A COMPUTER PROGRAM

FOR

PREDICTION OF TIDES

THESIS

Presented to the Graduate Council of
Southwest Texas State University
in Partial Fulfillment of
the Requirements

For the Degree of

MASTER OF SCIENCE

By

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EPIGRAPH

Discimus hinc tandem qua causa argentea Phoebe
Passibus haud aequis graditur; cur subdita nulli
Hactenus astronomo numerorum fraena recuset:
Cur remeant nodi, curque auges progrediuntur.
Discimus & quantis refluum vaga Cynthia pontum
Viribus impellit, fessis dum fluctibus ulvam
Deserit, ac nautis suspectas nudat arenas;
Alternis vicibus suprema ad littora pulsans.

At last we learn the reason why the silver Moon
Travels with unequal steps; and why she
Has refused to be subdued and has been subject to
The numbers of no astronomer until now.
And why the nodes regress, and why the apsides progress.
We learn also how the wandering Moon sets the tides into motion,
Whereby the surf now deserts the kelp along the shore,
Exposing shoals of sand suspected by sailors;
And then alternately drives the waves high upon the beach.

from the "Ode to Newton" by Edmond Halley, published in the preface to the *Principia* by Isaac Newton (1687)

translated by Donald Olson (1992)

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The author is also grateful to Ed Wallner (Wayland, Massachusetts) for assistance in obtaining harmonic constants for Boston and other ports. Certain parts of the present computer program are derived following the theoretical methods advocated by Wallner, in cases for which the author judges them to be superior to the methods employed by the NOS.

TABLE OF CONTENTS

I. INTRODUCTION

The ocean tides have been associated with the motion of the moon for a long time. The history of tidal observations has been traced back at least to Posidonius (135 B.C. - 51 B.C.):

> "Posidonius says that the movement of the ocean observes a regular series like a heavenly body, there being a daily, monthly, and yearly movement according to the influence of the moon." [2]

The moon's apparent influence upon the tides was explained in many ways in the centuries between Posidonius and Isaac Newton. It was Newton who, in his *Principia*, first provided a scientific basis for understanding the connection between the moon and the tides.[3] In order to begin to apply his theory of gravitation to an analysis of the tides he made the simplifying assumption that the surface of the earth was entirely covered by water, that is, there were no land masses (Figure 1). To avoid the complications of tidal currents, he assumed that the earth and moon, moving with their correct relative speeds, yet moved with indefinite slowness when compared to the ocean. Therefore, the ocean remained always in equilibrium and the tidal bulges remained beneath the moon. This theory is known as the Equilibrium Theory of Tides.[4]

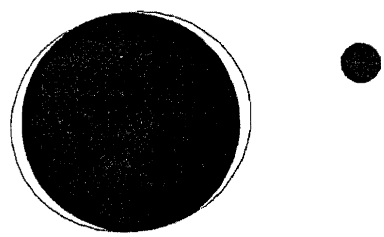


Figure 1

II. INTRODUCTION TO CELESTIAL MOTIONS WHICH AFFECT THE TIDES

As the height of the tide depends on the positions of the moon and sun it is required to describe those positions. The moon has an elliptical orbit about the earth, as does the earth about the sun. From a more convenient viewpoint, the geocentric point of view, the sun appears to orbit the earth in an elliptical orbit. A geocentric coordinate system defines the origin to be the center of the earth. For strictly positional astronomy, coordinate systems are used which project the orbital positions of bodies onto the so-called celestial sphere, which is considered to be at an infinite distance from the origin.

One such coordinate system, the equatorial coordinate system (Figure 2)[5], is defined as:

> ...[an] astronomical coordinate system in which the fundamental reference circle is the celestial equator and the zero point is the vernal equinox, ♈. The coordinates are right ascension, α , and declination, δ , which are measured along directions equivalent to those of terrestrial longitude and latitude, respectively.[6]

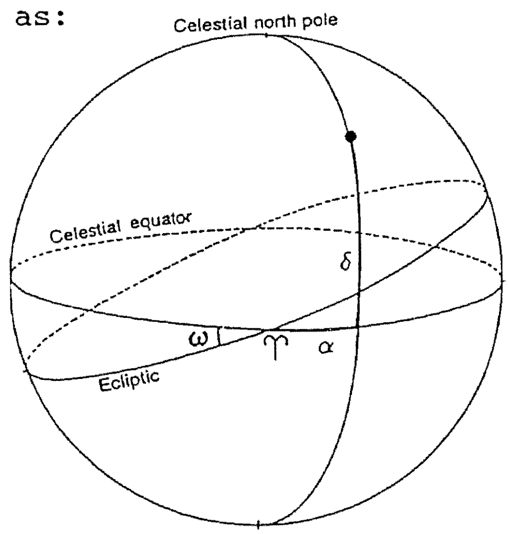


Figure 2

LUNAR ORBIT

The orbit of the moon about the earth is an ellipse having a very nearly constant eccentricity of 0.054900489 [9] The mean distance of the moon from the earth is given by the semi-major axis (384,400 x 10^3 m)[10]. However, since the orbit is an ellipse the moon-earth distance varies during each monthly orbit from a minimum (perigee) of about 360,000 km to a maximum (apogee) of about 410,000 km. The tide generating force due to gravity depends on distance and thus on where the moon is relative to perigee.

The position of lunar perigee relative to the vernal equinox changes with time. It has long been known that the apsides have a progressive motion.[11] Apsides are the two points in an orbit that lie closest to (periapsis) and farthest from (apoapsis) the center of gravitational attraction. The prefixes peri- and apo- are used in describing the apsides of specific orbits. The line of apsides is the straight line connecting the two apsides and is the major axis of an elliptical orbit.[12]

The eccentricity of the lunar orbit, while being very nearly constant, does alternately increase and decrease, being a maximum when the sun passes the moon's line of apsides and a minimim when the sun is at right angles to it.[13] This leads to a perturbation of the lunar orbit called *evection*.

SOLAR ORBIT

The sun's orbit about the earth has an eccentricity of 0.01675104[14] and a semi-major axis (mean distance) of 149,597,870 km.[15] The small eccentricity of this orbit makes for a small fractional variation in sun-earth distance. However, it must be taken into account for precision in the tide calculation.

The projection on the celestial sphere of the apparent orbit of the sun, called the ecliptic, is not co-planar with the celestial equator, but is at an inclination of about 23.5° relative to it. The two points at which the orbit of a celestial body intersects a reference plane, usually the plane of the ecliptic or celestial equator, are called nodes. When the body, such as the sun, moon, or a planet, crosses the reference plane from south to north it passes through the ascending node; crossing from north to south the body passes through its descending node. The line joining these two nodes is the line of nodes.[16]

LUNAR ORBIT RELATED TO SOLAR ORBIT (ECLIPTIC)

The lunar orbit has an almost constant inclination, i, of 5.1453964° relative to the ecliptic.[17] (Figure 4)[18] Therefore, during a draconic month (which is the time interval of mean length 27.2212 days between two successive passages of the moon through the ascending node) the moon's latitude ranges from about +5° to −5°.

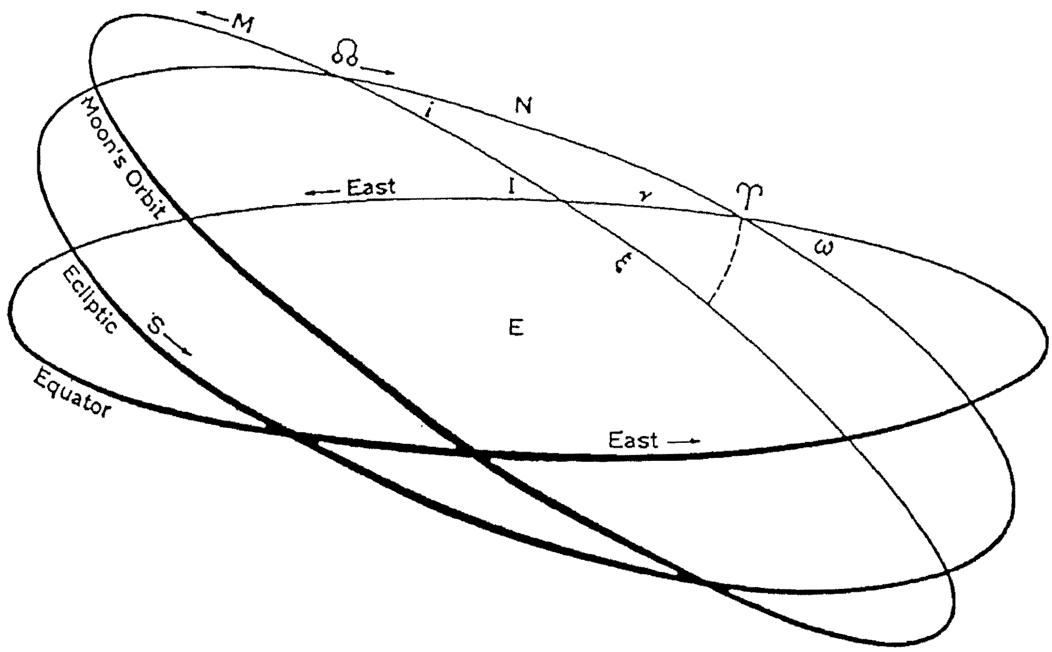


Figure 4

The line of nodes of the moon's orbit has a westward motion of about 19° per year. Therefore, in approximately 18.6 years the lunar line of nodes regresses completely one time around the ecliptic. At some point in this 18.6 year cycle the ascending node passes through the vernal equinox and then the moon's obliquity, its orbital inclination relative to the celestial equator, is a maximum. So, the range of declinations of the moon is maximized, having extreme values of +28° (+23° + 5°) and -28° (-23° - 5°). Approximately 9.3 years later, the node passes the autumnal equinox and the range of declinations of the moon is minimized, having extreme values of +18° (+23° - 5°) and -18° (-23° + 5°). This variation in the obliquity of the moon's orbit introduces an important inequality in the tidal movement which must be taken into account.[19]

ANOTHER METHOD OF DESCRIBING POSITION

The local hour angle of a celestial body is the angle measured westwards along the celestial equator from an observer's meridian to the hour circle of the celestial body. It is usually expressed in hours, minutes, and seconds from 0h to 24h. It is thus measured in the same units but in the opposite direction to right ascension which it can replace in the equatorial coordinate system. The angle measured eastwards along the equator from the meridian is sometimes called the meridian angle. Due to the daily apparent rotation of the celestial sphere, a star's hour angle increases continuously from 0h at the meridian; after six hours the hour angle is 6h and after 24 hours the star again crosses the meridian.[20]

III. HISTORY OF TIDE THEORIES AND PREDICTION

Before the 1860s tide prediction was done with simple methods involving lunitidal intervals. The mean high-water lunitidal interval is defined as the average time interval between the transit of the moon across the meridian of the place and the following high water.[21] It is an easy matter to estimate the time of moon's passage across the meridian.

Then, a crude estimate of the time of high tide can be done by setting

(time of high water) = (time of lunar transit) + (lunitidal interval).

Such methods were in use in the 1700s and 1800s. Popular almanacs of the day listed the time of lunar transit ("southing of the moon") and estimates of the time of high water.[22] This crude technique was only roughly accurate, since it did not take into account effects of lunar distance (perigee and apogee), lunar declination (moon north or south of the celestial equator), solar distance (perihelion or aphelion), and solar declination (sun north or south of the celestial equator).

HARMONIC ANALYSIS

The principles of harmonic analysis of the tides were actually first developed in 1773 by Pierre Simon Laplace and were published in his *Mécanique Celeste*. However, no widespread practical use was made at that time. Some studies were made of high and low waters, to compare theory with observation, but there was no attempt at general tide prediction or production of tide tables.[23]

William Thomson (Lord Kelvin) devised a method of harmonic analysis about 1867. The theory was further developed by G. H. Darwin and J. C. Adams and took a final form by 1883. The terminology developed then is still in use today with regard to the naming of the tide components, their theoretical forms, and the various quantities used in the analysis: node factors, equilibrium arguments, phases, and speeds. These quantities will be described in detail below for the harmonics used in the present program.

The first tide-prediction machine was constructed under the direction of William Thomson (Lord Kelvin) in 1873. This machine employed 10 harmonics and produced an analog curve of tide height versus time.

The United States Coast and Geodetic Survey built a tide-prediction machine in 1882. This analog machine was designed by William Ferrel and accounted for 19 harmonics. This mechanical device had dials from which times of high and low water could be read off, but no tide curve of height versus time was produced. The Ferrel machine was used for all Tide Tables produced from 1885 through 1911.[24]

An improved mechanical tide-prediction machine, employing 37 harmonics, was developed by R. A. Harris and E. G. Fischer of the U. S. Coast and Geodetic Survey in 1910. This machine produced an analog graph of tide height versus time and also could give figures for high and low waters. The machine stopped when the pen reached an extreme, and an operator could manually read the time of high or low water from dials on the side of the machine. An intricate gear system was required to mechanically sum 37 harmonics, each with a different amplitude, frequency, and phase angle. The Harris-Fischer machine produced all tide predictions in the United States for the period 1912 until 1965.[25] It has been stated that, during World War II, armed guards were placed in force around the machine, since it was irreplaceable to the war effort.[26] Manual summing of 37 harmonics, with pencil and paper and logarithm tables, was prohibitively time-consuming.

The first computer programs for tide prediction were developed in 1956. These codes, in machine language, were developed for the IBM 701, then for the IBM 704, and finally for the IBM 7094 mainframe computers.[27] An improved computer program was developed in 1966 in the FORTRAN IV language for the IBM 7094 mainframe.[28] Computer methods have been used for tide prediction in the Tide Tables from 1966 to the present.

IV. HARMONIC ANALYSIS AND HARMONIC PREDICTION OF THE TIDES

In the harmonic analysis method, the complicated behavior of the actual tide, due to the complicated motions of the moon and sun, is regarded as being made up of a combination of simple tides, each of which is due to a hypothetical tide-producing body which moves at a constant speed in a circular orbit in the plane of the equator.

In Newton's equilibrium tide, the tidal bulges follow the moon. Thus, to a first approximation, the principal hypothetical body moves along the equator at a constant rate which follows the actual moon. The daily motion of the actual moon is a combination of the rotation of the earth and the orbital motion of the moon around the earth. This results in a "tidal day" of 24 hours 50 minutes. This gives a period of 12 hours 25 minutes for the principal hypothetical body since there are two tidal bulges.[29] The tide due to this hypothetical body is a simple tide having only one high water and one low water per period. The range of this tide as well as the time interval between the meridian passage of this body and high water vary from place to place and can only be determined from actual tidal measurements.

However, actual tidal measurements are a combination of all the different tides due to different hypothetical bodies. For instance, since tides are a gravitational effect there must be a hypothetical body which makes a tide reflecting the elliptical nature of the orbit of the moon, that is reflecting the varying distance between the earth and moon. This body would cause a high water when the moon is at perigee, and a low water when the moon is at apogee. When the moon is at perigee, the high water would be in phase with the high water of the tide due to the principal hypothetical body. Likewise, when the moon is at apogee the low water would be in phase with the high water of the tide due to the principal hypothetical body. Therefore the so-called elliptic hypothetical body must have a period such that in an anomalistic month, the time interval from perigee to perigee of mean length 27.55 days, it loses one complete revolution on the principle hypothetical body.

Since 12 hours 25 minutes = 12.42 hours, then in 27.55 days or 661 hours the principal body will complete 661 ÷ 12.42 = 53.25 revolutions. So the elliptic body must complete 52.25 revolutions in 661 hours which means its period must be 12.65 hours or 12 hours 39 minutes.[30]

Since these are hypothetical bodies, it is not required to calculate the tide-producing forces due to these bodies, or to explain how these bodies could maintain stable orbits, but only to consider the constituent effect upon the tide due to each hypothetical body. This is called harmonic analysis, an explanation of which is beyond the scope of this paper, but the results of which will be used for tide prediction.

For the harmonic prediction of the tide it is necessary to calculate the sum of the constituent tides. The tide height at any time is given by the superposition of 37 constituents which represent periodic variations in the relative positions of the sun, moon, and earth.[31]

Thus, the height of the tide at any time is represented by the equation

$$h = Z_0 + \sum_{n=1}^{37} f_n H_n \cos (V_n + u_n - \kappa_n)$$

where

h = *height of the tide at time* t
Z_0 = *mean height of water level above local chart datum*
f_n = *node factor in the* 18.6 *year lunar nodal cycle*
H_n = *mean amplitude of the constituent* n
V_n = *argument of the constituent* n
u_n = *argument of the constituent* n
κ_n = *epoch (phase) of the constituent* n

Certain quantities (f_n, V_n, u_n) are determined astronomically and are therefore the same for every port and place in the world. These variables depend only on the position of the sun and the moon. The effect of the declination of the moon upon the tide is accounted for by the nodal factors (f_n) of the constituents. The factor f_n depends upon the longitude of the moon's ascending node and is introduced in order to reduce the mean amplitude of the constituent, H_n, to the true amplitude on the date used for the tide prediction.[32] V_n and u_n are arguments which taken together give the theoretical phase of the constituent in the equilibrium tide. V_n is an angular quantity which involves multiples of the hour angle of the mean sun, the mean longitudes of the moon and sun, and the mean longitude of the lunar or solar perigee. u_n is an angle which depends upon the longitude of the moon's ascending node.[33] The equations for such quantities involve only astronomical variables.

Other variables (Z_o, H_n, κ_n) are port-dependent. They are determined by empirical observation of tide levels in the port. In principle, observations of the water level are made hourly for as long a period as is practicable. For the highest accuracy, measurements should extend for at least one 18.6 year nodal cycle. Within this period, all significant astronomical modifications of tides will occur.[34]
Therefore, nearly all possible configurations of the moon have been sampled if observations are made for the full 18.6 years. Such a situation is considered optimal for estimating the station constants for a port.

In practice, observations often are restricted to shorter time periods. There are methods for deriving estimates of the station constants (reference level Z_o, amplitudes H_n, and phases κ_n) from observations over time periods of: 14, 15, 29, 58, 87, 105, 134, 163, 192, 221, 250, 279, 297, 326, 355, and 369 days. These periods are related to astronomical periods. For example, the shortest periods (14 or 15 days) are such that tide ranges at a new moon or a full moon, and at a perigee or an apogee will be observed.

During World War II, the station constants for Tarawa (Gilbert Islands) given in Appendix B were derived from a 134-day series of observations beginning on December 1, 1943.

At other more established ports, of course, observations over the full 18.6-year period have been performed.

V. ASTRONOMICAL FACTORS

Any tidal constituent must have a frequency which is simply derived from six fundamental frequencies, which are the inverses of the following periods:

1 mean solar day

(period of the earth's rotation relative to the mean sun, an abstract reference point that follows a circular orbit around the celestial equator moving eastward at a constant speed and completes one circuit in the same time, one tropical year, as the apparent sun takes to orbit the ecliptic.)[35]

1 tropical month

(period of the moon's orbital motion around the earth)

1 tropical year

(period of the earth's orbital motion around the sun)

8.85 years

(period for the progression of the lunar line of apsides)

18.6 years

(period for the regression of the lunar line of nodes)

20900 years

(period for the progression of the line of apsides of the earth's orbit around the sun)

Every tide component has a characteristic frequency and period, which must be a simple combination of these six basic frequencies.

To make this more precise, the equations will be given for the six fundamental astronomical variables. These equations are taken from Jean Meeus' *Astronomical Formulae for Calculators*.[36]

The mean longitude of the sun can be given by

$$h = 279.69668 + 36000.76892\,T + 0.0003025\,T^2 \quad ,$$

where T here represents the number of Julian centuries reckoned from 12h ET on December 31, 1899, that is:

$$T = \frac{JD - 2415020.0}{36525} \quad ,$$

where JD is the Julian date, the number of days and fractions of a day that have elapsed since noon Greenwich Mean Time on Jan 1, 4713 BC. (The consecutive numbering of days makes the system independent of the length of month or year and the JD is thus used to calculate the frequency of occurrence or the periodicity of phenomena over long periods. The system was devised in 1582 by the French scholar Joseph Scalinger.)[37]

The mean longitude of the solar perigee can be given by

$$p_1 = h - M \quad ,$$

where h is the mean longitude of the sun (given above) and M is the sun's mean anomaly which can be given by

$$M = 358.47583 + 35999.04975\,T - 0.00015\,T^2 - 0.0000033\,T^3 \quad .$$

The mean longitude of the moon can be given by

$$s = 270.434164 + 481267.8831\,T - 0.001133\,T^2 + 0.0000019\,T^3 \quad .$$

The mean longitude of the lunar perigee, derived in analogy to the mean longitude of the solar perigee, can be given by

$$p = 334.329556 + 4069.034\,T - 0.010325\,T^2 - 0.0000125\,T^3 \quad .$$

The longitude of the lunar ascending node can be given by

$$N = 259.183275 - 1934.142\,T + 0.002078\,T^2 + 0.0000022\,T^3 \quad .$$

In addition to the primary variables listed above, several additional quantities are needed.

