each of you toss the thumbtack 200 times today. [class chuckle] In fact, in the past 10 minutes, in this class we already ran this experiment 400 times, didn't we? Let's gather our results and put it together, starting from this group...

Ms. Xu used a spreadsheet to calculate the result and drew a line chart to show that the frequency of getting a "point up" tended to become steady as the number of experiments got significantly larger. When asked why she spent so much time on that class-wise experiment, Ms. Xu replied in the post-observation interview:

It's not meaningless...I do want to embed all the key points that I want to cover today into that experiment. Think about it...we talked about the general process of doing statistics—raising a hypothesis, run the experiment, collect data, analyze data, and verify the hypothesis. In addition, students

will have a much stronger impression on the definitions that are related to that experiment. Some of the facts were also introduced in that experiment, such as that we need to increase the number of experiments to make it more convincing. We didn't do the second experiment suggested in the textbook which is about tossing a coin, but that one (tossing a thumbtack) was reviewed by our math group and is kept.

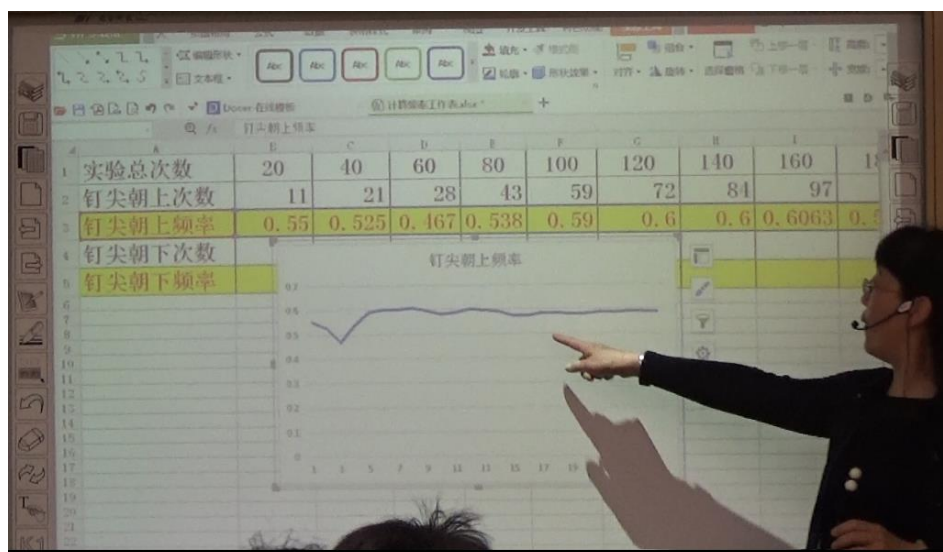


Figure 9. Ms. Xu used a line chart to show the behavior of the frequency when the number of experiments got significantly larger

After the discussion of the result, Ms. Xu posed some further questions on the screen and asked students to think after class:

- (1). From the experiment above, do you think the probabilities of getting a “point up” and a “point down” are the same?

(2). Two students did the experiment for 1000 times, from which there were 640 times of “point up.” According to this result, they claimed that the probability of getting a “point up” is higher than the probability of getting a “point down.” Do you agree with it?

(3). Another two students did this experiment 20 times, from which there were nine times of “point up.” According to this result, they claimed that the probability of getting a “point up” is higher than the probability of getting a “point down.” Do you agree with it?

For the last 15 minutes of the second class, Ms. Xu asked the class to do work on some simple practice problems in probabilities. As it was the first class of that chapter, the problems involved minimum amount of computation, such as “what is the probability of drawing a red ball from the bag, if the bag contains two red balls and three white balls?” and “what is the probability of drawing a heart from a standard deck of playing cards?”

In the third observation, the class went back from the multimedia classroom to their daily classroom. At the beginning of the class, Ms. Xu emphasized the keyword of the topic for the day:

Ms. Xu: We’ll go over section 6.3 today. Class, please tell me the topic of this section.

Class: [together] The Probabilities of Equally Possible Events.

Ms. Xu: [writes while speaking] The Probabilities of Equally Possible Events...What is the keyword of this topic?

Class: Probability... [pause for a second] no, equally possible!

Ms. Xu: That's alright, we are studying probabilities anyways. [underlines the word "probability"] But what's more important, is that we are focusing on the *equally possible* events. Now let's look at your textbook...

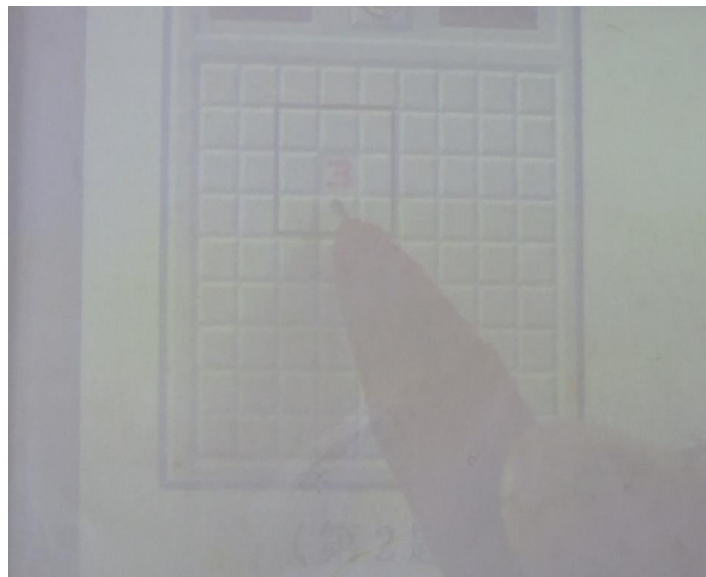


Figure 10. Ms. Xu used "Minesweeper" as an example of finding probabilities

In that class, Ms. Xu did not spend too much time on the definitions; instead, she assigned a significant amount of practice problems from the textbook and the practice book in which various real-life situations were involved, including classic playing cards, prize wheels, and pick-a-ball-from-the-bag problems. She even used classic Minesweeper game (see Figure X) as an example and asked students about the probability of losing the game if the first tile was a 3, where the

objective of the game is to clear the “mines” on the board with the clues about the neighboring mines in the field. During that time, she asked several students to work on the board, and frequently called students to answer questions as she usually did. While students were working on the problem set in four-student groups, Ms. Xu walked around and checked for students’ mistakes. She prompted a whole class discussion about the problems and summarized the key concept of finding the probability for equally-likely events.

Lesson summary. In her three observations, Ms. Xu finished the leftover of some geometry problems and started the introductory unit of probability. She used the setting of four-student groups very frequently to assist her in her teaching. In addition, she allowed students to conduct an actual experiment to better understand concepts in introductory probability, instead of showing the result theoretically.

Pedagogical content knowledge in teaching

Table 8.
Ms. Xu's PCK in teaching

Categories	Ms. Xu	
	Instances	Percent
PCK in teaching		
Prior Knowledge	14	13.86
Concept or definition	13	12.87
Rule and procedure	6	5.94
Draw Picture or table	8	7.92
Give example	14	13.86
Connect to concrete model	6	5.94
Students who do not understand prior knowledge	5	4.95

Table 8. Continued.

Provide student opportunity to think and respond	10	9.90
Manipulative activity	2	1.98
Attempts to address students' misconceptions	13	12.87
Use questions or tasks to correct misconceptions	3	2.97
Use questions or tasks to help students' progress in their ideas	4	3.96
Provide activities and examples that focus on student thinking	3	2.97
Unintelligible response	0	0.00
Incorrect	0	0.00
Total	101	100.00

The frequencies of Ms. Xu's instances during observations were relatively even among categories. She gave many examples (14 times) after going over the basic concepts of probabilities; the geometry problems she reviewed in the first class were not counted as the problems were done previously by students. During those two geometry problems, Ms. Xu addressed students' misconception several times, in addition to the emphasis of prior knowledge. In addition, she used several activities and concrete models when introducing the concepts in probabilities. She made no mistake or unintelligible responses during the observations.

Connections between models and abstract thinking. In her classes, Ms. Xu conducted experiments of tossing thumbtacks, a number game, and sweepstake

turntables with students in the introductory probability class before talking about probabilities of equally-likely events.

Use of questions. Ms. Xu was not only able to pose various questions to prompt students thinking and address their misconception, she, in the interview, also provided a point of view that encourage the use of questions in her four-student group setting:

I do ask a lot of questions in some classes. However, what I do more often is to let students think within the four-student group. For instance, when I finished an example in class, I would give another one and ask the class to discuss it. I will let the group leaders lead discussions, and it is up to them to prompt questions to their group leaders. On the other hand, if the group members have questions, they could ask their group leaders...by doing that, it stimulates both the leaders and the members of the group to think about that question. For members, they need to understand the prerequisite knowledge at least and know what the questions are asking; for the group leaders, on the other hand, it prompts them to understand the material in a deeper meaning...you have to understand it before you can teach someone else.

After the second observation, Ms. Xu pointed out that it is a fundamental skill for teachers to ask various levels of questions to different students: "...it is necessary for all teachers...to know when to ask which student what question. Today in the class I barely asked any good student any questions—I know that they know the answers. Since we just introduced the idea of probability and it's all about the concepts and definitions, I

asked those who didn't do so well in my class to answer questions. However, it is not going to work so well after teaching the same class for a long time, as students are aware of your pattern and they are not willing to raise their hands."

In addition to students' behavior at school, academic performance is one of the key factors for teachers in China to determine if a student succeeds. When Ms. Xu used the word "good student," she was referring to those who achieved academically, instead of implying that the students were behaving.

Furthermore, Ms. Xu said she encouraged students to seek help from outside, and she thought that it would promote student thinking. The word "expert" that Ms. Xu used in her quote below didn't necessarily mean an expert as a person. According to Confucius, "in a part of three there must be one whom I can learn from"; what Ms. Xu meant was to ask anyone who had the knowledge or any reliable resource that could answer that student's question.:

I used to struggle with the job itself when I was young and didn't have too much time to think about how students felt...but now I am a head teacher and have been teaching for so many years. I started to focus on these achieving students, and I wonder how I personalize my methods with those who achieve and those who underperform. Now I will guide those achieving students to utilize the internet. If you have a question, ask an expert. Go to the internet and find the answer. If a student came to me whenever he has a question, I could only say that I don't know everything, because I only teach middle school mathematics, and he will be much better than me in the future. Some of their classmates' parents might be

college professors—go ahead and ask them. I just provide them with a platform for learning. Also, I will use encouraging language and make the kids feel that they can deal with it themselves. It works out well.

Use of prior knowledge. Ms. Xu mentioned prior knowledge numerous times in her teaching, especially in solving the geometry problems in the first observation. Expecting that the students were supposed to know the answers, she posed questions about prior knowledge such as “what properties of this pair of symmetric figures do we need?” and “what was the important conclusion we discussed last time about isosceles triangles?” In the first post-observation interview, when asked about the question “I noticed that you mentioned lots of prior knowledge for those geometry problems. How important do you think it is that we keep referring to the prior knowledge?” Ms. Xu replied: “...Of course! Prior knowledge is important for not only middle schoolers but all stages in mathematics... Some students give everything back to the teacher once the topic is taught and try to learn it again when they need it. That is not going to work well, and I am almost certain that the student could not perform very well... It is like drawing water in the lake with a sieve.” She supplemented her answer by talking about how teachers can accomplish the use of prior knowledge: “To remind students about it, we must be cautious when we prepare for classes; that is, we need to find out where we could mention prior knowledge and use it. Certainly, the exercise problems reflect a lot of it, and I am not worried about those top students, but the average students need some reminder.”

Innovative teaching method. During the observations, Ms. Xu showed a rather innovative teaching method compared to other Chinese mathematics teachers' daily routines. What Ms. Xu did in her class was to divide the whole class into groups of four students, according to their seating at the beginning of the semester. By reviewing students' previous academic performance (mostly the test scores), she then assigned one group leader in each group, usually the best performing student in the group. The group leaders would shoulder a lot of responsibility in Ms. Xu's class—collecting and sometimes checking other group member's homework, leading discussions, and helping other group members with their work. After a major test, there could be another round of leader assignment so that every student could potentially be a group leader. When asked to elaborate on the idea of grouping, Ms. Xu said:

As I know, other teachers of my two classes don't use this grouping, nor do other math teachers in the school. So, students will need a long time to get used to this setting. If you only use it when you take over the 9th-grade class, students don't even have time to be familiar with it before they take the Entrance Exam at the end of 9th grade. It works so much better when I started with the 7th grade and did it slowly yet consistently. I got even better in this round. I found that my burden was lightened a lot. I have been teaching for almost thirty years, and frankly, I am not that energetic any longer. I used to take an eye on 50, even 100 students, but I can't do it anymore. Now I must let students watch each other. If I couldn't, I'll let the group leaders help me.

