

EXPLORING PROSPECTIVE ELEMENTARY TEACHER'S
PEER-INTERVIEWER VERBAL INTERACTIONS IN A
MATHEMATICS CONTENT COURSE
DISSERTATION

by

Teresa Lynn Salazar, M.S.

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Committee Members:

Max Leon Warshauer, Chair

Hiroko Kawaguchi Warshauer, Co-chair

Jessica Lynn Bishop

Luz Angélica Maldonado Rodriguez

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LIST OF ABBREVIATIONS

Abbreviation	Description
A	Acceptance
CBMS	Conference Board of the Mathematical Sciences
CGI	Cognitively Guided Instruction
F	Feedback
I	Initiation
NCTM	National Council of Teachers of Mathematics
PAR	Peer-Assisted Reflection
PI	Peer-Interviewer
PIRT	Peer Interaction for Responsive Teaching
PS	Problem-Solver
PST	Elementary Prospective School Teacher
PUFM	Profound Understanding of Fundamental Mathematics
R	Response

ABSTRACT

Responsive teaching interactions, an effective teaching practice, are difficult to develop. To train prospective elementary teachers of mathematics to interact responsively with students completing mathematics tasks, we first need to know more about how prospective teachers interact. This study explored the peer-interviewer interactions of three prospective elementary teachers during three peer task interviews in a one-semester mathematics content course. The study found that the peer-interviewers used more elicitations connected to mathematics during task reflection phases. The study includes a discussion on considering prospective elementary teachers' perspectives of understanding mathematics and how to develop interviewer interaction skills further.

I. RATIONALE

Children begin formal schooling with intuitive number sense. A child's number sense develops with influences from their family, cultures, and communities. Elementary mathematics teachers can support and extend children's intuitive number sense to build and extend mathematical thinking (see Carpenter, Fennema, Franke, Levi, & Empson, 2015). Supporting and extending children's intuitive number sense benefits children's conceptual mathematical understanding (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996; Pierson, 2008).

Teachers supporting and extending children's number intuition contrasts with traditional mathematics instructional practices. In traditional mathematics classrooms, students follow a teacher's mathematical thinking in the pattern of "review, demonstration, and practice" (National Council of Teachers of Mathematics [NCTM], 2014, p. 9) instead of building on children's mathematical thinking.

Learning to listen and respond to children's ways of mathematical thinking is necessary to build and extend on children's understanding (Empson & Jacobs, 2008). Jacobs and Empson (2016) explain that teaching should be *responsive* to children's mathematical thinking with adaptive teaching moves. To build and extend a child's mathematical thinking, a teacher needs to learn to elicit and explore the child's ideas and thoughts. I conceptualize responsive interactions in my study as interactions that elicit, explore, support, and extend another person's mathematical thinking, and I refer to teaching that is responsive to children's mathematical reasoning as responsive teaching.

During responsive teaching, children are encouraged to build explanations of their understanding as they work on mathematical tasks. In a classroom setting, responsive

teachers encourage other children to respond to the original child's mathematical ideas. Children explain or justify their mathematical claims in sense-making interaction exchanges. Children learn to analyze tasks, explain their solution methods, check their solutions, and possibly extend or generalize their solutions in responsive teaching environments.

Developing Responsive Teaching

Mathematics education researchers and teacher educators suggest that responsive teaching shaped by listening to student's mathematical ways of thinking with spontaneous responsive interactions is challenging to put into practice (Jacobs & Empson, 2016; Pierson, 2008; Jacobs, Empson, Krause, & Pynes, 2015; Schoenfeld, 2016; Empson & Jacobs, 2008). Jacobs et al. posit that "despite widespread enthusiasm for this approach [responsive teaching], expertise has proven challenging to develop" (2015, p. 15). Empson and Jacobs posit that "these skills [listening responsively] are not usually acquired in teacher preparation programs" (2008, p. 267) Although responsive listening and teaching often take years to develop, the change is fundamental in the teacher's conceptualization of their role (Empson & Jacobs, 2008). The issue is the development of responsive teaching practices. In what ways can elementary prospective school teachers (PSTs) begin to develop responsive teaching practices?

Research advancements for the development of responsive teaching of mathematics include studies of classroom teacher responsiveness to student's mathematical thinking in the classroom and child task interviews (e.g., Pierson, 2008; Jacobs & Ambrose, 2009; Jacobs & Empson, 2016). While the research is advancing in

areas that support the development of responsive teaching expertise, the question remains on how to apply this research in teacher preparation programs.

In teacher development, *approximations of practice* are practices that are close to teaching in the classroom. An approximation of practice allows PSTs the opportunity to experiment with teaching practices within the university classroom (Grossman et al., 2009). One approximation of practice used in the curriculum of some mathematics content courses is responding to children's mathematical thinking presented on videos or copies of written student work (e.g., Beckmann, 2014; Sowder, Sowder, & Nickerson, 2017). The advantage of using videos or written student work of children's mathematical reasoning is PSTs exposure to children's ways of reasoning. PSTs have opportunities to think as a teacher who observes children working on mathematical tasks or assess written work.

To respond to children, teachers often think rapidly in a classroom as they observe and analyze children's mathematical thinking. Jacobs, Lamb, and Philipp (2010) summarize this rapid thinking practice as "the hidden practice of in-the-moment decision making when teachers must respond to children's verbal- or written-strategy explanations" (p. 192). They termed this hidden teaching "professional noticing of children's mathematical thinking" (p. 192), where children's mathematical thinking is based on researched ways that children reason about mathematics. Per Jacobs et al. (2010), professional noticing of children's mathematical thinking includes: (a) "attending to children's strategies" (p. 194), (b) "interpreting children's understandings" (p. 195), and (c) "deciding how to respond based on children's understanding" (p.195). Blömeke, Gustafsson, and Shavelson (2015) label these skills as "situation-specific skills."

Mathematics teacher educators can ask PSTs to attend, interpret, and decide how to respond to artifacts of children's mathematical work. Artifacts of children's mathematical work in mathematics content courses provide opportunities for PSTs to approximate the practice of situation-specific skills used in teaching mathematics while deepening their mathematical knowledge. The issue is that while PSTs decide how to respond to videos or written work, there is not the opportunity to practice interacting with the person working on a mathematical task. Thus, while valuable and necessary *planning* for responsive interactions is possible, practiced responsive interactions require human interactions.

One way for PSTs to practice one-to-one interactions is to conduct child task-based interviews as outlined by Ginsburg (1997) in *Entering the Child's Mind* (e.g., Groth, Bergner, & Burgess, 2016; Philipp et al., 2007; McDonough, Clark, & Clark, 2002; Crespo & Nicol, 2003). A one-to-one task-based interview with a child allows a PST to concentrate on responsive interactions before adding the complexity of a few children in a conference or group setting or a whole classroom of children in classroom discussion.

Lesson rehearsals with peers and a mathematics teacher educator provide an approximation of practice of lessons to be taught later in the classroom with children (Lampert et al., 2013). Pre-service elementary teachers have the opportunity to practice responsive teaching among peers before using the lessons in the classroom.

The opportunity for child task-based interviews may not be available during many mathematics content courses, and PST's may not be preparing lessons for field-based experiences. Practicing situation-specific skills of noticing with children's mathematical

work is valuable, but can there be *additional* approximations of practice where PSTs respond to each other's mathematical thinking to prepare PSTs for interactions with children?

One approximation of a child mathematical task-based interview is a peer mathematical task-based interview. The interview environment is a place to explore the mathematical thinking of another person. In responsive teaching interactions, exploring another person's mathematical thinking is necessary to support and extend the person's mathematical thinking. In an interview, PSTs can practice eliciting another person's mathematical thinking by posing purposeful questions. The disadvantage of peer mathematical task-based interviews is that PSTs are not responding to children's mathematical reasoning; instead, PSTs are responding to the mathematical thinking of an adult, but the advantage is that PSTs have one-to-one personal interactions with their peers to develop and practice interactions in-the-moment. In addition to peer mathematical task interviews providing an environment to practice interactions with mathematical thinking, both the interviewer and interviewee have the opportunity to gain insight into deeper mathematical understanding by considering different ways of approaching the problems.

Exploring Peer-Interviewer Interactions

My study's exploration of peer-interviewer interactions considers peer task interviews as an approximation of a teacher working individually with a child. Before this study, I developed a cycle of activities to encourage PSTs to practice eliciting, supporting, and building on the other person's mathematical thinking called Peer Interaction for Responsive Teaching (PIRT). A cycle of PIRT activities includes: (a) an

interview and responsive interaction preparation assignment, (b) a mathematical task preparation assignment, (c) time in task teams to compare task preparations and strategies, (d) a peer task interview, and (e) peer-interviewer and problem-solver reflections of peer-interviewer interactions. Figure 1 provides a visual representation of PIRT activities. I designed the interview and interaction preparation assignment to foster the development of responsive interactions.

The interview and interaction preparation assignments are based on readings about mathematical task-based interviews, questioning, responsive moves, and strategies to encourage productive struggle. The core of this study is to understand the ways that PSTs interact as interviewers during peer mathematical task interviews in an environment of activities designed to encourage responsive interviewer interactions.

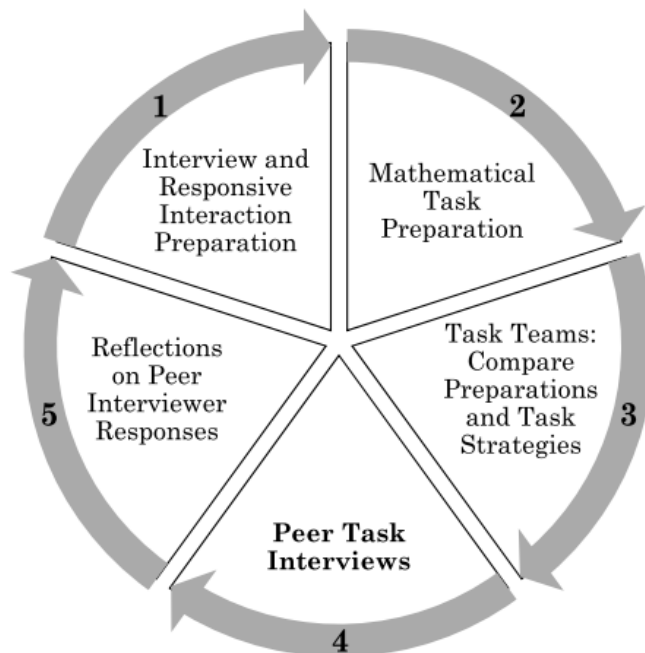


Figure 1. PIRT cycle of activities

The purpose of my study is to explore the PST peer-interviewer interactions during three peer task interviews in the context of PIRT cycles as part of a problem-solving based initial mathematics content course for future teachers at a southern university in the United States. I defined peer interview interactions as verbal responses that a peer-interviewer made during a peer mathematical task-based interview. I used a multiple case study design (Yin, 2014) focused on the PST interviewer interactions of three PSTs.

The research questions for my study were:

1. In what ways did the PST peer-interviewers verbally interact with a peer problem-solver during three peer mathematical task-based interviews?
2. How did the PSTs' interviewer verbal interactions during three peer task-based interviews develop over a semester?

This study contributes knowledge to the research community about the ways that PST interviewers interact and develop interviewer interactions in the context of peer interview cycles in a problem-solving based initial mathematics content course.

In the following chapters, I outline a conceptual framework, discuss my methods, present my findings, and conclude with a discussion of the findings, including implications and recommendations for developing responsive interactions. In Chapter Two, I present my conceptual framework of interview interaction within a continuum of competency with relevant research. I discuss the research setting and interpretive methods used in the exploration of PST interview interactions in Chapter Three. Chapter Three includes the participants and context of the study, data collection, data analysis, and trustworthiness of the study. In Chapter Four, I present significant PST interviewer

interaction findings within the context of an interview structure and discuss how the PST's common development theme. I conclude with Chapter Five by presenting a discussion of the findings, the study limitations, and recommendations to develop responsive PST interviewer interactions.

II. CONCEPTUAL FRAMEWORK

My research questions focused on PST interviewer interactions as observable performances. While my research focused on one type of behavior, I conceptualized this performance as part of a continuum of competence (Blömke et al., 2015). In this chapter, I present my conceptual framework of competence specifically related to interactions in the context of developing responsive interactions through mathematical task-based interviews. To study the development of interactions, I used a conceptualization of responsive teaching and contributing practices. I begin with my conceptualization of responsive interactions with students completing mathematical tasks, discuss corresponding mathematical practices for one-on-one interactions, and then present the conceptual framework of interaction competence as a continuum.

Conceptualization of Responsive Interactions

For this study, I conceptualize interactions as the interactions specifically between a teacher or more-knowledgeable other with a problem-solver about a mathematical task. Interactions are a means of being responsive with a problem-solver. I begin by considering literature related to responsive teaching. Responsive teaching has at its heart the mathematical sense-making of students (Schoenfeld, 2016). Robertson, Atkins, Levin, and Richards (2016) reviewed responsive teaching literature in both mathematics and science education and identified three common themes in conceptualizations and instantiations of responsive teaching including (a) “foregrounding the substance of student’s ideas” (p. 1), (b) “recognizing the disciplinary connections within student’s ideas” (p. 2), and (c) “taking up and pursuing the substance of student thinking” (p. 2).

These are not all the possible themes of responsive teaching, but they do provide insight into how teachers might respond productively to student ideas.

We may see responsive teaching themes as separate or as integrated. At times these themes may overlap as when a teacher who attends to the substance of a child's idea then pursues the substance by connecting the idea with a mathematical structure. An example would be a child who is adding $3 + 8$ and counts up from 8 as 8,9,10,11. The teacher may recognize the difference of counting up 8 counts from 3 and that the child is using the commutative property of addition. The teacher may pursue the substance to discuss with the student whether $3 + 8$ is the same as $8 + 3$, why that may be true, and if this would be true for any numbers. Here the attending, connecting, and pursuing may have individual instances and overlap. Before a teacher can respond to the substance of a student's mathematical idea, a teacher also needs to be able to elicit the student's mathematical thinking (NCTM, 2014).

This study focuses on the verbal interactions of PSTs in mathematical task-based interviews. I conceptualize responsive interviewer interactions in mathematical task-based interviews as interactions that elicit, support, or extend the problem-solver's mathematical thinking by pursuing the problem-solver's mathematical ideas or connecting the problem-solver's ideas with mathematics.

Components of a Competency Continuum

Blömeke et al. (2015) suggest that competency includes both the performance and criterion traits of the person that is performing. Blömeke and associates propose viewing competency as a continuum that includes dispositions (cognitive and affective traits) that

lead to situation-specific skills of perceiving, interpreting, and deciding how to act. Disposition and situation-specific skills work together in the observed actions.

Observable teacher interactions in mathematics involve aspects of what makes up a teacher dispositions such as cognitive knowledge (see Neubrand, 2018 for an overview of six theories of professional knowledge for teachers of mathematics) and affective dispositions (see Philipp, 2007 for a review of belief and affect) and hidden situation-specific practices of noticing based on the situation (Jacobs et al., 2010).

My conceptual framework for both the structure of peer mathematics task-based interviews and the study of peer-interviewer interactions is from the framework of competency as a continuum presented by Blömeke et al. (2015). I conceptualize teacher/interviewer interaction competency as a dynamic continuum of dispositions, situation-specific skills, and performance (Blömeke et al., 2015) functioning together as an interviewer or teacher interacts with a problem-solver about a mathematical task.

Cognitively Guided Instruction (CGI) is an area of well-documented mathematics education research impacting responsive teaching research in mathematics and science education (Richards & Robertson, 2016). From the CGI tradition, a model of responsive teaching has emerged where teachers are responsive to children's mathematical thinking that contains a minimum of three essential elements (Jacobs & Empson, 2016). These elements include: (a) "knowledge of children's mathematical thinking," (b) "noticing of children's mathematical thinking," and (c) "enacting moves to support and extend children's mathematical thinking" (Jacobs and Empson, 2016, p. 186). I see knowledge about the frameworks of children's mathematical thinking as a cognitive disposition, noticing as situation-specific skills, and teacher's moves that support or extend a

children's mathematical thinking as performance so that the three elements correspond with components of Blömeke et al.'s (2015) competency continuum.

Seminal CGI research was conducted by Carpenter et al. (1989) as an experimental study in which frameworks of children's mathematical thinking about addition and subtraction were presented to twenty randomly assigned first grade teachers in professional development without presenting instructional practices. The teachers in the experimental group encouraged children to use more problem-solving strategies and listened to the children's process of thinking more than the control teachers (Carpenter et al., 1989). Since the 1989 study, CGI researchers have continued research on frameworks of children's mathematical thinking and presenting these frameworks to teachers (see Carpenter et al., 2015).

Since CGI research has impacted responsive teaching research and research from this tradition includes components for teacher/interviewer interaction competency as a dynamic continuum, I highlight works from the CGI tradition. When I use literature referring to noticing, attending, or using children's mathematical thinking, the phrase *children's mathematical thinking* refers to research-based frameworks of children's mathematical thinking.

I considered three NCTM (2014) mathematical teaching practices (a) "pose purposeful questions" (p. 35), (b) "support productive struggle in learning mathematics" (p. 48), and (c) "elicit and use evidence of student thinking" (p. 53) that PSTs can begin to practice as part of responsive teaching. All eight NCTM (2014) mathematical teaching practices relate to responsive teaching, but I chose these three practices to begin the practice of interactions in a one-on-one setting.

Blömeke et al. (2015) include cognition and affective/motivational domains as part of dispositions. The conceptualization is that a person's cognition and affect-motivation impact situation-specific skills (noticing skills) and hence the resulting observable behavior or interactions in this study. I also consider how outside influences (Levin, Hammer, & Coffey, 2009) beyond a person's competencies can impact a person's actions. This chapter considers literature that relates to PSTs and the components of a continuum of competency (performance, situation-specific skills, and disposition) for one-on-one interactions involving a mathematical task and the three highlighted NCTM (2014) mathematical teaching practices.

Dispositions: affections, beliefs, and cognition. Blömeke et al. (2015) include cognitive abilities and the affective/motivational characteristics as dispositions. I define dispositions as qualities of the mind and character. I do not conceptualize the cognitive and affective domains as entirely separate. Philipp (2007) explains that affect is comprised of emotions, attitudes, and beliefs where beliefs are more cognitive than attitudes which are more cognitive than emotions. Dispositions are often divided into measurable traits that correlate with domain-specific performance (Blömke et al., 2015).

Connecting PSTs' affections with mathematics. If we consider a common definition of affection, we find words such as feeling, liking, caring, attachment, fondness (see <https://www.merriam-webster.com/dictionary/affection>). Philipp (2007) gives a working description of affect as “a disposition or tendency or an emotion or feeling attached to an idea or object” (p. 259). Philipp continues to describe emotion as “feelings or states of consciousness,” attitudes as “manners ... that show one's disposition or opinion,” and beliefs as “psychologically held understandings ... about the world that are

thought to be true” (p. 259). In this section, I present two studies where the researchers studied changes in the affective domain when connecting PSTs’ affections with mathematics.

Philippou and Christou (1998) studied the attitudes of a group of 162 Greek PSTs before, during, and after a preparatory mathematics education program for prospective teachers. At a university in Greece, a three-course mathematics education preparation program was designed to educate students in mathematics content and methods from a historical perspective. The researchers chose the historical perspective based on the assumption that the Greek PSTs would “show a special *interest* in reading mathematics, mostly developed by their ancestors in their own language, under the conditions of their genesis” (emphasis added, p. 193). Philippou and Christou measured the attitudes using three separate attitude scales focusing on attitude, self-rated feeling toward mathematics, and a judgment scale. Philippou and Christou’s study provides an example of a mathematics education program for PSTs that connected the mathematics content with a Greek PST affection toward Greek history. The study reported statistically significant changes in PST attitudes towards mathematics on 23 out of 28 items on two attitude measurement scales. Philippou and Christou found that a preparatory mathematics education program connected to a PST affection could change PST’s attitudes positively.

Philipp et al. (2007) performed a large-scale study using comparative PST groups in mathematics content courses to study the effects of different treatments on beliefs and mathematical content knowledge. The study tested the authors’ theory that PSTs who studied children’s mathematical thinking while learning mathematics would have certain advantages over other PST experiences. The authors’ theory rests on a model of growth

called “circles of caring” detailed in Philipp, Thanheiser, and Clement (2002). The idea is to connect with PST’s affection or care for children. The model contains three concentric circles. Children are in the inner circle of caring that connects to a concentric circle of “children’s mathematical thinking” that connects to an outer concentric circle of “mathematics” (Philipp et al., 2002, p. 197). The circle of caring model starts with a PST’s affection of caring for children that develops into a PST’s affection of caring about mathematics by way of children’s mathematical thinking. The authors’ theory was that experiences with children’s mathematical thinking at the same time as a first mathematics content course could impact beliefs and mathematics content knowledge.