One of these is the obliquity of the ecliptic, ω, which can be given by

$$\omega = 23.452294 - 0.0130125\,T - 0.00000164\,T^2 + 0.000000503\,T^3 \quad .$$

Certain intermediate angles must be defined to generate the correct node factors f_n and the arguments V_n and u_n. The equations are from USCGS Special Publication No. 98. Using the standard notation, we define the following angles: ν, the right ascension of the intersection, and ξ, the longitude in orbit of the intersection, are given by

$$\nu = AX - AY$$

$$\xi = N - AX - AY$$

$$AX = \tan^{-1} \frac{\cos\frac{(\omega - i)}{2}}{\cos\frac{(\omega + i)}{2}} \tan\frac{N}{2}$$

where

$$AY = \tan^{-1} \frac{\sin\frac{(\omega - i)}{2}}{\sin\frac{(\omega + i)}{2}} \tan\frac{N}{2}$$

$N =$ *the mean longitude of the lunar node*

The obliquity of the lunar orbit, I, can be found from the equation

$$\cos I = \cos i \cos \omega - \sin i \sin \omega \cos N \quad .$$

P, the mean longitude of the lunar perigee as measured from the intersection, is simply

$$P = p - \xi \quad .$$

A term, ν', which is in the argument of the K_1 constituent, can be given by

$$\nu' = \tan^{-1} \frac{\sin 2I \ \sin\nu}{\sin 2I \ \cos\nu + 0.3347} \quad ,$$

and ν'', which is a term in the argument of the K_2 constituent, can be given by

$$\nu'' = \frac{1}{2} \tan^{-1} \frac{\sin^2 I \ \sin 2\nu}{\sin^2 I \ \cos 2\nu + 0.0727} \quad .$$

The L_2 constituent includes two intermediate variables, an amplitude factor, R_a, and a term in the argument, R, which can be given by

$$R_a = \frac{1}{\sqrt{1 - 12 \tan^2 \frac{I}{2} \cos 2P + 36 \tan^4 \frac{I}{2}}}$$

$$R = \tan^{-1} \left[\frac{\sin 2P}{\left(\frac{1}{6 \tan^2 \frac{I}{2}} \right) - \cos 2P} \right]$$

The M_1 constituent also includes two intermediate variables, an amplitude factor, Q_a, and a term in the argument, Q_u, which can be given by

$$Q_a = \frac{1}{\sqrt{\frac{1}{4} + \frac{3 \cos I \cos 2P}{2 \cos^2 \frac{I}{2}} + \frac{9 \cos^2 I}{4 \cos^4 \frac{I}{2}}}}$$

$$Q_u = \tan^{-1} \left[\frac{\sin 2P}{\left(\frac{3 \cos I}{\cos^2 \frac{I}{2}} \right) - \cos 2P} \right]$$

Related to Q_u is another intermediate variable, Q, such that

$$Q_u + Q = P$$

where

$$Q = \tan^{-1}\left[\frac{5\cos I - 1}{7\cos I + 1}\right] \tan P \quad .$$

These angles will be incorporated into the equations of the astronomical factors for the harmonics used in the computer program. The following section tabulates and describes these harmonics.

VI. THE 37 HARMONICS USED IN TIDE PREDICTION

In theory, an infinite number of terms should be retained in the cosine expansion of the tide height versus time. In practice, only a finite number are retained. For most ports, it is standard in the system developed by the U. S. Coast and Geodetic Survey to tabulate 37 harmonics.

The 37 harmonics used in the USCGS system are listed on the "Form C&GS-444" for the port being studied. For the present study, copies of this form were obtained from the National Ocean Survey for the ports of Boston (Massachusetts), Los Angeles (California), San Diego (California), San Francisco (California), and Tarawa (Gilbert Islands).

Copies of the "Form C&GS-444" for Boston and several other ports will be given in Appendix B.

PROPERTIES OF THE 37 HARMONICS USED IN TIDE PREDICTION

The purpose of the next section is to explain in more detail each of the 37 harmonics used in the computer tide prediction program. For each of the harmonics, eight properties will be given, as follows.

1. NUMBER

The number of the constituent will be given, as an integer from 1 to 37. This number corresponds to the position on the rows of "Form C&GS-444," which contains 37 rows from #1, the M_2 tide, to #37, the $(MS)_4$ tide.

2. COMMON NAME

These well-known common names, which were originally assigned in the 1800s, are as given on "Form C&GS-444." The name can indicate properties of the tide, for example the M_2 tide is the principal lunar semidiurnal (2 high waters per day) tide. The S_2 tide is the principal solar semidiurnal tide.

3. NAME IN SPECIAL PUBLICATION NO. 98

These names are taken from Table 2 and Table 2a of Special Publication No. 98. In this system, tides are assigned to the A-series, the B-series, or to the "shallow-water constituent" series.

4. COMMENTS

For each of the 37 tide components, comments are provided. Definitions presented are those given in the standard Tide and Current Glossary published by the National Ocean Survey.

Some tides are related to the moon (lunar tides), some to the sun (solar tides), some are combinations (lunisolar tides), or they can have other causes (shallow-water overtides and meteorological tides). A constituent tide can be classified based on its speed. It can be diurnal (one high water and one low water per tidal day), or semidiurnal (two high waters and two low waters per tidal day), or terdiurnal (three high waters and three low waters per day), or it can have a different period.

Comments will also indicate the physical significance of the component. For example, some tides express the effects of the varying lunar distance, as the moon moves through perigee and apogee. Other tides represent declinational effects, as the moon or sun moves north or south of the celestial equator.

Indicated in the comments will be the "shallow-water constituents." These result from the fact that when a wave moves into shallow water near shore, the wave loses its simple harmonic form. Some of these components are called "overtides" by analogy with overtones in acoustics. These components have speeds which are exact multiples of one of the elementary constituents. For example, the principal lunar and solar tides are M_2 and S_2. The lunar overtides are called M_4, M_6, and M_8, while the solar overtides are denoted by S_4, S_6, and S_8. Their arguments and speeds are exact multiples of those of the fundamental tides.

Also worth mentioning are the "meteorological" tides. These include the tides called Sa (solar annual), Ssa (solar semi-annual), and S_1 (solar diurnal), with periods corresponding to the tropical year, the half tropical year, and the solar day. These constituents appear in the cosine series expansion of the tide height versus time. However, their effects cannot be separated from the general seasonal effects of the weather, seasonal variations in temperature, barometric pressure, direction and force of the wind, and other meteorological effects. All such effects with long time-scales are allowed for in the Sa and Ssa constituents. The S_1 tide (solar diurnal) also includes some meteorological effects, particularly in places where periodic land and sea breezes occur in different directions during the day and the night.

5. NODE FACTORS

For each tide component, the node factors are taken from the equations given in the text of Special Publication No. 98. The node factors are slowly changing periodic functions which modify the amplitudes of the constituents which depend on the longitude of the moon's node.[38]

Note that the exact formulae from Special Publication No. 98 are used in the present work and in the computer program given here. This represents one of the significant improvements of the present program, compared to the computer programs used by the NOS for tide prediction.

In the NOS computer programs, the node factors are evaluated only one time per year. The same node factor is used for every day of that year. The NOS programs were based on methods that pre-date the 1950s and the computer age and much of the NOS programming retains the philosophy of "look-up tables." In the early 1900s, the node factors were tabulated for the middle of every year from 1850 to 1999. These same tables are still used in the modern NOS program.

In the program developed here, the node factors are computed from the exact formulae, 1440 times per day. Even on a personal computer this computation runs very quickly, with an entire day's tide curve produced in less than 15 seconds on a PC 386-class computer.

In summary, our program computes node factors from exact formulae and makes no use of the simplified tables in Special Publication No. 98.

6. EQUILIBRIUM ARGUMENTS u

The equations for the arguments u are taken from the text of Special Publication No. 98 and from Table 2 and Table 2a in Special Publication No. 98. Again, our program makes use of the exact formulae and makes no use of simplified look-up tables.

7. EQUILIBRIUM ARGUMENTS V

The equations for the arguments V are also taken from the text of Special Publication No. 98 and from Table 2 and Table

2a in Special Publication No. 98. Again, the exact formulae are used.

8. SPEED

The constituent speeds are taken from Table 2 and Table 2a in Special Publication No. 98.

Any tides with speeds equal or nearly equal to 15 degrees per hour are considered "diurnal" in type. Such harmonics describe tidal bulges which travel around the earth at the cited speed and cause one high water and one low water per day.

Those tides with speeds equal or nearly equal to 30 degrees per hour are considered "semidiurnal" in type. Such harmonics describe tidal bulges which travel around the earth at the cited speed and cause two high waters and two low waters per day.

Any tides with speeds equal or nearly equal to 45 degrees per hour are considered "terdiurnal" in type. Such harmonics describe tidal bulges which travel around the earth at the cited speed and cause three high waters and three low waters per day.

HARMONIC

CONSTITUENTS

Name: M_2 tide

Designation in USCGS Special Publication No. 98: A_{39}

The M_2 tide is the principal lunar semidiurnal constituent. It represents the rotation of the earth with respect to the moon.

Equilibrium arguments:

$$V(1) = 2T - 2s + 2h$$

$$u(1) = 2\xi - 2\nu$$

Speed:

$$a(1) = 28.9841042\ °/\text{hour} \qquad (\text{ semidiurnal })$$

Node factor:

$$f(1) = \frac{\cos^4 \frac{I}{2}}{0.9154}$$

Name: S_2 tide

Designation in USCGS Special Publication No. 98: B_{39}

The S_2 tide is the principal solar semidiurnal constituent. It represents the rotation of the earth with respect to the sun.

Equilibrium arguments:

$V(2) = 2T$

$u(2) = 0$

Speed:

$a(2) = 30.0000000\ °/\text{hour}$ (semidiurnal)

Node factor:

$f(2) = 1$

Name: N_2 tide

Designation in USCGS Special Publication No. 98: A_{40}

The N_2 tide is the larger lunar elliptic semidiurnal constituent. This constituent, along with L_2 (#33 of 37), modulates the amplitude and frequency of M_2 for the effect of variation in the moon's orbital speed due to its elliptical orbit.

Equilibrium arguments:

$$V(3) = 2T - 3s + 2h + p$$

$$u(3) = 2\xi - 2\nu$$

Speed:

$a(3) = 28.4397295$ °/hour (semidiurnal)

Node factor:

$$f(3) = f(1)$$

Name: K_1 tide

Designation in USCGS Special Publication No. 98: A_{22} and B_{22}

The K_1 tide is the lunisolar diurnal constituent. This constituent, with O_1 (#6 of 37), expresses the effect of the moon's declination; they account for the diurnal inequality, which is the difference in height of the two high waters or of the two low waters of each day. In extreme cases, they can change tides which are normally semidiurnal (two high waters and two low waters each day) into diurnal tides (one high water and one low water each day). Also, K_1, taken together with P_1 (#30 of 37), expresses the effect of the sun's declination.

Equilibrium arguments:

$$V(4) = T + h - 90°$$

$$u(4) = -\nu'$$

Speed:

$$a(4) = 15.0410686 \text{ °/hour} \qquad (\text{ diurnal })$$

Node factor:

$$f(4) = \frac{C_1}{\langle C_1 \rangle} = \frac{\sqrt{A^2 + 2AB\cos\nu + B^2}}{(.5 + .75e^2)\sin(2\omega)(1 - 1.5\sin^2 i) + B}$$

where

$$A = (.5 + .75e^2)\sin 2I$$

$$B = (.5 + .75e_1^2)S'\sin(2\omega)$$

$e = 0.054900489$ (*eccentricity of moon's orbit*)

$e_1 = 0.01675104$ (*eccentricity of sun-earth orbit*)

$S' = 0.4602$

Name: M_4 tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The M_4 tide is a shallow water overtide of the principal lunar constituent M_2. Therefore, its speed is exactly twice as fast and its node factor is the square of the node factor of the M_2 tide. The equilibrium arguments are also exactly twice those of the M_2 tide.

Equilibrium arguments:

$$V(5) = 4T - 4s + 4h$$

$$u(5) = 4\xi - 4\nu$$

Speed:

$$a(5) = 57.9682084 \text{ °/hour}$$

Node factor:

$$f(5) = [f(1)]^2$$

Name: O_1 tide

Designation in USCGS Special Publication No. 98: A_{14}

The O_1 tide is a lunar diurnal constituent. Along with K_1 (#4 of 37) it expresses the effects of the moon's declination. They account for the diurnal inequality.

Equilibrium arguments:

$$V(6) = T - 2s + h + 90°$$

$$u(6) = 2\xi - \nu$$

Speed:

$$a(6) = 13.9430356 \text{ °/hour} \qquad (\text{ diurnal })$$

Node factor:

$$f(6) = \frac{\sin I \cos^2 \frac{I}{2}}{0.3800}$$

Name: M_6 tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The M_6 tide is a shallow water overtide of the principal lunar constituent M_2. Therefore, its speed is exactly three times as fast and its node factor is the cube of the node factor of the M_2 tide. The equilibrium arguments are also exactly three times those of the M_2 tide.

Equilibrium arguments:

$$V(7) = 6T - 6s + 6h$$

$$u(7) = 6\xi - 6\nu$$

Speed:

$$a(7) = 86.9523127 \ °/\text{hour}$$

Node factor:

$$f(7) = [f(1)]^3$$

Name: $(MK)_3$ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The $(MK)_3$ tide is a terdiurnal compound tide formulated by $M_2 + K_1$. Its arguments V and u are the sums of the arguments of the M_2 and K_1 constituents. Note that its node factor is the product of the node factors of M_2 and K_1.

Equilibrium arguments:

$$V(8) = 3T - 2s + 3h - 90°$$

$$u(8) = 2\xi - 2\nu - \nu'$$

Speed:

$$a(8) = 44.0251729 \text{ °/hour}$$ (terdiurnal)

Node factor:

$$f(8) = f(1) \times f(4)$$

Name: S_4 tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

S_4 is a shallow water overtide of the principal solar semidiurnal constituent. Its equilibrium arguments and its speed are exactly twice those of the S_2 constituent.

Equilibrium arguments:

$$V(9) = 4T$$

$$u(9) = 0$$

Speed:

$$a(9) = 60.0000000\ °/\text{hour}$$

Node factor:

$$f(9) = 1$$

Name: $(MN)_4$ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

$(MN)_4$ is a quarter diurnal compound tide originating from $M_2 + N_2$.

Equilibrium arguments:

$$V(10) = 4T - 5s + 4h + p$$

$$u(10) = 4\xi - 4\nu$$

Speed:

$$a(10) = 57.4238337\ °/\text{hour}$$

Node factor:

$$f(10) = [f(1)]^2$$

Name: ν_2 tide

Designation in USCGS Special Publication No. 98: A_{43}

ν_2 is the larger lunar evectional constituent. With λ_2 (#16 of 37), μ_2 (#13 of 37), and (S_2), this constituent modulates the amplitude and frequency of the principal lunar constituent, M_2, to account for the variation in solar attraction of the moon. This attraction results in a slightly pear-shaped lunar ellipse and a difference in lunar orbital speed between motion toward and away from the sun. Although (S_2) has the same speed as S_2, its amplitude is extremely small.

Equilibrium arguments:

$$V(11) = 2T - 3s + 4h - p$$

$$u(11) = 2\xi - 2\nu$$

Speed:

$a(11) = 28.5125831$ °/hour (semidiurnal)

Node factor:

$$f(11) = f(1)$$

Name: S_6 tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

S_6 is a shallow water overtide of the principal solar constituent, S_2, thus its equilibrium arguments and its speed are three times those of S_2.

Equilibrium arguments:

$$V(12) = 6T$$

$$u(12) = 0$$

Speed:

$$a(12) = 90.0000000\ ^\circ/\text{hour}$$

Node factor:

$$f(12) = 1$$

Name: μ_2 tide

Designation in USCGS Special Publication No. 98: A_{45}

The μ_2 tide is a variational constituent. Along with λ_2 and ν_2 it modulates M_2 for the effects of evection.

Equilibrium arguments:

$$V(13) = 2T - 4s + 4h$$

$$u(13) = 2\xi - 2\nu$$

Speed:

$a(13) = 27.9682084$ °/hour (semidiurnal)

Node factor:

$$f(13) = f(1)$$

Name: $(2N)_2$ tide

Designation in USCGS Special Publication No. 98: A_{42}

> The $(2N)_2$ tide is a lunar elliptic semidiurnal constituent. It is a second-order constituent.

Equilibrium arguments:

$$V(14) = 2T - 4s + 2h + 2p$$

$$u(14) = 2\xi - 2\nu$$

Speed:

$$a(14) = 27.8953548 \ °/\text{hour} \qquad (\text{ semidiurnal })$$

Node factor:

$$f(14) = f(1)$$

Name: $(OO)_1$ tide

Designation in USCGS Special Publication No. 98: A_{31}

> $(OO)_1$ is a lunar diurnal constituent. It is a second-order constituent.

Equilibrium arguments:

$$V(15) = T - 2s + h - 90°$$

$$u(15) = -2\xi - \nu$$

Speed:

$a(15) = 16.1391017$ °/hour (diurnal)

Node factor:

$$f(15) = \frac{\sin I \sin^2 \frac{I}{2}}{0.0164}$$

Name: λ_2 tide

Designation in USCGS Special Publication No. 98: A_{44}

The λ_2 tide is the smaller lunar evectional constituent.

Equilibrium arguments:

$$V(16) = 2T - s + p + 180°$$

$$u(16) = 2\xi - 2\nu$$

Speed:

$a(16) = 29.4556253$ °/hour (semidiurnal)

Node factor:

$$f(16) = f(1)$$

Name: S_1 tide

Designation in USCGS Special Publication No. 98: B_{71}

The S_1 tide is a solar diurnal constituent.

Equilibrium arguments:

$V(17) = T$

$u(17) = 0$

Speed:

$a(17) = 15.0000000$ °/hour (diurnal)

Node factor:

$f(17) = 1$

Name: M_1 tide

Designation in USCGS Special Publication No. 98: A_{16} and A_{23}

The M_1 tide is the smaller lunar elliptic diurnal constituent. This constituent, with J_1 (#19 of 37), modulates the amplitude of the declinational K_1 for the effect of the moon's elliptical orbit. A slightly slower constituent, designated (M_1), with Q_1 (#26 of 37), modulates the amplitude and frequency of the declinational O_1 for the same effect.

Equilibrium arguments:

$$V(18) = T - s + h + p - 90°$$

$$u(18) = -\nu - Q_u$$

Speed:

$a(18) = 14.4966939$ °/hour (diurnal)

Node factor:

$$f(18) = \frac{f(6)}{Q_a}$$

where

$$Q_a = \frac{1}{\sqrt{\frac{1}{4} + \frac{3\cos I \cos 2P}{2\cos^2 \frac{I}{2}} + \frac{9\cos^2 I}{4\cos^4 \frac{I}{2}}}}$$

and $P = p - \xi$

Name: J_1 tide

Designation in USCGS Special Publication No. 98: A_{24}

The J_1 tide is the smaller lunar elliptic diurnal constituent. This constituent, with M_1, modulates the amplitude of the declinational K_1 for the effect of the moon's elliptical orbit.

Equilibrium arguments:

$$V(19) = T + s + h - p - 90°$$

$$u(19) = -\nu$$

Speed:

$$a(19) = 15.5854433 \text{ °/hour} \qquad (\text{ diurnal })$$

Node factor:

$$f(19) = \frac{\sin 2I}{0.7214}$$

Name: Mm tide

Designation in USCGS Special Publication No. 98: A_2

The Mm tide is the lunar monthly constituent. It expresses the effect of irregularities in the moon's rate of change of distance and speed in orbit.

Equilibrium arguments:

$$V(20) = s - p$$

$$u(20) = 0$$

Speed:

$$a(20) = 0.5443747 \ ^\circ/\text{hour}$$

Node factor:

$$f(20) = \frac{\frac{2}{3} - \sin^2 I}{0.5021}$$

Name: Ssa tide

Designation in USCGS Special Publication No. 98: B_6

The Ssa tide is the solar semiannual constituent. This constituent, along with the Sa constituent (#22 of 37), accounts for the nonuniform changes in the sun's declination and distance. In actuality, they mostly reflect yearly meteorological variations influencing sea level.

Equilibrium arguments:

$$V(21) = 2h$$

$$u(21) = 0$$

Speed:

$$a(21) = 0.0821373\ °/\text{hour}$$

Node factor:

$$f(21) = 1$$

Name: Sa tide

Designation in USCGS Special Publication No. 98: B_{64}

The Sa tide is the solar annual constituent. (see #21 of 37, the Ssa constituent)

Equilibrium arguments:

$V(22) = h$

$u(22) = 0$

Speed:

$a(22) = 0.0410686$ °/hour

Node factor:

$f(22) = 1$

Name: MSf tide

Designation in USCGS Special Publication No. 98: A_5

The MSf tide is the lunisolar synodic fortnightly constituent.