Ms. Xu also mentioned that the group leaders could prompt student learning when she responded to another question about how to teach students to solve a quadratic equation: “There are some (non-achieving) students who just don’t study. Say, for my math class there are ten formulas that I required them to memorize...but they just won’t. However, for a four-student group, I can ask the group leaders to watch those students. I can’t follow all these students and checks their formulas again and again, but my group leaders can. The group leaders “chase” their members to study, and if we keep doing that for three years, in 9th grade they will at least know something to deal with tests.”

In addition, according to Ms. Xu, her four-student-group setting saved her a lot of time in class. Instead of going over a lot of practice test problems and checking every answer with the class, she had more time to lecture. What’s more, though, at a minor level, this setting seemed to, according to Ms. Xu, develop leadership as the group leaders were given authorities and responsibilities. In the quote below, Ms. Xu elaborated her setting in detail, about how she utilized the group leaders in correcting mistakes in assessments:

I prepare the test key and hand it to students and ask them to check answers by themselves. “Just write the correct answers aside if you made a mistake,” I usually say so that everyone could finish correcting in just a few minutes. After that, I just go over the ones that I planned to emphasize. I lead the class and spend only about ten minutes. Then I leave everything else to the group leaders. The group leaders will go over the rest of the test in the group. I used to need to go over seven, eight problems, but last time I only did three—the ones I think that are difficult or the ones I think worth talking about. I throw

the rest to the group leaders. If you are confused about a problem, ask your group leader. However, you must correct your mistakes. In addition, you need to write the steps when correcting multiple choices or filling-the-blanks. If one wants to do this, s/he must know exactly how to do that problem. The collaboration between the group leaders and group members is so good that they are really putting their work on it. I asked for it before, but they just wouldn't follow my instruction. In the last round when I taught 9th grade, they wouldn't listen and correct their answers. When I collected the papers, it was a mess. This time I just left it to the group leaders. When they turned in the paper, I asked each group to turn in their papers together with the group leaders' paper on top. It's the same that the group leaders had signed their names and promised that everyone in the group had corrected the mistakes. I then gave them four new worksheets. If the group leader couldn't teach everyone else well in the group, they couldn't get their new homework. In addition, I gave the group leaders the authority to remove problems for his/her members. If the leader felt that a problem is too hard for one of the members, he/she could remove that problem for the member. It doesn't have to be 100% completed. Amazingly, it worked way better than before.

Pedagogical content knowledge in curriculum

Knowledge of mathematical curricula in general. Though it has already been more than 15 years since the new Beijing Normal University Press (BNUP) textbooks were used, she was still able to compare the BNUP textbooks with older People's

Education Press (PEP) textbooks. When asked to compare the two versions of textbooks in the interview, Ms. Xu replied:

BNUP textbooks start almost every lesson with a scenario. Those textbooks used real-life situations; it is very different from the (PEP) textbooks before. After teaching for so many rounds and years, I found that some of these real-life situations are good, but some are not so good...a little bit farfetched. Another thing is that it is sometimes quite challenging to start a topic with a practical problem. If we only investigate the concept mathematically, it won't be too hard sometimes. If you throw out such a difficult real-life situation, students will have a headache and won't focus. It's better to say "let's talk about a simple concept, and we'll do practice problems over and over again when we learn it." When we understand the conceptual idea, we'll come back and solve this relatively hard problem. For students that are not so good, it's ok for them to give up on that; for those good students, we'll let them solve those problems. So, for this real-life introduction, sometimes I use them as an introduction, sometimes I use them as examples, and sometimes I skip the first and then come back for them.

She also mentioned that the teachers' ideas changed gradually after the new textbook was introduced:

Another thing is that the BNUP textbooks emphasize on...well, after the curricula reformation, our (the teachers') ideas changed. When we teach, we started to focus more on students. They are middle school kids in their

puberty; they are sometimes even more sensible (than adults). The textbooks I used when I was a student was the PEP ones. They were used for many years. If I recall the experience that when I was in middle school, and talking about, for instance, the development of the idea of shapes, PEP textbooks are relatively weak in that aspect. Even though we were so strong in plane geometry, much stronger than students now. We could add so many auxiliary lines on a proof problem, but the senses on shapes are not as strong as kids today. The reformed textbooks added the transformations of the shapes. They added these shifts, rotations, and axial symmetries. PEP only talked about centrosymmetry, which was very mathematical. But the textbook that we use now (BNUP) tells students that these figures are obtained by rotation, that I don't have to prove every single condition to show that they (the corresponding parts in the figure) are congruent. It looks like we sacrifice some reasoning, but students' geometric senses are better. It's more helpful. I think the reform was quite successful in this aspect. However, in plane geometry, BNUP textbooks focus on sensibilities and then rationality. It's like the idea of the spiral curriculum.

Ms. Xu then used two examples in geometry to compare the two textbooks and shared her experience when she first taught using the new textbooks. She admitted that it was more of an experiment for both teachers and students to learn from the new textbooks since there were topics that she had little experience teaching and had no idea how to teach.

Curriculum coherence. During the interview, Ms. Xu expressed curriculum coherence when she talked about preparing a lesson with colleagues: "...For instance, if we talk about congruent triangles for our 7th grade, we would discuss how to set up the formula and rules (to be consistent). As students grow up, we face a new situation when we move on to the next grade. We ask students to write "for triangles this and that," and link all three necessary conditions together in 7th grade; in 8th grade, we open them up and no longer have such strict requirement, as long as they are aware of it. In 9th grade, because of the Entrance Exam, we need to tighten up the requirement again."

Cultural beliefs towards teaching

Belief in exams. Ms. Xu talked about her belief and experience of using test results to support her teaching. She believes that the reflection and analysis of the exams are of more importance for both teachers and students: "If you run analyses of students' responses, you can see very clearly about the topics that students are still confused about. You can adjust your teaching in later classes. The previous test statistics is also important; by analyzing it, you see where students could make mistakes, instead of relying on your experience. It helps you with the emphasis when you prepare for the lessons. That's why nowadays they have all those stats besides the problems on the practice books, such as the difficulty level and passing rate. These stats could also help students to practice over their weaknesses purposely."

Belief in practices. In the observations of Ms. Xu, she assigned an average of ten problems in each class. Regarding the practices, Ms. Xu mentioned the transition that students faced in 7th grade: “My students are indeed in the transition from elementary to middle school, both physiologically and psychologically,” she said. “But it’s their responsibility to adapt it. And they have to adapt it quickly...otherwise, it’s even more challenging later in 9th grade and even high school. ‘Practice makes perfect’ is not just a saying...I do give my students a lot of problems to practice, both in class and after class. They are facing the Entrance Exam in less than two years, and they need to get used to the pressure and workload as quickly as possible.”

Besides reinforcing the understanding of knowledge, Ms. Xu also believed that one of the main purposes of doing practices and homework was to find the errors and correct them as early as possible. “We can’t let the errors continue and try to fix it later. Even though there is a saying ‘it is never too late to mend,’ when we examine their work, we need to find out not only their errors but also how they came up with such errors as early as possible. The early students realize their errors, the faster they could eliminate the errors. If we leave it and try to fix it later right before exams, it would be very ineffective.”

Belief in collaborative lesson preparation. During the interview, Ms. Xu appreciated the collective lesson preparation and emphasized the importance of it: “We meet and prepare for the class together. Not for a whole semester—it’s too long. It’s not efficient for us to discuss the teaching plan for the whole semester. We meet every Monday during the second period in the afternoon, and three groups prepare for the class

collectively in the offices... Three prep groups, we have tasks for each teacher. The tasks are assigned a week before. For example, we have five classes next week and six for 9th grade. Then each of them is responsible for one class. You'll need to prepare the class in every detail—the textbook, exercise, and the supporting exercise booklet by the Bureau of Education of Shandong, and all the other exercises, several books, all of them, you'll need to do all these problems beforehand. Then have an overall understanding. Prepare the class in advance, including the slide shows. That's our requirement. Then on Monday, you'll need to talk to other teachers about how to teach this class. How to introduce, the emphasized examples, exercises, how to deal with the text and after-class problems, should students do problems as homework or they need to answer orally, so on and so forth, you'll need to speak out your opinion. And on the corresponding exercise, you are responsible for determining which question is good, which one is not so good, or even adding or deleting certain problems and correct any mistake. Everyone prepares the class in every detail; then other teachers can look at yours and use your idea.”

Though the idea of collective lesson preparation seems to lose the personalities of the teachers and make all the classes the same, Ms. Xu pointed out that it is not that unified as it looks:

However, we are not saying that you have to keep it in that way. The good thing about our math group, in my opinion, is that we emphasize the collective preparation, but we don't always stick with it. It's reasonable and necessary for teachers to be personalized. Someone is good at this, and someone is not, you can't force her or him to talk about a specific example.

However, when we prepare collectively, we mention the focus point of this class, and how well we want our students to achieve. We will communicate about that. All kinds of small issues. We gather and discuss them, including the slides. If I am preparing for this class, I will make the slide show and send it to others. If I cannot make it on Monday, I will send that later. For instance, if I am preparing for Thursday's class and I don't have a complete idea yet, I will send out the slides on Tuesday or Wednesday. Some teachers are not fond of these modern techniques, and they are still using chalk. We are all OK with that. We download them online and modify it by ourselves. They are quite complete now on the CD. The CD comes with the textbook. We use the ones on the CD, and we have online resources.

Ms. Xu also pointed out that it is important to use the corresponding practice problems for each prepared class. "Another highlight of our collective preparation is that we keep the tradition of assessing every chapter we go over," she said. "The second semester of 7th grade contains six chapters, and we had five major tests, likely once for every two weeks. For Chapter 1, the computation of polynomial expression, we even gave two tests on it. The key thing is that we always make our original test problems by ourselves. We kept updating the problem sets. The teachers who taught last year passed these assessments to the teachers next year, and the sets must be modified for at least 50%. We keep the other 50% to ensure consistency."

In addition, Ms. Xu indicated that it is especially important for new in-service teachers to prepare lessons with experienced teachers: "Another reason why I have been emphasizing our collective preparation in the schools that I have taught for more than 20

years is that it evens the teaching levels of our teachers. We have 19 teachers in our group, and of course, they have differences in their personalities. However, if you follow us and do these problems, examples, and preparation, even a novice will be well trained.”

Summary

Ms. Xu used the four-student groups throughout her teaching both inside and outside of the classroom. Such setting was the core of her teaching routine in checking students’ performance, enhancing student cooperation and engagement, clarifying student misconceptions, and developing leadership. Her teaching was very interactive as she constantly called students to stand up and answer questions.

Case 3: Ms. Zhang

Ms. Zhang had been teaching since 1994. She graduated from a university in Shandong with a Mathematics degree. During her 22 years of teaching, she was the head teacher for 18 years. Before she was assigned to School N in 2005, she had taught vocational high school and three years in another middle school. She had been teaching all three middle school grades, and she was teaching two classes of 8th grade and was the leader of the 8th-grade mathematics teachers’ group at the time of this study. She considered herself a person who is very good at mathematics; one reason why she became a mathematics teacher. She also admitted that she developed her mathematical thinking in her four years of studying mathematics in college, and it had a great effect on her for being a mathematics teacher in the future.

Lesson Description

The observations of Ms. Zhang's 8th-grade classes took place on May 20, May 23, and May 26 in 2016. Though the three classes were not back-to-back, the classes remained in the chapter of rational equations. The classroom follows the typical setting in most public schools in China, but instead of giving direct lectures and assigning practices, Ms. Zhang taught more interactively by using student presenter for most of her daily 45-minute lessons. In the first observation of Ms. Zhang, the student presenter started his slide with the topic of Rational Equations, of which was the new material Ms. Zhang needed to cover on that day. The student presenter started directly with the process of solving a rational equation, and Ms. Zhang intervened immediately.

Presenter: Good morning, everyone. Today I will introduce the Rational Equations to you. The first step in solving rational equations is...

Ms. Zhang: [presenter], sorry, but I must interrupt you for a moment. I know you have a rich knowledge of rational equations already, but perhaps the rest of the class doesn't yet. Can you explain to the class what a rational equation is?

Presenter: It is an equation with variables in at least one denominator.

Ms. Zhang: Can you give us an example of a rational equation?

Presenter: $1/x = 2/x$.

Ms. Zhang: Let's look at this one. Does this equation have a solution?

Class (chuckle): No.

Ms. Zhang (to the presenter): Can you give us another (example)?

Presenter: $1/x = 3$.

