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OSCILLATION OF SOLUTIONS FOR SYSTEMS OF HYPERBOLIC EQUATIONS OF NEUTRAL TYPE

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ABSTRACT. In this paper, we obtain sufficient conditions for the oscillation of solutions to systems of hyperbolic differential equations of neutral type. We consider such systems subject to two kinds of boundary conditions.

1. INTRODUCTION

We consider the following system of hyperbolic differential equations of neutral type

$$\begin{aligned} &\frac{\partial^2}{\partial t^2}\Big[u_i(x,t) - \sum_{j=1}^{m} c_{ij}u_j(x,t-\tau)\Big] \\ &= a_i(t)\Delta u_i(x,t) + b_i(t)\Delta u_i(x,t-\rho) - p_i(x,t)u_i(x,t) \\ &\quad - \sum_{k=1}^{m}\int_a^b q_{ik}(x,t,\xi)u_k[x,g(t,\xi)]d\mu(\xi), (x,t) \in \Omega\times\mathbb{R}_+ \equiv G, \end{aligned} \tag{1.1}$$

subject to either of the following boundary conditions

$$\frac{\partial u_i}{\partial n} + \nu_i(x,t)u_i = 0, \quad (x,t)\in\partial\Omega\times\mathbb{R}_+ \tag{1.2}$$

$$u_i(x,t) = 0, \quad (x,t)\in\partial\Omega\times\mathbb{R}_+, \tag{1.3}$$

where Ω is a bounded domain in R^n with a piecewise smooth boundary $\partial\Omega$, $R_+ = [0,\infty)$, Δ is the Laplacian operator in $\mathbb{R}^n$, c_{ij}, $\tau > 0$ and $\rho > 0$ are constants, n is the unit outward normal vector of $\partial\Omega$.

There has been considerable interest in obtaining sufficient conditions for oscillatory solutions of partial functional differential equations, as this type of equations arise frequently in many application fields (see for example the monograph [7]). Recently, several papers concerning systems of hyperbolic functional differential equations have appeared in literatures [1, 2, 3]. It is noted that previous work focused only on the cases where the neutral coefficient number lies between -1 and 0, that is $-1 \le c(t) \le 0$. To the best of our knowledge, very little work has been done for other cases.

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The aim of this paper is to study the oscillation problem for the above system of hyperbolic differential equations of neutral type, and give some oscillatory criteria for such systems. Throughout the paper, we assume that the following conditions hold:

- (H1) $a_i(t)$, $b_i(t) \in C(R_+, R_+)$;
- (H2) $p_i(x,t) \in C(G, R_+)$, $q_{ik}(x,t,\xi) \in C(G \times [a,b], R)$;
- (H3) $\nu_i(x,t) \in C(\partial\Omega \times \mathbb{R}_+, R_+)$;
- (H4) $g(t,\xi) \in C(R_+ \times [a,b], R)$, $g(t,\xi) \le t$ for $\xi \in [a,b]$, and $\liminf_{t\to\infty, \xi\in[a,b]}\{g(t,\xi)\} = \infty$;
- (H5) $\mu(\xi) \in ([a,b], R)$ is nondecreasing, and the integral in (1.1) is a Stieltjes integral.

Definition. A vector function $u(x,t) = \{u_1(x,t), \dots, u_n(x,t)\}^T$ is said to be a *solution* of (1.1) and (1.2) or (1.1) and (1.3) if it satisfies equation (1.1) in $G \equiv \Omega \times \mathbb{R}_+$ and the associated boundary condition on the boundary of G.

Definition. A vector solution $u(x,t)$ of the boundary value problem is said to be *oscillatory* in the domain G if at least one of its nontrivial components is oscillatory. Otherwise, the vector solution $u(x,t)$ is said to be *nonoscillatory*.

For the following neutral differential inequality

$$\frac{d^2}{dt^2}[y(t) - \lambda(t)y(t-\tau)] + p(t)y(t) + \int_a^b q(t,\xi)y[g(t,\xi)]d\mu(\xi) \le 0, \quad t \ge 0, \tag{1.4}$$

where $\lambda(t) \in C'(R_+, R_+)$, $p(t) \in C(R_+, R_+)$, $q(t,\xi) \in C(R_+ \times [a,b], R_+)$, we assume that the following conditions hold.