Equilibrium arguments:

$V(23) = 2s - 2h$

$u(23) = 0$

Speed:

$a(23) = 1.0158958$ °/hour

Node factor:

$f(23) = f(20)$

Name: Mf tide

Designation in USCGS Special Publication No. 98: A_6

The Mf tide is the lunar fortnightly constituent. This constituent expresses the effect of departure from a sinusoidal declinational motion.

Equilibrium arguments:

$$V(24) = 2s$$

$$u(24) = -2\xi$$

Speed:

$$\mathbf{a}(24) = 1.0980331\ °/\text{hour}$$

Node factor:

$$f(24) = \frac{\sin^2 \frac{I}{2}}{0.1578}$$

Name: ρ_1 tide

Designation in USCGS Special Publication No. 98: A_{18}

The ρ_1 tide is the larger lunar evectional diurnal constituent.

Equilibrium arguments:

$$V(25) = T - 3s + 3h - p + 90°$$

$$u(25) = 2\xi - \nu$$

Speed:

$a(25) = 13.4715145$ °/hour (diurnal)

Node factor:

$$f(25) = f(6)$$

Name: Q_1 tide

Designation in USCGS Special Publication No. 98: A_{15}

The Q_1 tide is the larger lunar elliptic diurnal constituent. (see the M_1 constituent, #18 of 37)

Equilibrium arguments:

$$V(26) = T - 3s + h + p + 90°$$

$$u(26) = 2\xi - \nu$$

Speed:

$a(26) = 13.3986609$ °/hour (diurnal)

Node factor:

$$f(26) = f(6)$$

Name: T_2 tide

Designation in USCGS Special Publication No. 98: B_{40}

> The T_2 tide is the larger solar elliptic constituent. This constituent, along with R_2 (#28 of 37), modulates the amplitude and frequency of S_2 for the effect of variation in the earth's orbital speed due to its elliptical orbit.

Equilibrium arguments:

$$V(27) = 2T - h + p_1$$

$$u(27) = 0$$

Speed:

$a(27) = 29.9589333$ °/hour (semidiurnal)

Node factor:

$$f(27) = 1$$

Name: R_2 tide

Designation in USCGS Special Publication No. 98: B_{41}

The R_2 tide is the smaller solar elliptic constituent. (see the T_2 constituent, #27 of 37)

Equilibrium arguments:

$$V(28) = 2T + h - p_1 + 180°$$

$$u(28) = 0$$

Speed:

$a(28) = 30.0410667$ °/hour (semidiurnal)

Node factor:

$$f(28) = 1$$

Name: $(2Q)_1$ tide

Designation in USCGS Special Publication No. 98: A_{17}

The $(2Q)_1$ tide is a lunar elliptic, second-order, diurnal constituent.

Equilibrium arguments:

$$V(29) = T - 4s + h + 2p + 90°$$

$$u(29) = 2\xi - \nu$$

Speed:

$a(29) = 12.8542862$ °/hour (diurnal)

Node factor:

$$f(29) = f(6)$$

Name: P_1 tide

Designation in USCGS Special Publication No. 98: B_{14}

The P_1 tide is a solar diurnal constituent. (see the K_1 constituent, #4 of 37)

Equilibrium arguments:

$$V(30) = T - h + 90°$$

$$u(30) = 0$$

Speed:

$a(30) = 14.9589314$ °/hour (diurnal)

Node factor:

$$f(30) = 1$$

Name: $(2SM)_2$ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The $(2SM)_2$ tide is a compound semidiurnal constituent. Its origin is given by the formula $2S_2 - M_2$.
Note that $V(31) = 2\ V(2) - V(1)$ and
$u(31) = 2\ u(2) - u(1)$

Equilibrium arguments:

$$V(31) = 2T + 2s - 2h$$

$$u(31) = -2\xi + 2\nu$$

Speed:

$a(31) = 31.0158958$ °/hour (semidiurnal)

Node factor:

$$f(31) = f(1)$$

Name: M_3 tide

Designation in USCGS Special Publication No. 98: A_{82}

The M_3 tide is a lunar terdiurnal constituent.

Equilibrium arguments:

$$V(32) = 3T - 3s + 3h$$

$$u(32) = 3\xi - 3\nu$$

Speed:

$$a(32) = 43.4761563\ °/\text{hour}$$ (terdiurnal)

Node factor:

$$f(32) = [f(1)]^{\frac{3}{2}} = \frac{\cos^6 \frac{I}{2}}{0.8758}$$

Name: L_2 tide

Designation in USCGS Special Publication No. 98: A_{41} and A_{48}

> The L_2 tide is the smaller lunar elliptic semidiurnal constituent. This constituent, along with constituent N_2 (#3 of 37), modulates the amplitude and frequency of M_2 for the effect of variation in the moon's orbital speed due to its elliptical orbit.

Equilibrium arguments:

$$V(33) = 2T - s + 2h - p + 180°$$

$$u(33) = 2\xi - 2\nu - R$$

Speed:

$$a(33) = 29.5284789 \text{ °/hour} \qquad \text{(semidiurnal)}$$

Node factor:

$$f(33) = \frac{f(1)}{R_a}$$

where

$$R_a = \frac{1}{\sqrt{1 - 12\tan^2\frac{I}{2}\cos 2P + 36\tan^4\frac{I}{2}}}$$

and $P = p - \xi$

Name: $(2MK)_3$ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

> The $(2MK)_3$ tide is a shallow water terdiurnal compound constituent. It is compounded according to the formula $2M_2 - K_1$.

Equilibrium arguments:

$$V(34) = 3T - 4s + 3h + 90°$$

$$u(34) = 4\xi - 4\nu + \nu'$$

Speed:

$$a(34) = 42.9271398 \text{ °/hour} \qquad \text{(terdiurnal)}$$

Node factor:

$$f(34) = [f(1)]^2 \times f(4)$$

Name: K_2 tide

Designation in USCGS Special Publication No. 98: A_{47} and B_{47}

> The K_2 tide is a lunisolar semidiurnal constituent. This constituent modulates the amplitude and frequency of M_2 and S_2 for the declinational effect of the moon and sun respectively.

Equilibrium arguments:

$$V(35) = 2T + 2h$$

$$u(35) = -2\nu''$$

Speed:

$a(35) = 30.0821373$ °/hour (semidiurnal)

Node factor:

$$f(35) = \frac{C_2}{\langle C_2 \rangle} = \frac{\sqrt{A^2 + 2AB\cos 2\nu + B^2}}{(.5 + .75e^2)\sin^2\omega\,(1 - 1.5\sin^2 i) + B}$$

where

$$A = (.5 + .75e^2)\sin^2 I$$

$$B = (.5 + .75e_1^2)\,S'\sin^2\omega$$

$e = 0.054900489$ (*eccentricity of moon's orbit*)

$e_1 = 0.01675104$ (*eccentricity of sun-earth orbit*)

$S' = 0.4602$

Name: M_8 tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

> The M_8 tide is a shallow water overtide of the principal lunar semidiurnal constituent, M_2. Its equilibrium arguments and its speed are four times those of M_2.

Equilibrium arguments:

$$V(36) = 8T - 8s + 8h$$

$$u(36) = 8\xi - 8\nu$$

Speed:

$$a(36) = 115.9364169 \text{ °/hour}$$

Node factor:

$$f(36) = [f(1)]^4$$

Name: $(MS)_4$ tide

Designation in USCGS Special Publication No. 98: Shallow Water Constituent

The $(MS)_4$ tide is a quarter diurnal shallow water compound constituent. Its origin is given by $M_2 + S_2$.

Equilibrium arguments:

$$V(37) = 4T - 2s + 2h$$

$$u(37) = 2\xi - 2\nu$$

Speed:

$$a(37) = 58.9841042\ ^\circ/\text{hour}$$

Node factor:

$$f(37) = f(1)$$

VII. COMPUTER PROGRAM

The computer program used in this thesis was written in the language of Microsoft Quickbasic 4.5. The code was developed to run on IBM-PC and compatible machines.

Quickbasic 4.5 automatically compiles before running the source code.

The compiled code runs efficiently, producing an entire tide curve for a day in a few seconds on a 386 machine with math coprocessor. The tide height is calculated for every minute of the day, producing 24 x 60 =1440 points on a day's graph.

The complete program listing is given in Appendix A.

VIII. CHECKING THE COMPUTER PROGRAM

The expected accuracy is plus or minus 5 minutes, or better, in the time of high or low water. The expected accuracy in tide height is plus or minus about 0.2 foot, formally. However, the actual tide height on any given day can differ from prediction because of the effects of wind, barometric pressure, and other meteorological variables.

The program was checked against the published tide tables of the NOS, as shown in the following table. Note that the present program evaluates astronomical factors 1440 times per day, while the NOS program evaluates certain factors (in particular, the nodal factors) only once per year, and uses those values for the entire year. Therefore, the present program should be as accurate, or more accurate, than the NOS program. Slight differences (plus or minus a few minutes, and plus or minus a few tenths of a foot) in the following table are attributable to the different algorithms.

COMPARISON OF PROGRAM PREDICTIONS WITH NOS TIDE TABLE
(for Boston, Massachusetts)

	Thesis Program		NOS Tide Table		
Date	Time	Height	Time	Height	
	hr min	feet	hr min	feet	
1/1/1992	01:59	1.233	01:59	1.2	Low Water
	08:22	10.048	08:22	10.1	High Water
	14:50	0.355	14:50	0.3	Low Water
	21:04	8.627	21:02	8.6	High Water
1/2/1992	02:50	1.265	02:50	1.2	Low Water
	09:11	10.095	09:09	10.1	High Water
	15:39	0.238	15:37	0.2	Low Water
	21:52	8.685	21:51	8.7	High Water
1/3/1992	03:37	1.222	03:36	1.2	Low Water
	09:56	10.153	09:55	10.2	High Water
	16:22	0.142	16:21	0.1	Low Water
	22:35	8.769	22:36	8.8	High Water
1/4/1992	04:21	1.141	04:20	1.1	Low Water
	10:38	10.203	10:37	10.2	High Water
	17:01	0.072	17:00	0.1	Low Water
	23:15	8.861	23:14	8.9	High Water
1/5/1992	05:02	1.055	05:01	1.0	Low Water
	11:17	10.221	11:17	10.2	High Water
	17:39	0.035	17:39	0.0	Low Water
	23:53	8.949	23:53	8.9	High Water
1/6/1992	05:43	0.990	05:43	1.0	Low Water
	11:56	10.184	11:55	10.2	High Water
	18:16	0.040	18:16	0.1	Low Water
1/7/1992	00:30	9.027	00:29	9.0	High Water
	06:23	0.963	06:23	1.0	Low Water
	12:34	10.072	12:34	10.1	High Water
	18:53	0.098	18:53	0.1	Low Water

IX. APPLICATION

The motivation for the present program was to produce tide calculations for use in several other projects at SWTSU.

Results from this program have already been used to calculate the time of high and low waters in Boston Harbor on April 18 and 19, 1775. April 18 was the night on which Paul Revere began his famous ride, and April 19 was the date of the battles at Lexington and Concord, which began the American Revolution.

Graphs of the Boston tides on those days and a preprint of a published article which included results from this program are included in Appendix D. The article appeared as "Paul Revere's Midnight Ride" in the April 1992 issue of *Sky & Telescope* magazine.[39]

X. CONCLUSION

A computer program was written based on the theoretical foundation delineated in Special Publication No. 98. The program's precision was enhanced by application of the techniques of Wallner. Careful verification of tide calculations from the program indicate that tide predictions can be extended, without appreciable accumulation of error, over a time interval of 2000 years.

The use of harmonic analysis in tide prediction is an excellent example of using astronomical theory and mathematical physics to produce results of interest.