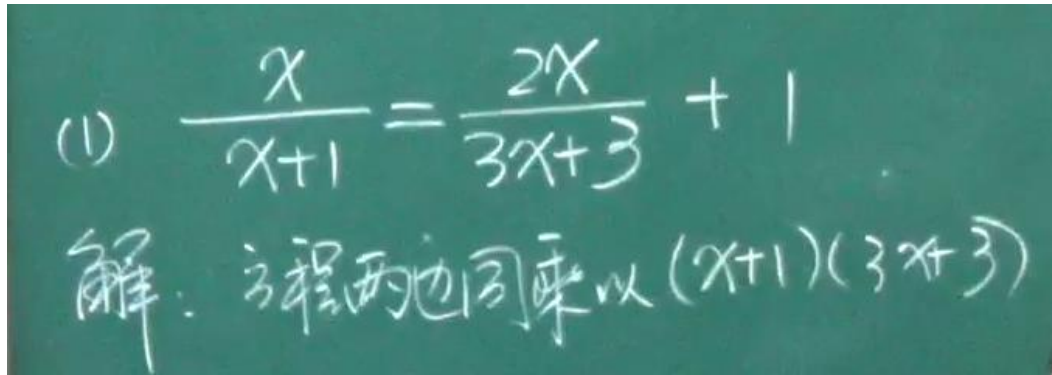
Ms. Zhang: Great. Now let's think about the definition again...



Figure 11. Ms. Zhang emphasized the importance of checking solutions in rational equations.

The student presenter then explained the procedure for solving rational equations. As the presenter went over each step, Ms. Zhang emphasized it again, writing it on the board. Later in the class, she kept recalling this procedure for almost all the examples. She particularly emphasized possible extraneous solutions in rational equations and the necessary step of checking. While the presenter was going through an example, Ms. Zhang kept asking the class questions such as “what is the next step according to our procedure?” and “how is (the presenter) doing?” Though the student presenter was standing in the front for almost the entire class, Ms. Zhang did not simply let him go

through his slide; instead, she did the examples on the slides from scratch again on the board, clearly explaining what happened within each step.



The image shows a green chalkboard with white chalk writing. The top line is the equation $(1) \frac{x}{x+1} = \frac{2x}{3x+3} + 1$. The bottom line is the Chinese text '解: 方程两边同乘以 $(x+1)(3x+3)$ '.

Figure 12. The first step of an example. Ms. Zhang wrote what student presenter said, “Solution: multiply both sides of the equation by $(x+1)(3x+3)$.”

During the class, Ms. Zhang paid close attention to the student presenter and was ready to raise small discussions according to student presenter’s performance. In an example where the student presenter posed the equation

$$\frac{x}{x+1} = \frac{2x}{3x+3} + 1$$

Ms. Zhang asked the student presenter to talk about the steps to solve this equation. Then we have the following scenario.

Ms. Zhang: OK, now [student presenter] please tell the class how to solve this equation. I’ll write down what you said on the board.

Presenter: Emmm...Solution: multiply both sides of the equation by $(x+1)(3x+3)$... (See Figure 12)

Ms. Zhang: [Paused for a second but still wrote down what the presenter said] OK...by $(x+1)(3x+3)$. Now I hear a lot of us down there have something different to say. What do you think? [to the whole class]?

Class: [having all kinds of answers in inaudible voices, but finally after a few seconds, some students spoke out] ... $3x+3$ is three times $(x+1)$.

Ms. Zhang: You want to transform this a bit? OK...I will rewrite this...
[wrote $\frac{x}{x+1} = \frac{2x}{3(x+1)} + 1$ on board] now what is the common denominator for these (expressions)?

Class: $3(x+1)$.

Ms. Zhang: It doesn't have to be this complicated, does it? [corrected the common denominator on the board]

After a few examples, Ms. Zhang emphasized the importance of checking the solution again: "Again... don't forget to write down 'After checking, (the number) is the solution of the original equation.' You'll lose one point for not writing it in the Entrance Exam. Sometimes the problem is very tedious but only worth four points. Losing one point out of four because of not writing down a sentence is not worthwhile."

At the end of the first class, Ms. Zhang handed out a pop-quiz about the operations on rational expressions—what they had learned from previous classes.

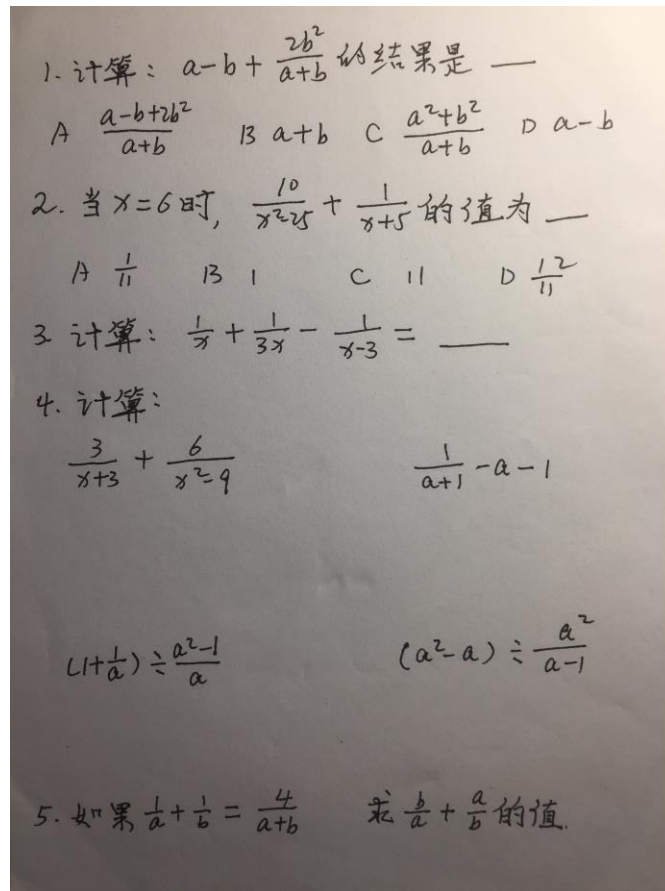


Figure 13. A copy of Ms. Zhang's quiz from the first observation

This quiz assessed operations of rational expressions as seen in Figure 13, the questions are related to performing the operations, simplifying, and for the last question, students were asked to find the value of $\frac{b}{a} + \frac{a}{b}$, provided that $\frac{1}{a} + \frac{1}{b} = \frac{4}{a+b}$. Ms. Zhang said that her classes were having such quizzes twice a week, if not more often. They are not part of the students' grade, according to Ms. Zhang, but more like a "self-reflection about how much you know and what you need to know." The quiz was not collected at the end of the class, but Ms. Zhang asked a few students to do board work and went over the answers with the class.

The second class of Ms. Zhang started with the presentation given by another student. The topic of the presentation was “The Application Problems Using Rational Equations.” In the very first slide, Ms. Zhang, with the presenter, went over the general process of solving application problems using the rational equation: read (the question)—set (the variable)—write (the equation)—solve and check. Such a process was repeated and emphasized in previous classes. The presenting student then showed the class with a problem:

A factory mixes ingredient A, which is worth 2000 yuan in total, and ingredient B, which is worth 4800 yuan in total, together. The average price of the mixture per 0.5 kg is 3 yuan less expensive than ingredient A, and 1 yuan more expensive than ingredient B. What is the mixture's price per 0.5 kg?

Once the student read the question, Ms. Zhang reviewed the general formula “income equals unit price times amount sold” in simple sales problems and asked a few students for its variations. The student presenter started solving the problem but quickly made a mistake when setting up the table.

Instead of pointing the mistake out, Ms. Zhang waited, in silence, and didn't comment until the student realized and corrected his mistake. The student was nervous and made a few more mistakes, but Ms. Zhang led him and the whole class to set up the correct table and solved the problem. She summarized, at the end of the problem, “this is not an easy problem that we usually see...an application sales problem could involve purchase price, sales price, profit rate, unit price, profit/loss, and other concepts, and we

must know what they mean under the situation of the questions. As the market economy grows, this type of questions becomes popular in our High School Entrance Exam."

The student presenter took the next 30 minutes talking about the other five types of application problems using rational equations: construction projects, traveling, upstream/downstream, the concentration of solution mixtures, and cargo transportation. Though all these application problems only involved using rational equations, most of them are quite challenging. For instance, the question that involved a construction project looks like the following:

A project can be done in 6 days by team A and team B. The company will need to pay 8,700 yuan to team A and B in total. If this work is done by team B and team C, it will take ten days, and the company needs to pay 9,500 yuan to team B and C in total. If team A and C work together for five days, they will complete $\frac{2}{3}$ of the project, and the company needs to pay 5,500 yuan to team A and C in total.

(1) How many days would it take for team A, B, and C to finish the project if they do the work individually?

(2) If the company requires the project to be done in no more than 15 days, which team should the company sign the contract with to spend the least amount of money? Explain your reasoning.

A question that requires students to set up and find the intricate relationships among the three variables is commented by Ms. Zhang as "not a typical problem in daily exercises, a bit challenging but completely doable" in class. When she collaborated with

the student presenter to go over this problem, she didn't teach as if it was a problem for performing students; on the contrary, she constantly called students to answer small but related questions, such as "what is the relationship between working efficiency, time, and total work?" or "what is the next step to finding the combined work efficiency for team A and B?" More students were involved and prompted to think about the problem, instead of waiting for Ms. Zhang to explain it.

Surprisingly, Ms. Zhang spent more than 40 minutes in a 45-minute class, going over slides prepared by the student, and did not teach any new material. Several students went to the front and did board work. In the post-observation of that class, Ms. Zhang said:

I did not just call students' name randomly and ask them to do board work. In those split seconds I have to think about quite a few things: how difficult is that problem? Who would know how to do it, who possibly knows, and who doesn't? Is anyone not focusing in the classroom? Then I call their names strategically; usually, I first ask those who got distracted [chuckles]. Then depending on the level of the question, I will either chose the top students to do a more challenging problem and maybe just a few steps for those who are not performing—we need to keep them confident.

Ms. Zhang only taught new material for about half an hour for three classes observed in total, where they started 9th grade's content (Parallelogram and Its Properties) in advance. Though the material was supposed to be covered at the beginning of the 9th grade, it is quite common for teachers, at least in the City of Qingdao, to start the material earlier in order to squeeze some time out for preparation of the Entrance Exam. Even for

the new material, Ms. Zhang asked the student presenter to prepare slides for the introductory part, and the practice problems that followed definitions immediately started to require prior knowledge.

Lesson summary. In these three observed lessons, Ms. Zhang taught rational equations and their applications, as well as the start of 9th-grade material in geometry. Instead of using a direct-instruction teaching method, Ms. Zhang used a rather innovative teaching method in her classes. Letting students prepare and present the material in turns, she left students much wider space for critical thinking about the knowledge they learned instead of sitting there and copying notes. In Ms. Zhang's classes, there was a significant amount of interactions, in which she clarified students' misconception and corrected their mistakes.

Pedagogical Content Knowledge in Teaching

Table 9.
Ms. Zhang's PCK in teaching

Categories	Ms. Zhang	
	Incidents	Percent
PCK in teaching		
Prior Knowledge	8	7.3
Concept of definition	10	9.1
Rule and procedure	15	13.6
Draw Picture or table	9	8.2
Give example	6	5.5
Connect to concrete model	3	2.7
Students who do not understand prior knowledge	6	5.5
Provide student opportunity to think and respond	16	14.5
Manipulative activity	0	0.0

Table 9. Continued.

Attempts to address students' misconceptions	18	16.4
Use questions or tasks to correct misconceptions	10	9.1
Use questions or tasks to help students' progress in their ideas	8	7.3
Provide activities and examples that focus on student thinking	1	0.9
Unintelligible response	0	0.0
Incorrect	0	0.0
Total	110	100.0

The most frequently observed instances for Ms. Zhang's observations were attempts to address students' misconceptions, provide students opportunities to think and respond, and rule and procedure. Almost all the opportunities she provided to prompt student thinking were given by utilizing students' presentations. She also utilized the presenters' mistakes to clarify students' misconceptions. Since the topics (rational equations and applications) involves mainly algebraic computation, she frequently reminded students about the procedure in general. She used no manipulative nor activities during the observations due to the nature of the topic (rational equations and its applications).

Students' thinking and misconceptions. Ms. Zhang did not use activities or manipulatives to prompt students' thinking or address their misconceptions during observations. Instead, she constantly asked questions throughout student presentations and kept interacting with students. In the three observations, Ms. Zhang used questions to address and correct misconceptions and help students' progress in their ideas a total of 36

times. Most of her questions were open-ended and provided students opportunities to think and respond.

Use of questions. When asked about the use of questions, Ms. Zhang explained:

The use of question is an art. It is one of the best ways that we teachers know whether students know the knowledge or if they are concentrating in class. Very often, I ask questions in a sudden, and if the student is not focusing, he wouldn't even know what I asked. I don't wait for volunteers to raise their hands, since almost all volunteers know the answers to the questions, and I know that these students are the achieving ones. What I care more are those who don't know (the answers), and I wonder in what way are they confused at. Therefore, it somehow forces them to follow my pace and to absorb what I teach.