- (A1) There exists a function $h(t,\xi) \in C(R_+ \times [a,b], R_+)$, such that $h(h(t,\xi),\xi) = g(t,\xi)$, and $h(t,\xi)$ is nondecreasing with respect to t and ξ, and also $t \ge h(t,\xi) \ge g(t,\xi)$;
- (A2) $\liminf_{t\to\infty} \int_{g(t,b)}^t \int_a^b q(s,\xi)d\mu(\xi)ds > 1/e$;
- (A3) $\liminf_{t\to\infty} \int_{h(t,b)}^t \int_a^b q(s,\xi)d\mu(\xi)ds > 0$.

We remark here that the existence of the function $h(t,\xi)$ has been proved in [4]. The following two Lemmas are derived from the known literatures and are useful to the proof of the main results of this paper.

Lemma 1.1 ([4])**.** *Under assumptions (A1)–(A3), the first-order differential inequality*

$$x'(t) + \int_a^b q(t,\xi)x[g(t,\xi)]d\mu(\xi) \le 0 \tag{1.5}$$

has no eventually positive solution.

Lemma 1.2 ([6])**.** *Assume that $\lambda(t) \le 1$, and (A1) holds. If for some $0 \le \varepsilon_0 \le 1$ and $\xi_0 \in [a,b]$,*

$$\liminf_{t\to\infty} \int_{h(t,b)}^t \int_a^b \varepsilon_0 q(s,\xi)g(s,\xi)d\mu(\xi)ds > 0, \tag{1.6}$$

$$\liminf_{t\to\infty} \int_{g(t,\xi_0)}^t \int_a^b \varepsilon_0 q(s,\xi)g(s,\xi)d\mu(\xi)ds > \frac{1}{e}\exp\Big[-\liminf_{t\to\infty}\int_{g(t,\xi_0)}^t \varepsilon_0 sp(s)ds\Big], \tag{1.7}$$

then inequality (1.4) *has no eventually unbounded positive solution.*

2. Main Results

We introduce the following notation:

$$\begin{gathered} p_i(t)=\min_{x\in\overline{\Omega}}\{p_i(x,t)\},\quad P(t)=\min_{1\le i\le n}p_i(t);\\ Q_{ii}(t,\xi)=\min_{x\in\overline{\Omega}}\{q_{ii}(x,t,\xi)\},\quad Q^*_{ik}(t,\xi)=\max_{x\in\overline{\Omega}}\{q_{ik}(x,t,\xi)\}. \end{gathered}\tag{2.1}$$

Theorem 2.1. *Suppose that $0\le C\le 1$ and (A1) holds. If for some constants $0\le\varepsilon_0\le 1$ and $\xi_0\in[a,b]$,*

$$\liminf_{t\to\infty}\int_{h(t,b)}^t\int_a^b\varepsilon_0 Q(s,\xi)g(s,\xi)d\mu(\xi)ds>0,\tag{2.2}$$

$$\liminf_{t\to\infty}\int_{g(t,\xi_0)}^t\int_a^b\varepsilon_0 Q(s,\xi)g(s,\xi)d\mu(\xi)ds>\frac{1}{e}\exp\Big[-\liminf_{t\to\infty}\int_{g(t,\xi_0)}^t\varepsilon_0 sP(s)ds\Big],\tag{2.3}$$

then each unbounded solution $u(x,t)$ of (1.1)–(1.2) *is oscillatory in the domain G, where*

$$C=\max_{1\le i\le n}\Big\{c_{ii}+\sum_{j=1,j\ne i}^m|c_{ji}|\Big\},\quad Q(t,\xi)=\min_{i\le i\le n}\Big\{Q_{ii}(t,\xi)-\sum_{j=1,j\ne i}^m Q^*_{ji}(t,\xi)\Big\}.$$