Philipp et al. (2007) worked with twelve sections of approximately 30 students per section with 159 PSTs participating in the study. The researchers used a modified-random participant selection based on PST schedules to place the PSTs into five different groups. The five groups consisted of one control group that did not participate in any additional experience apart from the mathematics content course and four groups with different experiences in addition to the mathematics content course. The additional experiences were additional classes. Two of the classes focused on children’s mathematical thinking as laboratory experiences, and two classes focused on mathematics classroom observations as apprenticeship experiences. There were two types of children’s mathematical thinking laboratory experiences. One group included one-on-one child interviews, and the other group focused the whole time on children’s mathematical thinking using primarily videos. Two different groups observed mathematics classrooms. One group observed teachers known for using reform-oriented

practices, and the other group observed classrooms that were conveniently close to the university.

Philipp et al. (2007) did pre/post belief and content knowledge assessments to measure seven beliefs about mathematics, mathematics teaching, and mathematics learning. The mathematics content assessment concentrated on place value and rational number tasks. The children's mathematical thinking groups showed more change in beliefs (38.6% and 36.1% for a large change in beliefs) than the groups observing mathematics classrooms (18.6% and 6.9% for a large change in beliefs) and the control group (13.3% for a large change in beliefs). The researchers made seventy pairwise comparisons between the groups. All the belief change pairwise comparisons were higher for the children's mathematical thinking groups. Eighteen of the comparisons showed significantly higher rates in the change of beliefs for children's mathematical thinking groups (p-values ranged from .000 to .007 for significantly higher rates). The children's mathematical thinking groups also improved more on the content test, but the change was not statistically significant. Philipp et al. found that an additional laboratory experience of children's mathematical thinking connecting to PST's affection for children with a first mathematics content course could change PST beliefs about mathematics teaching and learning.

Connecting to PST affections during mathematics teacher preparation has been shown to impact both PSTs' attitudes and beliefs about mathematics and mathematics education. My conclusion is that the alignment of PST's affection for children, attitudes, and beliefs is an integral part of a PST's interaction development toward responsive teaching. Philipp, Siegfried, and Thanheiser (2019) continue to promote connecting to

PST's interest in children to see mathematics through the lens of children's mathematical thinking. One additional component of the affective domain that I discuss next is that a teacher's emotions, attitudes, or beliefs about student learning and hence responsiveness may not be the same for all students.

Beliefs about students impact teaching practices. Philipp (2007) explains beliefs as “psychologically held understandings ... about the world that are thought to be true” (p. 259). Belief can be considered to overlap the affective and cognitive domains. Teachers can change teaching practices based on beliefs about student capabilities (Schoenfeld, 1988; Schoenfeld, 2016). Beliefs about what teaching practices are appropriate for different groups of students is a critical component to a teacher's interaction practices.

Consider the typical good teaching of traditional mathematics in an article by Schoenfeld (1988) entitled, “When Good Teaching Leads to Bad Results.” The case study shows how aspects of teacher interactions in a high school geometry course encourage memorization and practice of skills. The teacher did not interact with students to foster student mathematical thinking (Schoenfeld, 1988).

Schoenfeld (2016) shares in “Making Sense of Teaching” that he asked the teacher in Schoenfeld (1988) during the research one day if he ever considered allowing the students to explore a problem on their own. The teacher's reply was, “No, that would just confuse them. I do that with my honors students” (p. 243). Schoenfeld visited the honors class and found that the teacher did change his teaching practices for a different group of students.

Schoenfeld (2016) explains, “He [the teacher] possessed the relevant pedagogical content knowledge, but he only used it in contexts where he felt it was appropriate” (p.243). Schoenfeld’s article illustrates that a teacher can change his or her practices from students mimicking the teacher’s mathematical thinking by shifting the responsibility for mathematical learning to the students. From Schoenfeld (2016), we see that teacher’s attitudes and beliefs impact their teaching practices, which can change under different contexts.

Mathematical knowledge for teaching. Mathematical knowledge in Blömeke et al.’s (2015) model is part of the person’s disposition, specifically cognition. In this section, I address the knowledge necessary to teach elementary mathematics. In 1986, Shulman reviewed trends of knowledge testing to become a schoolteacher in the United States. Until the time of Shulman’s article, tests for teachers concentrated on content knowledge and pedagogical knowledge. Shulman presents another perspective that adds pedagogical content knowledge as a kind of knowledge necessary for teachers. Pedagogical content knowledge extends beyond the subject content knowledge to include the knowledge needed to teach mathematics effectively. Shulman includes representations that foster understanding the content, understanding strategies that make the content learning easy or difficult, and curricular knowledge.

Mathematics content courses are a place to develop the mathematical knowledge necessary for elementary mathematics. The Conference Board of the Mathematical Sciences (CBMS, 2012) states that mathematical content courses “should highlight connections between topics at the elementary and middle levels” (p. 25). As mathematics teacher educators are highlighting connections between mathematical topics, the

mathematics content course is a place where PSTs can begin practicing the connections of student ideas with mathematics. Ball, Thames, and Phelps (2008) include knowledge of content and students as one of the domains of pedagogical content knowledge.

Ball (1993) examined her teaching in elementary school mathematics to investigate the challenges of performing mathematical teaching practices where students were actively involved in experimenting with ideas. One of the dilemmas that Ball identified related to being honest to the mathematics while honoring the children's ideas about mathematics. She used the example of children who believe and agree that zero is not a number. The dilemma is the teacher's role. How is the mathematics connected to the teaching in ways that a teacher provides opportunities to learn while respecting the child's thinking? Ball (1993) presents these insights: that "mathematical knowledge is helpful is obvious ...the same is true for knowledge about students and learning ... And I am increasingly aware that there are many resources beyond knowledge that contribute to wise practice: patience, respect, flexibility, humor, imagination, and courage, for instance" (p. 395).

Ball (1993) emphasized knowledge about mathematics, along with the pedagogy of students and learning. Later, Ball et al. (2008) presented domains of mathematical knowledge for teaching. I consider two additional perspectives on knowledge to teach elementary school mathematics.

Ma (1999) coined the phrase – Profound Understanding of Fundamental Mathematics (PUFM). Fundamental mathematics is the mathematical substance of elementary school mathematics. The mathematical substance includes a structure of connected conceptual knowledge related to different aspects of elementary school

mathematics. Ma (1999) elaborated on PUFM: “By profound understanding I mean an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough. Although the term profound is often considered to mean intellectual depth, its three connotations, deep, vast, and thorough, are interconnected” (Ma, 1999, p. 120). I conceptualize PUFM as connected mathematical knowledge needed to notice problem-solver’s mathematical ideas and connect these ideas with the mathematics.

CGI researchers have emphasized knowledge about researched methods of children’s mathematical thinking shown in children’s strategies (see Carpenter et al., 2015). Jacobs and Empson (2016) label the knowledge teachers need as *knowledge of children’s mathematical thinking* in their model of teaching that is responsive to children’s mathematical thinking.

The short response time factor in a complex classroom environment makes real-time interactive decisions that are faithful to the mathematics discipline, a challenge. Noting that adapting instruction requires specialized content knowledge related to teaching, as well as knowing how students understand the mathematics shows that responsive interactions are additionally challenging if the needed content knowledge is unknown to the teacher (Jacobs et al., 2015). Instructors of mathematics content courses can combine mathematical knowledge with teaching practice (Ball & Cohen, 1999). Responding to each other’s mathematical reasoning serves as an approximation of practice (Grossman, et al., 2009) to responsive teaching.

PST dispositions of attitudes, beliefs, and content knowledge can be cultivated by connecting with their existing affections (Philippou & Christou, 1998; Philipp et al., 2007). The pedagogical content knowledge of content and students is necessary for

mathematical knowledge for teaching (Ball et al., 2008). To interact with students about a mathematical task that respects the problem-solver's thinking and the mathematics, PST's need to develop knowledge of content and students with the belief that all students can reason about mathematical concepts.

Situation-specific skills: noticing mathematical thinking. This study is concerned with PST's verbal interactions with problem-solvers working on a mathematical task in preparation to interact as teachers with students. Responsive interactions in a classroom are challenging since the classroom is a complex environment where a teacher cannot respond to all things happening at once, so choices are necessary (Jacobs et al., 2015). Before making choices as to where to focus responses, the teacher should notice the mathematical thinking of the child (Jacobs et al., 2015). Jacobs et al. (2010) analyzed various aspects of teacher noticing based on previous research. They presented a theoretical conceptualization for professional noticing of children's mathematical thinking including three interrelated skills: (a) "attending to children's strategies," (b) "interpreting children's mathematical understandings," and (c) "deciding how to respond based on children's understandings" (p. 172). Attending, interpreting, and deciding how to respond correspond with Blömeke et al.'s (2015) situation specific-skills of perception, interpretation, and decision making. Noticing mathematical thinking skills are foundational to how teachers respond to children.

Jacobs et al. (2010) studied 131 prospective and practicing elementary teacher's professional noticing of children's mathematical thinking, specifically the three areas of attending, interpreting, and deciding how to respond. They showed that noticing expertise can be learned. While the noticing framework of Jacobs et al. relates directly to

researched ways of children's mathematical thinking, I consider noticing skills in a general sense of a problem-solver's mathematical thinking.

Attending to the substance of student ideas. Attending to the substance of student ideas involves attending to the meaning behind what a student may say or do. There is a genuine interest in listening to the core of student mathematical ideas. Attending to student's ideas involves listening that goes beyond matching a student's thinking to what a teacher expects, which Empson and Jacobs (2008) call "directive listening" (p. 268). When a teacher begins to become curious about a student's thinking, they may use "observational listening" (p. 268) or "responsive listening" (p. 269) to draw out extended details of student understandings (Empson & Jacobs, 2008). The depth of attention is not just for students to share their thinking, but to understand the perspective of the student (Robertson et al., 2016). This theme matches well with a clinical child interview where the goal is to learn from the child (Ginsberg, 1997). An interview setting can be a beginning environment for exploring the perspective and substance of a student's mathematical thinking rather than evaluating or correcting the student's mathematical thinking.

Attending to the substance of student ideas implies a focus on the mathematical meaning of what students say or do. Teachers "try to understand what students are saying, from the student's perspective" (Robertson et al., 2016, p. 2). To attend to the substance of student ideas, a teacher needs to *listen to understand* the student's thinking in contrast to *listening to evaluate* a student for the correctness of the student's procedural thinking. This attention to the mathematical thinking in student's ideas is challenging. Attending to the substance of student ideas involves learning to listen and attending to

children's strategies. Attending to student's mathematical thinking benefits teacher growth in both knowledge and instruction (Fennema et al., 1996).

Empson and Jacobs (2008) in "Learning to Listen to Children's Mathematics" considered a synthesis of research to present a set of benchmarks of trends in the development of learning to listen to children's mathematical ideas. These benchmarks show a progression towards trends in which children's mathematics is more central to the teacher. There are three benchmarks: (a) directive listening to see if child's reasoning matches the teacher's reasoning, (b) observational listening to explore (observe) the child's thinking, and (c) responsive listening to build on a child's mathematical reasoning.

Empson and Jacobs (2008) suggest ways that mathematics teacher educators can provide learning experiences for teachers in learning to listen. Their suggestions include discussions of artifacts of children's written work and video showing mathematical thinking and opportunities for teachers to interact with children about their thinking and reflect on these interactions (Empson & Jacobs, 2008).

Star and Strickland (2008) studied what pre-service teachers attended to after watching videotaped classroom lessons. They note that pre-service teachers' ability to notice and interpret depends on what they attend to first. Jacobs et al. (2010) build on Star and Strickland. Specifically, Jacobs et al. consider how teachers attend to the mathematical details of children's strategies in a study of written responses to student artifacts (written and video). This data, collected from 131 practicing and prospective teachers, showed that attending to children's strategies is a difficult skill for both pre-service and in-service teachers to develop (Star & Strickland, 2008; Jacobs et al., 2010).

Jacobs et al. (2010) note that attending to children's strategies is foundational but requires time and guided support to learn.

Fennema et al. (1996) conducted a longitudinal study of 21 primary grade teachers over four years. During these years, the teachers participated in a Cognitively Guided Instruction (CGI) professional development program that focused on how children's mathematical thinking develops. The study found that 18 of the 21 teachers grew in beliefs and instruction over the four-years. The participant teacher's instruction changed from demonstrating procedures to supporting children's mathematical thinking (Fennema et al., 1996). Interest in both the student's thinking and mathematics grew generatively. Teachers began to see teaching mathematics as an on-going learning process about students, their thinking, and mathematics. The CGI research (Carpenter et al., 2015) summarizes that teacher change "is a slow process, with changes in knowledge and instruction building upon one another" (p. 206). Teacher preparation can begin this slow process instead of waiting until teachers have formed classroom habits.

As teachers focus on understanding student's mathematical thinking from the student's perspective, there is a distinction between emphasizing directive listening and observational and responsive listening. Jacobs et al. (2015) posit that attention to children's mathematical thinking takes time to develop but concentrating on children's mathematical thinking produces teacher growth toward responsive teaching.

Interpreting children's strategies. Jacobs et al. (2010) consider to what extent a prospective or practicing teacher's interpretation of a child's work related to the details of a child's specific strategy and the knowledge about the child's mathematical thinking. They found that less than half of the prospective teacher provided evidence of

interpreting children's understanding. Jacobs et al. explain that "to interpret children's understandings, one must not only attend to children's strategies but also have sufficient understanding of the mathematical landscape to connect how those strategies reflect understanding of mathematical concepts" (Jacobs et al., 2010, p. 195). They concluded that interpretation issues might be related to either not attending to children's strategies first or a lack of the mathematical knowledge necessary to make sense of children's strategies.

Deciding how to respond to student's ideas. Jacobs and Philipp (2010) identified the following four teacher reasoning categories when deciding how to respond to children's mathematical thinking: (a) the child's mathematical thinking, (b) the teacher's mathematical thinking, (c) the child's affect, and (d) general teaching moves. We can assist PSTs in learning to decide how to respond to student's mathematical thinking by presenting them with research-based options. Their article "Supporting Children's Problem Solving" (2010) in NCTM's practitioner journal for elementary school teachers (*Teaching Children Mathematics*) provides a video transcript and questions to consider in an open presentation of differentiating between the teacher reasoning categories. It includes thoughtful suggestions on how to move toward decision making based on the child's mathematical thinking.

Situation specific skills of noticing student's mathematical thinking by attending, interpreting, and deciding how to respond to a problem's solver's thinking happen quickly as a teacher performs the observable action of an interaction response. A teacher's dispositions affect their noticing skills, which all are a part of a teacher's interactions with a problem-solver working on a mathematical task.

Performance: responsive teaching and interactions. Verbal interactions of PSTs with problem-solvers working on a mathematical task is the focus of this study. To simplify the classroom environment, I consider interactions in a one-to-one environment where a PST interacts with one problem-solver. The one-to-one environment provides a setting that encourages PSTs to explore the mathematical thinking of the problem-solver. The goal of studying PST's interviewer interactions is a beginning step toward studying PST's development of interactions toward responsive teaching. The following section covers common themes of responsive teaching, three teaching practices, and PSTs as interviewers.

Responsive teaching. Common themes of responsive teaching in mathematics include placing importance on student's mathematical thinking, recognizing mathematical thinking in student's ideas, and following up on student mathematical thinking by supporting and extending the mathematical essence of student thinking (Robertson et al., 2016).

The foregrounding of a student's idea(s) involves interactions where prominence is placed on the thinking of students while they are working on a mathematical task. The focus is on listening to understand instead of evaluating the problem-solver's thinking (Robertson et al., 2016). Focus on the student's mathematical ideas requires an interest (an affective disposition) in another person's ways of mathematical thinking. Focusing on a student's mathematical ideas involves perceiving or attending (a situation-specific skill) to the mathematical ideas of another person from the perspective of the other person (Robertson et al., 2016). The situation-specific skill of attending to the student's

mathematical thinking ties to interactions where the student's mathematical reasoning and ideas are prominent.

Recognizing the mathematical connections in the substance of student mathematical ideas involves listening for connections between student ideas and the mathematics (Robertson et al., 2016). Teachers have the opportunity to highlight mathematical kernels from student's ideas that have the potential to deepen the mathematical understanding of those involved in the discussion (see Ball, 1993 for examples from an elementary mathematics classroom). Ball (1993) explored her teaching of elementary mathematics and described this dual attention to children and mathematics.

My practice is also honest in its respect for third graders as mathematical thinkers.

...My ears and eyes must search the world around us, the discipline of mathematics, and the world of the child with both mathematical and child filters.

(p. 394)

Connecting student's ideas to the mathematics discipline involves abilities of attention and interpretation (situation-specific skills) of the mathematical ideas of student's work from the perspective of the student (Robertson et al., 2016). Attending to the substance of student's mathematical ideas includes profound knowledge of both the mathematical content knowledge about the topic of study and ways that people think about the mathematical topic of study (cognitive dispositions).

Pursuing the meaning of student thinking involves interactions that are responsive to student's mathematical ideas and supportive of the mathematical substance of student thinking. The act of following up on student's thinking requires decision making, a situation-specific skill. Teachers can build upon and make connections between student's

ideas and the mathematics in-the-moment interactions or in the long-term by building lessons around student mathematical ideas (Robertson et al., 2016).

Three mathematical teaching practices. I highlight three mathematical teaching practices from *Principles to Actions* (NCTM, 2014). Eliciting and using student thinking, posing purposeful questions, and supporting productive struggle are practices that involve interactions. These three practices apply to both classroom situations and one-on-one interactions with a mathematical task and can be incorporated into a task-based interview.

The mathematical teaching practice “elicit and use evidence of student thinking” (NCTM, 2014, p. 53) can be decomposed into eliciting student thinking and using student thinking. These twin practices are essential parts of formative assessment (NCTM, 2014). Formative assessment is defined as “all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged” (Black & Wiliam, 1998, pp. 7-8). As students share mathematical ideas and sense-making, the teacher has the opportunity to gather evidence of student learning to adjust instruction not just in the moment, but also in lesson planning. Responsive teaching and formative assessment foci can be different. Formative assessment may be done with a focus to evaluate student thinking, but the focus of responsive teaching is to build on student thinking (Richards & Robertson, 2016). Formative assessment of student mathematical thinking provides a base for responsive teaching (Pierson, 2008).

One way to generate evidence of student mathematical thinking is the NCTM (2014) practice of “posing purposeful questions” (p. 35). Eliciting the meaning of student thinking involves the skill of posing purposeful questions. Boaler and Brodie (2004)

explored the questions that teachers asked across three schools with approximately 1000 students to develop teacher question codes. Boaler and Brodie (2004) presented nine teacher question codes. The pose purposeful questions practice section of *Principles to Actions* presents a framework of four types of questions. The question types in the framework are: (a) “gathering information [for recall of vocabulary and procedures], (b) probing thinking, (c) making the mathematics visible, and (d) encouraging reflection and justification” (NCTM, 2014, pp. 36-37). The gathering information category relates to quick checks for formative instruction to check areas to explore and elicit mathematical thinking. The probing moves allow a teacher to elicit or pursue more student thinking. Teachers can use making the mathematics visible and encouraging reflection and justification categories to help students connect their sense-making to the mathematics discipline while building and extending mathematical understanding. The NCTM (2014) framework for question types is a concise framework for posing purposeful questions. The categories of “making mathematics visible” and “encouraging reflection and justification” complement responsive teaching themes.

The CGI tradition provides an emerging framework of teaching moves that includes moves to elicit and use student thinking with purposeful questions. Jacobs and Ambrose (2009) first presented an initial teaching moves framework in *Teaching Children Mathematics*, a practitioner journal for elementary teachers of mathematics. Jacobs and Ambrose (2009) analyzed videos of 65 teachers in one-to-one interviews with 231 children solving story problems in the domain of whole number operations. There were 1,018 story problems used in the interviews. From their analysis, Jacobs and Ambrose (2009) identified eight categories of teaching moves that supported and

extended children's mathematical thinking. They divided the eight categories into two sets: one set of four teaching moves for when an answer is incorrect, and the other four teaching moves for when an answer is correct.

Jacobs and Empson (2016) studied videos of both one-on-one interviews and classroom interactions of an expert teacher's moves that supported and extended children's mathematical thinking in the domain of fractions. The work of Jacobs and Empson (2016) builds on the teacher moves categories of Jacobs and Ambrose (2009). Jacobs and Empson (2016) refer to the concept of a teaching move as "a unit of teaching activity that has coherence with respect to a purpose" (p. 2). Their findings included an adapted framework of four categories of teacher moves to support and extend children's mathematical thinking. One category contains four subcategories. The article describes each category and provides a concise picture of the emerging framework. Jacobs and Empson (2016) provide examples of responses showing the flavor and variety of fundamental concepts of each category and subcategory.