Another intriguing point of view was put forward by Ms. Zhang during the interview. She pointed out that not only do teachers need to design the questions carefully but also need to treat students' questions with caution. For some questions raised by students, she explained, teachers should not give out answers directly, and sometimes students' questions should be utilized by teachers back to students:

...Students just asked all kinds of questions. Very often, they asked questions regarding algebra, and most of the times it was their own careless mistakes. Some questions are indeed very constructive and worth a detailed explanation. For those questions that were related to careless mistakes, I do not give out answers directly. Instead, I ask them to recheck their work. If the

issue persists, I will put that student's work on the screen and let the whole class examine it. Some said it ashamed that student, but I think not. It is much better to ask the whole class to help you find out the problem than to lose points on the exam because of the same mistake. On the other hand, if the question is conceptual and prompts a deeper understanding, I will probably call for a discussion. I always want my students to find out the answers themselves than just waiting for me to "feed" them. The level of understanding will not be profound if we just "feed" them.

The use of prior knowledge. Ms. Zhang mentioned prior knowledge eight times when she taught in the observations. It mostly occurred when she reminds students about the material covered from previous classes to solve the current problem, such as "from what we learned last class, how do we deal with these denominators?" and "After you set up the equation, what did we say about the first thing to do?" The quiz after the first class was also an emphasis of prior knowledge; "students must be very fluent in the operations in rational expressions before they do rational equations and applications. This quiz is to let them see how good they are now," Ms. Zhang commented.

It is intriguing to see that Ms. Zhang mentioned the connection between the curriculum and prior knowledge, pointing out that students could follow the curriculum to learn. "The textbook itself also has such spiral escalation," she said, "you can learn it again in 8th grade if you did not learn well in 7th grade. Unlike the older version we used to have, it somehow gives you that opportunity to learn it again. I also told my students, that the topic I am talking about is similar to what we had learned before, and now is the

time to catch it up if you fell behind earlier. Maybe there is only 50% of students who learned it well the first time, but it will be 80% after teaching it for the second time. "

Innovative teaching method. What Ms. Zhang did in her class was to ask the students to present at the beginning of each class. The topics were not limited to relate to the lecture of that day as long as the presentation was about mathematics. When Ms. Zhang responded to my question that asked how to teach the idea of slope in the interview, she first explained using her presentation method:

...I could also ask that student to give a presentation before the lecture. The topics of our presentations at the beginnings of classes have a vast range. They can talk about whatever they'd like, as long as it is related to mathematics. It would be great if those students could find something that is related to slope from the internet and gives a presentation. If he couldn't, I'll see if any other can. The student that gives the presentation might not truly understand everything that he presents, but I can finish what he needs.

The duration of presentations varied in the three observations of Ms. Zhang's classes. The shortest one only took about 10 minutes, while the longest one occupied the entire class period. Ms. Zhang said:

I try to provide my students with some opportunities to speak in front and to communicate with their classmates. This is also a way to show off their knowledge and talents. I had a student who only spent a few minutes and told a story about Gauss—but it was brilliant! Having student presentations also supports my teaching. What I need to teach of that day is sometimes right in

the presentation slides. If there were something that students couldn't explain well, it would be my turn to take it over and try to relate it to our curriculum.

On the other hand, however, Ms. Zhang pointed out that the disadvantage of her teaching approach was apparent as well, probably not feasible for all the mathematics classes in China: "One of the reasons why some other teachers are reluctant to do this is because it's too time-consuming and very often, you can't manage the time as you will. It is common to see that a student presenter spends almost half a class to present. Remember the second class you came in? It was one of the days that the presentation took the whole class period. I arranged this deliberately, as it was a practice class anyway—but the student presenter was staying in the front for the whole class. Another reason is that the teachers need to think a lot, as the topics can be anything related to mathematics. If neither the students nor the teachers could explain clearly, it would be quite embarrassing. I am OK if I am not able to answer questions, not from the textbook; I even check out the answers after class and let them know. But if they have such questions every class and you can't solve them, it would be time-consuming and slow the overall pace for the following classes."

Moreover, Ms. Zhang pointed out that a teacher needs to have a very rigorous system of knowledge in mathematics in order to handle all the situations in the student presentations: "Clearly, you won't have time to check every student's presentation and give him advice every day. So, if you are doing this, you can't just prepare for the class that you teach today. Your 'web of knowledge' needs to be so finely woven, so that you can face all the situations that they could have during the presentations."

Pedagogical content knowledge in curriculum

Knowledge in the mathematical curriculum. During the interview, when Ms. Zhang was asked to compare the two versions of textbooks, she replied:

I had used PEP textbooks for three years and then went to teach high school. When I came back, the textbooks were changed to BNUP ones. That (PEP) series of textbooks is so rigorous in terms of its logic and procedures that we can understand the material by just carefully reading it. (PEP) Textbooks have almost every step in the proofs. One characteristic of this textbook is that each problem has a background. From each background, a mathematical problem is extracted and being investigated. There will be a lot of thinking and questions during the investigation: why so? What happened to this and that? The textbooks keep prompting students' thinking, and concepts and knowledge will be introduced during this process. The disadvantage of that series, however, is that it doesn't have a rigorous deduction, and not too many examples to look from. Many parents are not able to help their kids anymore, as they don't know how to do those problems either. They might have the answer to that problem in their head, but they do not know the steps in between. This version of textbooks has advantages and disadvantages as well.

As a very experienced teacher who had been teaching many levels in the education system of China, Ms. Zhang was not only able to compare the two versions of textbooks, but also provided some suggestions in improving the quality of mathematics textbooks: "...Even for the newer ones (BNUP), the textbooks still tend to keep traditional problems that have been used for decades. Those traditional problems are heavily based on using

certain procedures. We could have more innovative and non-traditional problems of various types, and such problems could be contributive to our students' understanding.”

Ms. Zhang also suggested, that not only educators need to analyze and research more on textbooks, but teachers should also study the textbooks to be clear about the strengths and weaknesses of the books so that we can prepare and teach classes wisely: “It is one of the most basic requirements for teaching. You don’t have to recite the entire series of textbooks, but as a teacher, you need to be capable of referring to any knowledge point in the curriculum when answering questions. When you prepare lessons, textbooks will be the most important reference, so you need to study it in depth.”

Cultural beliefs towards teaching

Belief in practices. Though there were student presenters in all three of Ms. Zhang’s classes, the main objective of those classes was to practice and get familiar with the topic. She spent nearly two and a half classes for the students to practice solving rational equations and their applications. In her interview, Ms. Zhang explained:

I do believe ‘practice makes perfect,’ but we have to make sure that students practice with correct guidance. We cannot just throw them one hundred problems and ask them to turn it in tomorrow. You’ll need to explain if there is a mass confusion...on the other hand, only explaining is not enough. Likely, students would not understand with only lectures and without practice...even sometimes they seem to. They must practice a considerable amount of problems and see various types of problems so that they can be flexible during the tests. Students would know the process with a minimal

lecture—you need to tell them—but to understand when and how to use it takes a lot of practice.

She further added, “For my two classes that are stepping to 9th grade, there are a lot of hard problems in front of them. They need to have what we called ‘mathematical mind’ to solve them. In that stage, only explaining one or two examples is not enough...they have to see a lot of the problems with the same types to get the idea.”

Ms. Zhang also believed that the practice problems in which students did incorrectly play a much more important role in learning mathematics. She pointed out during the interview: “If a student understands the certain concept and can solve problems for that topic consecutively, I wouldn’t recommend him/her to keep practicing. You have to practice with a purpose, and focus on those that you did incorrectly. Some students feel embarrassed when going over wrong answers—but nothing is more embarrassing compared to missing points in the Entrance Exam. Only if students go over their incorrect responses will they know what weakness do they have.”

Collaborative work. Ms. Zhang was very affirmative about the collaborative work between her and her colleagues. As the head teacher of the 8th-grade mathematics teachers, she needs to lead the group to prepare lessons weekly. Ms. Zhang pointed out that a typical lesson plan consisted of several items, including objectives, teaching notes, teaching approaches, examples, practice problems with solutions, and summary. It would usually take a teacher two to four hours to prepare a lesson for the group, but Ms. Zhang thought that such time is worth spending, as it provided an opportunity for teachers to better understand the content as well as teaching by the thorough study of the textbook and related material. “With enough preparation time,” Ms. Zhang explained, “our

teachers would be able to study the material and understand the mathematical concept fully. Later, by studying these detailed lesson plans, (teachers) would improve their teachings and make it more effective.”

During the interview, Ms. Zhang emphasized, in addition to collaborative lesson preparations, the importance of being observed and having critiques from colleagues and educators. She believed that it is critically important for the new teachers not only to receive feedback from the observers but also to learn how to teach effectively by observing experienced teachers. Though she had been teaching for over twenty years, she admitted that she was still being observed quite frequently by her apprentices. Moreover, she talked about her experience of having a mentor teacher who observed her for a whole month and summarized some teaching methods for her to use when she first started: “Even today I am grateful to my mentoring teacher at that time. I feel like this is important and necessary for us experienced teachers to pass on our experience to our apprentices. Only if we show them how to teach, generation by generation, will we be able to refine our teaching over the years.”

Summary

Ms. Zhang used student presentations in her classes and utilized such a setting as a tool to assist her teaching. She emphasized the use of prior knowledge to develop mathematical ideas and kept a high level of student engagement by constantly providing comments to the presenter and posing related questions to the class. She used in-class assessment and numerous practice problems right after going over the procedure and concepts to fortify the students’ understanding of the material. As an experienced teacher,

she was affirmative about the collaborative work between teachers to improve the quality of teaching.

Case 4: Ms. Liu

Ms. Liu graduated from Hainan University in Hainan, the very southern part of China, in 2010. With the bachelor's degree of pure mathematics, she came to Qingdao, Shandong after graduation, obtained her teaching certificate, and had been teaching mathematics in School C since then. At the time of this study, she had taught mathematics for nearly six years. She didn't plan to become a teacher at first, but it was rather an expectation from her parents. After a few months, she started to enjoy the surroundings and working with kids. She had taught 9th grade three times before the study, and it was her fourth time teaching 9th grade.

Lesson Description

When Ms. Liu was first observed on May 16, 2016, there were only 26 days left for her 9th-grade classes to take the High School Entrance Exam. All the new material, including content for 9th grade, was taught even before the first semester started, and students had rounds of reviews to prepare for the test. In the first observation of Ms. Liu, she reviewed most problems from a practice test with the class two days before. For almost 40 minutes, she went over the following geometry problem on the exam:

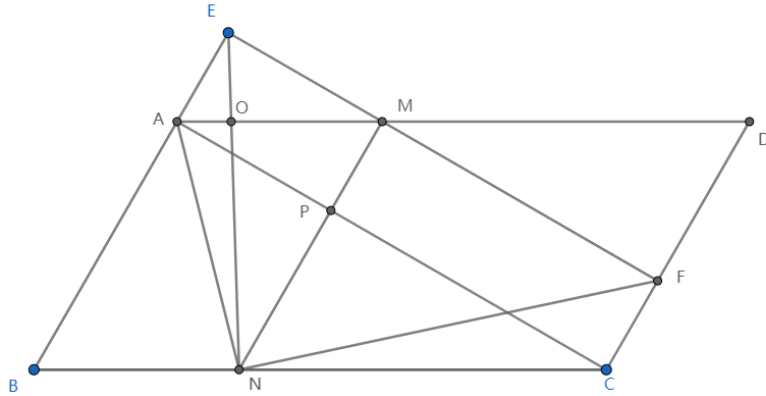


Figure 14. Practice exam problem in Ms. Liu's class

A parallelogram $ABCD$ is shown above. $AB = 4$, $BC = 8$, and $\angle B = 60^\circ$.

Point E starts from A and moves along the extension of AB with the speed of 1 unit/sec. Draw the line EF such that EF is perpendicular to CD , intersecting CD at F and intersecting AD at M . Passing point M , draw a line that is parallel to AB , intersecting BC at N .

- 1) After t seconds, write the length of line segment AM using an algebraic expression in terms of t .
- 2) Does it exist a point of time t , such that EN is perpendicular to BC ? Find the value of t if it exists.
- 3) Find the area of quadrilateral $AEFN$.

This problem is a typical problem that involves geometry, functions, and trigonometry for 9th grade in China. Though it was not the hardest problem on the practice test, later in the after-observation interview, Ms. Liu admitted that it was not a question for everyone, and only the top students are able to solve the last part of the

question: “It’s so close to the Entrance Exam, and I don’t expect everyone on getting full 12 points on that problem...I’ll be very satisfied if all of my students are able to get the first part correctly.”

The overall teaching strategy of Ms. Liu was inclined heavily to test-related strategies and learning abilities. “...Had everyone done the test again after class yesterday? At this moment, I’m not asking you just to copy what I wrote on the board—even a 3rd-grader can do that. I want you to think. I want you to understand the concept and idea of this problem. So after class, even with a scratch paper... you should do this problem again, from the beginning to the end of it. Only when you write it down and think it through will it become yours.” Ms. Liu started the lesson by asking if everyone had done the problem, emphasizing the importance of understanding.