APPENDIX A

Program Listing: TIDES

Microsoft QUICKBASIC 4.5

for IBM PC (or compatible)

```
'       TIDES
'
'  based on
'  MANUAL OF HARMONIC ANALYSIS AND PREDICTION OF TIDES
'  Special Publication No. 98
'  Coast and Geodetic Survey
'  Revised (1940) edition
'
   DEFDBL A-Z
   DIM AMPL(37), KAPPA(37), NAMECONSTIT$(37)
   DIM F(37), V(37), U(37)
   SCREEN 9: CLS
   PI = 4# * ATN(1#)
   TWOPI = 2# * PI

100 'READ IN PORT AND STATION CONSTANTS
   READ CITYNAME$
   READ LONGITUDEDEG, LONGITUDEMIN, LONGITUDEEW$
   READ LATITUDEDEG, LATITUDEMIN, LATITUDENS$
   READ TIMEMERIDIAN, TIMEMERIDIANEW$
   READ Z0, HTMIN, HTMAX
   FOR J = 1 TO 37
   READ AX, NAMECONSTIT$(J), AMPL(J), KAPPA(J)
   KAPPA(J) = KAPPA(J) * PI / 180#
   LONGITUDE = LONGITUDEDEG + LONGITUDEMIN / 60
   IF LONGITUDEEW$ = "E" THEN LONGITUDE = -LONGITUDE
   IF TIMEMERIDIAN$ = "E" THEN TIMEMERIDIAN = -TIMEMERIDIAN
   NEXT J
'  INCREASE CONSTITUENT AMPLITUDES FOR 1987 TIDE TABLES
'   (EXCEPT Sa and Ssa)
   FOR J = 1 TO 37
   IF J <> 21 AND J <> 22 THEN AMPL(J) = AMPL(J) * 1.024
   NEXT J

200 'INPUT Y/M/D FROM KEYBOARD
   GOSUB 10000    ' CALENDAR TO JD
   JDUT0HUT = JD
   TAPPROX = (Y - 1900) / 100
   DELT = (.4# + 1.2053# * TAPPROX + .5# * TAPPROX * TAPPROX) / 1440#
   JDET0HUT = JDUT0HUT + DELT

300 'ASTRONOMICAL DATA
   JDET = JDET0HUT
   T = (JDET - 2415020#) / 36525#
   GOSUB 20000 'COMPUTE S, H, P, P1, N
   GOSUB 30000 'COMPUTE N, I, NU, XI, ..., Q

     CLS
     PRINT "ASTRONOMICAL DATA"
     PRINT MO; "/"; DA; "/"; IY;
     PRINT TAB(50); USING "JDUT #######.####"; JDUT0HUT
     PRINT USING "TIME T = ###.#########"; T;
     PRINT TAB(50); USING "JDET #######.####"; JDET0HUT
     PRINT USING "          MEAN LONG. OF MOON (s)  ###.#### "; S * 180# / PI
     PRINT USING "          MEAN LONG. OF  SUN (h)  ###.#### "; H * 180# / PI
     PRINT USING "MEAN LONG. OF LUNAR PERIGEE (p)  ###.#### "; P * 180# / PI
```

```
PRINT USING "MEAN LONG. OF SOLAR PERIGEE (p1) ###.#### "; P1 * 180# / PI
PRINT USING "   LONG. OF LUNAR ASC. NODE (N)  ###.#### "; N * 180# / PI
PRINT
PRINT USING "         OBLIQUITY OF LUNAR ORBIT (I)       ###.#### "; I * 180# / PI
PRINT USING "            OBLIQUITY OF ECLIPTIC (OMEGA) ###.#### ";OMEGA * 180#/PI
PRINT USING "       INCLINATION OF LUNAR ORBIT (INCL)  ###.#### ";INCL * 180#/ PI
PRINT USING "    RT. ASCENSION OF INTERSECTION (NU)      ###.#### "; NU * 180# / PI
PRINT USING "   LONG. IN ORBIT OF INTERSECTION (XI)      ###.#### "; XI * 180# / PI
PRINT USING "TERM IN ARGUMENT OF K1 CONSTITUENT (NUP)   ###.#### "; NUP * 180#/PI
PRINT USING "TERM IN ARGUMENT OF K2 CONSTITUENT (NUPP)  ###.#### "; NUPP * 180#/PI
PRINT USING "MEAN LONG. LUNAR PERIGEE FROM INT. (CP)     ###.#### "; CP * 180# / PI
PRINT USING "FACTOR IN AMPL.  OF L2 CONSTITUENT (RA)     ###.#### "; RA
PRINT USING "TERM IN ARGUMENT OF L2 CONSTITUENT (R)      ###.#### "; R * 180# / PI
PRINT USING "FACTOR IN AMPL.  OF M1 CONSTITUENT (QA)     ###.#### "; QA
PRINT USING "TERM IN ARGUMENT OF M1 CONSTITUENT (QU)     ###.#### "; QU * 180# / PI
PRINT USING "TERM IN ARGUMENT OF M1 CONSTITUENT (Q)      ###.#### "; Q * 180# / PI

330  LOCATE 24, 1: PRINT "HARD COPY ASTRONOMICAL DATA ? (Y/N)";
   WHILE INKEY$ < > "": WEND
340  HCAD$ = INKEY$
   IF HCAD$ = "" THEN GOTO 340
   IF HCAD$ = "Y" OR HCAD$ = "y" THEN
     LPRINT "ASTRONOMICAL DATA"
     LPRINT MO; "/"; DA; "/"; IY;
     LPRINT TAB(50); USING "JDUT #######.####"; JDUT0HUT
     LPRINT USING "TIME T = ###.########"; T;
     LPRINT TAB(50); USING "JDET #######.####"; JDET0HUT
     LPRINT USING "         MEAN LONG. OF MOON (s)  ###.#### "; S * 180# / PI
     LPRINT USING "         MEAN LONG. OF  SUN (h)  ###.#### "; H * 180# / PI
     LPRINT USING "MEAN LONG. OF LUNAR PERIGEE (p) ###.#### "; P * 180# / PI
     LPRINT USING "MEAN LONG. OF SOLAR PERIGEE (p1) ###.#### "; P1 * 180# / PI
     LPRINT USING "   LONG. OF LUNAR ASC. NODE (N)  ###.#### "; N * 180# / PI
     LPRINT
     LPRINT USING "         OBLIQUITY OF LUNAR ORBIT (I)       ###.#### "; I * 180# / PI
     LPRINT USING "            OBLIQUITY OF ECLIPTIC (OMEGA) ###.#### ";OMEGA* 180#/PI
     LPRINT USING "       INCLINATION OF LUNAR ORBIT (INCL)  ###.#### ";INCL * 180#/PI
     LPRINT USING "    RT. ASCENSION OF INTERSECTION (NU)      ###.#### ";NU * 180#/PI
     LPRINT USING "   LONG. IN ORBIT OF INTERSECTION (XI)      ###.#### ";XI * 180#/PI
     LPRINT USING "TERM IN ARGUMENT OF K1 CONSTITUENT (NUP)   ###.#### ";NUP * 180#/PI
     LPRINT USING "TERM IN ARGUMENT OF K2 CONSTITUENT (NUPP)  ###.#### ";NUPP * 180#/PI
     LPRINT USING "MEAN LONG. LUNAR PERIGEE FROM INT. (CP)     ###.#### ";CP * 180# / PI
     LPRINT USING "FACTOR IN AMPL.  OF L2 CONSTITUENT (RA)     ###.#### ";RA
     LPRINT USING "TERM IN ARGUMENT OF L2 CONSTITUENT (R)      ###.#### ";R * 180# / PI
     LPRINT USING "FACTOR IN AMPL.  OF M1 CONSTITUENT (QA)     ###.#### ";QA
     LPRINT USING "TERM IN ARGUMENT OF M1 CONSTITUENT (QU)     ###.#### ";QU * 180# / PI
     LPRINT USING "TERM IN ARGUMENT OF M1 CONSTITUENT (Q)      ###.#### "; Q * 180# / PI
     LPRINT CHR$(12)
     GOTO 330
   END IF


500  'NODE FACTORS AND ARGUMENTS
   THA = PI - LONGITUDE * PI / 180#
   GOSUB 40000  'COMPUTE NODE FACTORS F(J), ARGUMENTS U(J)
   GOSUB 50000  'COMPUTE ARGUMENTS V(J)
```

```
550   CLS
      LL = 1: UL = 13
      PRINT "NODE FACTORS AND ARGUMENTS"
      PRINT CITYNAME$; "   ";
      PRINT LONGITUDEDEG; CHR$(248); LONGITUDEMIN; "'"; LONGITUDEEW$; " ";
      PRINT LATITUDEDEG; CHR$(248); LATITUDEMIN; "'"; LATITUDENS$
      PRINT MO; "/"; DA; "/"; IY; "  Oh UT"
      PRINT USING "JDUT #######.####"; JDUT0HUT
      PRINT USING "JDET #######.####"; JDET0HUT
      PRINT USING "LONG. OF LUNAR ASC. NODE (N)  ###.#### "; N * 180# / PI
      PRINT
      PRINT " J CONSTIT   F    AMPL  F*AMPL    V      U     V+U   KAPPA  V+U-K"
    FOR J = LL TO UL
      PRINT USING "## "; J;
      PRINT NAMECONSTIT$(J);
      PRINT USING "###.###"; F(J);
      PRINT USING "###.###"; AMPL(J);
      PRINT USING "###.###"; F(J) * AMPL(J);
      PRINT USING "####.##"; V(J) * 180 / PI;
      PRINT USING "####.##"; U(J) * 180 / PI;
      VPLUSU = V(J) + U(J)
      VPLUSU = VPLUSU - TWOPI * INT(VPLUSU / TWOPI)
      PRINT USING "####.##"; VPLUSU * 180 / PI;
      PRINT USING "####.##"; KAPPA(J) * 180 / PI;
      VUKAPPA = V(J) + U(J) - KAPPA(J)
      VUKAPPA = VUKAPPA - TWOPI * INT(VUKAPPA / TWOPI)
      PRINT USING "####.##"; VUKAPPA * 180 / PI
    NEXT J
      WHILE INKEY$ <> "": WEND
      PRINT : PRINT "(HIT SPACE BAR TO CONTINUE)";
      WHILE INKEY$ <> CHR$(32): WEND
      IF UL = 13 THEN LL = 14: UL = 26: GOTO 550
      IF UL = 26 THEN LL = 27: UL = 37: GOTO 550

560 LOCATE 23, 1: PRINT "HARD COPY NODE FACTORS AND ARGUMENTS? (Y/N)";
    WHILE INKEY$ <> "": WEND
570   HCNFA$ = INKEY$
      IF HCNFA$ = "" THEN GOTO 570
      IF HCNFA$ = "Y" OR HCNFA$ = "y" THEN
           LPRINT "NODE FACTORS AND ARGUMENTS"
           LPRINT CITYNAME$; "   ";
           LPRINT LONGITUDEDEG; CHR$(248); LONGITUDEMIN; "'"; LONGITUDEEW$; " ";
           LPRINT LATITUDEDEG; CHR$(248); LATITUDEMIN; "'"; LATITUDENS$
           LPRINT MO; "/"; DA; "/"; IY; "  Oh UT"
           LPRINT USING "JDUT #######.####"; JDUT0HUT
           LPRINT USING "JDET #######.####"; JDET0HUT
           LPRINT USING "LONG. OF LUNAR ASC. NODE (N)  ###.#### "; N * 180# / PI
           LPRINT
           LPRINT " J CONSTIT   F    AMPL  F*AMPL    V      U     V+U   KAPPA  V+U-K"
           FOR J = 1 TO 37
           LPRINT USING "## "; J;
           LPRINT NAMECONSTIT$(J);
           LPRINT USING "###.###"; F(J);
           LPRINT USING "###.###"; AMPL(J);
           LPRINT USING "###.###"; F(J) * AMPL(J);
           LPRINT USING "####.##"; V(J) * 180 / PI;
```

```
        LPRINT USING "####.##"; U(J) * 180 / PI;
        VPLUSU = V(J) + U(J)
        VPLUSU = VPLUSU - TWOPI * INT(VPLUSU / TWOPI)
        LPRINT USING "####.##"; VPLUSU * 180 / PI;
        LPRINT USING "####.##"; KAPPA(J) * 180 / PI;
        VUKAPPA = V(J) + U(J) - KAPPA(J)
        VUKAPPA = VUKAPPA - TWOPI * INT(VUKAPPA / TWOPI)
        LPRINT USING "####.##"; VUKAPPA * 180 / PI
        NEXT J
        LPRINT CHR$(12)
        GOTO 560
    END IF

600  'MENU FOR OUTPUT OF TIDE HEIGHTS
    CLS : HCHILO$ = "N": HCTHVST$ = "N"
    LOCATE 15, 1
    PRINT "SPACEBAR.....GRAPH ONLY"
    PRINT : PRINT "  1    .....GRAPH & HARD COPY HIGH AND LOW WATERS"
    PRINT : PRINT "  2    .....GRAPH & HARD COPY TIDE HEIGHT vs. TIME"
    PRINT "                          AND HIGH AND LOW WATERS"
    PRINT : PRINT "  3    . ...ANOTHER DATE"
    WHILE INKEY$ <> "": WEND
620  A$ = INKEY$
    IF A$ = CHR$(32) THEN GOTO 700
    IF A$ = "1" THEN HCHILO$ = "Y": GOTO 680
    IF A$ = "2" THEN HCTHVST$ = "Y": GOTO 650
    IF A$ = "3" THEN
        CLS
        LOCATE 15, 1
        PRINT "LAST DATE WAS: ";
        PRINT MO; "/"; DA; "/"; IY
        PRINT
        GOTO 200
    END IF
    GOTO 620
650  PRINT : INPUT "TIME STEP (in minutes) = "; TIMESTEP
680  'HEADER FOR HARD COPY OUTPUT (either 1 or 2 )
    LPRINT : LPRINT MO; "/"; DA; "/"; IY;
    LPRINT CITYNAME$; "  ";
    LPRINT "(TIME MERIDIAN "; TIMEMERIDIAN; CHR$(248); TIMEMERIDIANEW$; ")";
    LPRINT " (Z0"; : LPRINT USING "##.##"; Z0; : LPRINT ")"

700  'DRAW GRAPH
    'DRAW FRAME AND LABELS
    CLS
    XL = 100: XR = 400: YT = 25: YB = 215: LT = 4: EXTRCOL = 55
    LINE (XL, YT)-(XR, YB), , B
    FOR J = 0 TO 24
    XP = XL + (XR - XL) * (J / 24)
    LINE (XP, YB)-(XP, YB - LT): LINE (XP, YT)-(XP, YT + LT)
    IF J / 6 = INT(J / 6) THEN
        AX = 2 * LT
        LINE (XP, YB)-(XP, YB - AX): LINE (XP, YT)-(XP, YT + AX)
        XC = 80 * (XL / 640) + (J / 24) * (XR - XL) * (80 / 640)
        YC = 25 * (YB / 350)
        LOCATE YC + 2, XC - 1
```

```
        PRINT J; "h";
    END IF
    NEXT J
    FOR J = HTMIN TO HTMAX
    YP = YB - (YB - YT) * (J - HTMIN) / (HTMAX - HTMIN)
    LINE (XL, YP)-(XL + LT, YP): LINE (XR, YP)-(XR - LT, YP)
    IF (J / 5) = INT(J / 5) THEN
        AX = 2 * LT
        LINE (XL, YP)-(XL + AX, YP): LINE (XR, YP)-(XR - AX, YP)
        XC = 80 * XL / 640
        YC = 25 * YB/350 - 25*((YB - YT)/350) * (J - HTMIN)/(HTMAX - HTMIN)
        LOCATE YC + .5, XC - 5
        PRINT J;
    END IF
    NEXT J
    LOCATE 1, 24
    PRINT MO; "/"; DA; "/"; IY;
    LOCATE 21, 1
    PRINT CITYNAME$; "  ";
    PRINT LONGITUDEDEG; CHR$(248); LONGITUDEMIN; "'"; LONGITUDEEW$; " ";
    PRINT LATITUDEDEG; CHR$(248); LATITUDEMIN; "'"; LATITUDENS$;
    PRINT "  (TIME MERIDIAN "; TIMEMERIDIAN; CHR$(248); TIMEMERIDIANEW$; ")";
    PRINT " (Z0"; : PRINT USING "##.##"; Z0; : PRINT ")"
    'HIGH AND LOW WATERS TO SCREEN
    AY = .5 + .5 * (YT + YB) * 25 / 350
    AX = XL * 80 / 640
    LOCATE AY, .2 * AX
    PRINT "ft";
    AY = YT * 25 / 350
    LOCATE AY + 2, EXTRCOL
    PRINT "HIGH AND LOW WATERS";
    LOCATE AY + 4, EXTRCOL
    PRINT " h  m   ft";

1000 'TIDE HEIGHT VS. TIME
    HTPP = 0: HTP = 0
    LL = -1: UL = 1441
    FLAGMAX = TIMESTEP
    FLAG = FLAGMAX + LL
    JDUT0HLOCAL = JDUT0HUT + TIMEMERIDIAN / 360#
    FOR MINUTEOFDAY = LL TO UL STEP 1
    JDUT = JDUT0HLOCAL + MINUTEOFDAY / 1440#
    JDET = JDUT + DELT
    T = (JDET - 2415020#) / 36525#
    GOSUB 20000 'GIVEN: TIME T  COMPUTE: S, H, P, P1, N
    THA = PI + (MINUTEOFDAY * PI / 720#) + (TIMEMERIDIAN - LONGITUDE) * PI / 180#
    GOSUB 50000 'GIVEN: THA,S,H,P,P1  COMPUTE: V(J)
    'COMPUTE TIDE HEIGHT
    HT = Z0
    FOR J = 1 TO 37

    HT = HT + F(J) * AMPL(J) * COS(V(J) + U(J) - KAPPA(J))
    NEXT J
    'PLOT POINT ON GRAPH
    XP = XL + (XR - XL) * (MINUTEOFDAY / 1440#)
    YP = YB - (YB - YT) * (HT - HTMIN) / (HTMAX - HTMIN)
```

```
     PSET (XP, YP)

1500  'CHECK FOR HIGH OR LOW WATER
     IF MINUTEOFDAY < 1 THEN 1600
     EXTR$ = ""
     IF (HTP - HTPP) * (HT - HTP) < 0 AND (HT - HTP) < 0 THEN EXTR$ = "HIGH WATER": GOTO
1550
     IF (HTP - HTPP) * (HT - HTP) < 0 AND (HT - HTP) > 0 THEN EXTR$ = " LOW WATER": GOTO
1550
     GOTO 1600
1550  HOUREXTR = INT(.0001# + (MINUTEOFDAY - 1#) / 60#)
     MINEXTR = MINUTEOFDAY - 60# * HOUREXTR - 1
     HTEXTR = HTP
     HR$ = RIGHT$(STR$(HOUREXTR), 2)
     IF HOUREXTR < 10 THEN HR$ = "0" + RIGHT$(HR$, 1)
     MN$ = RIGHT$(STR$(MINEXTR), 2)
     IF MINEXTR < 10 THEN MN$ = "0" + RIGHT$(MN$, 1)
     Y = CSRLIN: LOCATE Y + 2, EXTRCOL
     PRINT HR$; ":"; MN$;
     PRINT USING "####.#"; HTEXTR;
     'LPRINT TIDE HEIGHT AND/OR HIGH AND LOW WATERS
     IF HCHILO$ = "Y" OR HCTHVST$ = "Y" THEN
          LPRINT HR$; ":"; MN$;
          LPRINT USING "####.###"; HTEXTR;
          LPRINT " <-- "; EXTR$
     END IF
1600  IF FLAG = FLAGMAX AND HCTHVST$ = "Y" THEN
          HOUR = INT(.0001# + (MINUTEOFDAY) / 60#)
          MIN = MINUTEOFDAY - 60# * HOUR
          HR$ = RIGHT$(STR$(HOUR), 2)
          IF HOUR < 10 THEN HR$ = "0" + RIGHT$(HR$, 1)
          MN$ = RIGHT$(STR$(MIN), 2)
          IF MIN < 10 THEN MN$ = "0" + RIGHT$(MN$, 1)
          LPRINT HR$; ":"; MN$;
          LPRINT USING "####.###"; HT
          FLAG = 0
     END IF

1980  HTPP = HTP: HTP = HT
     FLAG = FLAG + 1
1990  NEXT MINUTEOFDAY
     LOCATE 23, 1, 0
     WHILE INKEY$ <> "": WEND
2000  A$ = INKEY$
     IF A$ = CHR$(32) THEN GOTO 2100
     GOTO 2000
2100  PRINT "Press D for another date or ESC to quit";
2200  A$ = INKEY$
     IF A$ = CHR$(27) THEN GOTO 9000
     IF A$ = "D" THEN
          CLS
          LOCATE 15, 1
          PRINT "LAST DATE WAS: ";
          PRINT MO; "/"; DA; "/"; IY
          PRINT
          GOTO 200
```

```
      END IF
      GOTO 2200

9000   END

10000 'CALENDAR TO JD
     PRINT "INPUT CALENDAR DATE"
     WHILE INKEY$ <> "": WEND
     PRINT : INPUT "YEAR = "; A$
     IY = VAL(A$)
     IF IY <> INT(IY) THEN 10000
     PRINT : INPUT "MONTH (1-12) = "; A$: MO = VAL(A$)
     IF MO <> INT(MO) THEN 10000
     PRINT : INPUT "DAY = "; A$: DA = VAL(A$)
     IF DA <> INT(DA) THEN 10000
     D = 0   ' ZERO HOURS, MINUTES, AND SECONDS
     IF MO = 1 OR MO = 2 THEN Y = IY - 1: M = MO + 12
     IF MO > 2 THEN Y = IY: M = MO
     C = 0
     IF Y < 0 THEN C = -.75
     A = INT(.001 + Y / 100#)
     B = 2# - A + INT(.001 + A / 4#)
     IF IY < 1582 THEN B = 0
     IF IY = 1582 AND MO < 10 THEN B = 0
     IF IY = 1582 AND MO = 10 AND DA < 10 THEN B = 0
     AX = 365.25# * Y + C
     IF AX >= 0 THEN JD = INT(AX + .01#)
     IF AX < 0 THEN JD = INT(AX - .01#)
     JD = JD + INT(30.6001# * (M + 1))
     JD = JD + DA + D + 1720994.5# + B
     WHILE INKEY$ <> "": WEND
     RETURN

20000 'GIVEN: TIME T    COMPUTE: S, H, P, P1, N
     '                  (ASTRONOMICAL FACTORS)
     S = 270.434164# + 481267.8831# * T - .001133# * T * T + .0000019# * T * T * T
     S = S * PI / 180#
     S = S - TWOPI * INT(S / TWOPI)
     H = 279.69668# + 36000.76892# * T + .0003025# * T * T
     H = H * PI / 180#
     H = H - TWOPI * INT(H / TWOPI)
     P = 334.329556# + 4069.034# * T - .010325# * T * T - .0000125# * T * T * T
     P = P * PI / 180#
     P = P - TWOPI * INT(P / TWOPI)
     M = 358.47583# + 35999.04975# * T - .00015# * T * T - .0000033# * T * T * T
     M = M * PI / 180#
     M = M - TWOPI * INT(M / TWOPI)
     P1 = H - M
     P1 = P1 - TWOPI * INT(P1 / TWOPI)
     N = 259.183275# - 1934.142# * T + .002078# * T * T + .0000022# * T * T * T
     N = N * PI / 180#
     N = N - TWOPI * INT(N / TWOPI)
     RETURN

30000 'GIVEN: TIME T    COMPUTE: N, I, NU, XI, ..., Q
     '                  (ASTRONOMICAL FACTORS)
```

```
N = 259.183275# - 1934.142# * T + .002078# * T * T + .0000022# * T * T * T
N = N * PI / 180#
N = N - TWOPI * INT(N / TWOPI)
OMEGA = 23.452294# - .0130125# * T - .00000164# * T * T + .000000503# * T * T * T
INCL = 5.1453964#
OMEGA = OMEGA * PI / 180#: INCL = INCL * PI / 180#
CI = COS(INCL) * COS(OMEGA) - SIN(INCL) * SIN(OMEGA) * COS(N)
SI = SQR(1 - CI * CI)
I = ATN(SI / CI)
AX = ATN(COS(.5# * (OMEGA - INCL)) * TAN(N / 2#) / COS(.5# * (OMEGA + INCL)))
AY = ATN(SIN(.5# * (OMEGA - INCL)) * TAN(N / 2#) / SIN(.5# * (OMEGA + INCL)))
NU = AX - AY
XI = N - AX - AY
IF SIN(N) < 0 THEN XI = XI - TWOPI
NUP = ATN((SIN(2# * I) * SIN(NU)) / ((SIN(2# * I) * COS(NU)) + .3347#))
NUPP = .5# * ATN(SIN(I) ^ 2 * SIN(2# * NU) / (SIN(I) ^ 2 * COS(2# * NU) + .0727#))
P = 334.329556# + 4069.034# * T - .010325# * T * T - .0000125# * T * T * T
P = P * PI / 180#
P = P - TWOPI * INT(P / TWOPI)
CP = P - XI
CP = CP - TWOPI * INT(CP / TWOPI)
RA = 1# / SQR(1# - 12# * TAN(I/2#) * TAN(I/2#) * COS(2# * CP) + 36# * TAN(I/2#) ^ 4)
R = ATN(SIN(2# * CP) / ((1# / (6# * TAN(I / 2#) ^ 2) - COS(2# * CP))))
QA = 1#/SQR(.25#+1.5#*COS(I)*COS(2#*CP)/(COS(I/2#)^2)+2.25#*COS(I)^2/(COS(I/2#)^4)
QU = ATN(SIN(2# * CP) / (3# * COS(I) / (COS(I / 2#) ^ 2) + COS(2# * CP)))
Q = ATN((5# * COS(I) - 1#) * TAN(CP) / (7# * COS(I) + 1#))
IF COS(CP) < 0 THEN Q = Q + PI
Q = Q - TWOPI * INT(Q / TWOPI)
RETURN

40000 ' GIVEN: N,I,NU,XI,...,Q    COMPUTE: NODE FACTORS F(J), ARGUMENTS U(J)
  F(1) = (COS(I / 2#) / (COS(OMEGA / 2#) * COS(INCL / 2#))) ^ 4
  F(2) = 1#
  F(3) = F(1)
  SPRIME = .4602#: ECCM = .054900489#: ECCS = .01675104#
  AK1 = (.5# + .75# * ECCM ^ 2) * SIN(2# * I)
  BK1 = (.5# + .75# * ECCS ^ 2) * SPRIME * SIN(2# * OMEGA)
  CK1 = SQR(AK1 ^ 2 + 2# * AK1 * BK1 * COS(NU) + BK1 ^ 2)
  AVGCK1 = (.5# + .75 * ECCM^2) * SIN(2# * OMEGA) * (1# - 1.5# * SIN(INCL)^2) + BK1
  F(4) = CK1 / AVGCK1
  F(5) = F(1) * F(1)
  F(6) = SIN(I) * COS(I/2#)^2 / (SIN(OMEGA) * COS(OMEGA / 2#)^2 * COS(INCL / 2#)^4)
  F(7) = F(1) ^ 3
  F(8) = F(1) * F(4)
  F(9) = 1#
  F(10) = F(1) * F(1)
  F(11) = F(1)
  F(12) = 1#
  F(13) = F(1)
  F(14) = F(1)
  F(15) = SIN(I) * SIN(I/2#)^2 / (SIN(OMEGA) * SIN(OMEGA / 2#)^2 * COS(INCL / 2)^4)
  F(16) = F(1)
  F(17) = 1#
  F(18) = F(6) / QA
  F(19) = SIN(2# * I) / (SIN(2# * OMEGA) * (1# - 1.5# * SIN(INCL) ^ 2))
  F(20) = (2# / 3# - SIN(I)^2)/((2#/3# - SIN(OMEGA)^2)*(1# - 1.5# * SIN(INCL) ^ 2))
```

```
F(21) = 1#
F(22) = 1#
F(23) = F(20)
F(24) = SIN(I) ^ 2 / (SIN(OMEGA) ^ 2 * COS(INCL / 2#) ^ 4)
F(25) = F(6)
F(26) = F(6)
F(27) = 1#
F(28) = 1#
F(29) = F(6)
F(30) = 1#
F(31) = F(1)
F(32) = F(1) ^ 1.5
F(33) = F(1) / RA
F(34) = F(1) * F(1) * F(4)
SPRIME = .4602#: ECCM = .054900489#: ECCS = .01675104#
AK2 = (.5# + .75# * ECCM ^ 2) * SIN(I) ^ 2
BK2 = (.5# + .75# * ECCS ^ 2) * SPRIME * SIN(OMEGA) ^ 2
CK2 = SQR(AK2 ^ 2 + 2# * AK2 * BK2 * COS(2# * NU) + BK2 ^ 2)
AVGCK2 = (.5# + .75 * ECCM^2) * SIN(OMEGA)^2 * (1# - 1.5# * SIN(INCL) ^ 2) + BK2
F(35) = CK2 / AVGCK2
F(36) = F(1) ^ 4
F(37) = F(1)
U(1) = 2# * XI - 2# * NU
U(2) = 0#
U(3) = 2# * XI - 2# * NU
U(4) = -NUP
U(5) = 4# * XI - 4# * NU
U(6) = 2# * XI - NU
U(7) = 6# * XI - 6# * NU
U(8) = 2# * XI - 2# * NU - NUP
U(9) = 0#
U(10) = 4# * XI - 4# * NU
U(11) = 2# * XI - 2# * NU
U(12) = 0#
U(13) = 2# * XI - 2# * NU
U(14) = 2# * XI - 2# * NU
U(15) = -2# * XI - NU
U(16) = 2# * XI - 2# * NU
U(17) = 0#
U(18) = -NU - QU
U(19) = -NU
U(20) = 0#
U(21) = 0#
U(22) = 0#
U(23) = 0#
U(24) = -2# * XI
U(25) = 2# * XI - NU
U(26) = 2# * XI - NU
U(27) = 0#
U(28) = 0#
U(29) = 2# * XI - NU
U(30) = 0#
U(31) = -2# * XI + 2# * NU
U(32) = 3# * XI - 3# * NU
U(33) = 2# * XI - 2# * NU - R
U(34) = 4# * XI - 4# * NU + NUP
```

```
      U(35) = -2# * NUPP
      U(36) = 8# * XI - 8# * NU
      U(37) = 2# * XI - 2# * NU
      RETURN

50000  ' GIVEN: THA,S,H,P,P1        COMPUTE: ARGUMENTS V(J)
      V(1) = 2# * THA - 2# * S + 2# * H
      V(2) = 2# * THA
      V(3) = 2# * THA - 3# * S + 2# * H + P
      V(4) = THA + H - PI / 2#
      V(5) = 4# * THA - 4# * S + 4# * H
      V(6) = THA - 2# * S + H + PI / 2#
      V(7) = 6# * THA - 6# * S + 6# * H
      V(8) = 3# * THA - 2# * S + 3# * H - PI / 2#
      V(9) = 4# * THA
      V(10) = 4# * THA - 5# * S + 4# * H + P
      V(11) = 2# * THA - 3# * S + 4# * H - P
      V(12) = 6# * THA
      V(13) = 2# * THA - 4# * S + 4# * H
      V(14) = 2# * THA - 4# * S + 2# * H + 2# * P
      V(15) = THA + 2# * S + H - PI / 2#
      V(16) = 2# * THA - S + P + PI
      V(17) = THA
      V(18) = THA - S + H + P - PI / 2#
      V(19) = THA + S + H - P - PI / 2#
      V(20) = S - P
      V(21) = 2# * H
      V(22) = H
      V(23) = 2# * S - 2# * H
      V(24) = 2# * S
      V(25) = THA - 3# * S + 3# * H - P + PI / 2#
      V(26) = THA - 3# * S + H + P + PI / 2#
      V(27) = 2# * THA - H + P1
      V(28) = 2# * THA + H - P1 + PI
      V(29) = THA - 4# * S + H + 2# * P + PI / 2#
      V(30) = THA - H + PI / 2#
      V(31) = 2# * THA + 2# * S - 2# * H
      V(32) = 3# * THA - 3# * S + 3# * H
      V(33) = 2# * THA - S + 2# * H - P + PI
      V(34) = 3# * THA - 4# * S + 3# * H + PI / 2#
      V(35) = 2# * THA + 2# * H
      V(36) = 8# * THA - 8# * S + 8# * H
      V(37) = 4# * THA - 2# * S + 2# * H
      FOR J = 1 TO 37
      V(J) = V(J) - TWOPI * INT(V(J) / TWOPI)
      NEXT J
      RETURN

60000  DATA "BOSTON"
      DATA 71,3,"W"
      DATA 42,21.3,"N"
      DATA 75,"W"
      DATA 5.22,-4,14
      DATA 1,"M2    ",4.470,327.6
      DATA 2,"S2    ",.704,3.8
      DATA 3,"N2    ",.984,295.5
```

```
DATA 4,"K1    ",.480,134.8
DATA 5,"M4    ",.072,101.0
DATA 6,"O1    ",.380,115.3
DATA 7,"M6    ",.110,219.0
DATA 8,"(MK)2 ",.019,14.4
DATA 9,"S4    ",0,0
DATA 10,"(MN)4 ",.033,90.6
DATA 11,"NU2   ",.217,304.9
DATA 12,"S6    ",0,0
DATA 13,"MU2   ",.036,258.6
DATA 14,"(2N)2 ",.133,275.2
DATA 15,"(OO)1 ",.016,154.3
DATA 16,"LMBDA2",.068,4.3
DATA 17,"S1    ",0,0
DATA 18,"M1    ",.027,125.0
DATA 19,"J1    ",.031,140.3
DATA 20,"Mm    ",0,0
DATA 21,"Ssa   ",.054,96.2
DATA 22,"Sa    ",.085,142.6
DATA 23,"MSf   ",0,0
DATA 24,"Mf    ",0,0
DATA 25,"Rho1  ",.014,106.9
DATA 26,"Q1    ",.067,95.4
DATA 27,"T2    ",.063,337.1
DATA 28,"R2    ",.006,3.8
DATA 29,"(2Q)1 ",.010,95.8
DATA 30,"P1    ",.153,133.1
DATA 31,"(2SM)2",0,0
DATA 32,"M3    ",0,0
DATA 33,"L2    ",.190,18.8
DATA 34,"(2MK)2",.023,354.0
DATA 35,"K2    ",.202,5.9
DATA 36,"M8    ",.019,29.9
DATA 37,"(MS)4 ",.029,148.6
```