Proof. Suppose to the contrary that there is a nonoscillatory solution $u(x,t)=(u_1(x,t),\dots,u_n(x,t))^T$ of the problem (1.1) and (1.2), and that $|u_i(x,t)|>0$ for $t\ge 0$, $i=1,2,\dots,n$. From (H4), there exists a $t_1\ge 0$, for $j=1,2,\dots,m$, $k=1,2,\dots,m$, such that

$$|u_j(x,t-\tau)|>0,\quad |u_i(x,t-\rho)|>0,\quad |u_k[x,g(t,\xi)]|>0,\quad t\ge t_1,\xi\in[a,b].$$

Let

$$\begin{gathered} \delta_i=\operatorname{sgn}u_i(x,t),\quad \delta_k=\operatorname{sgn}u_k[x,g(t,\xi)];\\ Y_i(x,t)=\delta_iu_i(x,t),\quad Y_k[x,g(t,\xi)]=\delta_ku_k[x,g(t,\xi)], \end{gathered}\tag{2.4}$$

then $Y_i(x,t)>0$, $Y_j(x,t-\tau)>0$, $Y_i(x,t-\rho)>0$ and $Y_k[x,g(t,\xi)]>0$ for $t\ge t_1$, $\xi\in[a,b]$, $i=1,2,\dots,n$, $j=1,2,\dots,m$ and $k=1,2,\dots,m$.

Integrating equation (1.1) with respect to x over the domain Ω, for $t\ge t_1$, we obtain

$$\begin{aligned} &\frac{d^2}{dt^2}\Big[\int_\Omega u_i(x,t)dx-\sum_{j=1}^m\int_\Omega c_{ij}u_j(x,t-\tau)dx\Big]\\ &+\int_\Omega p_i(x,t)u_i(x,t)dx+\sum_{k=1}^m\int_\Omega\int_a^b q_{ik}(x,t,\xi)u_k[x,g(t,\xi)]d\mu(\xi)dx\\ &=a_i(t)\int_\Omega\Delta u_i(x,t)dx+b_i(t)\int_\Omega\Delta u_i(x,t-\rho)dx. \end{aligned}\tag{2.5}$$

Furthermore, it follows from (2.4) that

$$\begin{aligned}&\frac{d^2}{dt^2}\Big[\int_\Omega Y_i(x,t)dx-\sum_{j=1}^m\int_\Omega c_{ij}\frac{\delta_i}{\delta_j}Y_j(x,t-\tau)dx\Big]\\&+\int_\Omega p_i(x,t)Y_i(x,t)dx+\sum_{k=1}^m\int_\Omega\frac{\delta_i}{\delta_k}\int_a^b q_{ik}(x,t,\xi)Y_k[x,g(t,\xi)]d\mu(\xi)dx\\&=a_i(t)\int_\Omega\Delta Y_i(x,t)dx+b_i(t)\int_\Omega\Delta Y_i(x,t-\rho)dx\,.\end{aligned}\tag{2.6}$$

It is clear that

$$\int_\Omega\int_a^b q_{ik}(x,t,\xi)Y_k[x,g(t,\xi)]d\mu(\xi)dx=\int_a^b\int_\Omega q_{ik}(x,t,\xi)Y_k[x,g(t,\xi)]dxd\mu(\xi).\tag{2.7}$$

From the Green's formula and the boundary condition, we have

$$\int_\Omega\Delta Y_i(x,t)dx=\int_{\partial\Omega}\frac{\partial Y_i(x,t)}{\partial n}d\omega=-\int_{\partial\Omega}\nu_i(x,t)Y_i(x,t)d\omega\le 0,\tag{2.8}$$

$$\int_\Omega\Delta Y_i(x,t-\rho)dx=-\int_{\partial\Omega}\nu_i(x,t-\rho)Y_i(x,t-\rho)d\omega\le 0,\tag{2.9}$$

where $d\omega$ is the surface integral element on $\partial\Omega$. Moreover, it follows from (2.1) that