Ms. Liu was extremely rigorous about details of the steps in a proof. “Even though it looks very obvious and it is common sense, but you have to say ‘since ABCD is a parallelogram...therefore $AB=CD=4$ ’”, she reminded the students about the first steps of the proof. “You can’t take it as granted. I don’t want to see anyone fails to go to his dream school because he lost one point from not writing the proposition in a proof step.”

However, when Ms. Liu talked about the knowledge needed to write down these steps, she did not go into details or refer to the textbook but simply used it as if she just grasped it from her web of knowledge. The following scenario took place took less than five minutes, yet Ms. Liu covered several points of knowledge in geometry and trigonometry:

Ms. Liu: ...the fourth part, it says during its movement, can't we make $\angle MNF$ 45° ? This angle here is 45° . $\angle NMF$ is a fixed right angle, yes? If we want to make this one ($\angle MFN$) 45° ...

Class: $MN=MF$.

Ms. Liu: Right...[writes the step on the board] then these two segments will be the same! This is easy, isn't it? Then if $MN = MF$ and MF is 4 [writes on the board at the same time], and if $\angle MNF$ is 45 degrees, can we use trigonometry to make things easier? Otherwise, you'll need to find another angle of 45 degrees and two angles being the same? Then in the right $\triangle NMF$, $\tan \angle MNF$ equals to?

Class: MF/MN .

Ms. Liu: [writes on the board at the same time] MF/MN , that is...what is this? You can just write MF/MN equals 1, right? Can we write it in this way? Is $\tan 45^\circ$ equal to 1? So then MF equals to MN ? ...Since MN equals 4, and where did you get it? Here, from (3) isn't it? MF equals to...we get it from step (3) as well? It equals to 4, MF equals to...here, $\sqrt{3}(8-2t) / 2$, parentheses, proven from step (3), right? Then we get MF that corresponds to $\sqrt{3}(8-2t) / 2$ which is equal to 4. Solve for t and what is that?

S: $(12 - 4\sqrt{3}) / 3$.

Ms. Liu: $(12 - 4\sqrt{3}) / 3$, or we can say $4 - (4\sqrt{3} / 3)$, can't we? Answer: when t equals to this [circles the number], it is 45 degrees.

This is one of the several scenarios where Ms. Liu provided evidence of the use of various mathematical knowledge in a relatively short period.

Two weeks before Ms. Liu was observed, the entire 9th grade in School C took the second official practice test as part of the preparation for the High School Entrance Exam. The official practice tests were taken by almost all middle schools in the city so that students can rank themselves with respect to the entire city. In our second observation of Ms. Liu on May 18, 2016, the mathematics test was just graded and returned, and the main objective of that class was to review the test.

Figure X below shows the first two pages of the practice exam.

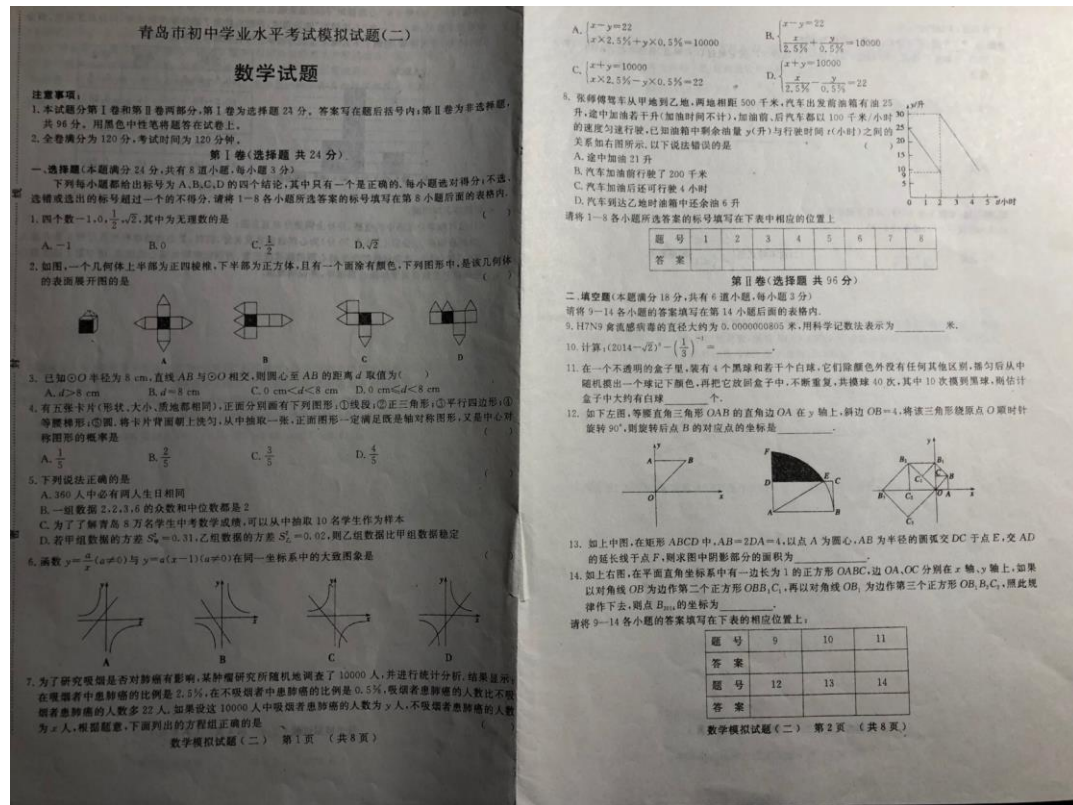


Figure 15. The first two pages of the second practice exam

Teaching 9th-graders who were going to take their first major test in their life, Ms. Liu paid extra attention to the details of problems that she went over. She constantly used reminders such as “this tiny statement is worth 2 points, don’t forget it!” “we have to be rigorous about this!” or “pay attention to the places I mentioned!” to remind students about the completeness of their steps. “It is really easy for a lot of them to lose a few points on the actual exam due to many reasons; the result—could be catastrophic,” She added later in the post-observation interview, “I’ll be thrilled if they could get a few of these points back just because I have reminded them in class.”

Ms. Liu went over most of the questions in the test rather quickly, only answering questions if most of the class did it incorrectly or if she thought a student’s mistake was “typical”; for instance, we have the following scenario:

Ms. Liu: Number 14. Did I cover a similar question in class a few days ago? What do we have to pay attention to?

Class: [varies answers]

Ms. Liu: These are all the places that you need to pay attention, what is the most important among them? The most important one!

Class: Parentheses.

Ms. Liu: Parentheses! (chuckled) I felt so bad on [Student] on such a difficult problem number 14. He did everything else correctly but missed the parentheses and got 0 on it. [to that student] did you remember this lesson?

After a quick walk-through for the first 23 problems, Ms. Liu finally reached number 24, which is a very similar problem to the one she explained in previous observation:

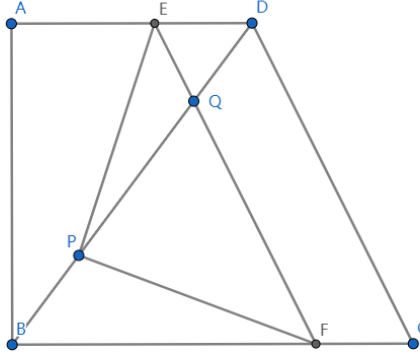


Figure 16. Last question on the second practice exam

As the figure is shown above, in right trapezoid $ABCD$, $AD \parallel BC$, $\angle A = 90^\circ$, $AB = 8$ cm, $AD = 6$ cm, $BC = 10$ cm. Point P starts from B and moves along BD , with a constant speed of 1 cm/s; meanwhile, line segment EF starts from CD and moves along DA , with a constant speed of 2 cm/s. EF and BD intersect at point Q . Connect PE and PF . When P meets Q , all the motion will stop. Let the time of motion be t (s).

- 1) What is the value of t , when $PE \parallel AB$?
- 2) Let the area of $\triangle PEQ$ be y (cm^2). Find the function relationship between y and t .
- 3) Does it exist a moment t , such that $S_{\triangle PEQ} : S_{\text{Trapezoid } ABCD} = 1 : 10$? If yes, find the value of t ; if not, please explain why not.

- 4) *Does it exist a moment t , such that PE is perpendicular to PF ? If yes, find the value of t ; if not, please explain why not.*

It is again the last problem on the test that Ms. Liu wanted to go over in detail. The last problem on the middle school mathematics exam, according to Ms. Liu, is usually “the hardest and only for those who want to go to the best high schools,” and she considered the audience to be only the top performing students in her class. “I can’t improve everyone’s grade now; I can only try my best to improve a few points for a small group (her top students),” she added in the post-observation interview.

Student: We can use $ED = FC$ as a fact...

Ms. Liu: Hold on a second, can we use $ED = FC$ directly as a fact? [looked around the class and paused for a second] We cannot. Be very careful about your work! Both you and I know it is a fact, but if you don’t write it down on the paper, the grading teacher will assume you do not know it. You’ll have to show that [figuring quadrilateral $EFCD$] is a parallelogram before saying ED and FC are of equal length. That’s what we called “inaccurate” when we grade papers. So what do we first need now?

Class: Since ED is parallel to FC , and EF is parallel to CD , therefore $EFCD$ is a parallelogram...

Ms. Liu: Hang on again...is ED parallel to FC a given?

Class: Yes...no! You have the trapezoid; that’s why (they are) parallel.

Ms. Liu: So what should the actual first step be?

Class: Since ABCD is a trapezoid, therefore AD is parallel to BC.

Ms. Liu: There you go... [wrote the step on the board and continued to teach]

In addition to the detailed explanation of problems, Ms. Liu constantly mentioned tips and strategies for the exams. When she was going over the last question which involved multiple parts, she said:

So you see, when you are in a test, if you don't know how to do the first part, then look at the second; if you can't do the second part, look at the third. There is nothing we can so when you get to the last part and still have no idea, but it is possible that there is a part that you know how to solve. Sometimes the parts at the end are simpler than the parts at the start, do whatever that you can, and don't give up easily.

Or a tip she mentioned for the compass-and-straightedge construction problem, to make the result look nicer:

...Now we have determined the center, point P, of our desired circle, and the radius of that circle is OP. Remember it will also need to pass point C as the question required. Now, if you would just use your compass and measure the distance... [demonstrate on the projector] before you draw the circle, it would look more accurate.

Lesson summary. As the only teacher who was teaching 9th grade in this study, Ms. Liu's teaching focus was different than the other three teachers. Instead of

emphasizing definition and procedures, she emphasized more on the details, completeness, and accuracy of steps. Another focal point of hers was the strategies for taking the tests, where other teachers had only mentioned very slightly in their classes. Ms. Liu didn't mention the coherence between mathematical topics in her lessons; instead, she just used the knowledge assuming that her 9th-graders have some sense of it.

Pedagogical content knowledge in teaching

Table 10.

Ms. Liu's PCK in teaching

PCK in teaching	Ms. Liu	
	Incidents	Percent
Prior Knowledge	11	17.46
Concept of definition	3	4.76
Rule and procedure	6	9.52
Draw Picture or table	3	4.76
Give example	2	3.17
Connect to concrete model	1	1.59
Students who do not understand prior knowledge	2	3.17
Provide student opportunity to think and respond	14	22.22
Manipulative activity	0	0.00
Attempts to address students' misconceptions	12	19.05
Use questions or tasks to correct misconceptions	7	11.11
Use questions or tasks to help students' progress in their ideas	2	3.17
Provide activities and examples that focus on student thinking	0	0.00

Table 10. Continued.

Unintelligible response	0	0.00
Incorrect	0	0.00
Total	63	100.00

The most frequently observed instances from Ms. Liu's classes were "use questions or tasks to correct misconceptions," "provide student opportunity to think and respond," and "prior knowledge." Ms. Liu mainly used. There were no unintelligible or incorrect responses during Ms. Liu's observations. Also, because of the nature of the problems she went over, there was no manipulatives nor activities that corresponded to her instruction.

Use of prior knowledge. In Ms. Liu's two classes, the use of prior knowledge was mentioned 11 times in total. In almost all the instances, she used prior knowledge as necessary steps of solving the problems. Since the solutions of those two main problems involved mathematical topics in functions, geometry, and trigonometry, it required students to have a coherent understanding between topics. Ms. Liu emphasized the use of prior knowledge both in classroom observation and in the interview, pointing out that it is critically important for middle school students, especially for 9th-graders, to establish a "web of knowledge":

It is so essential to weave your mathematical knowledge into a web [crossing fingers together to show a "web"]. We must realize, both as a student and as a teacher, that mathematics is a continuous process and all these topics we learn in middle school are somehow connected—but it's not seen in the textbook. So we teachers need to come up with it and emphasize

it to students in the classroom. We need to let them form a web of knowledge. For my 9th graders, it is almost impossible to get a high score without a well-established knowledge structure.

She then mentioned the first classroom observation of her: “Do you remember the geometry problem we did in the first class? Out of 12 total points, students will get only the first part (2 points) if they don’t have relevant knowledge in functions, similar triangles, and trigonometry.”