Jacobs et al. (2015) used the emerging framework in a multiple case study of three grade 4-5 teachers from different geographical areas skilled in responsive teaching of fractions. The researchers collected videos of two days of classroom instruction and videos of teacher-student interviews for 5-7 students for each teacher. This research added a category as well as a fifth subcategory to describe details of the child's existing strategy and changes some of the wording of categories.

Jacobs and Empson (2016) have another important conclusion from their study. They note that there was an alignment of teacher moves in both interview and classroom

environments. The alignment of teacher moves in interview and classroom settings shows that PSTs future classroom interactions may benefit from interview practice.

A third teaching practice from *Principles to Actions* is supporting students' learning of mathematics by encouraging productive struggle (NCTM, 2014). A central idea of productive struggle is student responsibility for their mathematical reasoning instead of a correct answer focus (NCTM, 2014). Research about supporting productive struggle complements the development of responsive interactions by providing PSTs with strategies that support students in struggling productively with mathematics (Warshauer 2015a, 2015b).

One may question if responding to a student's mathematical reasoning is wise since students' thinking can be full of confusing explanations, mistakes, and misunderstandings (Pierson, 2008). When a student is confused, it may take more time to guide them from the confusion to sound understanding than to step in and fix the student's confusion (Schoenfeld, 2016). Dealing with inadequate and faulty reasoning can be especially challenging, particularly if one wants to respond in a way that supports the student struggling productively with mathematical concepts (Warshauer, 2015a). Warshauer developed a summary of teacher responses to student struggles with mathematical tasks by studying 327 students in sixth and seventh-grade mathematics. She categorized the teacher's responses into four categories (telling, directed guidance, probing guidance, and affordance) (Warshauer, 2015a). Probing guidance and affordance, in particular, are interactions that teachers can use to elicit, support, and build mathematical thinking. In "Strategies to Support Productive Struggle," Warshauer (2015b) presents strategies that teachers can use to support productive struggle.

Warshauer (2015a) found specific strategies of what a teacher should do besides showing the student how to do the task. She outlined the following four teaching strategies: (a) question, (b) encourage, (c) give time, and (d) acknowledge. The article “Strategies to Support Productive Struggle” (2015b) in NCTM’s practitioner journal for middle school teachers (*Mathematics Teaching in the Middle School*) provides an example of a task and two teacher vignettes showing possible teacher responses to student struggle with the given task. The first vignette is an example of a teacher who guides the student to the correct procedural process to solve the task. The second vignette shows another teacher who uses the four teaching strategies to support a student productively through his struggle. These strategies, along with the teacher reasoning categories, can assist PSTs to consider their conceptualization of how to assist struggling students during peer task interviews.

From the literature, recent research has shed light on the teacher’s foci in deciding how to respond, the response categories, and strategies to support productive struggle. Sharing with PSTs a framework for deciding how to respond to student struggle, can provide PSTs with opportunities to consider their perspective of student struggle and areas that they may desire to develop as future teachers.

PSTs as task-based interviewers. Clinical child task-based interviews can be traced to Piaget’s methods of interviewing and examining children (Groth et al., 2016; Schorr, 2001). Ginsburg (1997) wrote a book, *Entering the Child’s Mind*, which discusses the child task-based interview and how teachers and researchers can use interviews to learn more about children’s mathematical thinking. This section of the

literature review presents five studies where PSTs in their education preparation conducted task-based interviews with children.

Schorr (2001) presents an early study of PSTs given opportunities to learn about clinical task-based interviews with children. Some PSTs also decided to interview adults. The study was part of a mathematics methods course with 23 PSTs over 15 weeks. The researcher was also the instructor of the course. In this study, PSTs conducted child (or adult if access to children was limited) interviews at least every other week for a minimum of seven interviews. The researcher reviewed written interview logs and journals of the PST's interviews and interview reflections. Schorr (2001) found three consistent themes. The interview practice with children helped PSTs with the following: (a) to change their mathematics teaching views or beliefs, (b) to notice that children (and adults) invent their problem-solving strategies, and (c) to see that a person who gets a correct answer does not necessarily understand the mathematics involved (Schorr, 2001, pp. 155-158). The conclusions of Schorr are insightful in showing that while interviewing children in one class is not sufficient, clinical interviewing helped in two major ways: (a) "to consider alternative approaches to teaching" and (b) to give motivation for "challenging and thought provoking course discussions about mathematical ideas" (Schorr, 2001, p. 160). This study shows that PSTs interviewing children encouraged PSTs to consider their views about teaching mathematics and served as a catalyst for rich discussions about mathematical ideas.

In 2002, two other studies were published about PST's interviews with children. McDonough et al. (2002) investigated PSTs from two universities using interview protocol formative assessments with children in Australia. In another study, Philipp et al.

(2002) studied PSTs in a course focused on children's mathematical thinking with individual child interview practice as a companion course to a mathematics content course.

McDonough et al. (2002) analyzed PST's comments about interviewing. They found two themes: (a) PSTs noticed differences between student ways of thinking, strategies, confidence, and expression of mathematical ideas and (b) PSTs experienced children using the same children's strategies that the PSTs had learned in preparation courses. PSTs gained an appreciation of children's mathematical abilities and thinking (McDonough et al., 2002).

Philipp et al. (2002) explored companion education courses to a first mathematics content course. The companion education courses were to provide more integration for students in content and pedagogical preparation. A random group of students participated in a course concentrating on children's mathematical thinking with opportunities to both interview and tutor children with discussions about these experiences and the mathematical content. The PSTs in this companion course increased more in beliefs toward reform mathematics than students in a companion course that observed reform-oriented classrooms. The PSTs in the course that included child interviews also discussed how their experiences motivated them to learn mathematics in the mathematics content course. Philipp et al. (2007) noted in a later published quantitative study that the PSTs in the companion course with child interviews showed higher gains in mathematics content, but the gains were not statistically significant. Overall, Philipp et al. (2007) concluded that the emphasis on children's mathematical thinking is effective for PSTs in mathematics content courses. They noted that many universities may not be able to add

additional courses, but that professors could include video clips of student mathematical thinking in their classrooms.

Crespo and Nicol (2003) investigated using child interviews with 18 elementary PSTs in a mathematics methods course to learn about questioning, listening, and responding. This study showed PSTs openly expressing their surprises and frustrations with the interview experiences. One surprise area for PSTs was how students performed computations mentally versus on paper. PSTs were surprised that elementary children could do subtraction more easily mentally than on paper and blamed the interview process for making children seem nervous (Crespo & Nicol, 2003). PSTs were trained for the child interviews, but they had two distinct patterns of evaluation or inquiry in the interviews (Crespo & Nicol, 2003). The study showed that some PSTs, even with training, may be resistant to an inquiry approach in interviews. Crespo and Nicol (2003) conclude that the child interviews provided opportunities for PSTs to learn questioning techniques and analyze student work, but that more support is necessary to move more PSTs toward an inquiry approach (Crespo & Nicol, 2003).

More recently, Groth et al. (2016) followed four PSTs in the process of learning to interview with two cycles of training, interviews, and reflection. This study included mock interviews where PSTs practiced the child interviews with each other before interviewing children. Groth et al. (2016) found that communicating the purpose of the interview to children and asking probing questions in-the-moment, were difficult aspects for PSTs. The interview cycle process did help PSTs realize that they were guiding the children too often and that there were opportunities where they could have explored the children's thinking more (Groth et al., 2016). This study showed some of the challenges

for PSTs interviewing children and shed encouraging light of PST's perception of needed improvement in exploring a child's thinking. This study provides further evidence that giving PSTs opportunities to practice interactions may help develop interactions that encourage PSTs to move from telling and guiding questions to eliciting and probing student's mathematical thinking more.

In summary, mathematics education researchers are encouraged with the potential of PSTs conducting child task-based interviews. The benefits focus on new realizations about children's mathematical strategies and thinking. The challenges are to provide more training and experiences to build techniques of eliciting mathematical thinking. Peer task interviews provide an opportunity for PSTs to begin to build elicitation techniques, in addition to supporting student struggle as appropriate.

Influences beyond a person's competency continuum. Influences outside of the teachers' dispositions and situation-specific skills can impact teachers' actions. Teaching responsively to student thinking is more challenging in educational environments that focus on classroom management and the amount of material covered in class, excluding the importance of student thinking (Levin et al., 2009). Alan Schoenfeld notes that he "has seen beginning teachers who were remarkably good at classroom management, because they came from a teacher preparation program that focused so intently on individual student learning that the teachers were remarkably attuned to their students' understandings, and thus had very few issues with classroom management" (Schoenfeld, 2016, p. 244). Schoenfeld's observation gives hope that university-based teacher preparation programs focusing on student thinking will produce teachers that focus on student thinking in their future teaching.

Levin et al. (2009) focused on nine novice-intern science teachers in an alternative certification program at a northeastern university. As part of the alternative certification program, the teachers attended seminars emphasizing student thinking. The researchers analyzed whether the interns attended to student thinking in classrooms with videos, seminar discussions, or seminar written work. The researchers found that eight of the nine intern teachers attended to student thinking in their first teaching year. Two of the interns were in a school environment that did not encourage them to attend to student thinking and instead emphasized course content coverage and classroom management skills. One of these interns was not part of the university seminar program in the summer before the internship and was the only intern teacher that did not show evidence of attending to student thinking while teaching, although she did focus on the school's intended curriculum and student behavior.

The other intern at this school entered the internship with the desire to attend to student thinking but struggled with classroom management. She was not able to attend to student thinking until the spring semester. Levin et al. (2009) highlight the intern's tension who desired to focus on student thinking but was in an environment that counseled her to stay on target with the curriculum and focus on student behavior. The authors reported that despite the environment, the intern did persist in attending to student thinking and was able to improve classroom management in her first year of teaching practice as an intern.

University preparation programs cannot pick the school environments where PSTs teach, but Schoenfeld's (2016) observations and the Levin et al. (2009) show that an emphasis on student thinking at the university can influence future teacher's teaching

practices. There is evidence that novice teachers may persist in attempting to attend to student thinking even when they are working in environments that emphasize other teaching practices to the exclusion of attending to student thinking. Therefore, activities that include interactional practice can encourage PSTs to attend to student thinking at the beginning of their teaching careers, even in the most challenging of circumstances.

From the literature presented, I propose that the development of the continuum of competency toward responsive interactions is critical for PSTs. My research is a beginning step to see how PST interviewers interact with a problem-solver at the beginning of their mathematics education coursework.

Conclusion

Responsive teaching interactions are beneficial, but challenging to develop (Jacobs & Empson, 2016; Jacobs et al., 2015; Schoenfeld, 2016; Jacobs et al., 2014; Pierson, 2008; Levin et al., 2009). CBMS (2012) recommends twelve hours of mathematics content course for PSTs. Twelve hours may be an ambitious goal, but many universities require one to four mathematics content courses for PSTs. These courses provide opportunities for PSTs to develop mathematical content knowledge for teaching while encouraging approximations of teaching practice (Ball & Cohen, 1999; Grossman, et al., 2009).

Since aspiring responsive teaching themes are challenging to develop (Jacobs & Empson, 2016; Pierson, 2008; Jacobs et al., 2015; Schoenfeld, 2016; Empson & Jacobs, 2008), this study considers *incremental change* toward developing responsive teaching. Star (2016) used the phrases “incremental improvements” and “incremental change” concerning policy. I propose incremental changes in teacher education for the

development of responsive teaching interactions. For an incremental change, I considered a one-on-one setting instead of the complex classroom, with a focus on interactions. In keeping with the idea of incremental change, I chose three NCTM (2014) mathematical teaching practices (a) “pose purposeful questions” (p. 35), (b) “support productive struggle in learning mathematics” (p. 48), and (c) “elicit and use evidence of student thinking” (p. 53) to present to PSTs. This research explored how PST interviewers interacted in peer interviews. Even though the study was about PST performance of interaction, my conceptual framework of a continuum of competency for responsive interactions was the basis of the materials used in the PST preparations and reflections of the peer interviews. The following chapters explain my study methods, findings, and conclusions.

III. METHODS

My research uses a multiple case study (Yin, 2014) to examine peer-interviewer interactions in peer task interviews. The rationale for using a case study was to provide an in-depth exploratory study of pre-service elementary teacher's interviewer interactions during mathematical task-based interviews in a beginning mathematics content course. A PST's interviewer interactions for three peer mathematical task-based interviews was a case. The purpose of multiple case studies was to explore the interviewer interactions with contrasting cases based on PST's different initial perceptions about teaching interactions in mathematics classrooms. The multiple case study was designed to provide information about themes and variations in peer-interviewer interactions. The research questions I addressed in my study were:

1. In what ways did the PST peer-interviewers verbally interact with a peer problem-solver during three peer mathematical task-based interviews?
2. How did the PSTs' interviewer verbal interactions during three peer task-based interviews develop over a semester?

The multiple case study goal explored PSTs interviewer interactions in the context of prepared materials provided to the PSTs before the interviews. The prepared materials included research-based readings about interviewing and questioning, eliciting student thinking, and productive struggle. The unit of analysis for the study was peer-interviewer interactions. The objective was to classify interviewer interactions in the context of mathematical task interviews to identify interaction patterns, variations, and development patterns. The peer task interview recordings and transcripts of three interview cycles were the primary sources of data used to describe and analyze both the

peer interactions and the development of their interactions. I triangulated the primary sources of data with the following secondary data sources: (a) the interviewer's written analysis of interview and responsive interaction preparation and goals, (b) written mathematical task preparation assignments, (c) peer interview and problem-solver analyses of their interview, and (d) participants interviews. My study contributes towards an understanding of PST's development toward responsive teaching practices in a one-on-one mathematical task-based interview setting during the beginning stages of their teaching preparation. In this chapter, I describe the study participants and setting, data collection, and data analysis. The chapter concludes with a discussion on the study's trustworthiness.

Participants and Setting

The participants of the study were from one section of a mathematics content course for pre-service teachers for early childhood through middle school grades from a large southern university in the United States. Interdisciplinary majors working toward six different certifications were required to take this course, which is the first of two to four required mathematics content courses depending on the teaching certification area.

The mathematics content course on *numbers and operations* had four goals: (a) understanding the underlying concepts of number systems and operations from the perspective of a grade school teacher; (b) knowing how to do mathematics; (c) learning to articulate why the mathematics operates as it does; and (d) applying the mathematics to real-world situations to become competent elementary and middle school teachers (taken from course syllabus). The instructor had over thirty years of experience teaching mathematics content courses for future early childhood through grade eight teachers.

Mathematics for Elementary Teachers with Activities (Beckmann, 2014) served as the course textbook. The textbook pedagogy was inquiry-based, emphasizing student thinking. The course met two days a week, with 80 minutes per class for a total of 28 days during the spring 2017 semester.

Thirty students completed the course who signed informed consent to participate in the study. All the students began the semester as self-reported teaching majors, but one student was a nursing student at the end of the semester. Table 1 shows the students by certification area or major. The biological sex of one student was male. The rest of the students were female. I used feminine pronouns throughout this report to protect the identity of the male.

Table 1

Participants by Teaching Certification Area or Major

Certification Area	Participant
EC – 6 ESL Generalist	23 (76.7%)
EC – 6 Bi-lingual Generalist	1 (3.3%)
4 – 8 Generalist	1 (3.3%)
4 – 8 Mathematics	3 (10%)
All – Level Special	1 (3.3%)
Other (Nursing)	1 (3.3%)

All of the participants participated in peer interviews as peer-interviewers and problem-solvers. The study focused on interviewer interactions for nine peer interviews. For the nine peer interviews, I selected three peer-interviewer participants from all the participants to explore interviewer interactions with those who had contrasting initial perceptions of mathematics teacher responses. All of the other students in the course were possible problem-solver participants for the nine interviews.

Peer-interviewer participant selection. To capture participant differences in the initial perception of mathematics teachers, the participants participated in a regular writing activity before any assignments related to the mathematics task-based interviews. The activity used an illustration of how two classroom teachers responded differently to similar student struggles on the same task, taken from *Principles to Actions* (NCTM, 2014, p. 51). The *showing* teacher directed a task through telling, directed guidance, and demonstration to help students be successful by finding the right answer. The *affordance* teacher, in contrast, focused on children's mathematical thinking by probing guidance and affordance (following Warshauer's (2015a) categories of teacher responses). For this study, participants responded to the illustration with two prompts as follows:

1. Which teacher's response most closely represents how you think you would respond as a teacher in this situation? Explain why you choose this teacher, including what would be similar and different in how you would respond.
2. Which teacher's response is better for the children's understanding of the mathematics? Explain why the response of the teacher that you choose is better for the children's understanding of mathematics.

The thirty students who completed the course, all completed the initial assignment as requested. After the initial assignment, I did initial coding to categorize the participants by their preference for the showing, affordance, or a mixture of teaching responses. For each question, I gave each participant a code for the first question and a code for the second question. The codes were for the showing, affordance, or mixture. I classified each participant, considering their two codes. The three category types were (a) the participant chose the showing teacher for both questions (type 1), (b) the participant

chose the affordance teacher for both questions (type 2), and (c) the participant chose different teachers for the two questions or chose a mixture of teachers for at least one question (type 3). Table 2 shows the different types of classifications.

Table 2

Category Types for Preferences of Contrasting Teacher Responses

Represents PST's Interactions	Interactions are Better for the Children		
	Showing	Affordance	Mixture
Showing	Type 1	Type 3	Type 3
Affordance	Type 3	Type 2	Type 3
Mixture	Type 3	Type 3	Type 3

After data collection, I re-coded the responses for reliability. The codes for ten participants were checked by a mathematics education assistant professor and a mathematics education doctoral student for reliability. The ten selected assignments included at least two of each type, including four participants who had favorable comments about both teaching styles. Each intercoder agreed with my codes by 95%, and they agreed with each other by 90%. Of the 30 participants, 12 were type one, 9 were type two, and 9 were type three. Table 3 shows the breakdown of the codes.

Based on the initial coding, I selected 11 potential participants that would receive the same mathematical tasks for the three interviews. My goal was to have one or two peer-interviewer participants from each category type to study their interviewer interactions in depth. Even though the goal was three or six peer-interviewer participants, I began with a pool of eleven participants due to possible interview absences. Eight of the 11 participants completed the mathematical task preparations and participated in the three interviews. Of the eight, I interviewed five participants in participant interviews after the

third peer interview. One of the five participants did not complete all of the pre-interview assignments. Of the remaining four participants, there was one of each type with an extra type three PST. I selected the type three participant because she had more variation of interviewer interactions, especially responses to interview questions. The three peer-interviewer participants were all prospective elementary generalist teachers. I gave pseudonyms to all the peer-interviewer participants and problem-solver participants that worked with them. Paige (type one), Reese (type two), and Beth (type three) were the peer-interviewer participants based on a combination of purposive and convenience sampling (Merriam, 2009) as described above.

Table 3

Preferences of Contrasting Teacher Responses

Represents PST's Interactions	Better Interactions for Children's Understanding			Totals
	Showing	Affordance	Mixed	
Showing	12 (40.0%)	2 (6.7%)	1 (3.3%)	15 (50.0%)
Affordance	2 (6.7%)	9 (30.0%)	2 (6.7%)	13 (43.4%)
Mixed	0 (0.0%)	2 (6.7%)	0 (0.0%)	2 (6.7%)
Totals	14 (46.7%)	13 (43.4%)	3 (10.0%)	30 (100%)

Note. Because of rounding, the final total is 100.1%.

Peer interview cycle context. I developed the peer interview cycle context during the two semesters before this exploratory study on PSTs' interviewer interactions. I named the peer interview cycle: Peer Interaction for Responsive Teaching (PIRT). Initially, I used Peer-Assisted Reflection (PAR) to encourage PSTs to explore each other's mathematical thinking with verbal interactions. PAR is a learning and assessment approach that complements student learning in university STEM courses (Reinholz, 2015b). PAR gives an iterative structure for students to engage in open-ended problems

through a cycle of activities. The cycle of activities involved (a) solving an open-ended homework task with a self-reflection form, (b) assessing a peer's initial solution attempt with an oral peer conference of the student and peer's initial solution attempts in class, and (c) revising the initial solution based on the peer's written assessment and oral conference (Reinholz, 2015a; Reinholz, 2015b; Reinholz, 2016). PAR inspired me to consider the idea of how to include human interactions to practice responsive interactions in a mathematics content course.