$$\int_\Omega p_i(x,t)Y_i(x,t)dx\ge p_i(t)\int_\Omega Y_i(x,t)dx.\tag{2.10}$$

Combining (2.7)–(2.10) and noting (2.1), we obtain

$$\begin{aligned}&\frac{d^2}{dt^2}\Big[\int_\Omega Y_i(x,t)dx-\sum_{j=1}^m\int_\Omega c_{ij}\frac{\delta_i}{\delta_j}Y_j(x,t-\tau)dx\Big]\\&+p_i(t)\int_\Omega Y_i(x,t)dx+\int_a^b Q_{ii}(t,\xi)\Big[\int_\Omega Y_i[x,g(t,\xi)]dx\Big]d\mu(\xi)\\&-\sum_{k=1,k\ne i}^m\int_a^b Q^*_{ik}(t,\xi)\Big[\int_\Omega Y_k[x,g(t,\xi)]dx\Big]d\mu(\xi)\le 0.\end{aligned}\tag{2.11}$$

Let

$$V_i(t)=\int_\Omega Y_i(x,t)dx,\qquad i=1,2\dots,n,\tag{2.12}$$

then $V_i(t)>0$, and it follows from (2.11) that

$$\begin{aligned}&\frac{d^2}{dt^2}\Big[V_i(t)-\sum_{j=1}^m c_{ij}\frac{\delta_i}{\delta_j}V_j(t-\tau)\Big]+p_i(t)V_i(t)\\&+\int_a^b Q_{ii}(t,\xi)V_i[g(t,\xi)]d\mu(\xi)-\sum_{k=1,k\ne i}^m\int_a^b Q^*_{ik}(t,\xi)V_k[g(t,\xi)]d\mu(\xi)\le 0\,.\end{aligned}\tag{2.13}$$

Furthermore, let $V(t)=\sum_{i=1}^m V_i(t)$, then $V(t)>0$, and it follows from (2.1) that

$$\begin{aligned} &\frac{d^2}{dt^2}\Big\{\sum_{i=1}^m\Big[V_i(t)dx-\sum_{j=1}^m c_{ij}\frac{\delta_i}{\delta_j}V_j(t-\tau)\Big]\Big\}+P(t)V(t)\\ &+\int_a^b\sum_{i=1}^m\Big\{Q_{ii}(t,\xi)V_i[g(t,\xi)]-\sum_{k=1,k\neq i}^m Q^*_{ik}(t,\xi)V_k[g(t,\xi)]\Big\}d\mu(\xi)\le 0. \end{aligned}\tag{2.14}$$

Noting that

$$\begin{aligned} &\sum_{i=1}^m\Big[V_i(t)-\sum_{j=1}^m c_{ij}\frac{\delta_i}{\delta_j}V_j(t-\tau)\Big]\\ &=\sum_{i=1}^m\Big[V_i(t)-c_{ii}V_i(t-\tau)-\sum_{j=1,j\neq i}^m c_{ij}\frac{\delta_j}{\delta_i}V_j(t-\tau)\Big]\\ &\ge\sum_{i=1}^m\Big[V_i(t)-c_{ii}V_i(t-\tau)-\sum_{j=1,j\neq i}^m |c_{ij}|V_j(t-\tau)\Big]\\ &=\Big[V_1(t)-c_{11}V_1(t-\tau)-\sum_{j=1,j\neq 1}^m |c_{1j}|V_j(t-\tau)\Big]\\ &+\dots+\Big[V_n(t)-c_{nn}V_n(t-\tau)-\sum_{j=1,j\neq n}^m |c_{nj}|V_j(t-\tau)\Big]\\ &=\sum_{i=1}^m V_i(t)-\Big(c_{11}+\sum_{j=1,j\neq 1}^m |c_{j1}|\Big)V_1(t-\tau)\\ &\quad-\dots-\Big(c_{nn}+\sum_{j=1,j\neq n}^m |c_{jn}|\Big)V_n(t-\tau)\\ &\ge\sum_{i=1}^m V_i(t)-\max_{1\le i\le n}\Big\{c_{ii}+\sum_{j=1,j\neq i}^m |c_{ji}|\Big\}\sum_{i=1}^m V_i(t-\tau)\\ &=V(t)-CV(t-\tau), \end{aligned}$$