Use of questions. Ms. Liu mainly led the class during my observations. Though she posed questions frequently and provided students opportunities to think, most of her questions were answered by the class together, and she mainly used the student’s responses to move her instruction forward. The questions Ms. Liu posed focused on the steps of the proofs and connections with prior knowledge. With the guidance from Ms. Liu, the students stated the steps of the solution aloud, and Ms. Liu wrote them on the board. According to Ms. Liu later in the post-observation interview, doing this is helpful for the less-performing students as they could “catch up by the unspoken help from others.” “Those students are not capable of doing this challenging problem, but at least they can listen to others and hopefully get some sense from it. I’ll be grateful if they could get points from a part or two,” said Ms. Liu. In the interview, she provided a point of view regarding the different levels of questions:

We must admit that students’ capabilities differ, especially for math classes, that you cannot take care of everybody’s need. However, it also depends on what topics you are teaching. It is possible that most of the class do not understand an extremely difficult concept for the first time. Then, as a

teacher, you need to pose questions strategically. If I had that situation, what I usually do for regular teaching is to pose the “ultimate” question first for the best students. Then I explain the ideas in layman’s language and ask simple—maybe connected to prior knowledge—questions for less-performing students. They need to participate in class. Finally, the main body of the lecture focuses on most of the class. Hopefully, everybody learns something at the end.

Ms. Liu’s questions, however, were mostly pro forma. The questions she posed mainly focused on prior knowledge. Though these questions did provide students opportunities to think, it was expected that the questions were to be answered correctly by performing students. There were some questions directly related calculations, and such questions were expected to be answered correctly as well.

Pedagogical content knowledge in curriculum

Knowledge of curriculum in general. As a 9th-grade teacher, Ms. Liu did show a profound knowledge about the mathematical curriculum during her teaching. Because the last problems on the practice tests usually assessed students’ understanding in multiple major topics in middle school mathematics (functions, geometry, and trigonometry), teachers need to quickly retrieve the corresponding knowledge while guiding the class to solve the problems. In the interview, Ms. Liu was asked to talk about the various teaching methods she would apply for a quadratic equation, $2x^2 - 5x = 3$. Ms. Liu was not only able to explain the regular methods, such as completing squares and quadratic formula, she also mentioned a method that is not adopted in the textbook. Named as “cross multiplication”, it works almost the same as the ac-method in the U.S. curriculum.

Though for some equations, it made the problem solving much faster, it is the method that students are not allowed to use in official test paper because it is absent in the textbook. For this, Ms. Liu explained:

This method is not in our current version of the textbook. These [pointing to the quadratic formula and completing squares on the paper] are (the methods in the textbook). But cross multiplication is what teachers teach as supplementary. We use cross multiplication a lot. A lot more than the trivial one. Since it is supplementary material when students are doing tests, we don't ask them to write the steps of cross multiplication on the paper; instead, we ask them to write the factored form on it.

She further added: "As middle school math teachers, we need to be aware of the alternative or supplementary methods. Even though some methods are not considered 'orthodox,' sometimes they work extremely well on some very difficult problems. Students can use them to obtain the answer quickly, and do some reverse-engineering to come up with the steps (chuckles)."

Knowledge of curriculum. Ms. Liu was able to compare the BNUP and PEP textbooks though she has not been using the PEP textbooks for more than 15 years:

"Both textbooks have advantages and disadvantages. The BNUP version focuses more on tricks, expansions, and math stories, while PEP version has more examples that include more details, such as the standard format, where do those theorems come from. However, I don't think those are that important. Students will learn with the guidance from teachers.

“But as a teacher, I think, if I were to teach BNUP version, there is so much room for teachers. Tons of things need to be supplemented by the teachers out of the textbooks. If students didn’t learn them, they might have some difficulties doing certain problems. One can solve a quadratic equation by cross multiplication in seconds, but you are still struggling using the quadratic formula. If a teacher knows how to utilize (this textbook), she will have a lot of free space for her teaching.

“PEP textbooks are quite rigorous. They have all the examples that they should have, as well as the derivations of all those formulas, especially on teachers’ books. Teachers’ books are very thick and very rigorous. BNUP textbooks give teachers some hard time, but leave a lot of space for teachers.

“They now made another slight change. When I first started teaching in Qingdao, we were using smaller books; now they use larger ones. The smaller-size version was a little bit *too* close to real life. Like the one we said, the volume of a cylinder. A cylinder of this thickness and height, if you melt it and make a thinner but higher cylinder, they will have the same volume. The smaller book would probably ask you ‘what is the height now,’ with the base radius given. For this section, the earlier BNUP book named the section *I Got Bigger*. I wonder how I should introduce this topic to students. If the topic was called *Volume of Cylinders*, then I know how to introduce it, but how am I supposed to introduce *I Got Bigger*? If you don’t use the name, what should the objective of this section be? It doesn’t look like a topic in math class. So, it’s a little bit over on getting close to real life, a little bit exaggerating.”

Cultural beliefs about teaching

Belief in examinations. As a teacher who was teaching 9th grade in the study, Ms. Liu expressed her opinion resignedly, when she was asked the question “how do you think of the effect of exams that students have today, and how do exams affect your teaching?” She believed that those endless tests are helpful for students to better prepare for the Entrance Exam, but the tests burdened students at the same time:

Honestly, the students are overwhelmed by tests today, but what teachers can control is very limited. They (9th graders) have been taking a formal mock exam every month for the past three months, a midterm test, and there are more informal tests almost twice a week...and that’s just for my math class. They have six subjects in total to prepare for the Entrance Exam. Since we finished all the new material last October, they have been taking endless tests, just to get them more used to that final one. There is not much we teachers can do, but to go over these tests again and again. The bad news is that there are too many tests for students and the pressure is tremendous. However, what’s beneficial for students by having this number of tests is that it is the most direct way to see where you need improvement; also, you can see your current condition by checking the rank in your class or even the whole grade. Some may argue that showing students the ranks is cruel, but only doing so can students see precisely where they are, and it stimulates themselves to study harder.

During the interview, Ms. Liu addressed the pressure of the High School Entrance Exam very frequently, and she admitted that this final exam affected her teaching

strategies: “If you teach a 9th-grade class, every problem you go over has to serve for the Entrance Exam. Before you come to class each day, you need to gather the information about the results from the previous practice exam and plan accordingly.”

Belief in practices. Ms. Liu was affirmative about having practice problems, but she also mentioned that all those practice problems are serving for the very last exam: “Definitely. By this point in time, my 9th-graders should all have practiced thousands of problems already. It is helpful to them. You know...they to be fast and flexible in the test room, and there is not enough time for them to relearn and understand. Therefore, we the teachers need to make sure that they can see as many types of problems as possible so that they can deal with all these tests.”

Ms. Liu also believed that the purpose of doing practice problems was not only to review and practice but also to reinforce the understanding of knowledge. Nevertheless, Ms. Liu did not believe that students’ logical thinking could be developed via practice problems: “The logic we are talking about does not exactly follow the fixed steps. We always seek new and better methods for math problems. I think it doesn’t and should not follow the exact steps. You have your logic, and I have mine. There are no fixed steps. No matter how you go will you solve this problem, just follow your own way. I don’t think it’s a very good idea to say that it follows certain steps... Logic is one of the most significant characteristics of mathematics, but most practice problems do not promote students’ logical thinking. They learn the steps or routine from the problems, but we always say that you can’t be too constrained; otherwise, you will be constrained on all science subjects...I don’t feel very comfortable when I see ‘follow the certain steps’”.

Belief in collaborative work. In her interview, Ms. Liu described the teaching research that served as the daily routine in School C: “The teaching-research is a seminar inside or outside the school. There are major seminars which are district-wise or city-wise, and all the math teachers will gather then. More often in the school, the math teachers in each grade gathered and discussed a particular class, the current pace, or the important content. The regular teaching research in our school does not occur only at the beginning of the semester but each month. Sometimes it happens every week or even every day within a smaller group. We usually discuss what should be done tomorrow. Also, what I taught today, and what happened during the classes, and what students discussed. Besides, whether we finished the tasks and what’s next tomorrow, we will all talk about it. What Mr. Qu (head teacher of mathematics) assigns is the formal written plan. He wants us to talk about the plan and how you want the school to assist you. For instance, if you need back-to-back classes, the arrangement of test times, evening self-study sessions, and so on. When you request those, he will contact the school and coordinate it.”

In the interview, Ms. Liu appreciated the collaborative work between teachers: “it’s helpful for new teachers like me to get the ideas of how to improve teaching from those experienced teachers. In the mathematics group of School C, all but two teachers are not very experienced—for me, those who had taught for more than 20 years is considered as ‘experienced.’ Those two teachers are extremely knowledgeable about mathematical content as well as teaching in general, so we need to learn from them. When we discuss the lessons casually in the office, they often point out something you’ve

never thought about in teaching. Such ideas enlarged my vision and helped me a lot in my teaching.”

Belief in parental support. As a 9th-grade teacher, Ms. Liu stated that parental support is especially vital for 9th-graders: “At this point, everything that a family does need to better serve for the Entrance Exam. Parents need to know how their kids perform and do certain things to improve their grades; for instance, sign their kids up for after-school tutoring. It is true that it burdens the kids—but it is worth it if you want to have a brighter future.”

Summary

As a 9th-grade mathematics teacher who was facing the very same pressure with students, the teaching of Ms. Liu mainly served for the Entrance Exam. During the observations, her instruction focused on the very details of practice problems. She emphasized the comprehensive understanding of mathematical topics and the tricks to deal with the tests. She posed many questions to keep the student’s concentration and thinking, but most of the questions were pro forma. The pressure from the Entrance Exam also drove her cultural beliefs about teaching. Even though she was a bit reluctant in giving students an overwhelming amount of practice problems and exams, she was affirmative about the efforts of such practices.

V. DISCUSSION

This study was a qualitative case study concerning middle school mathematics teachers' pedagogical content knowledge and their cultural beliefs towards teaching in Shandong, China. Four middle school mathematics teachers in Qingdao, Shandong, China participated in this study. Three research questions guided this research study:

1. What is the pedagogical content knowledge of middle school mathematics teachers from Shandong, China? In particular,
 - a) What is their knowledge of teaching mathematics,
 - b) What is their knowledge of mathematical curriculum?
2. What are selected middle school mathematics teachers' cultural beliefs towards teaching?
3. How are selected teachers' cultural beliefs towards mathematics teaching related to their pedagogical content knowledge?

To answer these research questions, I used data collected from during summer 2016, mainly ten observations of mathematics classes, at least two per teacher. Each teacher was interviewed in person during which teachers further explained their knowledge in teaching, knowledge in curriculum, and their cultural beliefs towards teaching. The findings from these data sources were presented in Chapter IV. In this chapter, I present a discussion of these findings and recommendations for future research.

Discussion of Findings

Pedagogical content knowledge in teaching

Use of prior knowledge and the connections between mathematical topics. In the introduction of new concepts, using prior knowledge not only helps students to review and reinforce the knowledge taught but also helps them to picture mathematics as an integrated whole rather than as separate knowledge (An et al., 2004). In this study, all three teachers who introduced new concepts mentioned prior knowledge. Also, using prior knowledge develops generalizations and helps students to solidify what they have learned and allow them to transfer the knowledge to new situations (Suydam, 1984). The connections between new knowledge and prior knowledge were suggested in the standard of the U.S. as well; for instance, National Council of Teachers of Mathematics (2000) pointed out that, “Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know” (p. 18). In this research study, all four teachers showed their knowledge of using prior mathematical knowledge to promote students’ mathematical thinking. Ms. Wang and Ms. Xu kept reminding students about the properties of axial-symmetric figures and logic in writing up proofs during their classes, and Ms. Liu focused more on the comprehensive connections between mathematical topics. Ms. Zhang and Ms. Liu both mentioned the “web of knowledge” during the interview and pointed out that the mathematical topics in middle school are not linearly progressive but interwoven together.

Mathematics education in China has been valuing the idea of using prior knowledge for centuries. According to Confucius, “If a man keeps cherishing his old knowledge, so as continually to be acquiring new, he may be a teacher of others.” All

four teachers valued the use of prior knowledge as evidenced in the observations as well as their interviews. In their teaching, they frequently used questions to recall prior knowledge and keep students focused. They repeatedly emphasized the importance of mathematical connections in class and the rigorous practice problems themselves reflected such connections. Consistent with other research (e.g., Cai et al., 2014) in the field, all four teachers emphasized the inner connection between the nature of mathematical knowledge beyond their teaching routines.