I piloted one PAR cycle in a mathematics content course during Spring 2016. The piloted task worked well with multiple solution paths, but not as well with the revised solutions. The nature of the task needed to provide opportunities for revised solutions (a feature of PAR not in PIRT) to be different from initial solutions. While PAR showed promise for showing multiple solution paths, the tasks needed adjustments that would allow for significant revisions.

A more critical issue was that I sought to develop interactions in a way that approximated teaching practice. The idea of peer task interviews began to replace PAR. PAR has students complete the same task as an assignment and then exchange task work for a peer to assess. PIRT has one student preparing a task to give to another student in a task-based interview where the student can interact with the problem-solver during and/or after the problem-solver works on the task.

In Fall 2016, I piloted peer task-based interviews in one mathematics content number and operation course. As a result of the peer interview pilot, I decided to not only vary tasks but also to vary the interviewer preparations for the task. Using varied interviewer preparations allowed PIRT to introduce different mathematical practices to

PSTs as part of the peer interview cycle. I included feedback and reflective practice with peer interview analysis forms. This process of fostering an environment to develop responsive peer-interviewer interactions led to the PIRT cycle that I used for this study.

PIRT was developed to use peer task interviews as an approximation of a teacher working individually with a child while introducing PSTs to the following NCTM (2014) research-based practices of mathematics teaching: (a) pose purposeful questions, (b) elicit and use student mathematical thinking, and (c) support productive struggle. Philipp et al. (2002) used a circles of caring model, which inspired the conceptual framework for PIRT. The circles of caring model considers elementary PST's interest in children and develops outward from that interest to children's mathematical thinking to mathematics knowledge for teaching. My conceptual framework connected a PST's caring for children to teaching children, and finally to mathematics teaching practices. Figure 2 shows a pictorial representation of the conceptual framework for PIRT.

I developed PIRT to facilitate the development of responsive interactions in peer task-based interviews. Figure 1 is a visual representation of the PIRT cycle of activities. PIRT cycles include:

1. interview and responsive interaction preparation,
2. preparation of a mathematical task,
3. peer collaboration to compare preparation and strategies approximated to teacher teams,
4. implementation of the prepared task with a classmate in a peer task interview,
and

5. written analysis of peer-interviewer interactions related to the peer's mathematical thinking and goals for the next PIRT cycle.

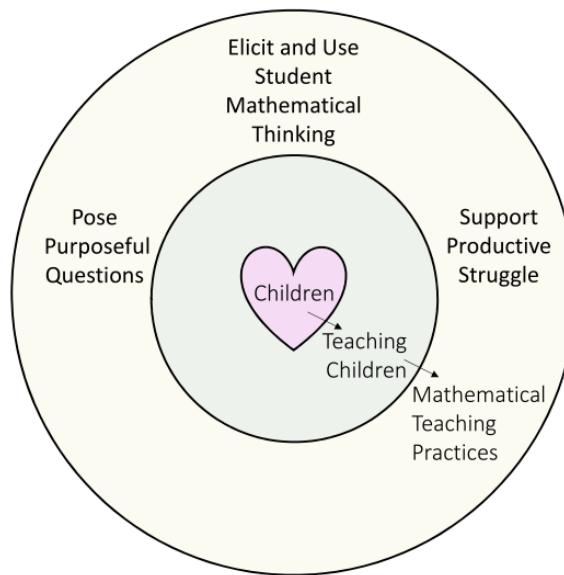


Figure 2. PIRT framework informed by NCTM (2014) and the Circles of Caring model in Philipp et al. (2002).

Each PST received an interview and interaction preparation assignment, which included material to read with a questionnaire to analyze the material. The questionnaire included prompts for the PSTs to make personal goals for peer interview interactions in the next peer task interview. PSTs in a mathematics content course were divided into task groups. Each group was given a different mathematical task from the curriculum materials to prepare for the peer interview. The mathematical task preparation approximates a teacher planning the use of a mathematical task in the classroom or with an individual student including (a) initially solving the mathematical task, (b) solving the mathematical task with additional strategies, and (c) listing possible difficulties a student may have in solving the mathematical task.

In the same class session as the peer interviews, each task group met in teams to compare preparations and solution strategies. The course instructor could visit task groups during this time to advise and answer questions before the individual task interviews. The task groups approximated teacher teams in schools.

Next, PSTs were assigned a peer so that each one had a different mathematical task. I prepared the peer pair assignments before the interviews, but I made changes in class to accommodate for absent students. A PST served as a peer-interviewer giving her task to the assigned peer as the problem-solver. As the problem-solver worked on the mathematical task, the peer-interviewer had opportunities to interact with the problem-solver about the task work and to ask questions of the problem-solver as an interviewer might explore the interviewee's mathematical thinking in a clinical mathematical task-based interview to understand her mathematical reasoning.

After the peer interview, the peer-interviewer and problem-solver answered analysis prompts about the interview. The PSTs then changed roles, so each person had the opportunity to be a problem-solver and peer-interviewer.

Interview and responsive interaction preparation assignments. The interview and responsive interaction preparation assignments concentrated on the teaching practice of interactions with students who are working on a mathematical task. The interview and responsive interaction preparation assignments each focused on one of the mathematical teaching practices from *Principles to Actions* (NCTM, 2014) included in the PIRT framework (see Figure 2).

The first assignment introduced PSTs to the interview environment and the practice of posing purposeful questions. After reading “Assessing for Learning” by Ed

Lavinowicz (1987), PSTs answered questions about interviewing, compared interviewing and teaching, and analyzed the aspects of interviewing that would be the easiest and most difficult to develop. As part of the first assignment, PSTs received a purposeful question framework with example questions and made goals for the upcoming interview.

Elicit and use student mathematical thinking was the theme of the second interview and interaction preparation assignment. PSTs were provided with the article: “Making the Most of Story Problems” by Victoria Jacobs and Rebecca Ambrose (2009) and a framework of supporting and extending moves used in one-on-one interactions taken as an excerpt of the *Responsive teaching with fractions* paper by Jacobs, Empson, Krause, and Pynes (2015). The article presented categories of teacher moves to support a child’s thinking before obtaining a correct mathematical answer and categories of moves to extend a child’s thinking after a child answered correctly. The teaching moves framework had five general categories, with sample teaching moves for each category. In the questionnaire, PSTs considered the categories of teacher moves and decided which type of move category they would like to develop in the next peer interview.

The last interview and interaction assignment focused on supporting productive struggle. The reading for this assignment was “Strategies for Productive Struggle” by Hiroko Warshauer (2105). Warshauer wrote about a teacher who supported the productive struggle of a student who worked on a task from a middle school curriculum, *Mathematics Explorations* (McCabe, Warshauer, & Warshauer, 2009). This third assignment began by having the PSTs use their mathematical knowledge to work on a ratio task in this article and then read the article. The PSTs then reflected on the teacher’s approach to the student who was struggling with the task. Next, the assignment

invited PSTs to consider examples from their personal experiences of mathematical struggle and whether the struggle was productive. Lastly, PSTs were prompted to consider how they could prepare for possible problem-solver struggle in the next peer interview.

The interview and interaction preparation assignments all focused on different teaching practices involving interactions with students working on mathematical tasks. The third assignment included proportional understanding as a component of mathematical content knowledge and tied that knowledge to pedagogical content knowledge of working with students struggling with mathematical tasks. Each task invited the PSTs to consider their role as an interviewer in the next peer interview and how she could apply the teaching practice in the upcoming interview. The three interview and interaction preparation assignments are in Appendix A.

Mathematical task preparation assignments. The mathematical task preparation assignments concentrated on PST's content and pedagogical content knowledge to interview a peer with the same task. Each PIRT cycle included two tasks. Half the class received an assignment for one task, and the other half received an assignment for another task. I selected each task from the course curriculum (Beckmann, 2014) and aligned the task topic with scheduled course topics. The three topics were fractional understanding, multi-digit addition or subtraction in a base other than ten, and proportional reasoning.

I selected two tasks for each topic. Each selected task had multiple solution methods to encourage PSTs to see multiple solution possibilities. In selecting the two tasks, I considered the context of each task. I selected the tasks in such a way that one of

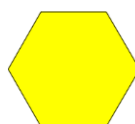
the two tasks had a context that was more familiar to the PSTs. The goal of having one task in a more familiar context was to allow PSTs to conduct interviews in settings where the problem-solver may struggle and where the problem-solver may solve the task directly. The goal was for the interviewer to practice eliciting and supporting the problem-solver both before and after they obtained a correct answer (Jacobs & Ambrose, 2009). The interactions of the PST interviewers I studied each completed the same three tasks. In the first and third interviews, the selection PST peer-interviewer participants prepared the mathematical task whose context was perceived to be less familiar. The task with a more familiar context was for the second interview for the three selected participants. I describe each of the tasks used in this study.

Fractional understanding. Beckmann (2014) presents a contrast between a part to whole perspective of fractions with a perspective of a “fraction as a collection of equal-sized parts” (Fierro, 2013, p. 244). I chose one of the tasks in Beckmann’s class activities that used pattern tile blocks to find the whole pattern if one pattern tile block is less than the whole. The task was as follows:

This assignment uses pattern tiles (blocks). Here are visual representations of the pattern tiles.



The hexagon pattern tile is $\frac{2}{3}$ of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design.



When I used this task in a previous course, I noticed that some of the students would begin the task by focusing on the denominator “3” and divide the hexagon into three equal sections. The task emphasizes that the *two-thirds* of the pattern tile design signifies that there are 2 equal parts each of size “one-third.” Once a problem-solver found what size was “one-third,” the problem-solver needed to add a third to the given two-thirds to complete the whole design.

The wording of the pattern tile task often allowed interviewers to work on interactions concerning understanding the task. I had noticed in previous courses that a common question about the task was if all the provided tiles needed to be used or could a tile be used more than once. The interviewer could make sure the problem-solver knew that the task meant that the tiles were options, not all tiles had to be used, and a tile could be used more than once.

In choosing the task, I appreciated that the pattern blocks allowed for various task solutions. Four solutions included: a hexagon with a trapezoid, three trapezoids, a hexagon with three triangles, and a hexagon with a rhombus and a triangle. The task is also easily adjusted by changing the fractional amount from two-thirds to one-half to aid in understanding the task or from two-thirds to three-halves to extend fractional understanding.

The mathematical task preparation assignment focused on PST’s mathematical content knowledge and some pedagogical content knowledge. The assignment asked PSTs to solve and describe how they solved the task. Next, the assignment directed PST to go beyond common knowledge and solve the task another way. Solving tasks in more than one way broadens the mathematical knowledge base of a teacher to relate to more

student's ways of reasoning. Table 4 shows the ways that Paige, Reese, and Beth solved the pattern tiles task on their mathematical task preparation assignment. Beth's work illustrates possible PST thinking of dividing the hexagon into thirds to find two-thirds of the hexagon, instead of recognizing that the whole hexagon is two-thirds of the pattern design. Beth's work also shows the correct response, possibly added during the task team time. Beth's additional solution was not adjusted.

Table 4

Mathematical Task Preparation Pattern Tile Task Solutions

Peer-Interviewer Participant	First Response (number one)	Additional Response (number four)
Paige (Type 1)	A drawing of three trapezoids	A written explanation describes a student calculating the surface area of the hexagon, dividing by two, and then multiplying by three get the total area. The student would then work to find a shape that matched the area.
Reese (Type 2)	A drawing of three trapezoids A drawing of a hexagon and a trapezoid	A drawing of a hexagon, rhombus, and triangle
Beth (Type 3)	The original hexagon was divided into three rhombi. Each rhombus was labeled as one-third. Two-thirds (two rhombi) were shaded with $= \frac{2}{3}$ written outside the hexagon. A drawing of a hexagon was labeled two-thirds with three attached rhombi, each labeled one-third. A drawing of a hexagon was divided into six triangles. A drawing of three trapezoids	A drawing of a hexagon was divided into six triangles, each labeled one-sixth. A written explanation compares the division of the hexagon by triangles with trapezoids and rhombi.

The assignment asked PSTs what concept about fractions was necessary to solve the task. Considering the conceptual knowledge necessary to solve the task is another teacher aspect of mathematical knowledge. The assignment, at the end, asked the PST to solve adapted tasks by changing the fractional amount from two-thirds to one-half and then three-halves and analyze how the adapted tasks changed the difficulty of the task.

The mathematical task preparation assignment also required PSTs to use pedagogical content knowledge to assess potential student difficulties and prepare questions in the case that a problem-solver had difficulties solving the task. A blank pattern tiles mathematical task preparation assignment and the three peer-interviewer's pattern tiles mathematical task preparation assignments are in Appendix B.

Multi-digit addition in base sixty. Beckmann (2014) provides some tasks in a practical context that can be solved using another base. The task was as follows:
Ruth runs around a lake two times. The first time takes 1 hour, 43 minutes, and 38 seconds. The second time takes 1 hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes, and seconds.

I choose an addition task in hours, minutes, and seconds since time is a familiar context that PSTs might solve intuitively using base sixty. I expected that some PSTs would prefer to work in base ten by converting amounts to base ten, finding an answer in seconds, and then converting the seconds into hours, minutes, and seconds.

The mathematical task preparation assignment again focused on PST's mathematical content knowledge and some pedagogical content knowledge. This second mathematical task preparation assignment, like the first mathematical task preparation assignment, required solving and explaining the solution in at least two ways and listing

potential student difficulties. The assignment asked PSTs to identify the addition and subtraction problem type (Add to, Take from, Part-part-whole, and Compare, Beckmann (2014) pgs. 94-95). For this assignment, instead of asking PSTs to solve adjusted tasks, the PST was asked to lower the difficulty of the task by adjusting the task.

Table 5 provides Paige, Reese, and Beth's responses to the task work and the additional way a student could work on the running times task on their mathematical task preparation assignments. All three ladies solved the task by adding and regrouping in base sixty. None of the assignments presented conversions to base ten as a possibility. Beth presents a task adjustment, and Paige suggests that the student may work the task as a subtraction task. Both Paige and Reese consider math drawings as an additional strategy. The running times mathematical task preparation assignment with Paige, Reese, and Beth's running times mathematical task preparation assignments are in Appendix C.

Table 5

Mathematical Task Preparation Running Times Task Solutions

Peer-Interviewer Participant	First Response (numbers one and two)	Additional Response (number five)
Paige (Type 1)	Added with regrouping in base sixty	Explains that a student may change the task from a join (add to) task to a separate (take from) task, but no strategy is provided. States that a student may use a base – sixty math drawing.
Reese (Type 2)	Added with regrouping in base sixty	A drawing of different objects is used for seconds, minutes, and hours. The minutes uses two different objects for base ten.
Beth (Type 3)	Added with regrouping in base sixty	An attempted task adjustment was given to change the task to a separate (take from) task.

Proportional reasoning. The third task involving proportional reasoning started with a ratio that changed. One could use not only algebraic proportional equations but other techniques such as strip diagrams, ratio tables, and double number lines (Beckmann, 2014) to solve the task. The strip diagram provided an elegant solution with the change in ratio. The task was as follows:

The ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?

The mathematical task preparation assignment focused on proportional reasoning, inviting students to solve the tasks using any method and then to use one of the techniques (strip diagrams, ratio tables, and double number lines) taught in the curriculum (Beckmann, 2014). Table 6 lists the strategies used by Paige, Reese, and Beth in the sharing cards mathematical task preparation assignment. Each peer-interviewer participant used a different strategy in their first response. All used ratio tables in one of the solution strategies. Only Beth used a double number line, and only Reese used a strip diagram. Paige initially solved the task with an algebraic equation. Appendix D contains the original assignment and the complete work of the peer-interviewer participants for the sharing cards mathematical task preparation assignment.

The third mathematical task preparation assignment was similar to the second, requiring a different mathematical area of content knowledge and pedagogical content knowledge. The main difference between the second and third assignments was that while the second assignment asked PST to consider the task type, the third assignment

asked PSTs to consider different mathematical representations that a problem-solver could use as solution techniques in their solution methods.

Table 6

Mathematical Task Preparation Sharing Cards Task Solutions

Peer-Interviewer Participant	First Response (number one)	Additional Response (number three)
Paige (Type 1)	Algebraic equation	Horizontal ratio table with adjustment rows.
Reese (Type 2)	Strip diagram	Proportion tables Horizontal ratio table with adjusted ratio table
Beth (Type 3)	Vertical ratio table with adjustment notes	Double number line

Each mathematical task preparation assignment focused on a different area of mathematical understanding. The assignments required PSTs not only to solve the task but to solve the task in more than one way and to use pedagogical content knowledge in considering potential difficulties of a problem-solver might encounter.

Reflection forms. Both the peer-interviewer and problem-solver completed reflection forms following each peer interview. The reflection forms focused on the problem-solver's mathematical understanding and the interviewer's interactions.

The first peer-interviewer reflection form asked interviewers what they attended to in the problem-solver's work. Each peer-interviewer reflection form asked the interviewer about how the problem-solver's work or mathematical thinking connected to mathematical ideas. The peer-interviewer's reflection forms also asked the interviewer to reflect on their responses. In the first and second interviews, the reflection form asked PST interviewers to consider how they would respond in a future peer interview.

The problem-solver reflection forms asked the problem-solvers about the questions that the peer-interviewer used during the interview. The problem-solver reflection form asked the problem-solver to recall if the interviewer asked questions that encouraged her to explain or think more deeply about her mathematical thinking. The last two reflection forms asked the problem-solver about the interviewer's strengths. The problem-solver could share what would have helped them in supporting and eliciting their mathematical thinking to solve the task.

The problem-solvers could share their insights from their forms with the interviewer after both of them completed the reflection forms. The purpose of the reflection form was for both the peer-interviewer and problem-solver to think about how interviewer responses impacted the problem-solver's mathematical thinking. The reflection forms for the three interviews are in appendix C.

Data Collection

The data collection corresponded to three PIRT cycles during the Spring 2017 semester. PSTs were asked to use an assigned number for all reflections. The primary source of data was the peer task-based interview recordings. Each pair of PSTs was given an audio recorder to record each of their interviews. Each PST began the recording with "Number x is interviewing number y " so that the participant's identification in the recordings was by numbers instead of participant names. A transcriber transcribed the audio recordings of the six peer-interviewers for a total of nine interviews. I reviewed and edited the transcriptions for an in-depth study.

I collected data from the three PIRT cycles in three ways. The first component (see Figure 1 for each PIRT cycle component) of the interview cycle was completed one

to two weeks before its corresponding peer interview. PSTs completed and uploaded the interview and preparation assignment to the course learning management system. The course instructor made these assignments available to me to mask with a participant number, copy, and scan for data analysis.

The second, fourth, and fifth components (see Figure 1 for each PIRT cycle component) of the interview cycle contained written data. After each set of interviews, the PSTs turned in a packet or written documents with the mathematical task preparation that they completed as an out of class assignment, the problem-solver's task work paper for the task, the peer-interviewer reflection of the mathematical task-based interview, and the problem-solver reflection of the mathematical task-based interview. These documents were grouped by interview for data analysis. I copied and then returned the original copies of the mathematical preparation task assignments to the course instructor.

The third type of data collection was the interview recordings. PSTs were requested to record each interview separately. The PSTs returned the recorders to me after the peer interviews. I was present for peer interview class sessions and wrote field observation memos for the three sessions

One additional source of data was the participant interviews. Five PSTs participated in a 30 to 45-minute semi-structured participant interview. The primary purpose of the interview was to receive direct PST feedback on the peer-interviewer interactions and development of interactions in the context of the PIRT cycle activities and mathematics content course. I conducted the individual participant interviews after the PIRT cycles. I reviewed the written documents and listened to the corresponding peer task interviews before the participant interview to discuss points of interest.

The peer task-based interview recordings and transcripts were the primary data source to identify and classify peer-interviewer interactions. I used written artifacts for triangulation. Triangulation with multiple sources “means comparing and cross-checking data collected” (Merriam, 2009, p. 216). Participant interviews for the three peer-interviewers were a primary source of data to identify an emic (Merriam, 2009) perspective of each interviewer’s interaction development during the three interviews.

Data Analysis

The goal of my exploratory data analysis was to identify and describe peer-interviewer interactions and interaction development during three peer task interviews that took place in a beginning mathematics content course. I used a multiple case study design with peer-interviewer interactions, specifically a move act, as the unit of analysis.

Interaction coding. To gain an overall understanding of the data, I began by reviewing the data for each PIRT interview following the interview. I organized the data by interview. First, I read an interviewer’s pre-interview written assignments (interview and interaction assignment and mathematical preparation assignment). I listened to the interview with the problem-solver’s task work. Lastly, I read the problem-solver and peer-interviewer reflections. I listened to the interviews in the context of the written artifacts to get a gestalt of the peer interview interactions (Bernauer, 2015).