and

$$\begin{aligned} &\sum_{i=1}^m\Big\{Q_{ii}(t,\xi)V_i[g(t,\xi)]-\sum_{j=1,j\neq i}^m Q^*_{ij}(t,\xi)V_j[g(t,\xi)]\Big\}\\ &=\Big[Q_{11}(t,\xi)V_1[g(t,\xi)]-\sum_{j=1,j\neq 1}^m Q^*_{1j}(t,\xi)V_j[g(t,\xi)]\Big]\\ &+\dots+\Big[Q_{nn}(t,\xi)V_n[g(t,\xi)]-\sum_{j=1,j\neq n}^m Q^*_{nj}(t,\xi)V_j[g(t,\xi)]\Big]\\ &=\Big[Q_{11}(t,\xi)-\sum_{j=1,j\neq 1}^m Q^*_{j1}(t,\xi)\Big]V_1[g(t,\xi)]\\ &\quad+\dots+\Big[Q_{nn}(t,\xi)-\sum_{j=1,j\neq n}^m Q^*_{jn}(t,\xi)\Big]V_n[g(t,\xi)] \end{aligned}$$

$$
\begin{aligned}
&\ge \min_{1\le i\le n}\Big\{Q_{ii}(t,\xi)-\sum_{j=1,j\ne i}^{m}Q^*_{ji}(t,\xi)\Big\}\sum_{i=1}^{m}V_i[g(t,\xi)]\\
&=Q(t,\xi)V[g(t,\xi)],
\end{aligned}
$$

we have from (2.14) that

$$
\frac{d^2}{dt^2}[V(t)-CV(t-\tau)]+P(t)V(t)+\int_a^b Q(t,\xi)V[g(t,\xi)]d\mu(\xi)\le 0. \tag{2.15}
$$

From Lemma 1.2, inequality (2.15) has no eventually positive solutions, which is in contradiction with $V(t)>0$. This completes the proof of Theorem 2.1. □

To investigate the boundary-value problem (1.1), (1.3), we consider the following Dirichlet problem

$$
\begin{gathered}
\Delta u+\alpha u=0,\quad (x,t)\in\Omega\times\mathbb{R}_+\\
u=0,\quad (x,t)\in\partial\Omega\times\mathbb{R}_+,
\end{gathered} \tag{2.16}
$$

where α is a constant. It is well-known [5] that the least eigenvalue α_0 of problem (23) is positive and the corresponding eigenfunction $\varphi(x)$ is positive for $x\in\Omega$.

We further introduce the following notation

$$
A(t)=\min_{1\le i\le n}\{a_i(t)\},\quad B(t)=\min_{1\le i\le n}\{b_i(t)\} \tag{2.17}
$$

Theorem 2.2. *Suppose that* $0\le C\le 1$, *(A1) and* (2.2) *hold. If for some constants* $0\le\varepsilon_0\le 1$ *and* $\xi_0\in[a,b]$,

$$
\begin{aligned}
&\liminf_{t\to\infty}\int_{g(t,\xi_0)}^{t}\int_a^b \varepsilon_0 Q(s,\xi)g(s,\xi)d\mu(\xi)ds\\
&>\frac{1}{e}\exp\Big[-\liminf_{t\to\infty}\int_{g(t,\xi_0)}^{t}\varepsilon_0 s[\alpha_0A(s)+P(s)]ds\Big],
\end{aligned} \tag{2.18}
$$

then each unbounded solution $u(x,t)$ *of the boundary-value problem* (1.1) *and* (1.3) *is oscillatory in the domain* G.

Proof. Suppose to the contrary that there is a non-oscillatory solution $u(x,t)=(u_1(x,t),\dots,u_n(x,t))^T$ of the problem (1.1) and (1.3), and $|u_i(x,t)|>0$ for $t\ge 0$, $i=1,2,\dots,n$. Proceeding as the proof of Theorem 2.1, there exists a $t_1\ge 0$ such that $Y_i(x,t)>0$, $Y_j(x,t-\tau)>0$, $Y_i(x,t-\rho)>0$ and $Y_k[x,g(t,\xi)]>0$ for $t\ge t_1$, $\xi\in[a,b]$ and $j=1,2,\dots,m$, $k=1,2,\dots,m$.