Dealing with students' misconceptions. In the observations, four teachers used various ways to deal with students' misconceptions. Ms. Xu demonstrated expertise (An et al., 2006) at posing a different level of questions depending on student's levels, while Ms. Zhang would utilize student presentations to catch and clarify students' misconceptions. These two approaches require teachers to have a clear idea about individual students' levels, and it matches the result in (An et al., 2006). The approach that Ms. Wang used was giving students different levels of practice problems and explaining when the misconception surfaced. For novice teachers like Ms. Wang, it seems to be hard for her to manage students with different level because she had to deal with 45 students in the class. The practice problems she picked, however, consisted of different levels and covered student's needs. On the other hand, based on the general interpretation of the observations and interview, Ms. Liu was too busy going over the practice test problems because of the imminent Entrance Exam; she had no choice but only to take care of those who could digest what she explained.

Use of questions. All four teachers in this research study used questions and responses from students to some degree to develop mathematical ideas and move

instruction forward. Ms. Wang mainly used questions to keep her 7th graders focused, as for most of the questions she posed, very little wait time was provided between the questions and the answers. While Ms. Liu's questions mainly played as reminders of prior mathematical knowledge, most of her questions are considered as pro forma, for the responses of Ms. Liu's questions were calculation-related and expected to be answered by some students. The similar situation happened in Ms. Wang's case, where she asked questions mostly for checking answers in practice problems. On the other hand, Ms. Xu and Ms. Zhang, the two experienced teachers, interacted with students at a very high level. They received what students answered or presented and responded accordingly during their instructions. Such responses include commenting on student's mathematical ideas, asking other students for further ideas, and clarification and fortification of concepts or steps. The different levels of questions that Ms. Zhang and Ms. Xu posed also dealt with students' individual differences. This observed knowledge in teaching was also evident in Zhou et al. (2006).

Various teaching methods. The factor that I didn't take into account in the original research design was the teaching methods themselves, as I thought that middle school mathematics teachers in China, whether experienced or inexperienced, would, in general, follow the traditional "instruction-practice-assessment" routine of teaching. In China, teacher-centered instruction is considered to be heuristic and often used by middle school mathematics teachers (Zheng, 2006). Because of the large number of students in a typical class and the societal influence in teaching, traditional teacher-centered instruction is more popular (Wang & Cai, 2007). However, during the observations, Ms. Xu and Ms. Zhang taught innovatively with their unique class settings. Ms. Xu's four-student setting,

according to both her description and how she executed such setting in the classroom, fits with the research in complex instruction, in which teachers encourage students to help and be responsible for each other and value the perspectives of different students (Boaler, 2006; Cohen, Lotan, Scarloss, & Arellano, 1999). She partly relies on the assistance from group leaders to check completion and correctness of daily practice, which allows her to spend more time in lectures. Besides the privileges, the group leaders were also responsible for leading the communication and explaining some problems to their group members. Such move facilitates discussions, and her increased reliance on students connecting each other's idea keeps most students engaged.

Ms. Zhang's student presentation teaching method, on the other hand, could be theoretically traced back to the 1970s with the learning theory of student-centered constructivism (Barrett & Long, 2012). Through her teaching, Ms. Zhang appreciated how knowledge is constructed, and her instructional approach acknowledged the students' roles as active participants in the classroom. Though there was only one student presenter at a time, it was the whole class's job to keep track of the presenter's work and provide comments.

Ms. Xu and Ms. Zhang's innovative teaching methods reflect, from another perspective, that they have profound knowledge in teaching; though it seemed that the teachers did less work, both teachers admitted that it requires richer teaching experience, more knowledge in teaching and student psychology, and well-developed classroom management to handle such teaching methods. What's worth noting is that both Ms. Zhang and Ms. Xu pointed out that it took them at least a year to get students used to their setups. Although these two teaching methods could be very effective ways of

teaching, it seems that the experienced teachers in this study are more capable of handling such approaches.

Pedagogical content knowledge about the mathematical curriculum. All four teachers in this study were able to compare the BNUP textbooks with older PEP textbooks, even though the PEP textbooks had not been used in Qingdao for more than a decade. Not only were they able to point out the advantages and disadvantages of each version, Ms. Xu and Ms. Zhang even suggested a possible improvement. The deep and thorough understanding of the textbooks greatly helped them to teach more effectively, and all four teachers in this study believe that it is very important for a teacher to have an in-depth understanding in both mathematics and mathematics teaching. For them, having an in-depth understanding of textbooks is one of the keystones to teach effectively. With such an understanding of topics in the curriculum, they would have the ability to link the topics together and teach more systematically.

Ms. Wang, Ms. Xu, and Ms. Zhang used the curriculum-related material effectively in this study, mainly the practice book and problems from the textbooks, while Ms. Liu mainly explained questions on the practice exams. The choices of those problems, as I found in the interviews, were not made by a single teacher; instead, it was the collective wisdom. All four teachers emphasized the pre-design of teaching sequence and questioning based on the collaborative study of textbooks and perceptions about individual students beforehand. Moreover, they emphasized addressing student thinking and misconceptions and dealing with emerging events to achieve coherence. The coherence of instructions is almost natural when the teachers have a profound knowledge

of the curriculum itself and perception of knowledge coherence, and this is supported by X. Chen and Li (2010).

Cultural Beliefs towards teaching

All four teachers in this research study believed that the collaborative works of lesson preparations were very helpful in gaining experiences and improving teaching quality. One important thing I found in this study, is that all four teachers valued the collaborative work done by the teachers. They all have similar responses to the interview question “How would our teachers prepare for the lessons of a semester?” and appreciated the collective lesson preparations. During the collaborative lesson preparations, each class was prepared carefully with a thorough search of activities and practice problems from textbooks and other resources. The problems and questions were designed in order to enhance students’ mathematical thinking. Having the lesson plans at hand, the two experienced teachers, Ms. Xu, and Ms. Zhang, didn’t necessarily follow the exact steps in their actual teaching; instead, they used lessons plans as a general framework. For two novice teachers, Ms. Wang and Ms. Liu, the lesson plans were very detailed through the collaborative works, and they would follow the lesson plans as precisely as possible. The four teachers indicated that they improved their pedagogical knowledge by communicating with colleagues and observing others’ classes. Interestingly, this is contrary to the result in An et al. (2006), in which Chinese teachers would mainly plan instructions according to textbooks and students’ needs, and only 3% of teachers plan their lesson in a team.

Moreover, Ms. Zhang pointed out the collaborative work in another perspective: the observations. As an experienced teacher, she played the roles of both observer and observed teacher, providing models and suggestions in teaching for novice teachers. She mentioned that it was very helpful to reflect and enhance her teaching, especially at the early stage.

Despite the huge pressure from the Entrance Exam, Ms. Liu pointed out that she believed the main purpose of the exams is not only to rank students but to let students reflect themselves and find their “blind spots.” This point of Ms. Liu was put forward by the other three teachers as well. They all agreed that having exams would benefit the students in reflecting on their performance straightforwardly. In addition, Ms. Xu stated that she would analyze the results of tests to reflect students’ performance and reflect on her own teaching.

As the only 9th-grade teacher in this study, Ms. Liu tried her best in class to ensure the preciseness and accuracy of each tiny step for the upcoming Entrance Exam. “I am not 100% sure about if we can directly use the result of part a) for the rest of the problem—I will go check it for you when I go to teaching seminar tomorrow. We need to be rigorous.” Such pursuit of perfection was not observed in the other three teachers’ cases, and it indirectly showed the pressure that the Entrance Exam had brought to both students and teachers.

All four teachers in this study emphasized the belief of developing logical and mathematical thinking in middle school. Ms. Zhang and Ms. Xu considered having mathematical thinking as one of the key abilities for students to succeed in mathematics classes. As 7th-grade teachers, Ms. Wang and Ms. Xu mentioned the students’ transition

in their learning from elementary to middle school. They both believed, in the interviews, that students need to develop mathematical thinking to be challenged with more difficult concepts in the next two years or even high school. National Council of Teachers of Mathematics (2000) supports this view that “middle school students must progress through using reasoning to making conjectures and they must apply both inductive and deductive reasoning.”

Relationships between teachers' PCK and cultural beliefs

The result of this study showed that the four middle school mathematics teachers have their own beliefs about teaching under the influence of culture, which they translated into their unique teaching styles. Though the relatively non-traditional teaching approaches were observed, all four teachers valued, during the teaching and interviews, procedural development and a large number of practices in order for students to gain knowledge.

Under the nationalistic education, the goal of education in China is to help students fully develop in ideology, morality schooling, and discipline, and the purpose of learning is to become a useful person and to be able to contribute to the country (An et al., 2006). The rigorous examination system is consequently a key feature of this centralized education system. Because of the importance of the High School Entrance Exam, teaching in middle schools focuses much more heavily on test performance as students move to the next grade level, and it became almost the only force that guides the content and pace of teaching. Ms. Liu, who had taught 9th grade for years, admitted and appreciated the benefits that students got from huge amount of practice exams and problems, while Ms. Wang, who had only taught 7th grade, was reluctant about burdening

her students with overwhelming practices. Because of the pressure from the Entrance Exam, Ms. Liu's teaching strategies focused heavily on the rigor, accuracy, and comprehensive understanding of problems with tips and tricks to deal with the tests, while the other three teachers tend to emphasize the in-depth understanding of the topics being taught. Compared to Ms. Liu, Ms. Wang, Ms. Zhang, and Ms. Xu emphasized the tests relatively lightly. In their classes, they tended to value more on daily practices from textbooks and practice books, or other formative assessments. Furthermore, the pressure from the Entrance Exam also makes the teachers believe that education is not only to teach knowledge but also to help students to be successful in society in the future. Unfortunately, "being successful" is partly measured by if one can enter a prestigious college or high school (Zhu & Han, 2006). As a result, this belief directs four teachers' pedagogical content knowledge that focuses more on the proficiency and fluency in mathematical skills. This can be explained partly by the observation that teaching is a cultural practice (Gallimore, 1996).

One thing that both teachers and educators should realize is that being mathematically proficient is the premise of effective teaching (Hiebert et al., 2002). It is not realistic for teachers to teach effectively without being mathematically competent themselves. In this research study, all four teachers show that they have sufficient knowledge in mathematical content to teach middle school, as they all graduated with a degree in mathematics, identified themselves as ones who are strong in mathematics, and made no mistakes during observations. This matches with one of the goals that Hiebert, Morris, and Glass (2003) suggested in their model of the teacher preparation program.

It is important to point out that the findings of this study should not be simply applied to other education systems. Pedagogical content knowledge could emerge from different perspectives in different cultures. The collaborative work between groups of teachers and educators in lesson preparation and improvement of teaching quality should never be omitted in any culture or education system. Teachers should also realize that their own knowledge of the subject and understanding of the overall curriculum is the foundation of quality teaching. The teaching practices that Chinese teachers used can be partially adapted by other education systems to improve the rigor and perhaps the overall test scores; however, without the consideration of the cultural context, the direct graft of the Chinese education system to other cultures is not likely applicable.

Summary

This research project demonstrates how middle school mathematics teachers in Qingdao, Shandong, China exhibit their pedagogical content knowledge with their beliefs towards teaching to establish their effective teaching. Under the rather centralized education system, the classrooms seem to be homogeneous—same city, same textbooks, same language, even the same Han ethnic group. The classroom experiences, however, are intriguing from the ways that the teachers exhibit their pedagogical content knowledge in various ways. From direct lecture to a test-centered exercise class, from the consistent use of student presentation to the setting of the four-student group, the four teachers applied vastly different teaching strategies to pass on knowledge effectively. I think this study shows, as seen through the four cases, that even under a centralized education system where almost all the teaching serves for the High School Entrance

Exam, teachers' pedagogical content knowledge can be modeled differently to fit different classroom designs and needs accordingly.

Limitations

This research study has several limitations. First, it did not show teachers' quality of teaching quantitatively. It would be more visual if the quality of teaching were quantified so that it can be compared within the group and even compare with teachers in other regions or countries.

Second, even though the education system in China is relatively centralized, the four teachers in this study cannot represent the overall pedagogical content knowledge and beliefs for the entire group of teachers in China. Because Qingdao is considered as a more developed city economically, and with a fact that Shandong is known, almost notoriously, by its competitiveness in education, it is very likely to see different pedagogical content knowledge and beliefs towards teaching in other provinces. In addition, School Y and School N are considered, by local people, as in the first tier among middle schools in Qingdao in terms of teaching quality and high school enrollment rate. The descriptions of teachers' pedagogical content knowledge would be more representative if more middle schools at different levels were involved in this study. As with many other studies, this study is limited in analyzing only four Chinese teachers' pedagogical content knowledge and their cultural beliefs. Although Chinese teachers may share many similarities in their classroom instructions (Li & Li, 2009), it remains unclear about how teachers in other regions of China exhibit their pedagogical content knowledge.

Lastly, due to limited time and resources available, this study was conducted in a few days of May, where the spring semester was almost over at that time. I was only able to observe and study their teaching for those particular times and topics. The topics that teachers instructed on were based on that schedule and, because they taught all three grade levels, the topics were all different. In particular, the 9th-grade class was focusing on the comprehensive review, and no new material was taught during this study. Ms. Liu's teaching approach could be vastly different if she were teaching new material.