After considering an overview of the data as preparatory groundwork for detailed coding, I noticed that there were not many interviewer interactions matching the responsive interaction frameworks provided to the PSTs in the interview and responsive interaction preparation assignments. I was curious to understand PST interviewer’s interactions. I decided to use more exploratory interpretive coding to identify the verbal

interactions, so I used open coding (Corbin & Strauss, 2015) focused on process codes (Saldaña, 2016) for the nine peer-interviewer participant interviews. I coded peer-interviewer verbal interactions as move acts. I conceptualized a move act as a unit of oral interaction with coherence in functional activity. I used constant comparison analysis, incorporating memos and diagrams (Corbin & Strauss, 2015).

During open coding and constant comparison analysis, I noted variation in the interviewer interactions and the need to relate the interviewer interactions to the problem-solver's task work and interactions. I decided that the coding needed structure based on interaction turns and returned to the literature. Lineback (2016) identified methods used by researchers to assess teacher responsiveness. She noted discourse analysis as one research method "to determine how teachers' individual comments and/or questions respond to student ideas" (p. 205). Pierson (2008) used discourse analysis to identify responsiveness levels of teacher's follow-up interactions. I decided to use the classroom discourse works of Sinclair and Coulthard (1975/1978) and Mehan (1979) to develop an interaction coding structure for the mathematical task-based interviews in this study to identify the interviewer interactions.

Using discourse analysis based on Sinclair and Coulthard (1975/1978) and Mehan (1979), I added structure to the interaction coding. After adding structure to the coding system, I returned to the literature focusing on the frameworks given to the PSTs to check and modify move act codes. Patterns and themes of peer-interviewer move acts were found in the context of interaction exchanges between the peer-interviewer and problem-solver and in the context of the task flow. After coding the interviewer move acts, I

categorized the move acts into three categories. Next, I explain the interaction coding structure and the move act categories.

Interaction coding structure. I developed the coding structure to consider peer interviewer interactions in the context of interaction turns and in the context of task development. Each interview was divided into mathematical tasks. I divided tasks two ways to correspond with the two context foci: task phases and one or more interaction sequences. Interaction sequences were composed of one or more sequence exchanges, which were composed of one or more exchange moves. Exchange moves gave the context of interaction turns. For peer interviewer exchange moves, I coded move acts, which are the smallest unit of analysis. I considered move acts in the context of exchange moves and task phases. Figure 3 shows the coding structure. Next, I present coding decisions for each structural component.

For each interview recording, I considered the beginning of the interview to be when the interviewer read or gave the task to the problem-solver. The end of the interview was the end of the interview recording. Eight of the nine interviews had one task, and one interview had three tasks. For the interview with three tasks, the interview was divided into sections corresponding to the three tasks.

Each task was divided into mutually exclusive task phases. The task phases that occurred in all the tasks were task introduction, task response, and task closure. Optional task phases were task reflection and shift to adjust the task. The interview with three tasks is the only interview with the shift to adjust the task phase. The interview with three tasks had reflection exchanges about more than one task, which were coded as task reflection phases.

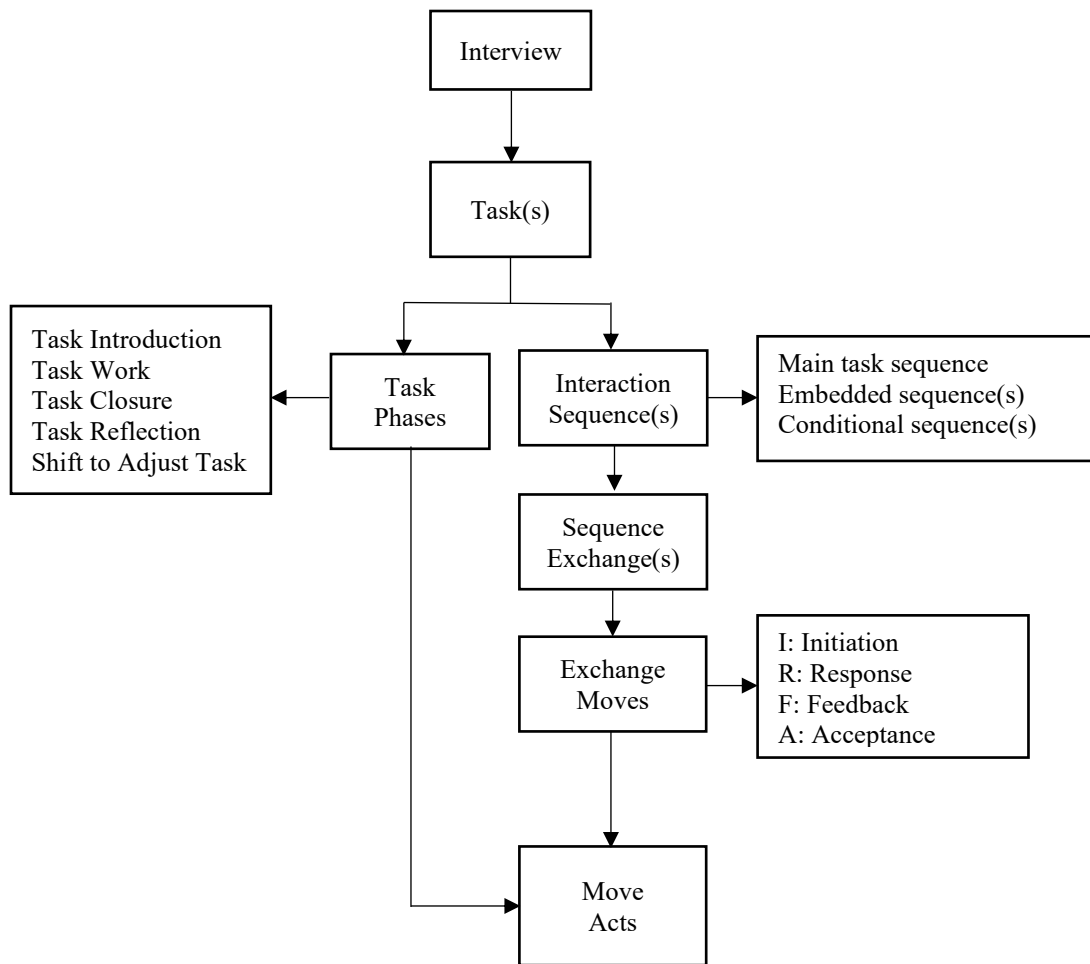


Figure 3. Coding structure flowchart. Downward arrows indicate the action of being composed of the group below the arrow. Sideways arrows indicate classification types.

The shift between each task phase was determined by the actions of the interviewer and problem-solver. In each task, the interviewer directed the problem-solver to work on a mathematical task, which was the task introduction phase of the interview. The task response phase was when actions focused on the problem-solver's task work. The task closure began when either the problem-solver sought feedback, or the interviewer gave feedback about at least one representation of the problem-solver's accepted solution. Task reflection phases contained at least one interaction sequence

about a problem-solver's accepted solution. The fourth column in Table 7 shows the coding of task phases for Beth's interview with Hoa using the running times task. Each task was also divided into interaction sequences where the same person initiated each sequence exchange. The fifth column in Table 7 contains the interaction sequence codes for Beth's interview with Hoa. Sequence exchanges began when either the peer-interviewer or problem-solver initiated an exchange by eliciting verbal information from the other person, directing action to be performed, or informing the other person in a new direction of the conversation. Each task had a main task interaction sequence consisting of a task initiation, task response, and task feedback. Some tasks contained additional exchanges in the main task interaction sequence called embedded sequences. An example of an embedded sequence is in lines 11 and 12 in Table 7, where Beth's interaction interrupts the main task sequence. After Hoa responds to Beth, she returns to the main task sequence by responding to the task. I followed Mehan (1979) in naming interaction sequences following the main task interaction sequences as conditional sequences, as seen in lines 15 through 18 in Table 7.

Each main task sequence consisted of a sequence task exchange that spread across three task phases. The interviewer initiated the task in the task introduction phase, the problem-solver responded to the task in the task work phase, and the interviewer provided feedback on the accepted solution in the task closure phase. See lines 1, 2, and 13 in Table 7 for examples of interactions in the main task sequence that span three task phases. For some tasks, additional interviewer feedback and problem-solver responses extended the main task exchange (Mehan, 1979). Embedded and conditional interaction sequences were contained within a task phase.

Table 7

Beth's Running Times Task-Based Coded Interview with Hoa

Line #	Speaker	Transcription	Task Phase	Interaction Sequence	Exchange Move	Move Act (Category)
1	Beth	The problem states that Ruth runs around the lake two times. The first time takes 1 hour 43 minutes and 38 seconds. The second time takes 1 hour 48 minutes and 29 seconds. What is Ruth’s total time for the two laps? Give the answer in hours, minutes, and seconds.	Task Introduction	Main Task	Initiation	Directive (D)
2	Hoa	Oh, okay. So, the first time takes an hour, 43 minutes, 38 seconds. The second time takes an hour, 48 minutes, and 29 seconds. The total two laps. Okay. So, one is eight plus nine, 29, 48, on out {unintelligible}. So, seconds, no, um. Um {unintelligible} that’s 17, 67 seconds.	Task Work		Response	
3	Beth	Um-hmm			Feedback	Acknowledgement (S)
4	Hoa	So, we have to make that a minute			Response	
5	Beth	Um-hmm			Feedback	Acknowledgement (S)
6	Hoa	So, we add a minute. Now, we have seven seconds.			Response	
7	Beth	Um-hmm			Feedback	Acknowledgement (S)
8	Hoa	So, then we have 49 plus 43. That’s {unintelligible}. Now we have 9s. So, we need to make that an hour.			Response	
9	Beth	Um-hmm			Feedback	Acknowledgement (S)
10	Hoa	Six ::			Response	

(Table 7 continued)

Line #	Speaker	Transcription	Task Phase	Interaction Sequence	Exchange Move	Move Act (Category)
11	Beth	So, minus 60 for the minutes.		Embedded: Beth	Initiation	Informative (D)
12	Hoa	Yeah, minus 60 is {unintelligible}.			Response	
		Yep, one, two, three. So, you would have 3 hours, 32 minutes {unintelligible}		Main Task	Response	
13	Beth	Yeah, that's perfect	Task Closure		Feedback	Evaluation (D)
14	Hoa	Good			Acceptance	
15	Beth	Is there another way you could have solved that problem?	Task Reflection	Conditional: Beth	Initiation	Encouraging another way (E)
16	Hoa	Um. Yeah, I guess I could've.			Response	
17	Beth	Uh, another way could be instead of having hours you can change the hours to minutes. And so that [Hoa: Oh] and then, so it would be {unintelligible} set {unintelligible} plus set {unintelligible} plus 43 plus 48. And if you wanted to do it that way. Or well as you can break up the length of seconds for the 30 plus 8 and 20 plus 9 if you wanted to break it down even farther.			Feedback	Information (D)
	Hoa	[see above]			Acceptance	
18	Beth	Oh, okay. That makes sense.			Acceptance	

Note. A speaker that speaks amid the other speaker is noted in brackets. A double colon signifies a prolongation of sound. The symbols for the categories of move acts are as follows: (a) D for Directional, (b) E for Elicitation, and (c) S for Support.

Each sequence exchange contained exchange moves. I followed Sinclair and Coulthard's (1975/1978) using the terms Initiation (I), Response (R), and Feedback (F).

The sixth column of Table 7 is for the coded exchange moves. A few tasks contained a four-part dialogue exchange. The fourth exchange move was termed Acceptance (A) for accepting the prior person's feedback or feedback of feedback. Lines 14 and 18 in Table 7 are examples of acceptance moves, one by Hoa, the problem-solver and one by Beth, the interviewer. The next paragraphs define each exchange move and provide examples.

An initiation was when a person began an interaction exchange where a reply was expected. Expected replies could be verbal or non-verbal actions. In her third interview with Brittany, Paige says, "So, yeah, add one, okay." directing Brittany to add one more column to a ratio table. Paige is initiating an interaction about doing something with the expectation that Brittany adds one more column to her ratio table. In her first interview with Rosa, Beth asks, "Do you need the shapes to help you?" Beth is initiating an interaction by offering manipulatives with the expectation that Rosa answers her question. Whenever either the interviewer or problem-solver began a new exchange topic, gave a direction, or elicited a response, I coded the exchange move as an initiation.

A response was a reply to an initiation that was not another initiation. One common response to an interviewer directing the problem-solver to complete a task was silence. Silence may or may not indicate that the problem-solver was working on the task. Answers to questions were another possible response. In Beth's first interview with Rosa, Rosa asks, "Do I keep going?" She then responds to herself with, "I guess, I just work through it." Beth, then also responds with, "Yes, you just work through it." If a person replies to an initiation with their own initiation, then the response is another initiation, and I coded these instances as initiations. For example, later in Beth's interview with Rosa, Beth initiates by directing Rosa to do something, and Rosa responds with a

question. Beth says, “Now you can just draw it.” and Rosa replies, “Oh, really?” Rosa initiates an elicitation for confirmation, so Rosa’s reply is coded as an initiation. One exception is when a response is in question format seeking an evaluation of a response such as when Rosa asks, “Like that?” Beth then evaluates her work with “Yes.” Responses followed initiations. Even though initiations expect responses, a person could choose not to respond to an initiation or reply with another initiation.

The turn pattern in the interviews was for one person to initiate, the other responds, and then the initiator has the option to provide feedback on the other person’s response. Feedback accepts, evaluates, or builds on a person’s response without being another initiation. Acceptance was a fourth option were the responder accepts feedback.

Sinclair and Coulthard (1975/1978) used follow-up as a class of moves that included acceptance, evaluation, and comments. Although it has been common to use IRF for initiation-response-follow-up (Wells, 1993), I choose to use feedback as opposed to follow-up in my elements of the structure since an elicitation (seeking a response) or directive (request to perform an action) could be a follow-up and an initiation. I decided to code all elicitations and directive actions as initiations to indicate whether the interviewer or problem-solver was leading the exchange. A shift in the person leading interactions indicated a change to a new interaction sequence.

Many changes in exchange moves were due to changes in speakers, but a speaker could also have more than one exchange move in a speech turn such as feedback to a response and then a new initiation. Another possibility found in Table 7, line 12, is where Hoa responds to an embedded initiation by Beth and then continues responding to the main task. Markers were words that functioned as boundaries in discourse (Sinclair and

Coulthard, 1975/1978). The markers “so,” “well,” and “okay” at the beginning of a phrase or sentence, were often indications of shifts in exchange moves. For sequence exchanges, both interviewer and problem solver exchange moves were coded, as seen in column six of Table 7. I coded periods of extended silence during a task work phase as problem-solver’s responses to the task initiation. Each exchange move, IRFA, was found for both the interviewer and problem-solver in at least one sequence exchange.

I coded peer-interviewer exchange moves into move acts, as seen in column seven of Table 7. Move acts described the type of interaction move. An initiation could be a directive act to perform an action, elicitation expecting a verbal response, or providing information, while a response could be an answer or informative. Feedback could be affirmation, acknowledgment, information, or evaluation. Acceptance moves were generally acknowledgments. I categorized move acts in the following three categories: elicitation, directional, and support. I provided a table of description and examples of move acts in Table 8.

In summary, each interview was divided into tasks and task phases. The main task interaction sequence consisting of one sequence exchange was identified with the peer-interviewer task initiation in the task introduction phase, response and feedback possibilities in the task work phase, and closing task feedback in the task closure phase. Initial exchange moves and move acts were coded for the main task sequence exchange. Next, additional interaction sequences and exchanges were identified in each task phase. The boundary for shifts in sequence exchanges were initiation exchange moves. When the person who was initiating the sequence exchange changed, this indicated a shift in an interaction sequence. The identification of additional interaction sequences in the task.

Table 8

Move Act Descriptions and Examples by Categories

Move Act	Description	Examples
Elicitation Act Category		
Requesting permission	The requesting permission act functioned to ask the other person for their input about interactions.	1:51 Reese: “ <i>May I ask you questions?</i> ”
Offering manipulatives	The offering manipulatives act functioned to ask the other person if they would like to use manipulatives.	1:7 Beth: “ <i>Do you need the shapes to help you?</i> ”
Checking knowledge	Checking knowledge elicitation checked the problem solver’s knowledge about a component of the task or task work.	1:53 Reese: “ <i>Okay, so, um, in the fraction of two-thirds, what does the three equal?</i> ” 1:122 Anita: “ <i>So if this whole thing represents three over two?</i> ” 1:123 Reese: “ <i>That means there are how many parts?</i> ”
Guiding procedures	A guiding procedures act functioned to direct the problem solver’s task work or thinking.	2:6 Amanda: “ <i>We have 2 hours, {unintelligible} 91 minutes, and 67 {unintelligible}</i> ” 2:7 Paige: “ <i>And then what can you subtract to:?</i> ”
Inquiring about procedures	Inquiring about procedures act functioned to ask the problem solver about a task procedure or process.	3:67 Brittany: “ <i>Oh, I know where I messed up.</i> ” 3:68 Paige: “ <i>What, did you skip a number?</i> ”
Focusing on representations	Focusing on representations were elicitation about a representation in the problem solver’s mathematical work.	1:5 Paige: “ <i>So, the one you just added would be the one-third?</i> ” 3:25 Reese: “ <i>So, if he, if Samuel had to give away one part [Robin: “Mh – hmm”] for them to have equal parts and it’s saying in the problem that he gave away 15 cards to make them equal, then what would one part represent?</i> ”
Adjusting the task	The adjusting the task act was an elicitation to shift to another task.	1:8 Reese: “ <i>Do you want me to start with an easier one?</i> ”

(Table 8 continued)

Move Act	Description	Examples
Encouraging another strategy	The encouraging another strategy act functioned to encourage the problem-solver to consider another strategy to solve the task.	2:15 Beth: <i>"Is there another way you could have solved that problem?"</i> 2:11 Reese: <i>"...Um, could you think of a different way to do it?"</i>
Encouraging another solution	The encouraging another solution act encourages the problem-solver to find another solution to the task.	1:35 Reese: <i>"...Um, so, say you didn't have those two symbols. How else would you represent the full design?"</i>
Probing thinking	The probing thinking act probed the thinking of the problem-solver's procedures.	2:4 Reese: <i>"...But how did you, uh, know that is would go from 67 to 7?"</i>
Encouraging adjustments	The encouraging adjustments elicitation act asked the problem-solver as a fellow PST about how the task could be adjusted for students.	2:32 Reese: <i>"...Um, so, say that you, um, didn't understand that like once you would hit 60, it would go to like 1 minute, instead of 60 seconds is equal to 1 minute, could you make it easier? Could you make the problem easier for the student that like didn't understand? Instead of a base 10, it would be base 60? By like keeping the same problem, but just like maybe changing the numbers or something?"</i>

Directional Act Category

Directive	A directive act functioned to request that the other person do some action. Each task began with a directive to work on the task. Directives functioned as initiation exchange moves.	3:18 Paige: <i>"And the number is higher up so you, you, you keep going."</i> 3:7 Reese: <i>"Flip them."</i> 1:19 Beth: <i>"...Now you, you can just draw it."</i>
Marker	Markers functioned as a discourse boundary. Marker acts were coded when another directional move was connected with the marker. Marker acts functioned as initiation and feedback exchange moves.	2:1 Reese: <i>"Okay."</i> 3:6 Brittany: <i>"So, then they both have 15? After [Paige: "So: "] Samuel gives Robert 15 of his cards, both boys have the same:."</i>
Informative	An informative act functioned to tell the other person some information. Informative acts functioned as initiations, responses, and feedback exchange moves.	2:2 Silence 2:3 Paige: <i>"Yeah, by grouping them it makes it easier to differentiate."</i> 2:9 Beth: <i>"So, that's two-thirds."</i>

(Table 8 continued)

Move Act	Description	Examples
Answer	An answer act provided an answer to an elicitation. The answer could be short or include information. Answers functioned as response exchange moves.	1:22 Rosa: "...Can I just change it?" 1:23 Beth: " <i>Yes.</i> " 1:13 Anita: "... So, do I draw it out?" 1:14 Reese: " <i>Yeah, just draw what you think the other half would be.</i> "
Evaluation	Evaluations went beyond recognition or encouragement to make a judgment of one or more responses. The intention of an evaluation was to bring closure. Evaluations functioned as feedback exchange moves.	1:9 Paige: " <i>Uh, that's what I got as well.</i> " 2:4 Reese: " <i>So, um, I mean you got it right.</i> " 3: 6 Beth: " <i>Yeah, that's correct.</i> "
Support Act Category		
Affirmation	An affirmation act functioned to show encouragement for verbal or non-verbal responses. An affirmation could confirm that the problem solver is correct, but an affirmation did not include closure. There were times that affirmations were spoken amid the other person's response. Affirmations functioned as response and feedback exchange moves.	2_8 Amanda: "...So, then you're left with 31 minutes [Paige: " <i>Good.</i> "] plus 1 so that's three hours. Again subtract 60." 1_65 Anita: "This is hard. It makes you think." 1_66 Reese: " <i>I know, right?</i> "
Acknowledgment	Acknowledgments showed recognition for verbal or non-verbal responses. Acknowledgments did not clearly show encouragement or emotional support. Acknowledgments did not include closure. There were times that acknowledgments were spoken amid the other person's response. Acknowledgments functioned as response, feedback, and acceptance exchange moves.	1:2 Maria: "Hmm. Ok. So, I think two-thirds would probably be the shape, because [Paige: <i>Yeah</i>] this is like the part of the hexagon." 1:3 Paige: " <i>Um – hmm.</i> "
Comment	A comment is a non-directional statement. Comments functioned as response and feedback exchange moves.	1: 55 Reese: " <i>Like let me think of another way to put it.</i> "

Note. Italicized interactions are the examples. Non-italicized interactions provide context. In the examples, the first number is the interview. 1 represents the pattern tiles task, 2 represents the running times task, and three represents the sharing cards task. The second number is the interview line number.

work phase clarified the response and feedback possibilities for the main task sequence. After interaction sequences and exchanges were identified, exchange moves and move acts were coded

Move act categories. Since the PSTs read and reflected on elicitations for the interview and interaction preparation assignments, I was interested in all the elicitation move acts in contrast to other move acts. I categorized move acts into three functional categories: Elicitation, Directional, and Support. Table 8 shows the move acts by move act category.