APPENDIX B

STATION SHEETS

BOSTON (MASSACHUSETTS)

AND

OTHER PORTS

FORM C&GS-444
(11-67)

U.S. DEPARTMENT OF COMMERCE
ESSA- COAST AND GEODETIC SURVEY

TIDES ~~CURRENTS~~ STANDARD HARMONIC CONSTANTS FOR PREDICTION

STATION 844-3970 Boston, MA.

Lat. 42° 21.3' N
Long. 71° 3.0' W
Long. 71° .05

COMPONENT	H AMPLITUDE	κ EPOCH	A κ' − κ	B 1.024×H	C 360° − κ'	D − κ'
	ft.	°	°	ft.	°	°
M_2	4.470	327.6	−2.8	4.577	+	−324.8
S_2	.704	3.8	−7.9	.721	+	−355.9
N_2	.984	295.5	−0.1	1.008	+	−295.4
K_1	.480	134.8	−4.2	.492	+	−130.6
M_4	.072	101.0	−5.6	.074	+	−95.4
O_1	.380	115.3	1.3	.389	+	−116.6
M_6	.110	219.0	−8.5	.113	+	−210.5
$(MK)_3$	.019	14.4	−7.0	.019	+	−7.4
S_4					+	−
$(MN)_4$	.033	90.6	−2.9	.034	+	−87.7
ν_2	.217	304.9	−.5	.222	+	−304.4
S_6					+	−
μ_2	.036	258.6	2.3	.037	+	−260.9
$(2N)_2$	.133	275.2	2.6	.136	+	−277.8
$(OO)_1$	(.016	154.3)	−9.6	.016	+	−144.7
λ_2	.068	4.3	−5.2	.070	+	−359.1
S_1					+	−
M_1	(.027	125.0	−1.4	.028	+	−123.6
J_1	.031	140.3	−6.9	.032	+	−133.4
Mm					+	−
Ssa	.054	96.2	−.4	.054	+	−95.8
Sa	.085	142.6	−.2	.085	+	−142.4
MSf					+	−
Mf					+	−
ρ_1	(.014	106.9)	3.7	.014	+	−110.6
Q_1	.067	95.4	4.1	.069	+	−99.5
T_2	.063	337.1	−7.7	.065	+	−329.4
R_2	(.006	3.3)	−8.1	.006	+	−355.7
$(2Q)_1$	(.010	95.8)	6.8	.010	+	−102.6
P_1	.153	133.1	−3.7	.157	+	−129.4
$(2SM)_2$					+	−
M_3					+	−
L_2	.190	18.8	−5.5	.195	+	−13.3
$(2MK)_3$	.023	354.0	−1.5	.024	+	−352.5
K_2	.202	5.9	−8.3	.207	+	−357.6
M_8	.019	29.9	−11.3	.019	+	−18.6
$(MS)_4$	.029	148.6	−10.7	.030	+	−137.9

REMARKS

Time meridian 75° W − +5 h.
Extreme range ______ ft./kn.
Dial ______
Marigram gear ______
Marigram scale ______
Z_0 4.88 (60-78) ft.
Permanent current ______ kn.
The DATUM is ~~a plane~~ ______ ft. below mean ~~low water springs~~ ~~lower~~ low water

Not used in the 1987 tide tables.

starting in 1989 tables:
Z_0 = 5.22
for a datum of
mean lower low water

Source of constants mean of six NOS one year L.S.H.A. from 1979-1983
Sa and Ssa from 20 years monthly MSL (1964-83)

Compiled by GF 2/5/85 (Date) Verified by [illegible] 2/7/85 (Date)

USCOMM-DC 50140-P67

FORM C&GS-444
(11-67)

U.S. DEPARTMENT OF COMMERCE
ESSA- COAST AND GEODETIC SURVEY

TIDES ~~CURRENTS~~ STANDARD HARMONIC CONSTANTS FOR PREDICTION

STATION Los Angeles (Outer Harbor), California ✓

Lat. 33° 43' N.
Long. 118° 16' W.
Long. 118° 27 W.

COMPONENT	H AMPLITUDE ft.	κ EPOCH °	A κ'−κ °	B XH ft.	C 360°−κ' °	D −κ' °
M_2	1.688	270.1	+4.7		+	−274.8
S_2	0.665	264.3	−3.5		+	−260.8
N_2	0.392	248.0	+9.0		+	−257.0
K_1	1.112	89.6	−2.1		+	−87.5
M_4					+	−
O_1	0.709	74.4	+6.7		+	−81.1
M_6					+	−
$(MK)_3$					+	−
S_4					+	−
$(MN)_4$					+	−
ν_2	0.083	256.4	+8.4		+	−264.8
S_6					+	−
μ_2	0.052	213.5	+12.8		+	−226.3
$(2N)_2$	0.052	225.9	+13.4		+	−239.3
$(OO)_1$	0.030	104.8	−10.8		+	−94.0
λ_2	0.012	267.4	+0.9		+	−268.3
S_1					+	−
M_1	0.041	83.6	+2.3		+	−85.9
J_1	0.056	97.2	−6.4		+	−90.8
Mm					+	−
Ssa	0.047	274.0	−0.7		+	−273.3
Sa	0.213	185.9	−0.3		+	−185.6
MSf					+	−
Mf					+	−
ρ_1	0.027	67.9	+10.5		+	−78.4
Q_1	0.127	65.7	+11.1		+	−76.8
T_2	0.039	264.3	−3.1		+	−261.2
R_2					+	−
$(2Q)_1$	0.018	59.2	+15.4		+	−74.6
P_1	0.356	86.0	−1.4		+	−84.6
$(2SM)_2$					+	−
M_3					+	−
L_2	0.032	253.0	+0.3		+	−253.3
$(2MK)_3$					+	−
K_2	0.196	259.4	−4.1		+	−255.3
M_8					+	−
$(MS)_4$					+	−

REMARKS

Time meridian 120° W. = +8 h.
Extreme range ___ ft. / kn.
Dial ___
Marigram gear ___
Marigram scale ___
Z_0 2.84 ~~2.81~~ ft.
Permanent current ___ kn.
The DATUM is ~~a plane~~ ___ ~~ft.~~ ~~below~~ mean ~~low water springs~~ lower low water

First used 1947 Tables (modified 1966)

1965 Z_0 = 2.84

ν_2 from 1930 series only

Source of constants From 3 - 369 day series (1930, 1938, 1940)
Sa & Ssa from 20 years (1924-1943)

Compiled by RCU 2-24-69 (Date) Verified by ___ (Date)

USCOMM-DC 50160-P67

FORM C&GS-444
(11-67)

U.S. DEPARTMENT OF COMMERCE
ESSA-COAST AND GEODETIC SURVEY

TIDES ~~CURRENTS~~ STANDARD HARMONIC CONSTANTS FOR PREDICTION

STATION San Diego, California

Lat. 32° 43.'0 N.
Long. 117° 10.'4 W.
Long. 117° .17 W.

Component	H Amplitude	κ Epoch	A κ′−κ	B 1.05×H	C 360°−κ′	D −κ′
	ft.	°	°	ft.	°	°
M_2	1.800	274.6	+2.5	1.890	+	−277.1
S_2	0.728	271.6	−5.7	0.764	+	−265.9
N_2	0.424	254.4	+6.8	0.445	+	−261.2
K_1	1.100	93.4	−3.2	1.155	+	−90.2
M_4					+	−
O_1	0.695	78.5	+5.6	0.730	+	−84.1
M_6					+	−
$(MK)_3$					+	−
S_4					+	−
$(MN)_4$					+	−
ν_2	0.084	260.0	+6.2	0.088	+	−266.2
S_6					+	−
μ_2	0.031	235.4	+10.6	0.033	+	−246.0
$(2N)_2$	0.056	234.4	+11.2	0.059	+	−245.6
$(OO)_1$	0.030	108.2	−11.9	0.032	+	−96.3
λ_2	0.013	273.2	−1.3	0.014	+	−271.9
S_1					+	−
M_1	0.035	77.1	+1.2	0.037	+	−78.3
J_1	0.055	100.8	−7.5	0.058	+	−93.3
Mm					+	−
Ssa	0.055	272.8	−0.7	0.055	+	−272.1
Sa	0.206	182.0	−0.3	0.206	+	−181.7
MSf					+	−
Mf					+	−
ρ_1	0.026	72.1	+9.4	0.027	+	−81.5
Q_1	0.128	71.6	+10.0	0.134	+	−81.6
T_2	0.043	271.6	−5.3	0.045	+	−266.3
R_2					+	−
$(2Q)_1$	0.018	63.6	+14.3	0.019	+	−77.9
P_1	0.341	91.7	−2.5	0.358	+	−89.2
$(2SM)_2$					+	−
M_3					+	−
L_2	0.040	247.4	−1.9	0.042	+	−245.5
$(2MK)_3$					+	−
K_2	0.204	265.0	−6.3	0.214	+	−258.7
M_8					+	−
$(MS)_4$					+	−

REMARKS

Time meridian 120° W. = +8 h.
Extreme range ______ ft. / kn.
Dial ______
Marigram gear ______
Marigram scale ______
Z_0 2.94 ~~2.93~~ ft.
Permanent current ______ kn.
The DATUM is ~~a plane~~ ______ ~~ft.~~ ~~below~~ mean ~~low water springs~~ / lower low water

First used 1943 Tables (modified 1966)

Amplitudes of short period constituents increased 5%.

M_2 from 1936 series only

New 1960-78 Z_0 first used in 1985 T.T.

Source of constants From 2-369 day series (1932 and 1936)
Sa & Ssa from 12 years monthly MSL (1922-1938)

Compiled by [signature] 2-24-69 (Date) Verified by ______ (Date)

USCOMM-DC 60160-P67

accepted 10-20-76
dcs

FORM C&GS-444
(11-67)

U.S. DEPARTMENT OF COMMERCE
ESSA-COAST AND GEODETIC SURVEY

#1813 TIDES ~~CURRENTS~~ STANDARD HARMONIC CONSTANTS FOR PREDICTION

(Golden Gate),
STATION San Francisco, California

Lat. 37° 48.4' N
Long. 122° 27.9' W
Long. 122° .46

	COMPONENT	H AMPLITUDE ft.	κ EPOCH °	A κ′−κ °	B 1.002×H ft.	C 360°−κ′ °	D −κ′ °
1	M_2	1.860	324.7	13.0	1.864	+	−337.7
2	S_2	0.446	330.9	4.9	0.447	+	−335.8
3	N_2	0.404	298.6	17.4	0.405	+	−316.0
4	K_1	1.218	103.6	2.1	1.220	+	−105.7
5	M_4	0.092	13.1	26.1	0.092	+	−39.2
6	O_1	0.756	86.5	10.9	0.758	+	−97.4
7	M_6					+	−
8	$(MK)_3$	0.073	124.4	15.2	0.073	+	−139.6
9	S_4					+	−
10	$(MN)_4$	0.031	344.2	30.4	0.031	+	−14.6
11	ν_2	0.076	300.2	16.8	0.076	+	−317.0
12	S_6					+	−
13	μ_2					+	−
14	$(2N)_2$	0.046	265.4	21.8	0.046	+	−287.2
15	$(OO)_1$	0.038	132.2	−6.6	0.038	+	−125.6
16	λ_2	(0.013	327.6)	9.3	0.013	+	−336.9
17	S_1					+	−
18	M_1	0.042	120.0	6.5	0.042	+	−126.5
19	J_1	0.058	115.8	−2.2	0.058	+	−113.6
20	Mm			[illegible]		+	−
21	Ssa	0.095	259.9	−0.7	0.095	+	−257.2
22	Sa	0.159	223.9	−0.3	0.159	+	−223.6
23	Msf	0.057	178.6	−8.1	0.057	+	−170.5
24	Mf	0.090	181.4	−8.8	0.090	+	−172.6
25	ρ_1	(0.029	79.1)	14.7	0.029	+	−93.8
26	Q_1	0.137	79.6	15.3	0.137	+	−94.9
27	T_2	(0.026	330.9)	5.2	0.026	+	−336.1
28	R_2					+	−
29	$(2Q)_1$	(0.020	69.4)	19.6	0.020	+	−89.0
30	P_1	0.389	99.2	2.8	0.390	+	−102.0
31	$(2SM)_2$					+	−
32	M_3					+	−
33	L_2	0.043	341.7	8.7	0.043	+	−350.4
34	$(2MK)_3$	0.048	99.6	24.0	0.048	+	−123.6
35	K_2	0.125	320.0	4.3	0.125	+	−324.3
36	M_8					+	−
37	$(MS)_4$	0.039	25.1	18.0	0.039	+	−43.1

REMARKS

Time meridian 120° W. +8 h.
Extreme range ft. / kn.
Dial
Marigram gear
Marigram scale
Z_0 ~~3.05~~ 3.13 ft.
Permanent current kn.
The DATUM is ~~ft. below~~ mean { ~~low water springs~~ / lower low water

Use beginning with 1979 [illegible] since 1977 [illegible] [illegible] [illegible]

New 1960-78 Z_0 first used in 1985 T.T.

Source of constants NOS H.A. of 365 days of tide observations beginning 74-1-1-0
Sa and Ssa from 19 years of [illegible] analysis 1956-1974.

Compiled by J.E.M. 8/19/76 (Date) Verified by [illegible] 8/20/76 (Date)

USCOMM-DC 50160-P67

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 444
Ed. May 1932

TIDES ~~CURRENTS~~ STANDARD HARMONIC CONSTANTS FOR PREDICTION

STATION *TARAWA, Gilbert Islands*

Lat. *1° 22' N*
Long. *172° 56' E*
Long. *172°.93 E*

COMPONENT	H AMPLITUDE (ft. kn.)	κ EPOCH (°)	A κ′ − κ (°)	B 4×H/1.03 (ft. kn.)	C 360° − κ′ (°)	D − κ′ (°)
M_2	1.944	125.3	+2.0	8.01	+232.7	−127.3
S_2	1.021	140.2	+14.1	4.21	+205.7	−154.3
N_2	0.405	123.3	−4.6	1.67	+241.3	−118.7
K_1	0.309	244.2	+7.6	1.27	+108.2	−251.8
M_4	0.005	134.7	~~+3.9~~	~~0.02~~	~~+221.4~~	~~−138.6~~
O_1	0.186	212.8	−5.6	0.77	+152.8	−207.2
M_6	~~0.011~~	~~280.2~~	~~+5.8~~	~~0.05~~	~~+ 74.0~~	~~−286.0~~
$(MK)_3$					+	−
S_4					+	−
$(MN)_4$					+	−
ν_2	(0.079	123.6)	−3.7	0.33	+240.1	−119.9
S_6					+	−
μ_2	~~(0.047~~	~~110.4)~~			+	−
$(2N)_2$	(0.054	121.3)	−11.1	0.22	+249.8	−110.2
$(OO)_1$					+	−
λ_2					+	−
S_1					+	−
M_1					+	−
J_1					+	−
Mm					+	−
Ssa					+	−
Sa					+	−
MSf					+	−
Mf					+	−
ρ_1					+	−
Q_1	(0.036	197.1)	−12.2	0.15	+175.1	−184.9
T_2	~~0.060~~	~~140.2~~			+	−
R_2					+	−
$(2Q)_1$					+	−
P_1	(0.102	244.2)	+6.6	0.42	+109.2	−250.8
$(2SM)_2$					+	−
M_3					+	−
L_2	(0.054	122.3)	+8.5	0.22	+224.2	−135.8
$(2MK)_3$					+	−
K_2	(0.278	140.2)	+15.1	1.15	+204.7	−155.3
M_8					+	−
$(MS)_4$					+	−

REMARKS

Time meridian *180° E* = *−12* h.
Extreme range ______ ft./kn.
Dial *10*
Marigram gear *3:2*
Marigram scale *1:20*
Z_0 *3.3* ft.
~~Permanent current~~ ______ ~~kn.~~
The DATUM is a plane *0.2* ft. below mean low water springs ~~lower low water~~

These constants first used for 1947 predictions.