Suggestions for Future Research

The term pedagogical content knowledge in this study consists of two elements: knowledge in teaching and knowledge in the mathematical curriculum. Because of the degree that teachers received, the assumption that the teachers have sufficient knowledge in mathematical content was made, and it was indirectly verified by the deep understanding of the textbook and self-reported background in mathematics. However, as the gap in pedagogical content knowledge was found between novice and experienced teachers, it could suggest that their levels of the content knowledge are also different. Future research could contain some forms of assessment to evaluate teachers' content knowledge and discuss with the other two aspects of their pedagogical content knowledge. Moreover, though the classroom observations and interviews provided in-depth evidence of how teachers exhibit their pedagogical content knowledge, all responses are qualitative. When further large-scale research is conducted, would similar responses appear again? Because of the rather centralized education system, would the teachers in different regions in China have similar pedagogical content knowledge? The scenarios could appear similar, but due to the local culture in regards to education and

overall economic status of the region, the classroom and the teaching may look drastically different in different regions. It begs the future research to compare and contrast these teachers' PCK and cultural beliefs towards teaching.

Furthermore, to improve the depth of the study, a mixed-method approach can be utilized in which both quantitative and qualitative data are analyzed. Like most of the other dissertations, this dissertation was conducted within the various limitation of time, money, and resources. I hope that future studies will recruit more teachers for a longer period of time with the same topics for them to teach in order to have a more thorough understanding of their pedagogical content knowledge.

APPENDIX SECTION

Appendix A. Consent Form to Participate in Research

Title of Project: CHINESE MIDDLE SCHOOL MATHEMATICS
TEACHERS' PEDAGOGICAL CONTENT
KNOWLEDGE AND CULTURAL BELIEFS TOWARDS
TEACHING

Principal Investigator: Zhaochen Song, M.S.
Graduate Teaching Assistant
Texas State University - San Marcos
Department of Mathematics
601 University Drive, Derrick 207
San Marcos, TX 78666
zs1044@txstate.edu
Office phone: (512) 245-4749
Cellular phone: (512) 216-9381

PURPOSE: You are being asked to participate in a research project that seeks to investigate the difference between the U.S. and Chinese middle school mathematics teachers about their pedagogical content knowledge. You are specifically being asked to participate in this study after another participant, a colleague of the investigator or the investigator himself identified you as a potential participant who might meet the criteria of the study. The intent of this research is to understand your views and experiences as a math teacher who is currently teaching or have been teaching mathematics in a middle school setting. Specifically, this study will examine your cultural beliefs towards teaching, your pedagogical content knowledge, and how and in what ways you have been successful or faced challenges in this environment.

PROCEDURES:

If you volunteer to participate in this research, you will participate in classroom observation, a survey, and an interview lasting for approximately 60 minutes each. In the survey, you will be asked to answer the questions evaluating your pedagogical content knowledge. In the interview, you will be asked to discuss your beliefs about teaching in mathematics classrooms. The classroom observation is mainly to examine your Mathematical Quality of Instruction to find a corresponding U.S. teacher to compare. The interview will be audio-recorded with your permission. Your participation is voluntary, and as such, you may withdraw from the study at any time without prejudice or jeopardy to your standing with Texas State University, San Marcos.

RISKS: In reflecting and talking about your beliefs as a middle school math teacher, you may become uncomfortable with unhappy experiences or memories recalled. However, you may elect not to answer any of the questions with which you feel uneasy and still, remain a participant in the research. There are no known psychological or physiological risks associated with participating in this research. However, some of the questions may

be considered sensitive. Participants are not required to respond to any question that they do not feel comfortable answering. All answers will remain confidential.

BENEFITS: You may not benefit directly from your participation in this research; however, this research may be beneficial to other teachers in understanding such difference and learning from it. In addition, the research may provide further insight into understanding the types of programs and policies.

COMPENSATION: You will receive a gift that is worth around 50 US dollars at the end of your participation if you finish all the items requested by the investigator.

CONFIDENTIALITY: Your name will never appear on any survey or research instruments. No Identity will be made in the data analysis. All written materials and consent forms will be stored in a locked file in the investigator's office and the principal investigator, Mr. Zhaochen Song, will have sole access. Your response(s) will appear only in statistical data summaries when the data are presented in written or oral form at scientific meetings. Your name will never appear in any publication of these data. All materials will be kept for three years.

RIGHT TO WITHDRAW: You are under no obligation to participate in this study. You are free to withdraw your consent to participate at any time without penalty. Your withdrawal will not influence any other services to which you may be otherwise entitled.

SUMMARY OF RESULTS: A summary of the results of this research will be supplied to you, at no cost, upon request.

QUESTIONS:

I understand that should I have any concerns about my participation in this study, I may call the investigator who is asking me to participate, Zhaochen Song, at (512) 245-6925.

The project 2016K3195 was approved by the Texas State IRB. Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to the IRB chair, Dr. Jon Lasser (512-245-3413 lasser@txstate.edu) or Monica Gonzales, IRB Regulatory Manager, ([512-245-2334](tel:512-245-2334) meg201@txstate.edu).

DOCUMENTATION OF CONSENT: I have read the above statements and understand what is being asked of me. I also understand that my participation is voluntary and that I am free to withdraw my consent at any time, for any reason, without penalty. On these terms, I certify that I am willing to participate in this research project.

Participant's Signature

Date

Investigator's Signature

Date

Appendix B. Interview Protocol

Part A. (This part is adopted from Bower (2016), Appendix A.)

Thank you very much for being interviewed. Before we get started, could you introduce yourself, such as your name, school that you are teaching, and the number of years that you have been teaching?

How many years have you been teaching mathematics? Teaching middle school mathematics?

Tell me about your journey towards becoming a middle school math teacher. When did you first start thinking that you might be a teacher? Why are you interested in teaching?

What event or people, if any, from elementary/middle/high school stand out to you in terms of your mathematics education and teacher? Please discuss what you consider to be high points, low points, and turning points of this time. Are there any challenges that you faced during these periods?

(Probe for specificity: What do you mean? Can you give me an example of that? Is there anything else you remember? You haven't mention____. Do you remember anything particular about that?)

How do you feel your own experience as a student have translated into what you do in your classroom? What things from your childhood do you still embrace? What have you cast aside?

Think of a person you know who is good at math. Who is he/she and why do you think that person is good at math? Then think of a person whom you know is not good at math and explain.

Part B. (This part is mainly adopted from Kennedy, Ball, & McDiarmid (1993))

For this part, I would like you to pick a grade you can imagine teaching...What grade is that? Now imagine that it is the beginning of the academic year, and the Math Group Leader meets with teachers to discuss the teachers' goals for their students.

When you meet with the math group leader, what would you say in describing the most important things you'd be trying to accomplish across the year with your pupil?

What would you say about things you'd be trying to accomplish in math with your pupils?

Given a particular topic (say, slope) (p. 41-42):

When is this topic taught? (Curriculum)

Ideas about how students learn this topic best.

Ideas about how to help students who have not learned this topic well.

Disposition toward pointing out errors to students.

Context: orientation toward the classroom

(p. 53) Suppose you have a pupil who asks you what 7 divided by 0 is. How would you respond? Why would you do that?

Probe to the following: If the teachers say it's undefined, ask, "What do you mean by undefined?"

If the teacher says, "You can't divide by zero," ask, "Why can't you divide by zero?"

What if this didn't seem to make sense to students? Is there something else that you would try? How would that help?

What if the student said, "It seems that if you divide by nothing, you don't divide and so you would still have 7". How would you respond to that, and why?

(p. 61) Helping students' concept and procedures: Solving Equations

Suppose you are teaching algebra. How would you help your students learn to solve equations like this:

$$2x^2 - 5x = 3$$

Why is that what you would do?

Many students find this hard. In your view, what makes this especially difficult?

If a student didn't get it, is there something you could do or show that would help the student make sense of it?

How would that help, and where did you get the idea?

(p. 65) Here is a textbook published by People's Education Press. This was the textbook we had been using in Qingdao until 2001. I've included the pages from the book as well as the teachers' guide notes. I'd like to use this as the basis for this part of the interview. Please take a few minutes to look it over, and then we will talk.

what are your initial reactions to this textbook section?

Are there things you think are quite good in here?

Some things you think are weaknesses or flaws? Why?

Now here's a section from another text series. Can you compare the two?

Does anything seem different?

Which do you prefer? And why?

Part C.

Today the one and foremost important goal for most middle school students in China seems to be “having a good grade in the High School Entrance Exam.” How do you think of that? Before taking the final Entrance Exam, students will take countless practice tests. How do you think of that phenomenon? Can you elaborate?

I noticed that you used (the number of practice problems) in your classes. Can you talk about your view about having practice problems? Do you think that students should take more/fewer exercise problems?

How do the teachers in your school prepare for the lessons? As an experienced/novice teacher, how do you think of the method of lesson preparation?

Besides lesson preparation, what other communications do our teachers have in the school? How do such communications help to improve your teaching?

Parents play the most important role in their children’s education out of class. Do you agree with such a statement? Why or why not? How do you think of parental support affect your teaching?

Any other comment that you would like to express about the education system in China in general?

Appendix C. Pre-Observation Interview Protocol (Kennedy et al., 1993, p. 101)

1. Could you tell me a little about what you plan to teach today?
2. Could you tell me about the activities that students will do?
3. How does the content of today's class connect to the other chapters/sections/curriculum in general?
4. Are you expecting to see anything that happens? Why?
5. Will the content today be difficult for your students? Why?
6. Is there anything I should especially pay attention to while I am observing?

Pre-Observation Interview Protocol (in Chinese)

课前采访稿

- 1、 您能简单描述下您今天计划的课程内容吗？
- 2、 您能简单描述下今天学生要做的活动吗？
- 3、 您今天的课程与整个教学大纲的其他部分有何练习？
- 4、 您对今天的课有何预期？ 为什么？
- 5、 今天的课程对您的学生来说有难度吗？ 为什么？
- 6、 今天的课程有需要我特别关注的地方吗？

Appendix D. Post-Observation Interview Protocol (Kennedy et al., 1993)

1. How did you feel things went in class? Did anything surprise you?
2. Did anything disappoint you? Were you pleased with anything?
3. How did you decide whom to call?
4. I noticed that you said/did ____ (example, tasks, explanation, etc.).
Where did it come from?
Why did you decide to do this?
How is it beneficial to students?
(Repeat for each representation identified)
5. I noticed _____. Why is that, or why did that occur?
6. Are there any questions that I haven't asked you that you think I should have?

Post-Observation Interview Protocol (in Chinese)

课后采访稿

- 1、 您对今天的课程感觉如何？有任何超出预期的情况发生吗？
- 2、 您对今天的课程的哪些部分感到满意/不满？
- 3、 您是如何挑选学生回答问题的？
- 4、 我注意到了您说了/做了_____（例题，活动，解释等）
 您是如何决定要做这个的？
 这对学生的益处是什么？
 （对于每一个发现的亮点，重复以上问题）
- 5、 您对于今天的课程还有什么别的想说的吗？

Appendix E. Observation Guide (Kennedy et al., 1993, pp. 102-103)

1. Observing and taking notes:
 - a) Arrive about half an hour before the scheduled observation time. Find a comfortable place to sit where you can see and hear well. Sketch a map of the physical arrangement of the classroom, labeling areas and displays.
 - b) Summarize the information of the class at the beginning of the write-up of the field notes.
 - c) Video- and audio- record the classroom sessions.
 - d) When taking notes, try to get as many direct quotes as possible, especially when the teacher talks about the subject matter (e.g., when s/he gives an explanation, answers, or asks questions, or give directions.
 - e) Refer to the teacher as T, and students as G1, G2, B1, etc. Assign numbers to as they are called on or speak in class.
 - f) Write up the notes as soon as possible after the observation while the memories are still fresh. Use the video to supplement handwritten notes.
2. Writing up field notes:
 - a) Write the teacher's name and the date of the observation at the top of the field notes.
 - b) Begin by describing the context:
 - i. A description of the classroom
 - ii. A description of the tasks in which students and teacher are engaged during the session.
3. Answer the questions of this observation guide. There are meant to integrate and be interpretive. Be sure to specify the evidence for your assertions giving the source and location of the data that support them.
 - a) (Agreement) To what extent did the observed lesson agree with what the teacher said in advance (i.e., in the pre-observation interview)?
 - b) (Classroom management) What is the teacher's approach to classroom management?
 - c) (Questions) What kind of questions did the teacher ask, and was there a particular pattern to the questions?
 - d) (Awareness of learners) How did the teacher seem to be aware of his/her students' strengths and weaknesses? How did the teacher find out what students knew?
 - e) (Student errors) What kinds of errors were made? How did the teacher respond to the errors?

- f) (Subject matter) How well did the teacher seem to know the subject matter at hand? (Draw inferences from the teacher's stated goals, the analogies, stories, and explanations used, questions asked, and responses were given.)
- g) Are there any other comments you have about this observation?

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