Elicitation acts functioned to request an oral response (Sinclair & Coulthard, 1975/1978). Elicitations often had the form of a question, but elicitation could also be the beginning of a statement for the other person to complete (Mehan, 1979). Directional acts functioned to provide direction without being an elicitation. Support acts functioned to show the other person that they had been heard or seen. The directional act category included giving directives, providing information, answering problem-solver elicitation, and signaling closure by evaluating oral or non-oral problem-solver responses. Directives differed from elicitation because they requested an action that is not necessarily an oral response (Sinclair & Coulthard, 1975/1978).

Support acts included affirmations, acknowledgments, and comments. Affirmations and acknowledgments related to what the other person was saying or doing and functioned as “keep going” or “I am with you” to the other person. Supportive and evaluation acts could use the same words or phrases such as “good” or “yes.” An evaluation act functioned differently from a support act because an evaluation act commented on quality with an intent of closure. In contrast, supportive acts functioned to

encourage what the other person was saying or doing without disrupting the other person's interaction moves.

Post-PIRT participant interview coding. Post-PIRT participant interviews provided a lens on each peer-interviewer participant's perspective of PIRT and her interaction development. Structural coding (Saldaña, 2016) was used to index interaction development sections of each interview. Themes were identified for each peer-interviewer participant, and then a theme was identified across the peer-interviewer participants.

Trustworthiness

My investigation, as an exploratory case study, did not align with a predetermined set of categories to use in my data analysis. I searched for themes, patterns, and structure through various iterations of the data guided by literature on case studies, coding, ground theory, and discourse analysis (Yin, 2014; Merriam, 2009; Saldaña, 2016; Corbin & Strauss, 2015; Sinclair & Coulthard, 1975/1978; Mehan, 1979). Since I approached the data as an interpretive process, I sought to reduce any researcher bias in making interpretations.

As a doctoral student and novice mathematics teacher educator, I was limited in familiarity with PSTs and mathematics content courses, but my greatest threat to bias was my perspective of a relational understanding of mathematics (Skemp, 1976/2006). When the data was not what I anticipated, I spent time reviewing all the data seeking to understand the data from the PST's perspective. I used reflexivity during the study by using memos and discussions with mathematics education doctoral students and professors, noting my reactions to the data collection and analysis (Merriam, 2009;

Corbin & Strauss, 2015). When I began to see that the PSTs could interpret PIRT materials from different perspectives than my own such as a calculational orientation (Thompson, Philipp, Thompson, & Boyd, 1994) or an instrumental processing understanding of mathematics (Skemp, 1976/2006), then I interpreted how their interactions and development of interactions aligned with self-reported reflections and participant interviews. Looking at the data from different perspectives provided theoretical triangulation (Yin, 2014).

By oral listening to all the collected interviews and reviewing all PIRT materials, I gained an overall understanding and breadth of PST interviewer interactions. I also examined the interview that I had transcribed and made necessary adjustments. Constant comparison (Corbin & Strauss, 2015) was used throughout the coding to ensure the internal validity of the interview structures and codes. Validity strategies also included triangulation (Merriam, 2009) of the interview recording and transcripts with written artifacts and participant interviews.

The participants were informed of the study's purpose and signed an informed consent form. The study was contained within the boundary of the PIRT activity cycles and focused on recorded peer-interviewer responses. A risk in peer interviews was that peer feedback could be rude or discouraging to peer's mathematical thinking. The instructor built a classroom environment that encouraged mutual respect and different mathematical ways of thinking before any peer interviews. Additionally, interview and interaction preparation included appropriate forms of peer interview interactions. University counseling was available in the case of harm done during peer interactions. No PST harm was evident or shown during the study.

Validity strategies, checking researcher reflexivity, and immersion in the data provided checks in developing the PST interviewer interaction structure and variations. In the following chapter, I present variations of PST peer-interviewer interactions within the interview interaction structure.

IV. FINDINGS

The purpose of this multiple case study was to explore the peer-interviewer verbal interactions of three PSTs during three mathematical task-based interviews in an initial mathematics content course at a large southern university in the United States. The research questions that guided my study were:

1. In what ways did the PST peer-interviewers verbally interact with a peer problem-solver during three peer mathematical task-based interviews?
2. How did the PSTs' interviewer verbal interactions during three peer task-based interviews develop over a semester?

I chose to examine PSTs' verbal interviewer interactions to discover the ways that PSTs interact with another person who is working on a mathematical task. Each case consisted of a PST who conducted three mathematical task-based interviews with a peer PST. The unit of analysis is the interviewer's verbal interactions called *move act* as defined in the previous chapter. I analyzed move acts within the context of each interview. Each interview had one or more mathematical tasks that the interviewer introduced to the problem-solver. To find different patterns and variations of interviewer interactions, I used constant-comparison (Corbin & Strauss, 2015) and discourse analysis (Sinclair & Coulthard, 1975/1978; Mehan, 1979). I then considered each case individually with the corresponding post-PIRT participant interview to answer the research question about PST peer-interviewer interaction development.

In this chapter, I present the patterns and variations of interview interactions. After describing the verbal interviewer interactions, I present the development of interviewer interactions over the semester for the three PST case studies.

PST Interviewer's Verbal Interactions

PST interviewer verbal interaction patterns were found within the interaction coding structure. The interaction coding structure is explained in chapter three. I found that all tasks contained a common main task sequence. There were three common exchange moves in each main task sequence. The rest of the interviewer interactions varied. After presenting the main task sequence with the shared exchange moves, I present the variations of interviewer verbal interactions in the context of the task phases and problem-solver's interactions.

Common interviewer interaction exchange moves. I segmented each interview by mathematical tasks. In eight of the nine interviews, the interviewer directed the problem-solver to complete one task. In one of the interviews, the interviewer presented three tasks for the problem-solver. I segmented tasks into task phases and found that each task contained three task phases. The characteristic task phases were *task introduction*, *task work*, and *task closure*. In the task introduction, the peer-interviewer (PI) initiated a task by directing the problem-solver (PS) to work on a mathematical task with a directive or marker verbal move act. During the task work phase, the problem-solver responded to the task. The interviewer provided feedback to the problem-solver using at least one task work evaluation in the task closure phase. An interview could have other interactions sequences, but these exchange moves were present for each task as a main task sequence. Figure 4 presents a visual illustration of the essential interview interaction structure where shaded figures represent peer-interviewer roles.

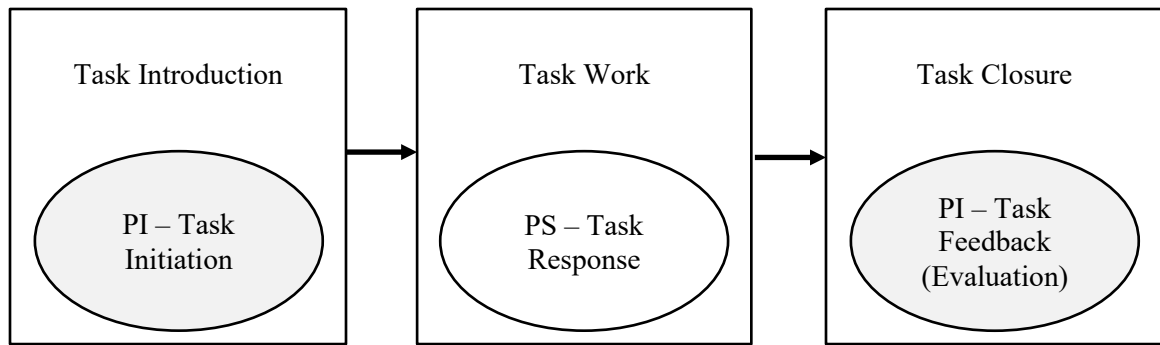


Figure 4. Common interview interactions flowchart.

Table 9 shows an excerpt from Paige’s running times task interview with Amanda as an example of the PST’s general PIRT interview task pattern (excerpt tables highlight the interviewer interactions emphasized in each section) As an introduction, Paige directs Amanda to work on the task by reading the task. The next nine transcript lines represent the task work phase of the interview. The task closure contains Paige’s task evaluation. In the example, Paige evaluates Amanda’s task solution with the word: “Good.”

Table 9

Common Interviewer Exchange Moves: Paige’s Second Interview Excerpt

Task Introduction	Task Work	Task Closure
I (Directive)		F (Evaluation)
2:1 Paige: “ Ruth runs around a lake two times. The first time takes 1 hour 43 minutes and 38 seconds. The second time takes 1 hour 48 minutes and 29 seconds. What is Ruth’s total time for the two laps? Give the answer in hours, minutes and seconds. ”	2:2-10 Amanda responds in silence and then talks through the task. Paige extends the interaction also.	2:11 Paige: “ Good. ”

Interviewer interaction exchange move variations. The variation of exchange moves was in the context of the task phases. In addition to the three task phases of task introduction, task work, and task closure, some interviews contained additional task phases. Two additional task phases were *task reflection* and *shift to adjust the task*.

Eight of the eleven tasks included a task reflection phase. A task reflection phase included interactions about the task work, solution, different solutions, different strategies, or elicitations beyond the answer. Task reflections occurred either before or after task closure. In the interview that contained more than one task, the original task work was interrupted by shifting to an easier task. Figure 5 shows a flowchart with all the possible task phases of the interview interaction structure.

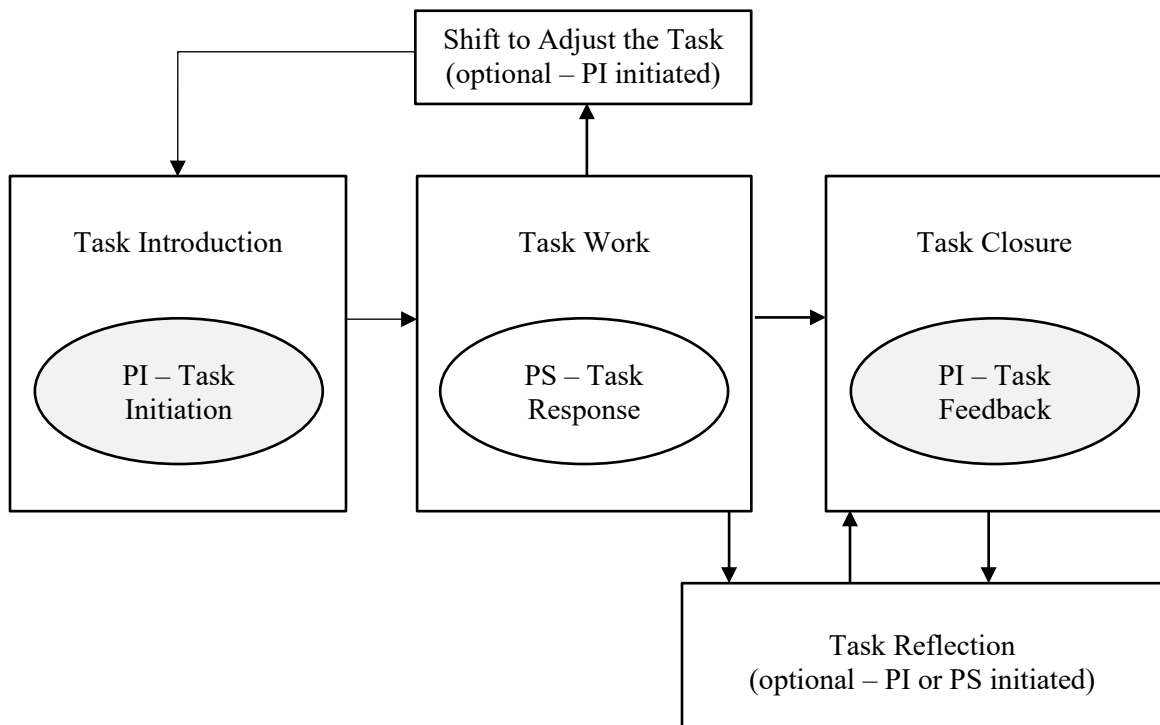


Figure 5. Task phase flowchart.

While each interview task had a task initiation, task response, and task feedback exchange move where the feedback was an evaluation move act, another theme was *dynamic variation*. The variations were dynamic because the interviewer interacted in-the-moment with different mathematical tasks, interview order, and problem-solver's interactions with the task and interviewer. The mathematical task was different for each interview, the order of being the first or second interviewer was different, and the problem-solver was different in each interview. Each interviewer had opportunities to consider different solution methods in a mathematical task preparation assignment and in teaching team groups. Since the PST interviewer could prepare for mathematical solution methods before the interview, the problem-solver's mathematical thinking and interactions could be anticipated, but not known before the interview. The changes in each interview allowed peer-interviewers to practice responding in-the-moment to another person working on a mathematical task. The peer-interviewer verbal interactions were in the context of the mathematical tasks and the problem-solver's interactions. Due to the variations between interviews, Table 10 shows the variation in task phases for each interview.

The findings showed that the peer-interviewers made at least one evaluation move act during a task closure phase and variations in additional verbal interactions. Verbal interviewer interactions during task work phases focused on the task work and solution, including affirming the problem-solver and guiding the problem-solver toward a solution. Verbal interviewer interactions in task reflection phases included additional types of elicitations focused on explanations of task work, task representations, and additional solution strategies.

Table 10

Task Phases for the Interviews

Patterns Task	Running Times Task	Sharing Cards Task
Paige		
Task Introduction	Task Introduction	Task Introduction
Task Work	Task Work	Task Work
Task Reflection	Task Closure	Task Closure
Task Closure		
Beth		
Task Introduction	Task Introduction	Task Introduction
Task Work	Task Work	Task Work
Task Closure	Task Closure	Task Closure
	Task Reflection	Task Reflection
Reese		
Task 1 Introduction	Task Introduction	Task Introduction
Task 1 Work	Task Work	Task Work
Shift to Adjust Task	Task Closure	Task Closure
Task 2 Introduction	Task Reflection	Task Reflection
Task 2 Work		
Task 2 Closure		
Task 1 Introduction		
Task 2 Reflection		
Task 1 Introduction		
Task 1 Work		
Task 1 Closure		
Task 1 Reflection		
Task 1 Closure		
Task 3 Introduction		
Task 3 Work		
Task 3 Closure		
Task 3 Reflection		
Overall Reflection		
Task 1 Reflection		
Task 2 Reflection		
Task 3 Reflection		
Overall Reflection		

I describe categories of interviewer interaction variations within the task phases. I followed Sinclair and Coulthard's (1975/1978) elements of interaction structure, termed Initiation (I), Response (R), and Feedback (F) to classify interaction moves in interaction sequences. Each interaction sequence did not need all of the structural elements. A fourth possible structural sequence element found in the peer interviews is Acceptance (A).

I include figures of flowcharts for variations of each task phase structure and excerpts of transcripts to illustrate variations. The first time I present a transcript, I include the reading of the task — afterward, transcripts with the same task state that the interviewer read the task. I placed the relevant interaction in bold for each transcript excerpt. The transcript excerpts illustrate the basis for my interpretations of the verbal peer-interviewer interactions.

Task introduction variations. Task introductions all included a directional move act from the interviewer giving a mathematical task to the problem-solver. Table 10 shows where the thirteen task introductions were during the interviews. In each of Paige and Beth's interviews, they read the task to the problem-solver, and the interview moved from task introduction phase to task work phase. Reese's pattern tiles and running times task interviews provided some variations, including beginning the task without reading the task and times were the problem-solver responded to the task initiation with acknowledgments or initiated sequences. Figure 6 shows the task introduction variations.

The PIRT interview instructions directed the interviewer to begin by reading the task to her peer, and this happened with ten of the eleven tasks. Reading the task was coded as a directive move act. Table 11 shows Reese's running times interview with Yesenia, where Reese does not read the task but gives the written task to the Yesenia. In this case, Reese's okay marked the beginning of the task introduction. Yesenia re-voiced the initiation and quietly worked on the task. A *marker move act* functioned to mark a boundary in the discourse (Sinclair & Coulthard, 1975/1978, p. 40). I coded marker move acts only when a boundary utterance was alone and not connected to another move act as Reese's verbal initiation in Table 11.

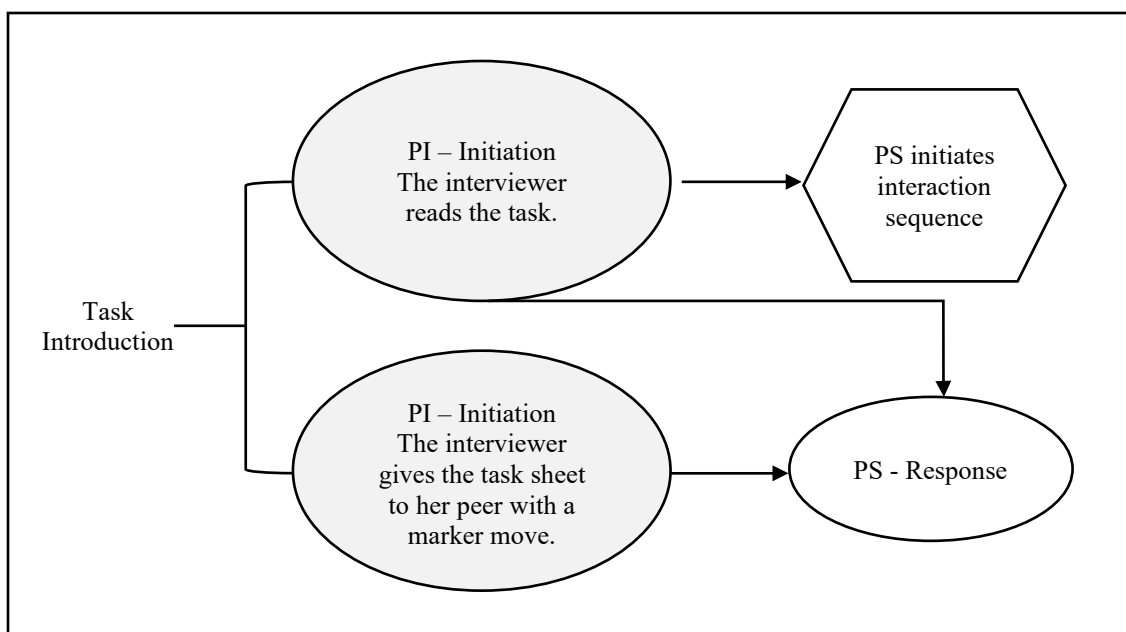


Figure 6. Task introduction interaction variations flowchart.

Table 11

Task Introduction Variation: Reese's Marker Initiation Example

Task Introduction	
I (Marker)	R
2:1 Reese: "Okay."	2:2 Yesenia: "Okay."

In Reese's pattern tile task interview with Anita, Reese presented Anita with three tasks, as seen in Table 10. Anita responded to some task directives with acknowledgments. Anita also initiated interaction sequences with questions. Reese responded to Anita's different initiations with three answer move acts. Table 12 excerpt is an example of Reese's answer response during an interaction sequence initiated by the problem-solver during the task introduction task phase.