Source of constants *analysis of 134-day series beginning 0h Dec. 1, 1943. Observations by U.S. Navy.*

Compiled by *L.E.W.* *4-17-46* (Date) Verified by *[illegible]* *4-18-46* (Date)

U. S. GOVERNMENT PRINTING OFFICE 595250

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 444

TIDES: STANDARD HARMONIC CONSTANTS FOR PREDICTION

STATION Port Adelaide (~~Government~~ [illegible] Wharf) Australia

Lat. 34° 51' S
Long. 138° 30' E
Long. 138.50

Component	H Amplitude	κ Epoch	A κ′−κ	B 4×H	C 360°−κ′	D −κ′
	ft.	°	°	ft.	°	°
M_2	1.70	120.0	−1.6	6.80	+241.6	−118.4
S_2	1.68	181.0	+8.0	6.72	+171.0	−189.0
N_2	0.09	246.1	−6.8	0.36	+120.8	−239.2
K_1	0.83	52.5	+4.4	3.32	+303.1	−56.9
M_4	0.02	173.5	−3.3	0.08	+189.8	−170.2
O_1	0.52	32.0	−6.0	2.08	+334.0	−26.0
M_6	0.01	259.0	−5.0	0.04	+106.0	−254.0
$(MK)_3$	—	—	—	—	+ —	− —
S_4	0.03	186	+16.0	0.12	+158.0	−202.0
$(MN)_4$	—	—	—	—	+ —	—
ν_2	0.06	76	−6.1	0.24	+298.1	−69.9
S_6	0.01	180	+24.0	0.04	+156.0	−204.0
μ_2	0.28	226	−11.3	1.12	+145.3	−214.7
$(2N)_2$	0.012	12.7	−12.0	0.05	+360	− 0
$(OO)_1$	0.026	73	+14.8	0.10	+272.2	−87.8
λ_2	0.012	148	+ 28	0.05	+209.2	−150.8
S_1	0.07	115.0	+4.0	0.28	+241.0	−119.0
M_1	0.02	16.0	−0.8	0.08	+344.8	−15.2
J_1	0.05	65	+9.6	0.20	+285.4	−74.6
Mm	0.07	200	+5.2	0.28	+154.8	−205.2
Ssa	0.225	88.5	+0.8	0.90	+270.7	−89.3
Sa	0.305	126.5	+0.4	1.22	+233.1	−126.9
MSf	0.12	255	+9.6	0.48	+ 95.4	−264.6
Mf	0.06	181	+10.4	0.24	+168.6	−191.4
ρ_1	0.020	23	−10.5	0.08	+347.5	−12.5
Q_1	0.07	31	−11.2	0.28	+340.2	−19.8
T_2	0.11	165	+7.6	0.44	+187.4	−172.6
R_2	0.04	138	+8.4	0.16	+213.6	−146.4
$(2Q)_1$	0.014	12	−16.4	0.06	+ 4.4	−355.6
P_1	0.22	56.5	+3.6	0.88	+299.9	−60.1
$(2SM)_2$	0.10	67	+12.6	0.40	+275.4	−84.6
M_3	0.06	99	−2.5	0.24	+263.5	−96.5
L_2	0.12	140	+3.5	0.48	+216.5	−143.5
$(2MK)_3$	—	—	—	—	+ —	—
K_2	0.465	178	+8.8	1.86	+173.2	−186.8
M_8	—	—	—	—	+ —	− —
$(MS)_4$	0.09	99	+6.4	0.36	+254.6	−105.4

REMARKS.

Time meridian 142 30 = 9.500 h. E
Approximate extreme range 8.6 ft.
Height dial 10 ft.
Marigram gear 3:2
Marigram scale 1:20

The DATUM is
a plane 1.00 ft. { ~~above~~ below
Mean { Low Water Springs ~~Lower Low Water~~
which is ~~[illegible]~~ * 4.48 ft. below
mean sea level, and is
approximately the datum of
the { ~~Coast and Geodetic Survey~~ Admiralty, ~~French, German~~ } charts.

* Datum changed to agree with Admiralty [illegible]
Time meridian 142° 30' E
first used in Tide Tables of 1924.

† [illegible] Statements to be furnished by Admiralty.

The constants are from a paper by R. W. Chapman and Capt. Inglis before Australasian Association Advancement Science Dec 13, 1895.

Harmonic constants are from hourly ordinates for 1 year beginning Mar 1, 1889 and 1 year beginning Jan 1, 1893.

Compiled by [illegible] Feb 8/1922 Date Verified by [illegible] 3/15/22 Date

11—5634

APPENDIX C

SAMPLE OUTPUT FROM COMPUTER PROGRAM

WITH GRAPHICAL RESULTS

In this section are some examples of the output of the computer program. The output includes graphs of the daily tide curve with a listing of high and low waters, listings of astronomical data, node factors and arguments, and times and tide heights (with high and low waters indicated).

The examples are chosen to elucidate the effects on the tide of different astronomical configurations. Also included is a one-month tide curve. In addition, the responses of different ports to the same astronomical configuration are shown.

The last illustration of this appendix is taken from the NOS publication *Our Restless Tides*. It shows some of the individual main semidiurnal and diurnal tidal constituents which combine to form the complete tidal prediction.

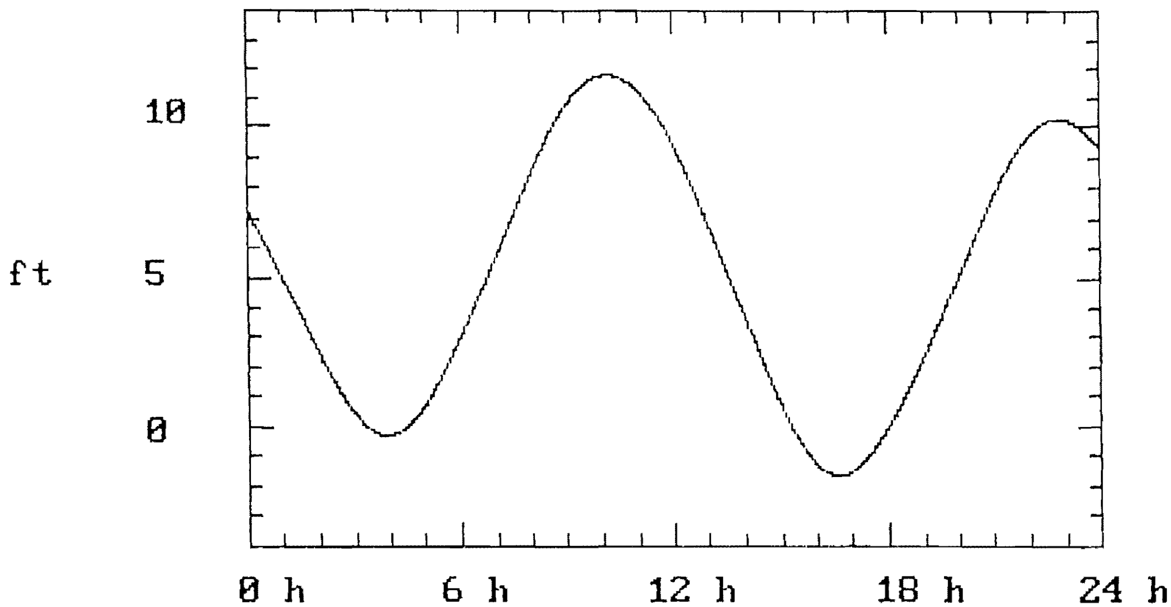
Output From Computer Program

PLACE: Boston Harbor

DATE: January 19, 1992

COMMENTS: Note that an augmented tide range [(+11.8) - (-1.6) = 13.4 feet] occurs because both a full moon and a lunar perigee fall on this date.

1 / 19 / 1992



HIGH AND LOW WATERS

h m	ft
03:54	-0.3
10:08	11.8
16:37	-1.6
22:49	10.2

BOSTON 71 ° 3 'W 42 ° 21.3 'N (TIME MERIDIAN 75 °W) (Z0 5.22)

ASTRONOMICAL DATA
1 / 19 / 1992 JDUT 2448640.5000
TIME T = 0.92047916 JDET 2448640.5013

MEAN LONG. OF MOON	(s)	107.4901
MEAN LONG. OF SUN	(h)	297.6545
MEAN LONG. OF LUNAR PERIGEE	(p)	119.7818
MEAN LONG. OF SOLAR PERIGEE	(p1)	282.8037
LONG. OF LUNAR ASC. NODE	(N)	278.8476

OBLIQUITY OF LUNAR ORBIT	(I)	24.7298
OBLIQUITY OF ECLIPTIC	(OMEGA)	23.4403
INCLINATION OF LUNAR ORBIT	(INCL)	5.1454
RT. ASCENSION OF INTERSECTION	(NU)	-12.2295
LONG. IN ORBIT OF INTERSECTION	(XI)	-11.1726
TERM IN ARGUMENT OF K1 CONSTITUENT	(NUP)	-8.4979
TERM IN ARGUMENT OF K2 CONSTITUENT	(NUPP)	-8.6725
MEAN LONG. LUNAR PERIGEE FROM INT.	(CP)	130.9544
FACTOR IN AMPL. OF L2 CONSTITUENT	(RA)	0.9268
TERM IN ARGUMENT OF L2 CONSTITUENT	(R)	-15.3414
FACTOR IN AMPL. OF M1 CONSTITUENT	(QA)	0.6921
TERM IN ARGUMENT OF M1 CONSTITUENT	(QU)	-20.0343
TERM IN ARGUMENT OF M1 CONSTITUENT	(Q)	150.9886

NODE FACTORS AND ARGUMENTS
BOSTON 71 ° 3 'W 42 ° 21.3 'N
1 / 19 / 1992 0h UT
JDUT 2448640.5000
JDET 2448640.5013
LONG. OF LUNAR ASC. NODE (N) 278.8476

J	CONSTIT	F	AMPL	F*AMPL	V	U	V+U	KAPPA	V+U-K
1	M2	0.994	4.577	4.552	238.23	2.11	240.34	327.60	272.74
2	S2	1.000	0.721	0.721	217.90	0.00	217.90	3.80	214.10
3	N2	0.994	1.008	1.002	250.52	2.11	252.63	295.50	317.13
4	K1	1.032	0.492	0.507	316.60	8.50	325.10	134.80	190.30
5	M4	0.989	0.074	0.073	116.46	4.23	120.69	101.00	19.69
6	O1	1.051	0.389	0.409	281.62	-10.12	271.51	115.30	156.21
7	M6	0.983	0.113	0.111	354.69	6.34	1.03	219.00	142.03
8	(MK)2	1.026	0.019	0.020	194.83	10.61	205.45	14.40	191.05
9	S4	1.000	0.000	0.000	75.80	0.00	75.80	0.00	75.80
10	(MN)4	0.989	0.034	0.033	128.75	4.23	132.98	90.60	42.38
11	NU2	0.994	0.222	0.221	246.27	2.11	248.38	304.90	303.48
12	S6	1.000	0.000	0.000	293.70	0.00	293.70	0.00	293.70
13	MU2	0.994	0.037	0.037	258.56	2.11	260.67	258.60	2.07
14	(2N)2	0.994	0.136	0.135	262.81	2.11	264.93	275.20	349.73
15	(OO)1	1.173	0.016	0.019	171.58	34.57	206.16	154.30	51.86
16	LMBDA2	0.994	0.070	0.069	50.19	2.11	52.31	4.30	48.01
17	S1	1.000	0.000	0.000	108.95	0.00	108.95	0.00	108.95
18	M1	1.518	0.028	0.042	328.90	32.26	1.16	125.00	236.16
19	J1	1.054	0.032	0.033	304.31	12.23	316.54	140.30	176.24
20	Mm	0.979	0.000	0.000	347.71	0.00	347.71	0.00	347.71
21	Ssa	1.000	0.054	0.054	235.31	0.00	235.31	96.20	139.11
22	Sa	1.000	0.085	0.085	297.65	0.00	297.65	142.60	155.05
23	MSf	0.979	0.000	0.000	339.67	0.00	339.67	0.00	339.67
24	Mf	1.110	0.000	0.000	214.98	22.35	237.33	0.00	237.33
25	Rho1	1.051	0.014	0.015	289.66	-10.12	279.55	106.90	172.65
26	Q1	1.051	0.069	0.072	293.92	-10.12	283.80	95.40	188.40
27	T2	1.000	0.065	0.065	203.05	0.00	203.05	337 10	225.95
28	R2	1.000	0.006	0.006	52.75	0.00	52.75	3.80	48.95
29	(2Q)1	1.051	0.010	0.011	306.21	-10.12	296.09	95.80	200.29
30	P1	1.000	0.157	0.157	261.30	0.00	261.30	133.10	128.20
31	(2SM)2	0.994	0.000	0.000	197.57	-2.11	195.46	0.00	195.46
32	M3	0.992	0.000	0.000	177.34	3.17	180.51	0.00	180.51
33	L2	1.073	0.195	0.209	45.94	17.46	63.39	18.80	44.59
34	(2MK)2	1.020	0.024	0.024	159.85	-4.27	155.58	354.00	161.58
35	K2	1.061	0.207	0.220	93.21	17.35	110.55	5.90	104.65
36	M8	0.978	0.019	0.019	232.92	8.46	241.37	29.90	211.47
37	(MS)4	0.994	0.030	0.030	96.13	2.11	98.24	148.60	309.64

1 / 19 / 1992 BOSTON (TIME MERIDIAN 75 °W) (Z0 5.22)

00:00	7.247	
01:00	4.940	
02:00	2.475	
03:00	0.436	
03:54	-0.293	<-- LOW WATER
04:00	-0.284	
05:00	0.753	
06:00	3.003	
07:00	5.730	
08:00	8.520	
09:00	10.779	
10:00	11.774	
10:08	11.790	<-- HIGH WATER
11:00	11.235	
12:00	9.398	
13:00	6.721	
14:00	3.660	
15:00	0.711	
16:00	-1.279	
16:37	-1.641	<-- LOW WATER
17:00	-1.497	
18:00	-0.054	
19:00	2.308	
20:00	5.090	
21:00	7.822	
22:00	9.719	
22:49	10.230	<-- HIGH WATER
23:00	10.207	
24:00	9.281	

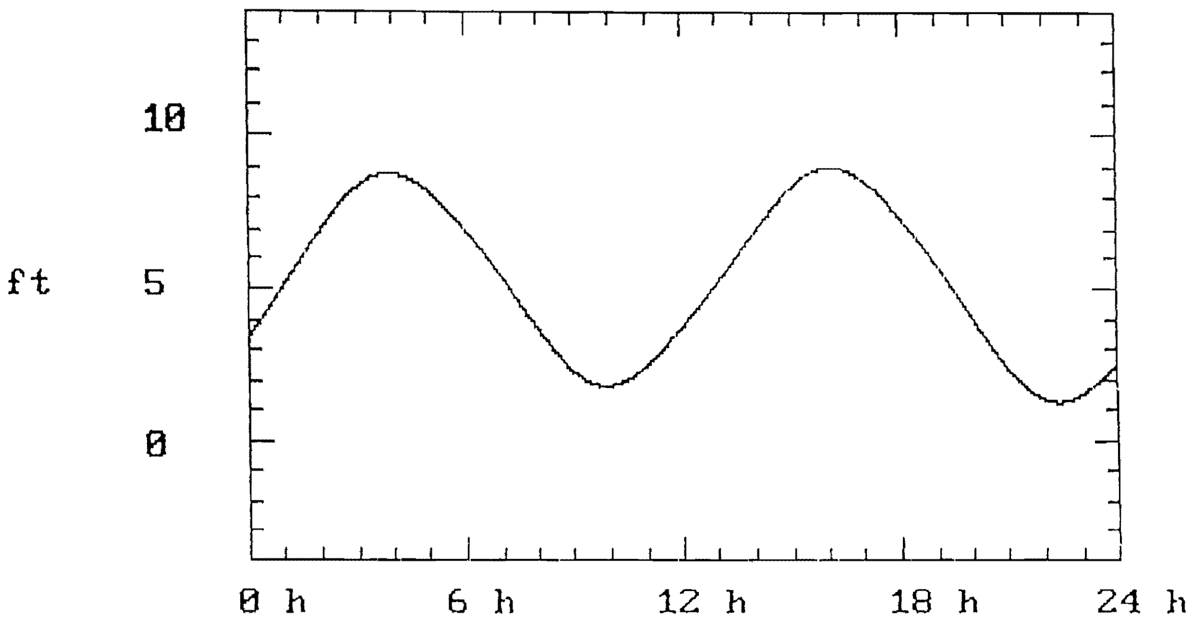
Output From Computer Program

PLACE: Boston Harbor

DATE: December 1, 1992

COMMENTS: Note that a decreased tide range [(+8.8) - (+1.8) = 7.0 feet] occurs because a first-quarter moon and a lunar apogee fall near this date.

12 / 1 / 1992



HIGH AND LOW WATERS

h m	ft
03:53	8.8
09:53	1.8
16:06	9.0
22:23	1.3

BOSTON 71 ° 3 'W 42 ° 21.3 'N (TIME MERIDIAN 75 °W) (Z0 5.22)

ASTRONOMICAL DATA

12 / 1 / 1992 JDUT 2448957.5000

TIME T = 0.92915815 JDET 2448957.5013

MEAN LONG. OF MOON (s)	324.4080
MEAN LONG. OF SUN (h)	250.1047
MEAN LONG. OF LUNAR PERIGEE (p)	155.0967
MEAN LONG. OF SOLAR PERIGEE (p1)	282.8186
LONG. OF LUNAR ASC. NODE (N)	262.0613

OBLIQUITY OF LUNAR ORBIT (I)	23.2624
OBLIQUITY OF ECLIPTIC (OMEGA)	23.4402
INCLINATION OF LUNAR ORBIT (INCL)	5.1454
RT. ASCENSION OF INTERSECTION (NU)	-12.9972
LONG. IN ORBIT OF INTERSECTION (XI)	-11.9407
TERM IN ARGUMENT OF K1 CONSTITUENT (NUP)	-8.9036
TERM IN ARGUMENT OF K2 CONSTITUENT (NUPP)	-8.9010
MEAN LONG. LUNAR PERIGEE FROM INT. (CP)	167.0374
FACTOR IN AMPL. OF L2 CONSTITUENT (RA)	1.2831
TERM IN ARGUMENT OF L2 CONSTITUENT (R)	-8.1990
FACTOR IN AMPL. OF M1 CONSTITUENT (QA)	0.5267
TERM IN ARGUMENT OF M1 CONSTITUENT (QU)	-6.6110
TERM IN ARGUMENT OF M1 CONSTITUENT (Q)	173.6484

NODE FACTORS AND ARGUMENTS
BOSTON 71 ° 3 'W 42 ° 21.3 'N
12 / 1 / 1992 0h UT
JDUT 2448957.5000
JDET 2448957.5013
LONG. OF LUNAR ASC. NODE (N) 262.0613

J	CONSTIT	F	AMPL	F*AMPL	V	U	V+U	KAPPA	V+U-K
1	M2	1.005	4.577	4.602	69.29	2.11	71.41	327.60	103.81
2	S2	1.000	0.721	0.721	217.90	0.00	217.90	3.80	214.10
3	N2	1.005	1.008	1.013	259.98	2.11	262.10	295.50	326.60
4	K1	0.999	0.492	0.491	269.05	8.90	277.96	134.80	143.16
5	M4	1.011	0.074	0.075	138.59	4.23	142.81	101.00	41.81
6	O1	0.997	0.389	0.388	160.24	-10.88	149.35	115.30	34.05
7	M6	1.016	0.113	0.114	207.88	6.34	214.22	219.00	355.22
8	(MK)2	1.004	0.019	0.020	338.35	11.02	349.36	14.40	334.96
9	S4	1.000	0.000	0.000	75.80	0.00	75.80	0.00	75.80
10	(MN)4	1.011	0.034	0.034	329.28	4.23	333.50	90.60	242.90
11	NU2	1.005	0.222	0.223	90.00	2.11	92.11	304.90	147.21
12	S6	1.000	0.000	0.000	293.70	0.00	293.70	0.00	293.70
13	MU2	1.005	0.037	0.037	280.69	2.11	282.80	258.60	24.20
14	(2N)2	1.005	0.136	0.137	90.67	2.11	92.78	275.20	177.58
15	(OO)1	0.982	0.016	0.016	197.87	36.88	234.75	154.30	80.45
16	LMBDA2	1.005	0.070	0.070	228.59	2.11	230.70	4.30	226.40
17	S1	1.000	0.000	0.000	108.95	0.00	108.95	0.00	108.95
18	M1	1.894	0.028	0.052	99.74	19.61	119.35	125.00	354.35
19	J1	1.006	0.032	0.032	78.37	13.00	91.36	140.30	311.06
20	Mm	1.017	0.000	0.000	169.31	0.00	169.31	0.00	169.31
21	Ssa	1.000	0.054	0.054	140.21	0.00	140.21	96.20	44.01
22	Sa	1.000	0.085	0.085	250.10	0.00	250.10	142.60	107.50
23	MSf	1.017	0.000	0.000	148.61	0.00	148.61	0.00	148.61
24	Mf	0.990	0.000	0.000	288.82	23.88	312.70	0.00	312.70
25	Rho1	0.997	0.014	0.014	180.94	-10.88	170.06	106.90	63.16
26	Q1	0.997	0.069	0.068	350.93	-10.88	340.04	95.40	244.64
27	T2	1.000	0.065	0.065	250.61	0.00	250.61	337.10	273.51
28	R2	1.000	0.006	0.006	5.19	0.00	5.19	3.80	1.39
29	(2Q)1	0.997	0.010	0.010	181.62	-10.88	170.73	95.80	74.93
30	P1	1.000	0.157	0.157	308.85	0.00	308.85	133.10	175.75
31	(2SM)2	1.005	0.000	0.000	6.51	-2.11	4.39	0.00	4.39
32	M3	1.008	0.000	0.000	103.94	3.17	107.11	0.00	107.11
33	L2	0.783	0.195	0.152	58.60	10.31	68.92	18.80	50.12
34	(2MK)2	1.009	0.024	0.024	229.53	-4.68	224.85	354.00	230.85
35	K2	0.976	0.207	0.202	358.11	17.80	15.91	5.90	10.01
36	M8	1.021	0.019	0.020	277.17	8.45	285.63	29.90	255.73
37	(MS)4	1.005	0.030	0.030	287.19	2.11	289.31	148.60	140.71