Multiplying both sides of equation (1.1) by the eigenfunction $\varphi(x)$ and then integrating the equation with respect to x over the domain Ω, for $t\ge t_1$, we obtain

$$
\begin{aligned}
&\frac{d^2}{dt^2}\Big[\int_\Omega u_i(x,t)\varphi(x)dx-\sum_{j=1}^{m}\int_\Omega c_{ij}u_j(x,t-\tau)\varphi(x)dx\Big]\\
&+\int_\Omega p_i(x,t)u_i(x,t)\varphi(x)dx+\sum_{k=1}^{m}\int_\Omega\int_a^b q_{ik}(x,t,\xi)u_k[x,g(t,\xi)]\varphi(x)d\mu(\xi)dx\\
&=a_i(t)\int_\Omega \Delta u_i(x,t)\varphi(x)dx+b_i(t)\int_\Omega \Delta u_i(x,t-\rho)\varphi(x)dx.
\end{aligned} \tag{2.19}
$$

Furthermore, we obtain

$$\begin{aligned}
&\frac{d^2}{dt^2}\Big[\int_\Omega Y_i(x,t)\varphi(x)dx-\sum_{j=1}^m\int_\Omega c_{ij}\frac{\delta_i}{\delta_j}Y_j(x,t-\tau)\varphi(x)dx\Big]\\
&+\int_\Omega p_i(x,t)Y_i(x,t)\varphi(x)dx+\sum_{k=1}^m\int_\Omega\frac{\delta_i}{\delta_k}\int_a^b q_{ik}(x,t,\xi)Y_k[x,g(t,\xi)]\varphi(x)d\mu(\xi)dx\\
&=a_i(t)\int_\Omega \Delta Y_i(x,t)\varphi(x)dx+b_i(t)\int_\Omega \Delta Y_i(x,t-\rho)\varphi(x)dx.
\end{aligned} \tag{2.20}$$

From the Green's formula and the boundary condition (1.3), we have

$$\int_\Omega \Delta Y_i(x,t)\varphi(x)dx=\int_\Omega Y_i(x,t)\Delta\varphi(x)dx=-\alpha_0\int_\Omega Y_i(x,t)\varphi(x)dx, \tag{2.21}$$

$$\int_\Omega \Delta Y_i(x,t-\tau)\varphi(x)dx=-\alpha_0\int_\Omega Y_i(x,t-\tau)\varphi(x)dx. \tag{2.22}$$

Combining (2.20)–(2.22), we obtain

$$\begin{aligned}
&\frac{d^2}{dt^2}\Big[\int_\Omega Y_i(x,t)\varphi(x)dx-\sum_{j=1}^m\int_\Omega c_{ij}\frac{\delta_i}{\delta_j}Y_j(x,t-\tau)\varphi(x)dx\Big]\\
&+p_i(t)\int_\Omega Y_i(x,t)\varphi(x)dx+\int_a^b Q_{ii}(t,\xi)\int_\Omega Y_i[x,g(t,\xi)]\varphi(x)dxd\mu(\xi)\\
&-\sum_{k=1,k\neq i}^m\int_a^b Q^*_{ik}(t,\xi)\int_\Omega Y_k[x,g(t,\xi)]\varphi(x)dxd\mu(\xi)\\
&=-\alpha_0 a_i(t)\int_\Omega Y_i(x,t)\varphi(x)dx-\alpha_0 b_i(t)\int_\Omega Y_i(x,t-\tau)\varphi(x)dx\,.
\end{aligned}$$

Let

$$U_i(t)=\int_\Omega Y_i(x,t)\varphi(x)dx,\qquad i=1,2\dots,n, \tag{2.23}$$

then the remainder of the proof is the same as that for Theorem 2.1 and thus is omitted here. This completes the proof. □

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