Table 12

Task Introduction Variation: Reese's Answer Response Example

Task Introduction	
I	
1:45 Reese: "...Okay, um, so now going back to that hexagon is two-thirds. Um, if that is equal to two-thirds, what would I need to make it a full three-thirds, three over three?"	
I	R (Answer)
1:46 Anita: "If this was two-thirds?"	1:47 Reese: "Yes, instead of one-half, now we're going back to the original problem of it equaling two-thirds."

In summary, during the task introduction phases, the interviewers used twelve directives and one marker move acts in initiation moves and three answer move acts in response moves during problem-solver initiated interaction sequences. All of the interviewer verbal interactions belonged to the directional act category of move acts.

Task work variations. The task work phases of the interviews began with a problem-solver's turn to interact about the task. The problem-solvers responded to the task by solving or attempting to solve the task in silence or talking through the task work. When the problem-solver was responding to the task, an interviewer could be silent, provide feedback to the problem-solver task work, or initiate an embedded interaction sequence. In some interviews, the problem-solver initiated embedded interaction sequences in the midst of task work. When the problem-solver initiated an interaction sequence, the interviewer had the role of responding to the problem-solver initiation. The only element in common for all the tasks was that the problem-solver responded to the interviewer's task initiation. I present the interviewer move act variations in the context of the task work interaction variations. Figure 7 provides a flowchart of the task work interaction variations found.

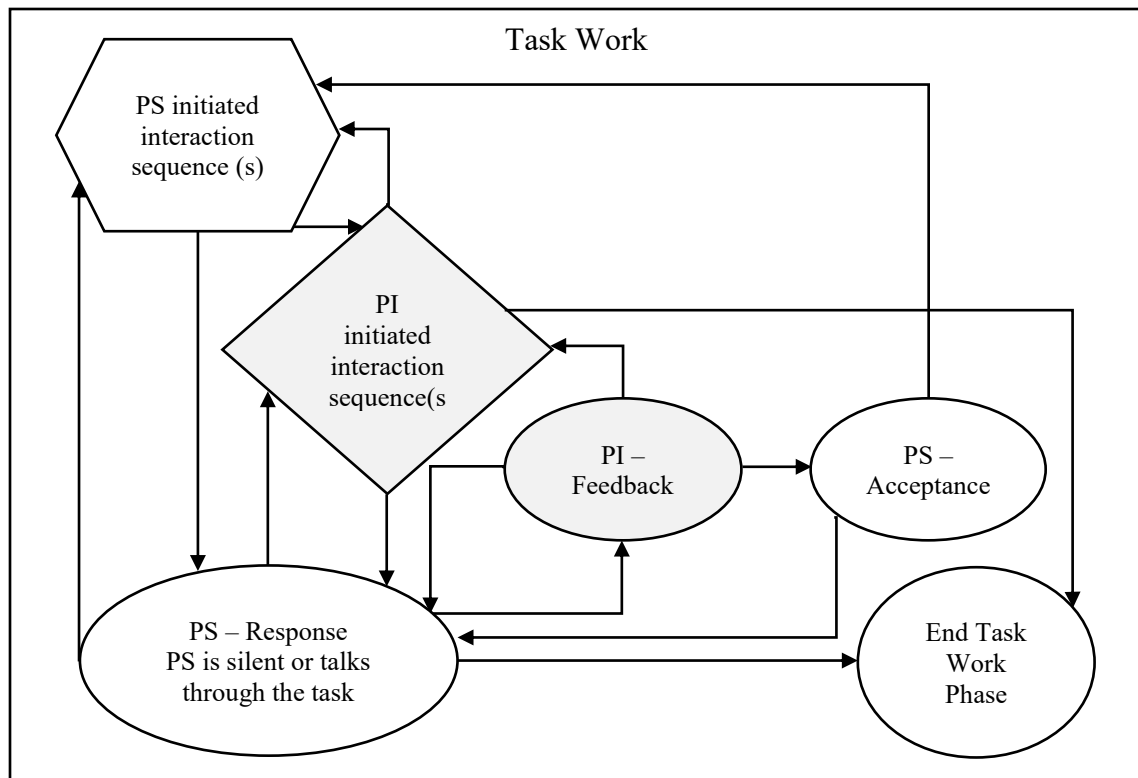


Figure 7. Task work interaction variations flowchart.

Peer-interviewer did not interact verbally. While the problem-solver worked through a task, the interviewer had a choice of interacting with the problem-solver about her task work or not verbally interacting during the task work portion. Reese is the only participant interviewer who was silent during one of the task work phases. Reese's running times task interview with Yesenia is an example of a silent task work element. Yesenia worked quietly on the task, and Reese waited until she finished. Reese then shifted to task closure with an evaluation move as feedback. Table 13 gives an example of no verbal interaction during the task work phase in the context of the essential task phases. Figure 8 displays the task work flowchart for when the interviewer had no verbal interactions.

Table 13

Task Work Variation: Reese's No Verbal Interaction Example

Task Introduction	Task Work	Task Closure
I	R	F (Evaluation)
Reese gives Yesenia the written task.	2:3 Silence	2:4 Reese: "So, um, I mean you got it right ..."

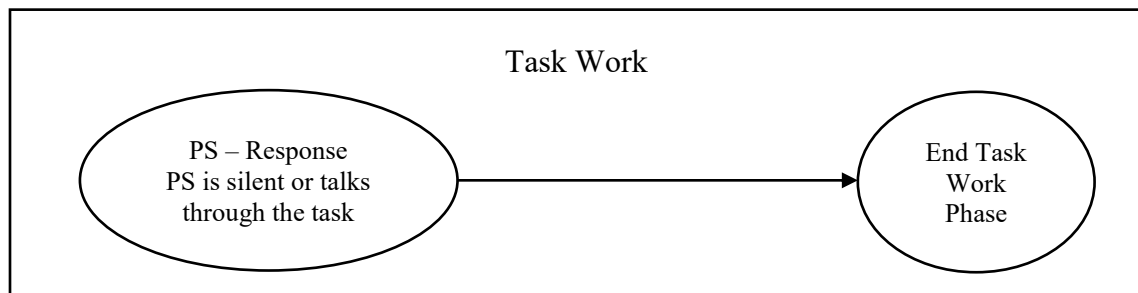


Figure 8. Task work variation flowchart: No PI verbal interactions.

Peer-interviewer extended the main task sequence with feedback. While the problem-solver responded to the task initiation, the interviewer could choose to interact verbally. One way that an interviewer interacted with the problem-solver as they talked was to provide feedback on what the problem-solver was saying or doing. Here, the interviewer's feedback moves extended the main task sequence. Feedback moves did not interrupt the problem-solver's task work. Figure 9 shows the portion of task work variation that corresponds to interviewer feedback.

Beth extended the main task sequence with feedback moves in her running times interview with Hoa. Lines two through nine in Table 7 show how Beth used acknowledgment move acts. Beth's feedback provided verbal interaction between the interviewer and problem-solver. According to the interview reflections, Beth's

interactions also functioned as encouragement. Hoa writes, “She encouraged me.” Hoa continued her task work following each instance of Beth’s feedback during the task work phase. Figure 9 shows the loop that occurred between Hoa’s main task response and Beth’s feedback.

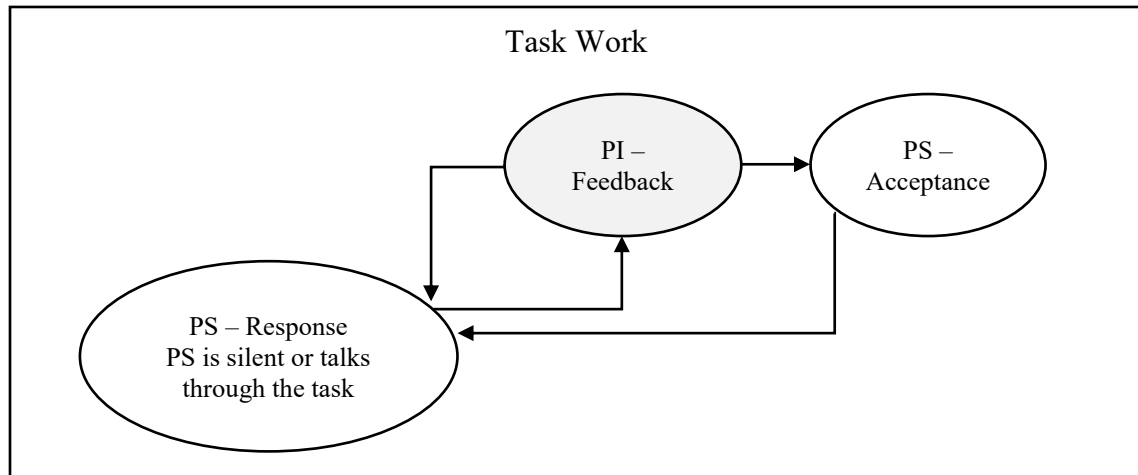


Figure 9. Task work variation flowchart: PI feedback.

Paige extended the main task sequence with feedback moves in all of her interviews. Paige used support and directional move acts as feedback. Table 14 provides an example of Paige using affirmation and informative move acts to reinforce Amanda’s task work. Although Paige restates what she sees in Amanda’s work, she only informs without interrupting Amanda’s work. Note that Amanda verbally accepted Paige’s informative feedback producing an acceptance exchange move. Amanda’s acceptance move aligns with the “PS – Acceptance” option in Figure 9.

Reese used support and directional move acts to extend the main task sequence during two task work phases in her pattern tiles task with Anita. In addition to acknowledgment and informative move acts, Reese used evaluation move acts. One way

some problem-solver's responded to task initiations was to ask for confirmation about a solution. If the solution was correct, the interview went to task closure. If the solution was incorrect, the interviewer could provide feedback. Evaluation is a move act possibility that Reese used after Anita responded with a possible solution. Table 15 shows that Reese responded with, "I don't know. You tell me." Although the feedback is not a direct "yes" or "no" evaluation, the move act functions to communicate to Anita that the solution is not right, and she needs to continue working on the task.

Table 14

Task Work Variation: Paige's Feedback Exchange Move Examples

Task Introduction		Task Work		Task Closure
I (Directive)	R	F (Informative)	A	
2:1 Paige: "Ruth runs around a lake two times. The first time takes one hour, 43 minutes, and 38 seconds. The second time takes one hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes and seconds"	2:2 Silence	2:3 Paige: "Yeah, by grouping them// it makes it easier to differentiate."	//Amanda: "Mm."	
		R	F (Affirmation)	
	2:4 Amanda: We know that 1-minute equals 60 seconds. {unintelligible} 60 minutes is equal to an hour.			
			2:5 Paige: "Good."	
	2:6-10 Continued interview interactions			
				F (Evaluation)
				2:11 Paige: "Good."

Table 15

Task Work Variation: Reese's Evaluation Feedback Example

Task Introduction		Task Work		Task Closure
I (Directive)	R	F (Evaluation)	A	
1:10 Reese: "Okay, um, so instead of the hexagon representing two-thirds, the hexagon represents half. If the hexagon represent half, what symbols could you use out of all these to represent the other half of it?"				
	1:11 Anita: "So, it would be this one? Right?"			
		1:12 Reese: "I don't know. You tell me."		
			1:13 Anita: "Oh"	
	1:14-28 Continued interview interactions			
				F (Evaluation) 1:29 Reese: "Good."

Each of the interviewers used feedback move acts to support and provide direction to problem-solvers during the task work phases in at least one interview. The support move acts consisted of eight acknowledgments and three affirmations, while the directional move acts consisted of two informatives and two evaluations. Each feedback exchange move extended the main task sequence.

Peer-interviewer initiated embedded interaction sequences. Paige, Reese, and Beth, each initiated at least one embedded interaction sequence during a task phase in two of their interviews. When the interviewer initiated an interaction sequence, her role was to initiate and possibly provide feedback to the problem-solver's response.

The participant interviewers used 52 move acts for initiation exchange moves and 20 move acts for feedback exchange moves in interviewer initiated interaction sequences during task work phases. Of the 52 initiation move acts, 20 were elicitations, and 32 were directionals. The directional move acts were directives and informative move acts. The interviewers used six types of elicitations. Paige and Reese used 20 feedback move acts in interaction sequences. The feedback move acts consisted of 13 support move acts and 7 directional move acts. Interviewer initiated interaction sequences connected in different instances to many elements of the task work variation flowchart. Figure 10 highlights the flow of interaction when the peer-interviewer initiates interaction during the task work portion.

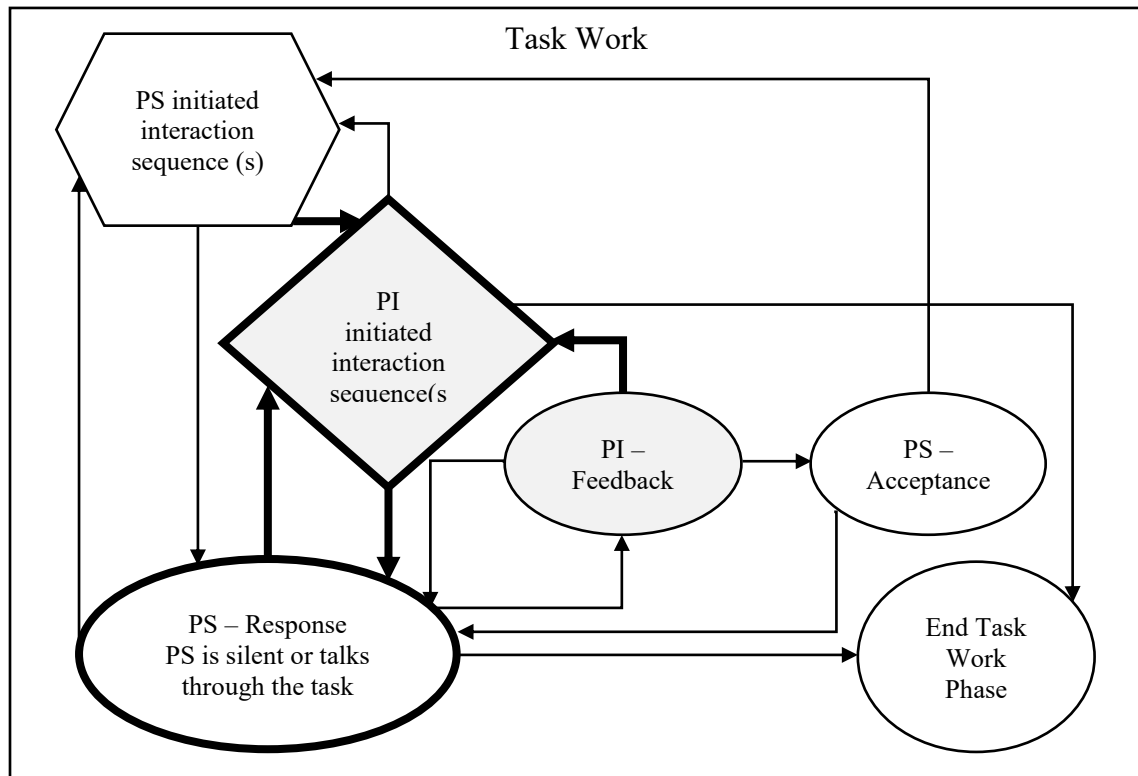


Figure 10. Task work variations highlighted flowchart: PI initiated interaction sequences.

Interviewers initiated interaction sequences with directional move acts, including directives and information in six of the eleven task phases. One example is Beth's informative move act during her running times interview (Table 7, line 11). An excerpt from Reese's sharing cards interview with Robin in Table 16 has examples of directive and informative move acts as initiations and acknowledgment and evaluation move acts as feedback.

The interviewers also choose to initiate interaction sequences during the task work phases with elicitation move acts. Seven task works phases had 20 interviewer elicitations as interaction sequence initiations. The seven types of elicitation move acts were (a) requesting permission, (b) offering manipulatives, (c) checking knowledge, (d) guiding procedures, (e) inquiring about procedures, and (f) focusing on representations. Table 17 displays an interview excerpt where Paige guides Amanda's procedures with an elicitation. After the interviewer-initiated sequence, Amanda returns to respond to the task elicitation with her work on the task.

Peer-interviewer initiated sequences embedded in the main task sequence during the task work phase included initiations and feedback. Interviewers initiated the twenty-one initiations based on the problem-solver work or verbal interactions. The 52 initiations took place following problem-solver main task responses, peer-interviewer initiated sequences, problem-solver initiated sequences, or peer-interviewer main task feedback (see highlighted portions of Figure 10). The peer-interviewers practiced elicitation during interviewer-initiated sequences. Approximately 38.46% of the initiations were elicitations. Half of the 20 elicitations were checking knowledge, and 30% were guiding procedures.

Table 16

Task Work Variation: Two of Reese's Initiated Sequence Examples

Task Introduction	Task Work	Task Closure
<u>I (Directive)</u>		
3:1 Reese: "Okay, the ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?"	3:2-6 Interview interactions	
	<u>I (Directive)</u>	R
	3:7 Reese: "Flip them."	
	3:8 Robin: "Hum?"	
	<u>I (Informative)</u>	<u>F (Acknowledgment, Evaluation)</u>
	R	
	3:9 Reese: "Samuel has five and Robert has three."	
	3:10 Robin: "Oh, I'm sorry."	
	3:11 Reese: "But, yeah."	
	3:12 Robin: "Like that?"	
	3:13 Reese: "Yeah."	
	R3:12-35 Continued interview interactions.	
		<u>F (Evaluation)</u>
		3:36 Reese: "You got it."

Paige and Reese utilized 20 feedback exchange moves. Support move acts were the most typical with 10 acknowledgment and three affirmation move acts. Five

evaluations and two informative move acts made up the directional move acts. Table 14, line 3:11, is an example of Reese using an acknowledgment move act to Robin's response. When Robin continues to respond to Reese's initiation, Reese evaluates her response line 3: 13.

Table 17

Task Work Variation: Paige's Initiated Sequence Example

Task Introduction	Task Work	Task Closure
I (Directive)		
2:1 Paige: "Ruth runs around a lake two times. The first time takes one hour, 43 minutes, and 38 seconds. The second time takes one hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes and seconds."		
2:2-6 Interview interactions		
I (Guiding Procedures)		R
2:7 Paige: " And then what can you subtract to? "		
2:8 Amanda: "You can subtract 60 minutes since that will equate to an hour."		
2:8-10 Continue interview interactions		
F (Evaluation)		
P2:11		
Paige: "Good."		

The participant interviewer used both initiations and feedback in peer-interviewer initiated sequences. Initiations contained elicitation, feedback moves were a place for support, and directionals were found in both initiations and feedback.

Problem-solver initiated embedded interaction sequences. When the problem-solver initiated an interaction sequence, the role of the peer-interviewer was to respond to

the problem-solver's initiation. Eight of the eleven tasks had at least one embedded sequence initiated by the problem-solver during a task work phase. Each interviewer had the opportunity to respond to a problem-solver initiation during a task work phase. Figure 11 highlights the flow of interaction when the problem-solver initiates interaction during the task work phase.

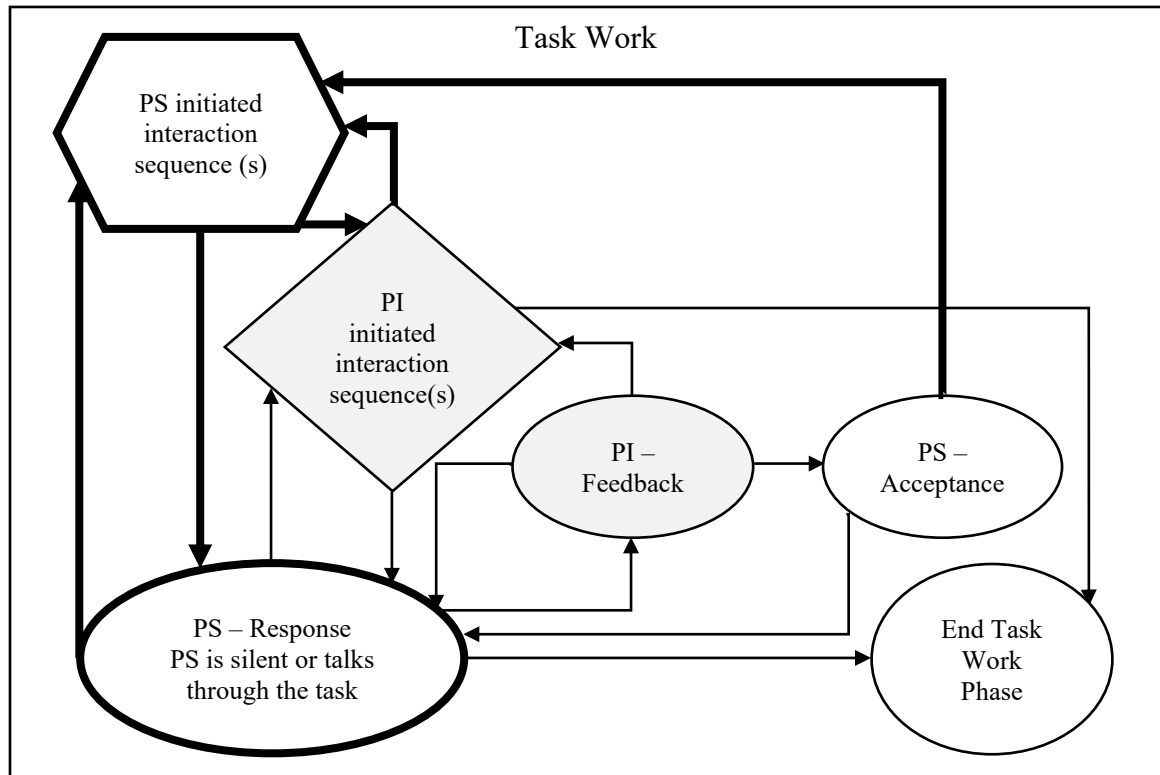


Figure 11. Task work variations highlighted flowchart: PS initiated interaction sequences.