12 / 1 / 1992 BOSTON (TIME MERIDIAN 75 °W) (ZO 5.22)

00:00	3.437	
01:00	5.148	
02:00	6.989	
03:00	8.363	
03:53	8.776	<-- HIGH WATER
04:00	8.769	
05:00	8.206	
06:00	6.977	
07:00	5.409	
08:00	3.717	
09:00	2.292	
09:53	1.803	<-- LOW WATER
10:00	1.810	
11:00	2.466	
12:00	3.730	
13:00	5.229	
14:00	6.910	
15:00	8.363	
16:00	8.985	
16:06	8.990	<-- HIGH WATER
17:00	8.592	
18:00	7.421	
19:00	5.830	
20:00	4.067	
21:00	2.376	
22:00	1.344	
22:23	1.254	<-- LOW WATER
23:00	1.475	
24:00	2.490	

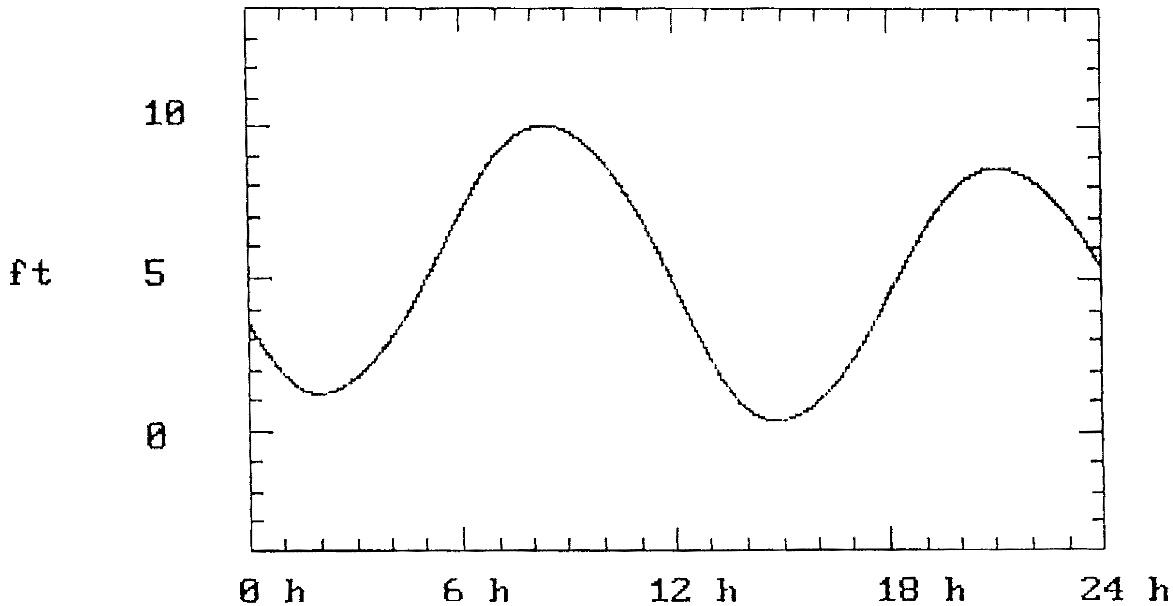
Output From Computer Program

PLACE: Boston Harbor

DATE: January 1, 1992

COMMENTS: This, the first day of 1992, was run simply to check that the program was operating correctly. Good agreement with the published NOS tide tables for 1992 was obtained.

1 / 1 / 1992



HIGH AND LOW WATERS

h m	ft
01:59	1.2
08:22	10.0
14:50	0.4
21:04	8.6

BOSTON 71 ° 3 ′W 42 ° 21.3 ′N (TIME MERIDIAN 75 °W) (Z0 5.22)

ASTRONOMICAL DATA

1 / 1 / 1992 JDUT 2448622.5000

TIME T = 0.91998635 JDET 2448622.5013

MEAN LONG. OF MOON	(s)	230.3149
MEAN LONG. OF SUN	(h)	279.9128
MEAN LONG. OF LUNAR PERIGEE	(p)	117.7765
MEAN LONG. OF SOLAR PERIGEE	(p1)	282.8028
LONG. OF LUNAR ASC. NODE	(N)	279.8008

OBLIQUITY OF LUNAR ORBIT	(I)	24.8099
OBLIQUITY OF ECLIPTIC	(OMEGA)	23.4403
INCLINATION OF LUNAR ORBIT	(INCL)	5.1454
RT. ASCENSION OF INTERSECTION	(NU)	-12.1582
LONG. IN ORBIT OF INTERSECTION	(XI)	-11.1040
TERM IN ARGUMENT OF K1 CONSTITUENT	(NUP)	-8.4544
TERM IN ARGUMENT OF K2 CONSTITUENT	(NUPP)	-8.6369
MEAN LONG. LUNAR PERIGEE FROM INT.	(CP)	128.8805
FACTOR IN AMPL. OF L2 CONSTITUENT	(RA)	0.9101
TERM IN ARGUMENT OF L2 CONSTITUENT	(R)	-14.9620
FACTOR IN AMPL. OF M1 CONSTITUENT	(QA)	0.7098
TERM IN ARGUMENT OF M1 CONSTITUENT	(QU)	-20.2932
TERM IN ARGUMENT OF M1 CONSTITUENT	(Q)	149.1737

NODE FACTORS AND ARGUMENTS
BOSTON   71 ° 3 'W 42 ° 21.3 'N
1 / 1 / 1992   0h UT
JDUT 2448622.5000
JDET 2448622.5013
LONG. OF LUNAR ASC. NODE (N) 279.8008

J	CONSTIT	F	AMPL	F*AMPL	V	U	V+U	KAPPA	V+U-K
1	M2	0.994	4.577	4.549	317.10	2.11	319.20	327.60	351.60
2	S2	1.000	0.721	0.721	217.90	0.00	217.90	3.80	214.10
3	N2	0.994	1.008	1.001	204.56	2.11	206.67	295.50	271.17
4	K1	1.034	0.492	0.508	298.86	8.45	307.32	134.80	172.52
5	M4	0.988	0.074	0.073	274.19	4.22	278.41	101.00	177.41
6	O1	1.054	0.389	0.410	18.23	-10.05	8.18	115.30	252.88
7	M6	0.982	0.113	0.111	231.29	6.33	237.61	219.00	18.61
8	(MK)2	1.027	0.019	0.020	255.96	10.56	266.52	14.40	252.12
9	S4	1.000	0.000	0.000	75.80	0.00	75.80	0.00	75.80
10	(MN)4	0.988	0.034	0.033	161.65	4.22	165.87	90.60	75.27
11	NU2	0.994	0.222	0.221	168.83	2.11	170.94	304.90	226.04
12	S6	1.000	0.000	0.000	293.70	0.00	293.70	0.00	293.70
13	MU2	0.994	0.037	0.037	56.29	2.11	58.40	258.60	159.80
14	(2N)2	0.994	0.136	0.135	92.02	2.11	94.13	275.20	178.93
15	(OO)1	1.184	0.016	0.019	39.49	34.37	73.86	154.30	279.56
16	LMBDA2	0.994	0.070	0.069	285.36	2.11	287.47	4.30	283.17
17	S1	1.000	0.000	0.000	108.95	0.00	108.95	0.00	108.95
18	M1	1.485	0.028	0.041	186.32	32.45	218.78	125.00	93.78
19	J1	1.056	0.032	0.034	51.40	12.16	63.56	140.30	283.26
20	Mm	0.977	0.000	0.000	112.54	0.00	112.54	0.00	112.54
21	Ssa	1.000	0.054	0.054	199.83	0.00	199.83	96.20	103.63
22	Sa	1.000	0.085	0.085	279.91	0.00	279.91	142.60	137.31
23	MSf	0.977	0.000	0.000	260.80	0.00	260.80	0.00	260.80
24	Mf	1.117	0.000	0.000	100.63	22.21	122.84	0.00	122.84
25	Rho1	1.054	0.014	0.015	229.97	-10.05	219.92	106.90	113.02
26	Q1	1.054	0.069	0.072	265.69	-10.05	255.64	95.40	160.24
27	T2	1.000	0.065	0.065	220.79	0.00	220.79	337.10	243.69
28	R2	1.000	0.006	0.006	35.01	0.00	35.01	3.80	31.21
29	(2Q)1	1.054	0.010	0.011	153.16	-10.05	143.11	95.80	47.31
30	P1	1.000	0.157	0.157	279.04	0.00	279.04	133.10	145.94
31	(2SM)2	0.994	0.000	0.000	118.70	-2.11	116.60	0.00	116.60
32	M3	0.991	0.000	0.000	115.64	3.16	118.81	0.00	118.81
33	L2	1.092	0.195	0.212	249.63	17.07	266.70	18.80	247.90
34	(2MK)2	1.021	0.024	0.024	335.33	-4.24	331.09	354.00	337.09
35	K2	1.066	0.207	0.221	57.73	17.27	75.00	5.90	69.10
36	M8	0.976	0.019	0.019	188.38	8.43	196.82	29.90	166.92
37	(MS)4	0.994	0.030	0.030	175.00	2.11	177.10	148.60	28.50

1 / 1 / 1992 BOSTON (TIME MERIDIAN 75 °W) (ZO 5.22)

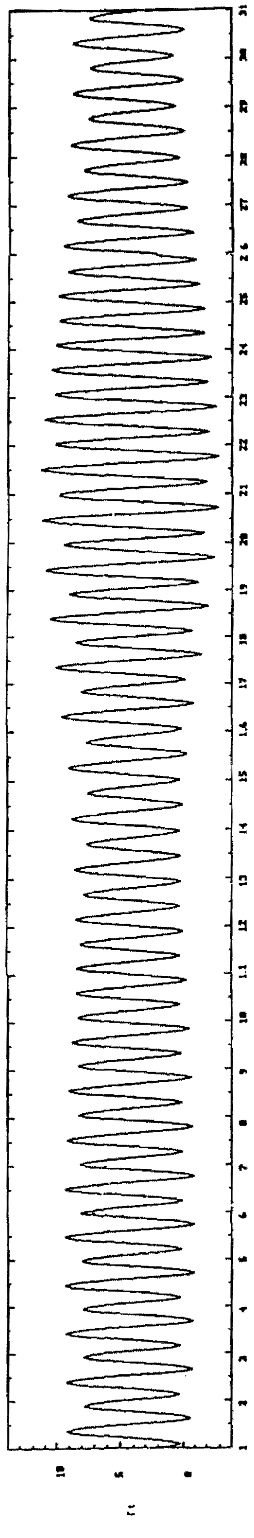
00:00	3.545	
01:00	1.835	
01:59	1.233	<-- LOW WATER
02:00	1.233	
03:00	1.745	
04:00	3.009	
05:00	4.922	
06:00	7.187	
07:00	9.062	
08:00	9.979	
08:22	10.048	<-- HIGH WATER
09:00	9.851	
10:00	8.837	
11:00	7.095	
12:00	4.761	
13:00	2.348	
14:00	0.746	
14:50	0.355	<-- LOW WATER
15:00	0.368	
16:00	0.951	
17:00	2.330	
18:00	4.406	
19:00	6.574	
20:00	8.085	
21:00	8.625	
21:04	8.627	<-- HIGH WATER
22:00	8.252	
23:00	7.131	
24:00	5.380	

ONE-MONTH TIDE CURVE

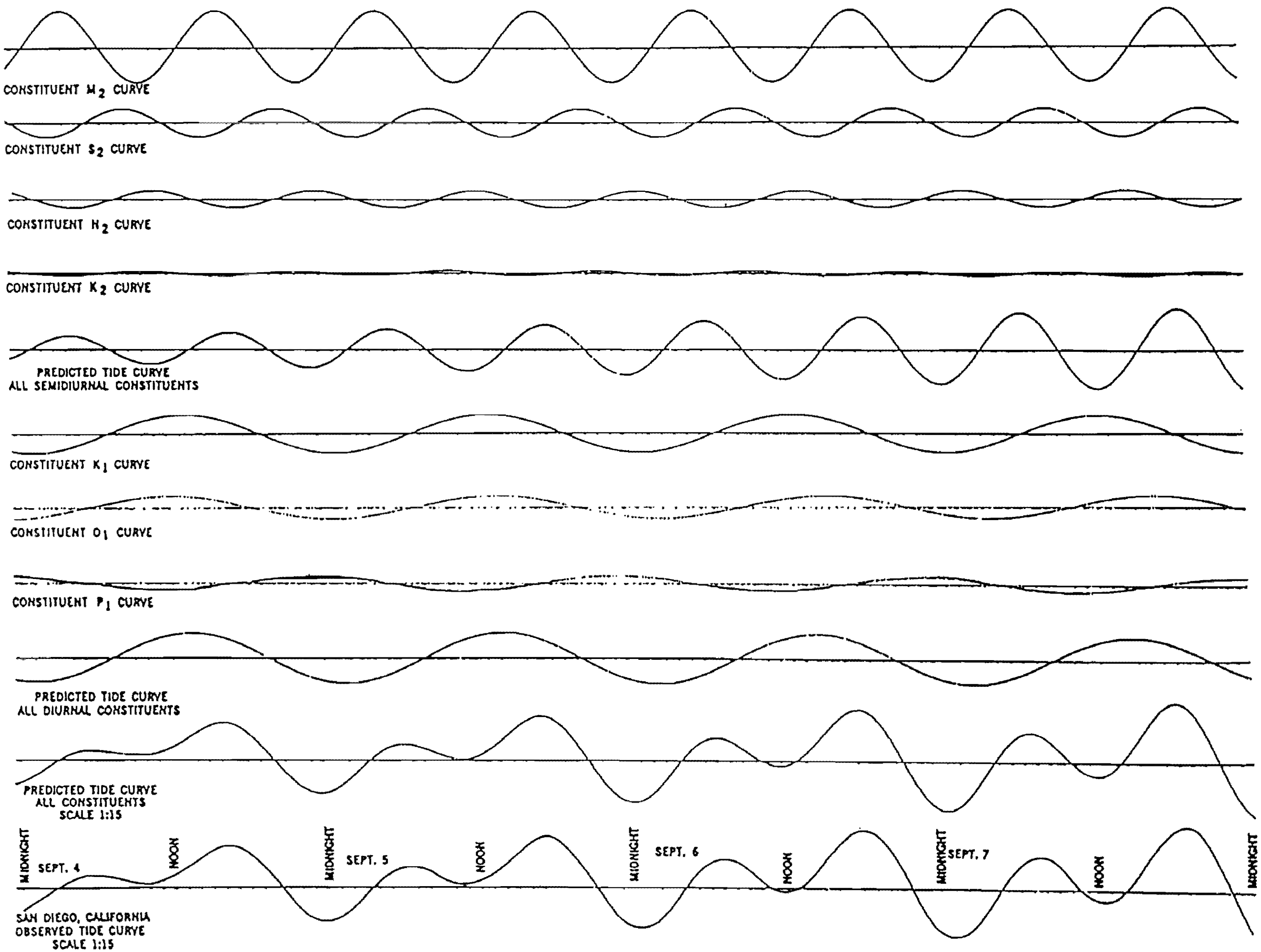
for January 1992

at

BOSTON, MASSACHUSETTS



GRAPH OF INDIVIDUAL TIDAL CONSTITUENTS


CONSTITUENT M2 CURVE
CONSTITUENT S2 CURVE
CONSTITUENT N2 CURVE
CONSTITUENT K2 CURVE
PREDICTED TIDE CURVE
ALL SEMIDIURNAL CONSTITUENTS
CONSTITUENT K1 CURVE
CONSTITUENT O1 CURVE
CONSTITUENT P1 CURVE
PREDICTED TIDE CURVE
ALL DIURNAL CONSTITUENTS
PREDICTED TIDE CURVE
ALL CONSTITUENTS
SCALE 1:15
MIDNIGHT
SEPT. 4
NOON
MIDNIGHT
SEPT. 5
NOON
MIDNIGHT
SEPT. 6
NOON
MIDNIGHT
SEPT. 7
NOON
MIDNIGHT
SAN DIEGO, CALIFORNIA
OBSERVED TIDE CURVE
SCALE 1:15

APPENDIX D

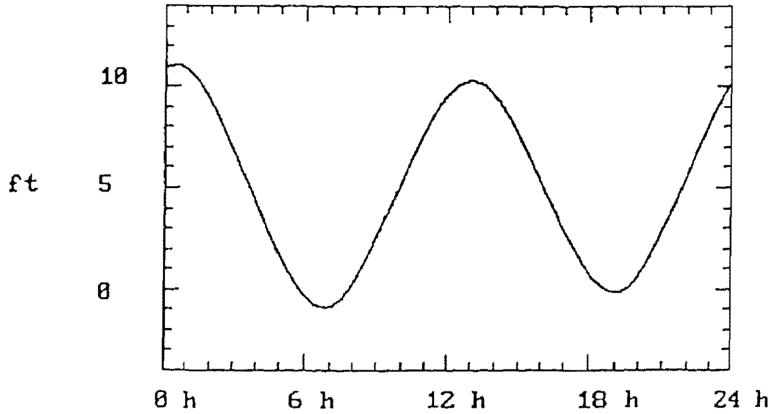
"PAUL REVERE'S MIDNIGHT RIDE"

This appendix contains, in addition to a pre-print of a published article[40], two tide curve graphs illustrating the tide of Boston Harbor on April 18, 1775 and April 19, 1775.

Paul Revere crossed the harbor in a small rowboat at about 10:30 p.m. on April 18, 1775. Note on the graph that the tide was rising at this time. The historical significance of this rising tide is discussed in the article.

Also note, on the graph for April 19, 1775, that high tide occurred at about 1:26 a.m. At this time British troops were in the final stages of their disembarkation from small boats onto the marshes on the Cambridge side of Boston Harbor. The historical significance of this high water is explained in the article.

4 / 18 / 1775

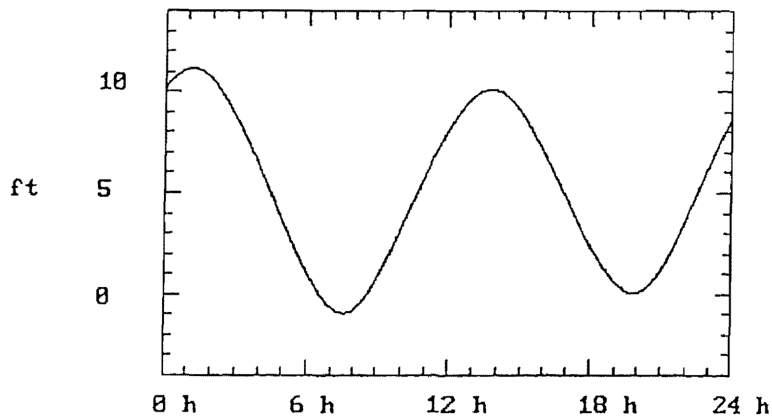


HIGH AND LOW WATERS

h m	ft
00:26	11.1
06:47	-0.9
12:58	10.3
19:02	-0.1

BOSTON 71 ° 3 'W 42 ° 21.3 'N (TIME MERIDIAN 75 °W) (Z0 5.22)

4 / 19 / 1775



HIGH AND LOW WATERS

h m	ft
01:10	11.2
07:33	-0.9
13:45	10.1
19:49	0.0

BOSTON 71 ° 3 'W 42 ° 21.3 'N (TIME MERIDIAN 75 °W) (Z0 5.22)

PAUL REVERE'S MOON AND TIDES

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Henry Wadsworth Longfellow, in his Tales of a Wayside Inn (1863), began the landlord's tale with the stirring lines:

> "Listen, my children, and you shall hear
> Of the midnight ride of Paul Revere,
> On the eighteenth of April, in Seventy-five;
> Hardly a man is now alive
> Who remembers that famous day and year."

The poem refers to the spring of 1775, when General Thomas Gage, British commander in Boston, received information that a considerable quantity of provincial military stores were concealed in Concord, a town about 18 miles west of Boston. On the night of April 18, Gage ordered a detachment of grenadiers and light infantry to proceed to Concord and destroy the depot. Boats began ferrying the troops from Boston to Cambridge across the tidal backwash of the Charles River, leading to the famous two-lantern signal ("two if by sea") from Old North Church. Paul Revere himself crossed the harbor sometime between 10 p.m. and 11 p.m. in a rowboat from Boston to Charlestown, from where he began his famous midnight ride towards Lexington and Concord.

It has long been recognized that Longfellow's poem has several historical errors. For example, the poem puts Revere on the wrong side of the harbor; actually, Revere was involved in sending the two-lantern signal and did not receive it. Also, the poem has Revere reach Concord; in reality, he was stopped just past Lexington, and it was another rider, Samuel Prescott, who reached Concord.

Longfellow's poem contains astronomical references; in particular, he mentions the Moon five times:

"...he...Silently rowed to the Charlestown shore,
Just as the moon rose over the bay..."

"...The Somerset, British man-of-war;
A phantom ship, with each mast and spar
Across the moon like a prison bar..."

"...he paused to listen and look down
A moment on the roofs of the town,
And the moonlight flowing over all..."

"...A hurry of hoofs in a village street,
A shape in the moonlight, a bulk in the dark..."

"...He saw the gilded weathercock
Swim in the moonlight as he passed..."

Are these astronomical references accurate, or do they contain errors like the historical errors which Longfellow made? Specifically, was the Moon rising as Revere crossed the river? Was there bright moonlight throughout the midnight ride?

MOON ON APRIL 18-19, 1775

Following the methods of Jean Meeus' Astronomical Formulae for Calculators (Willmann-Bell, 1982), we obtained the following lunar phases:

April 15, 1775 Full Moon (22h UT)

April 22, 1775 Last quarter Moon (20h UT)

In Boston on the night of April 18, 1775, there was indeed a bright waning gibbous Moon, 87 percent sunlit. Moonrise occurred at about 9:37 p.m. Eastern Standard Time. This corresponds to 9:52 p.m. in local mean time for the meridian of Boston (71 degrees 3 minutes west longitude, or 4h 44m west of Greenwich) and to 9:53 p.m. in apparent solar time. Apparent solar time is obtained from local mean time by making a correction called the equation of time, which was +1 minute on April 18 and varied between extreme values of about −15 minutes to +16 minutes over the course of 1775. Many astronomers are familiar with the equation of time as the difference between the time on a sundial (solar time) and the time on a clock (mean time). Regardless of how the time is expressed, it is clear that there was a bright Moon rising as Paul Revere crossed the harbor between 10 p.m. and 11 p.m.

We were able to locate several different colonial almanacs published in Boston for 1775, including those of Nathaniel Low, Isaac Bickerstaff, Isaiah Thomas, and Nathaniel Ames. Moonrise on April 18 was given as 9:45 p.m. (Low, Ames), 9:46 p.m. (Thomas), and 9:53 p.m. (Bickerstaff). These almanacs did not use standard time, as modern almanacs do, since the United States did not adopt standard time zones until 1883. But did the colonial almanacs use local mean time or apparent solar time? From a comparison of the almanac tables to computer calculations of sunrise for all 365 days of 1775, it became obvious that the colonial almanacs used apparent solar time.

PAUL REVERE'S ACCOUNTS

Paul Revere left three first-person accounts of the famous night. Two are in the Revere family papers; both describe how "the Moon shone bright." His third account, a letter sent in 1798 to Jeremy Belknap of the Massachusetts Historical Society, contains the following often-quoted lines:

> ...went to the north part of the town, where I had kept a boat; two friends rowed me across Charles River, a little to the eastward where the Somerset man-of-war lay. It was then young flood, the ship was winding, and the moon was rising.

It seems hard to understand how Paul Revere could have passed the Somerset man-of-war without being detected. If he was to the east of the ship, why didn't the British see his rowboat silhouetted against the rising Moon?

The computer simulation provides the answer. The Moon that night had a southern declination (-18 degrees) which, combined with Boston's northern latitude (42 degrees 22 minutes north), caused the Moon to rise considerably south of east. When Revere crossed the harbor, roughly 45 minutes after moonrise, the Moon was low (altitude 6 degrees) in the southeast (azimuth 121 degrees). As shown in the accompanying map, a sentinel on the Somerset would have seen the Moon rising over the city of Boston, not over the open water of the bay, and therefore he would not have seen the rowboat against the rising Moon. This calculation helps to explain why Revere was successful in reaching his horse on the opposite shore!

TIDES

In addition to Revere's reference to a "young flood" as he was crossing the harbor, Longfellow's poem mentions tides three times, including a description of the ferrying of the soldiers on "the rising tide, like a bridge of boats." We can check these statements with computer programs based on harmonic analysis of the tides. Our program (S&T: November, 1987, page 526) calculates the tide curve as the sum of 37 periodic constituents, with amplitudes and phases tabulated as the "station constants" for the port.

Boston Harbor: Tide calculations

(times are apparent solar time)

April 18, 1775	1:14 p.m.	high water
April 18, 1775	7:19 p.m.	low water
April 19, 1775	1:26 a.m.	high water

So, Revere's letter and Longfellow's poem are again both correct — there was a rising tide between 10 p.m. and 11 p.m. as Revere passed the Somerset.

Contemporary accounts also recorded the effects of the following high water which occurred early on April 19 and flooded the marshes on the Cambridge side of the river. A report by an anonymous British officer, recorded by Lieutenant Frederick Mackenzie of the Royal Welch Fusiliers, describes the events:

> The Grenadier & Light Companies of the Regiments in Boston were ordered to assemble on the Beach near the Magazine at 10 o'Clock last night...embarked, and landed at Phipps's farm. The boats then returned for the remainder, and it was near One oClock in the Morning before the whole were landed on the opposite shore...it was 2 oClock before they marched off. Their march across the marshes into the high road, was hasty and fatiguing, and they were obliged to wade, halfway up their thighs, through two Inlets, the tide being by that time, up. This should have been avoided if possible, as the troops had a long march to perform....

Lieutenant William Sutherland of the 38th Regiment described how he:

> ...embarked at the Magazine Guard & landed near Cambridge...here we remained for two long hours, partly waiting for the rest of the Detachment & for provisions, About 2 in the Morning on the 19th we marched...the Tide being in we were up to our Middles before we got into the road....

Colonial almanacs gave the time of morning high water, or "full sea", for April 19 as 2:34 a.m. (Low), 2:36 a.m. (Thomas), and 2:39 a.m. (Bickerstaff, Ames). All of these are somewhat later than the 1:26 a.m. computed here for that high water. Part of this difference could be explained by changes in the shoreline and the depths in Boston harbor through dredging and landfill. However, there is reason to believe that our computed times may be fairly accurate, even when projected back to 1775, since we can show that a certain time constant for the port has not changed much in the last two centuries.

The "high water lunitidal interval" is defined as the time interval between the Moon's transit across the meridian of the place and the following high water. A related quantity, called "High Water Full & Change," is the average value of the high water lunitidal interval on the days of full Moon or new Moon ("change"). We have located two series of navigation guides from the late 1700s; both state that HWF&C equals 11h 30m for Boston harbor. For comparison, we computed the times of high waters and then subtracted the times of the preceding lunar transits on the days of the 25 new and full Moons in 1992; the average difference, or HWF&C, is 11h 24m in 1992, in good agreement with the colonial value. This indicates that the harbor has not changed significantly, at least insofar as the times of high waters are concerned.

For the time of the high water on April 19, the colonial almanacs may be in error because of the relatively crude methods available then for tide prediction. The makers of colonial almanacs generally did not reveal their methods, but certain clues can be found in their tide tables. For example, on the full Moon day of April 15, 1775, our computer program gives a high water at 11:06 a.m., exactly the time listed in Nathaniel Low's almanac. However, Low gives the times of high waters as 8:06 on April 11th, 8:51 on the 12th, 9:36 on the 13th, 10:21 on the 14th, 11:06 on the 15th (full Moon), 11:58 on the 16th, 12:50 on the 17th, 1:42 on the 18th, and 2:34 on the 19th. Inspection of the differences in these numbers shows that Low is simply giving the tides later by 45 minutes per day before the 15th and by 52 minutes per day after the 15th. Similar patterns are seen in his almanac throughout 1775, with the sudden discontinuities in the time difference generally occurring on the days of new Moon, first quarter, full Moon, and last quarter. This suggests that Low computed tide times in detail on only a few days each month, and he obtained the other times by simple "uniform differencing," equivalent to linear interpolation. The large errors possible in such crude methods were noted in 1802 in the first edition of <u>The New American Practical Navigator</u> by the famous navigation expert Nathaniel Bowditch, who criticized rules "in which the tide is supposed to be uniformly retarded every day." Bowditch gave improved rules, but more accurate harmonic methods were not generally applied to ocean tides until the 1860s.

RELATION TO APRIL 18, 1992

On April 18, 1775, the Moon rose at about 9:37 p.m. EST, and 45 minutes later it stood 6 degrees above the southeastern horizon at azimuth 121 degrees. The Moon was in the waning gibbous phase and 87 percent sunlit.

On April 18, 1992, the Moon will rise at about 9:38 p.m. EDT, and 45 minutes later it will stand 6 degrees above the southeastern horizon at azimuth 128 degrees. The Moon will be in the waning gibbous phase and 96 percent sunlit. While the Moon in 1992 is somewhat closer to full, the expected difference in rise time is almost exactly cancelled by our use of daylight time.

If the sky is clear, the bright moonlight in 1992 will angle across Boston harbor, shine down the country roads, and illuminate Lexington Green and Concord Bridge, just as it did over two centuries ago, on the night before the first day of the American Revolution.

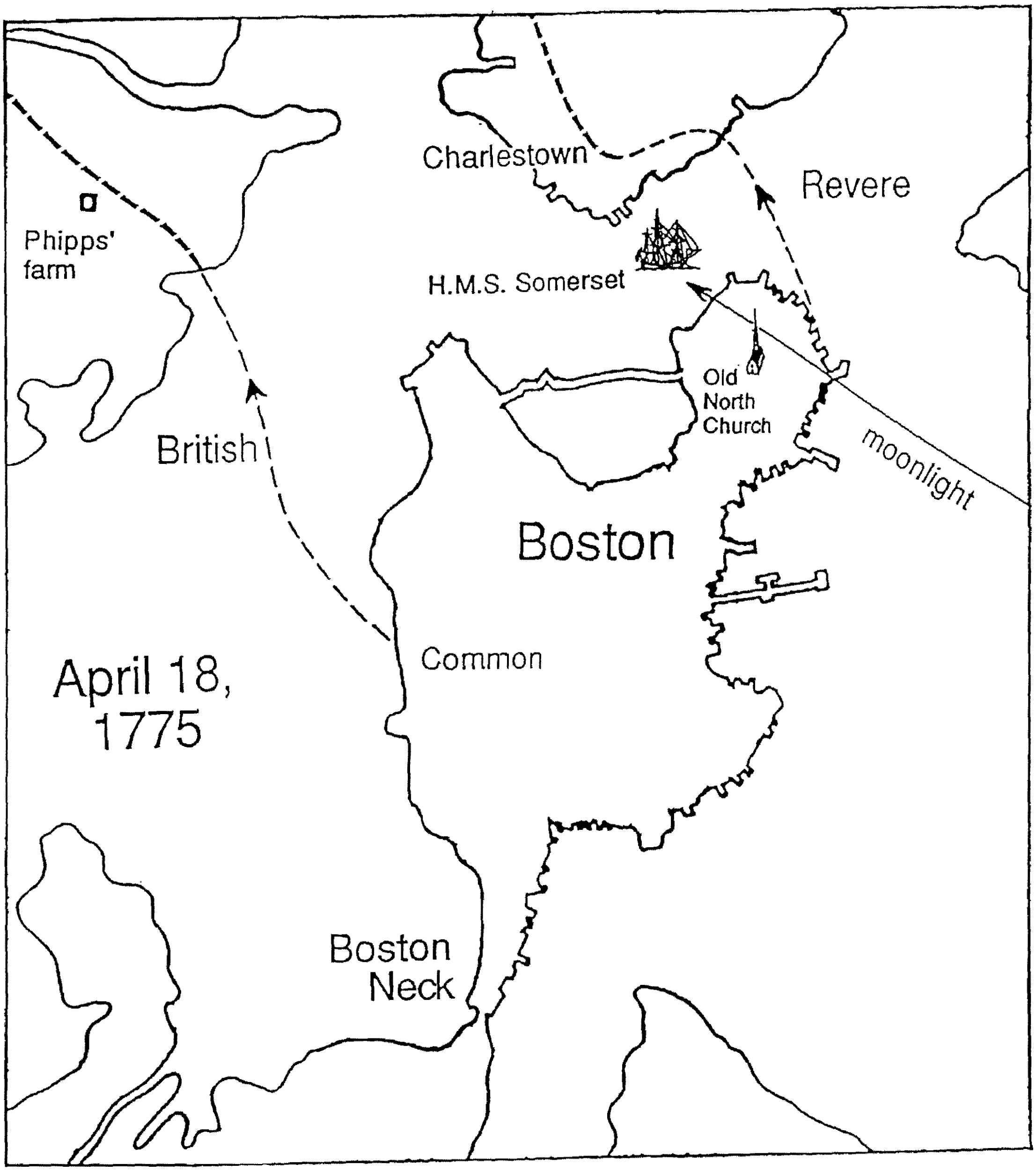
IV. APRIL hath 30 Days. 1775

BUT when the rage of winter's night is paſt,
(Kind heav'ns indulgences forever laſt.)
The vernal dawn of blooming ſpring appears,
And Sol's kind influence all nature chears :
Refreſhing ſhowers prepare the earth for ſeed,
While pleaſing verdure cloathes the fertile mead.

Firſt Quart. 7 Day 6½ Aft.	Laſt Quart 22 Day 2 Aftern
Full Moon 15 Day 5 Aft.	New Moon 29 Day 3 Aftern

M	W.	*Courts, Aſp. Weath. &c.*	*r.* ☉	*ſ.*	*F ſea.*	☽ *pl.*	*r.* ● *ſ.*	*O.S.*
1	7	St Philip & James. ☍ ☽ ♀	5 43	7	12 20	head	7 45	21
2	A	*Pleaſant.* ☌ ♃ ☽	5 42	7	1 8	neck	8 56	22
3	2	Sup. C. South Kingſton.	5 40	7	1 55	25	9 56	23
4	3	I.C.Barnſt.N Har.Amher	5 39	7	2 42	arms	10 56	24
5	4	*pleaſant* ☊ 2 ♍	5 38	7	3 29	20	11 52	25
6	5	*weather, for*	5 37	7	4 16	breaſt	Morn.	26
7	6	P.N. 9 41. *ſeveral*	5 35	7	5 3	15	0 46	27
8	7	*days together.*	5 34	7	5 49	26	1 35	28
9	A	● Apogee. Palm Sunday	5 33	7	6 35	heart	2 20	29
10	2	S. C. Briſtol. *warm,*	5 31	7	7 21	20	2 57	30
11	3	S.C. Charleſt. I.C. Plim.	5 30	7	8 6	belly	3 30	31
12	4	(Hartf & Charleſt. N.H.	5 28	7	8 51	14	4 1	*Ap*
13	5	*pleaſant,*	5 27	7	9 36	27	4 31	2
14	6	Good Friday ☌ ☽ ♄	5 26	7	10 21	reins	5 1	3
15	7	△ ☉ ♂ *thunder,* ☌ ♃ ☿	5 24	7	11 6	23	☾ riſe	4
16	A	Eaſter Sunday. *good*	5 23	7	11 58	ſecrets	7 46	5
17	2	S.C.E Greenw, *weather,*	5 22	7	12 50	20	8 38	6
18	3	S.C.Worc. I.C [illegible] Fal	5 20	7	1 42	thighs	9 45	7
19	4	Syrius ſets 9 40.	5 19	7	2 34	18	10 53	8
20	5	*foul* □ ♂ ♀	5 17	7	3 26	knees	11 57	9
21	6	*weather,* ☍ ♄ ☿	5 16	7	4 19	16	Morn.	10
22	7	*then* ● Per	5 15	7	5 12	legs	0 52	11
23	A	*fair,* Middling	5 13	7	6 4	14	1 40	12
24	2	Tides	5 12	7	6 55	29	2 23	13
25	3	S C. North. I C. Litchf.	5 11	7	7 46	feet	2 53	14
26	4	7*'s ſet 9 4. *perhaps a*	5 10	7	8 37	27	3 25	15
27	5	△ ♄ ♀ *cold* ☍ ☽ ♄	5 9	7	9 28	head	3 50	16
28	6	Yard L. ſets 9h. *rain,*	5 7	7	10 19	24	4 23	17
29	7	*then pleaſant*	5 6	7	11 10	neck	☽ ſets	18
30	A	*weather.* ☌ ☽ ♃	5 4	7	11 56	20	7 46	19

This page for April is from "An Astronomical Diary; Or, Almanack For the Year of Christian Era, 1775," written by Nathaniel Low and published in Boston by John Kneeland.



Paul Revere passed east of the H.M.S. Somerset shortly after moonrise on April 18, 1775. However, sentinels on the man-of-war would not have seen the rowboat silhouetted against the rising Moon, since the Moon rose considerably south of east that night.

REFERENCES

1. Paul Schureman, *Manual of Harmonic Analysis and Prediction of Tides*, Special Publication No. 98, Revised (1940) Edition (Washington, D.C. United States Government Printing Office, 1958, reprinted 1971)

2. George Howard Darwin, *The Tides and Kindred Phenomena in the Solar System* (San Francisco, W.H. Freeman and Co., 1962) p. 81

3. Harry Aaron Marmer, *The Tide* (New York, D. Appleton and Co., 1926) p. 20

4. Darwin, *Tides and Kindred Phenomena*, pp. 149 - 150

5. John M.A. Danby, *Fundamentals of Celestial Mechanics* (New York, The Macmillan Company, 1962), p. 9

6. Valerie Illingworth, editor, *The Facts on File Dictionary of Astronomy* (New York, Facts on File, Inc., 1979) p. 108

7. Danby, *Celestial Mechanics*, p. 10

8. Illingworth, *Facts on File Dictionary*, p. 98

9. *The Astronomical Almanac for the Year 1986* (Washington, D.C., United States Naval Observatory, U.S. Government Printing Office, 1985) p. F2

10. *The American Ephemeris and Nautical Almanac for the Year 1978* (Washington,D.C., U.S. Government Printing Office, 1976) p. 531

11. Isaac Newton, *A Treatise of the System of the World*, translated by I. Bernard Cohen (London, Dawsons of Pall Mall, 1969) p. 54

12. Illingworth, *Facts on File Dictionary*, p. 19

13. Paul Schureman (revised by Steacy D. Hicks), *Tide and Current Glossary* (Washington,D.C., U.S. Government Printing Office, 1975) p. 7

14. *The American Ephemeris and Nautical Almanac for the Year 1979* (Washington,D.C., U.S. Government Printing Office, 1977) p. 540

15. Roy L. Bishop, *Observer's Handbook 1992* (Canada, University of Toronto Press, 1991) p. 14

16. Illingworth, *Facts on File Dictionary*, p.221

17. *The Astronomical Almanac*, 1986, p. D2

18. Schureman, *Manual of Harmonic Analysis*, p. 6

19. Ibid.

20. Illingworth, *Facts on File Dictionary*, p. 153

21. Schureman, *Tide and Current Glossary*, p. 12

22. Donald W. Olson and Russell L. Doescher, "Lincoln and the Almanac Trial", *Sky & Telescope* **80** (No. 2), p. 184 (1990)

23. Fergus J. Wood, *The Strategic Role of Perigean Spring Tides* (Washington, D.C., U.S. Department of Commerce, National Oceanic and Atmospheric Administration, 1976) p. 111

24. Schureman, *Tide and Current Glossary*, p. 7

25. Ibid., p. 9

26. Donald W. Olson, 1992, private communication

27. D.L. Harris, N.A. Pore, and R.A. Cummings, "Tide and Tidal Current Prediction by High Speed Digital Computer," *International Hydrographic Review*, Vol. XLII, No. 1, Jan. 1965, pp. 95-103

28. N.A. Pore and R.A. Cummings, *A FORTRAN Program for the Calculation of Hourly Values of Astronomical Tide and Time and Height of High and Low Water*, Weather Bureau Technical Memorandum TDL-6 (Silver Spring,MD, Systems Development Office, 1975) p. 1

29. Marmer, *The Tide*, p. 181

30. Ibid., p. 183

31. Schureman, *Tide and Current Glossary*, p.3

32. Schureman, *Manual of Harmonic Analysis*, p. 124

33. Schureman, *Tide and current Glossary*, p. 7

34. *Our Restless Tides*, (U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Ocean Survey, n.d.) n.p.

35. Illingworth, *Facts on File Dictionary*, p. 199

36. Jean Meeus, *Astronomic Formulae for Calculators*, Third edition (Richmond,Virginia, Willmann-Bell,Inc., 1985)

37. Illingworth, *Facts on File Dictionary*, p. 169

38. Schureman, *Manual of Harmonic Analysis*, p. 25

39. Donald W. Olson and Russell L. Doescher, "Paul Revere's Midnight Ride", *Sky & Telescope* **83** (No. 4), p. 437 (1992)

40. Ibid.