All of the interviewer verbal interactions during problem-solver initiated interactions were responses. Interviewers responded to problem solver initiations with 27 directional and seven support move acts. Paige, Reese, and Beth all responded with directional move acts. Of the 27 directional move acts, 21 were answer move acts, 5 were informative move acts, and one was a marker move act.

During Beth's pattern tile task interview with Rosa, two of Rosa's initiated sequences about understanding the task are shown in Table 18. Beth responded to the initiation by giving information (line 1:9) that did not directly answer Rosa's elicitation. Beth responded to the second initiation example with an answer (line 1:10). Interviewers responded with answer move acts in eight task work phases.

Paige and Reese used support move acts to respond to a problem-solver's initiation in one task work phase. Six of the seven support move acts were affirmation move acts. One move act was a comment. The comment came from Reese. In her pattern tiles task with Anita, Anita initiated a statement in the middle of working on the first task. Anita said, "This is hard. It makes you think." Reese does not provide direction, but supports the challenge with "I know, right?" I coded Reese's response as a comment. Table 19 shows Paige's affirmative responses to two of Brittany's initiations.

Problem-solver initiated sequences gave the interviewers opportunities to practice response exchange moves. Only directional and support move acts were possible for response exchange moves; because I coded all interviewer elicitations as initiations. So, if an interviewer responded to a problem-solver initiation with an elicitation move act, the interaction flow, as seen in Figure 11, shifted from a problem-solver initiated sequence to a peer-interviewer initiated sequence.

Another variation was that the peer-interviewer did not verbally respond to a problem-solver's initiation. An excerpt from Reese's pattern tile task with Anita in Table 20 provides an example of Reese initiating a new interaction sequence in response to an initiation by Anita (line 1:124) and an instance where Reese did not verbally respond to an Anita's initiation in line 1:128. It is possible that Reese did respond to Anita's

Table 18

Task Work Variation: Beth's Directional Response Examples

Task Introduction	Task Work	Task Closure
I (Directive)		
1:1 Beth: "... The hexagon pattern tile is two-thirds of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design."		
1:2-8 Interview interactions		
I	R (Informative)	F
1:8 Rosa: "... So am I supposed to make::"		
	1:9 Beth: " So, that's two-thirds. "	
		1:10 Rosa: "Okay"
I	R (Answer)	F
1:11 Rosa: "And, I'm supposed to use all the shapes?"		
	1:12 Beth: " You don't have to use all of them. You can use anything that will get you to three-thirds; because, that would be the pattern. "	
		1:13 Rosa: "Oh, Okay."
2:14-17 Continue interview interactions		
		R F (Evaluation)
		1:18 Rosa: "Like that?"
		1:19 Beth: "Yes."

initiation with a non-verbal response that was not captured in the audio recording. Anita did not press for a response and became silent. I coded problem-solver's silence as responses in the main task sequence.

Table 19

Task Work Variation: Paige's Supportive Response Examples

Task Introduction	Task Work	Task Closure
I (Directive)		
3:1 Paige: "The ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?"	3:2-39 Interview interactions	
	I	R (Affirmation)
	3:40 Brittany: "So like for example, here if I was::, cause Samuel gave Robert 15."	3:41 Paige: "Um – hmm. Yes."
	I	R (Affirmation) F
	3:42 Rosa: "So, then it would be minus 15, \\ and that would give Robert 15, but they're not the same."	\\ Paige: "Yeah::"
	3:43 Paige: "Yeah, so yeah."	3:44 Rosa: "Oh. Okay."
	3:45-87 Continue interview interactions	
		F (Evaluation)
		3:88 Paige: "Whoa. Yes."

Table 20

Task Work Variation: Reese's Initiation and No Verbal Response Examples

Task Introduction	Task Work	Task Closure
I (Directive) 1:114 Reese: “Okay, um, we’ll do a more difficult one. Um, so, say instead of the hexagon representing two-thirds, it represents three over two. 1:115-122 Interview interactions I 1:123 Anita: “So, if this whole thing represents three over two?” I (Checking Knowledge) R F (Evaluation) 1:124 Reese: “That means there are how many total parts?” 1:125 Anita: “Three” 1:126 Reese: “Mh – hmm.” I 1:127 Anita: “So, you would be taking away a part?” 1:128 Silence 1:129-153 Continued interview interactions F (Evaluation) 1:154 Reese: “All right.”		

The three task interviews allowed each of the participant PST interviewers an opportunity to respond to a problem-solver initiation. Of the 34 responses, 27 were directional move acts. Reese also responded to problem-solver elicitations with another elicitation or no verbal response.

Task work peer-interviewer interaction variations summary. I designed PIRT to encourage PSTs to explore their peer's mathematical thinking, but none of the participant interviewers elicited information about their peer's relational mathematical thinking

during the task work phase. Directional move acts were 58% (70 out of 121) of the move acts. Directional move acts were included in initiations, responses, and feedback. Elicitations in initiations were 17% (20 out of 121) of the move acts, with 85% (17 out of 20) of these elicitations concentrated on procedural knowledge. The procedural knowledge elicitations included checking knowledge, guiding procedures, and inquiring about procedures. Support move acts were 26% (31 out of 121) of the task work interviewer move acts. I found support move acts in feedback and response exchange moves.

Shift to adjust the task. During Reese’s pattern tiles task with Anita, Reese asked Anita if she would like an easier task when she was struggling to start the task. The interview then flowed to a task introduction phase of a second task. After they interacted about the second task, then Reese returned to the original task. Figure 12 highlights the flow of interactions with interactions that shift to adjust the task. Table 21 shows Reese’s initiated interaction sequence to shift to adjust the task. Reese’s initiation with an adjusting the task elicitation was the only interviewer interaction in a shift to adjust the task phase.

Table 21

Shift to Adjust a Task Variation: Reese’s Adjusting Task Initiation

Task Introduction	Task Work
I	
1:1 Reese: “...The hexagon pattern tile is two-thirds of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design...”	
	1:2-7 Interview interactions
	I (Adjusting the Task)
	1:8 Reese: “Do you want me to start with an easier one?”
	1:9 Anita: “Yes.”
	R

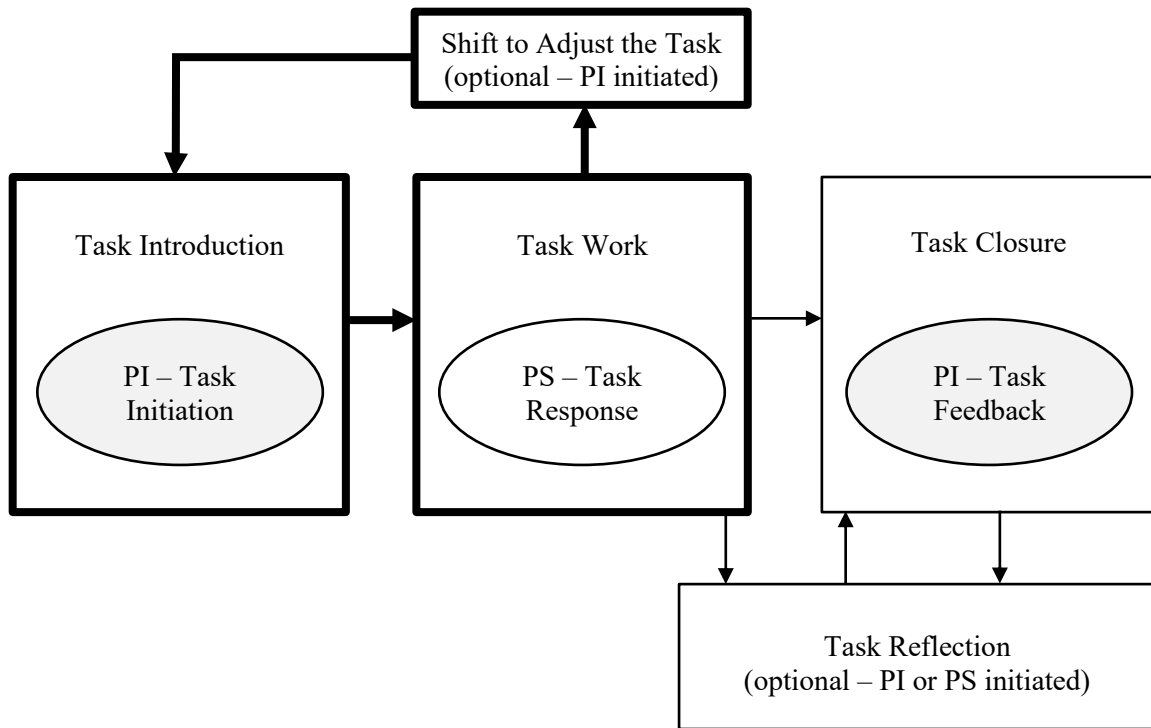


Figure 12. Shift to adjust the task highlighted flowchart.

Task closure variations. Each task closure had a main task sequence feedback exchange move that was an evaluation move act. The problem-solver or the peer-interviewer could transition into the task closure phase. When the peer-interviewer transitioned to the task closure, she gave a task solution evaluation. Four task closures had additional interaction sequences initiated by the interviewer or problem-solver after the task evaluation as part of the task closure. Each interviewer had task closures with and without additional interaction sequences. The different components in the task closure phase were the same as the task work phase with the same exchange move types. Figure 13 shows a flow chart for the task closure phases.

Peer-interviewer interacts with main task sequence feedback. For seven of the eleven task closures, the only peer-interviewer interaction was a task evaluation move act as a feedback exchange move (see Table 13, line 2:4; Table 14, line 2:11; Table 16, line

3:36; and Table 20, line 1:154). In five of these seven task closures, the interviewer's evaluation move act was the only move act during task closure.

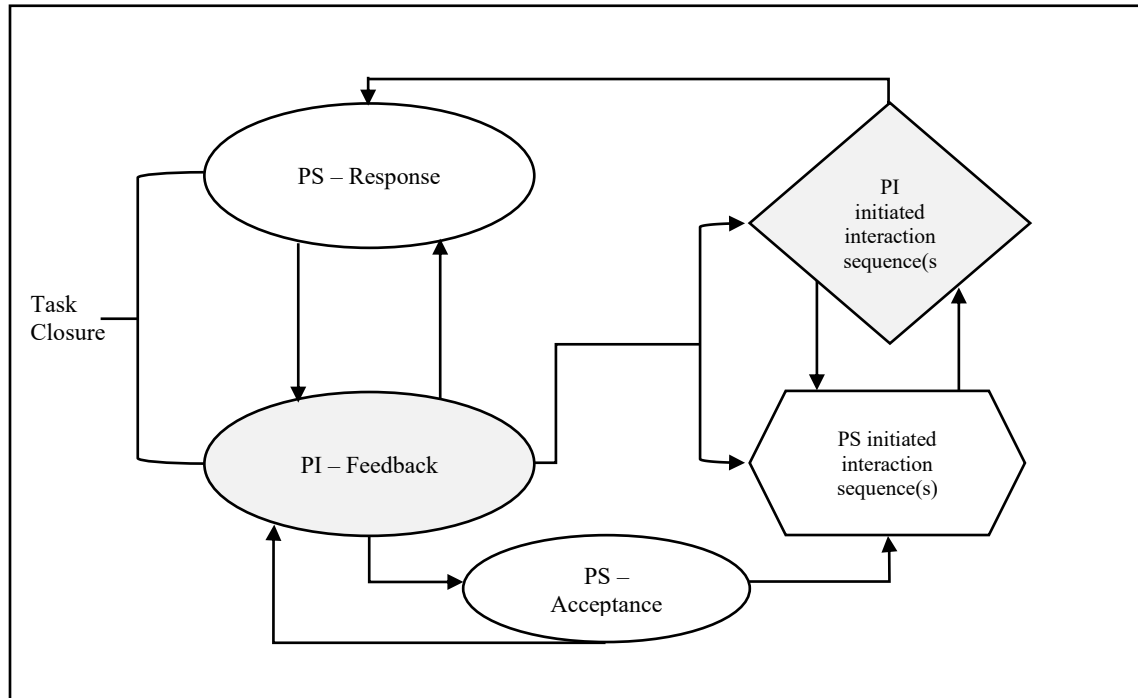


Figure 13. Task closure interaction variations flowchart.

In two of the seven task closures with one interviewer evaluation move act, the problem-solver added an acceptance exchange move (see Table 7, lines 13 and 14). The problem-solver's addition of acceptance does not alter the interviewer's interactions. The presence of four types of exchange moves (initiation, response, feedback, and acceptance) demonstrates a balance of interviewer and problem-solver interaction roles. The problem-solver was taking an active role in the interaction and had the freedom to balance the interaction. Figure 14 shows the task closure flowchart where the interviewer brings task closure with an evaluation with an optional problem-solver acceptance exchange move.

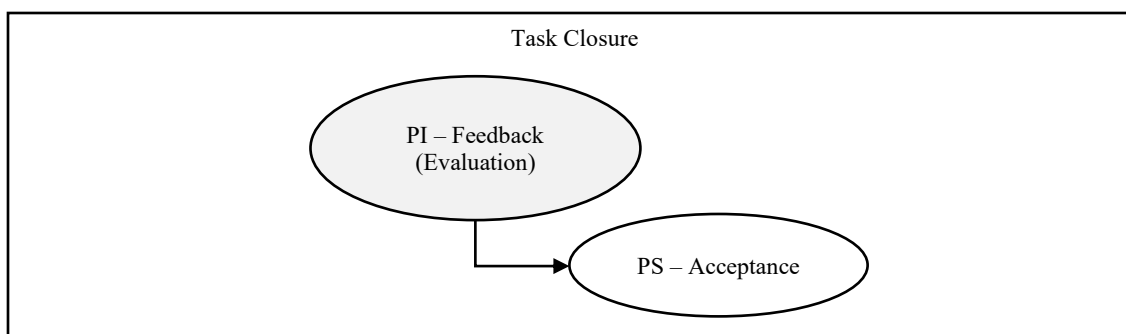


Figure 14. Task closure variation flowchart: PI feedback as one evaluation move act.

Additional main task interactions and interaction sequences. Each of the interviewers had at least one task closure phase that contained additional main task interactions and interaction sequences. There were two reasons for the additional interactions. One reason for additional interactions was that the problem-solver wanted confirmation of the task solution or did not understand the task solution that the peer-interviewer evaluated as correct. Other additional interactions dealt with how to represent the task solution of the pattern tiles task on the task sheet.

For the pattern tiles task, the problem-solver could solve the task with pattern block manipulatives and then draw the pattern. The pattern tiles interview task read:

This task uses pattern tiles or blocks. Here are visual representations of the pattern tiles. Work through the following mathematical task. Use more paper if needed. Please use pen. The hexagon pattern tile is $\frac{2}{3}$ of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design.

(Appendix B)

One reason for additional interaction sequences considered drawn representation of the pattern tile task solution. Beth's pattern tiles interview with Rosa is an example of additional interactions for the second pattern tile representation. After Beth evaluates the

manipulative solution representation, she directs Rosa to draw the representation on paper (see Table 22, line 1:19b). Beth's task closure with Rosa shows additional variations in task closure. Rosa began the task closure with a response to the task initiation. Rosa asks if her solution is correct. Beth follows with the evaluative feedback (see Table 22, lines 1:18 and 1:19a). Rosa also initiates additional interaction sequences after Beth's directive. Table 22 displays how Beth responds to Rosa's initiations with answer mover acts. Beth used four directional move acts in the task closure with Rosa, one evaluation in feedback, one directive as initiation, and two answers in responses. Figure 15 shows the task closure flow of interactions for Beth's task closure in the pattern tiles task with Rosa.

Table 22

Task Closure Variation: Beth's Directional Move Act Examples

Task	Task Work	Task Closure
Introduction		
I		
1:1 Beth reads the task.	1:2-17 Interview interactions.	
		R
		F (Evaluation)
		1:18 Rosa: "Like that?"
		1:19a Beth: "Yes, ..."
		I (Directive)
		1:19b Beth: "...now you can just draw it."
		I
		R (Answer)
		F
		1:20
		Rosa: "Oh, really?"
		1:21 Beth: "Yeah."
		1:22 Rosa: "Okay. ..."
		I
		R (Answer)
		1:22 Rosa: "... Can I just change it?"
		1:25 Beth: "Yes."

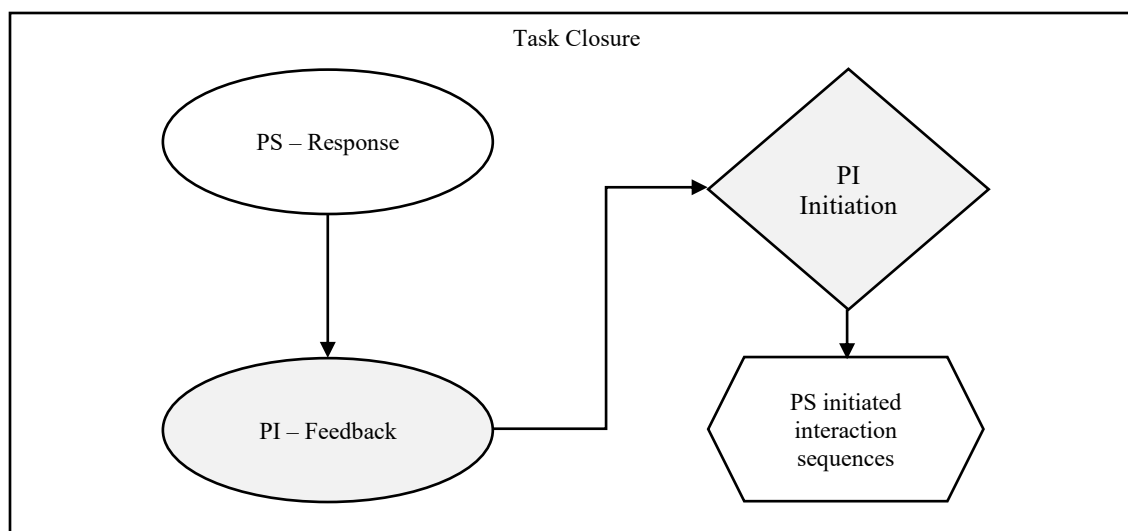


Figure 15. Task closure variation flowchart: Beth's pattern tiles task closure.

In Paige's sharing cards task interview with Brittany, Brittany continues to respond to the main task and seeks confirmation about the task solution after the task evaluation. Paige's sharing cards interview with Brittany provides a variation where the problem-solver was not clear about the task solution when Paige evaluated the task. Brittany arrives at the task solution, and Paige evaluates the solution by saying, "Whoa. Yes" (see Table 19), but Brittany continues to respond to the main task initiation. Paige's interactions show her concentration on Brittany's acceptance of the task solution. Table 23 illustrates the continued interactions and repeated evaluations until the problem-solver accepts the correct task solution. Paige's interactions show her concentration on Brittany's acceptance of the task solution. Table 23 continues the task closure of Table 19 to show Paige's directional and elicitation move acts focused on Brittany's acceptance of the task solution. Figure 16 presents the flow of interactions for Paige's sharing cards task closure with Brittany.

Table 23

Task Closure Variation: Paige's Directional and Elicitation Move Act Examples

Task Closure		
F(Evaluation)		
3:88 Paige: "Woah. Yes." R	F (Informative)	A
3:89 Brittany: "So, then it would be Samuel has 75 and Robert has 45, right? Would that be the answer?"	3:90 Paige: " Well, then when you think about the problem, what you have to do to get the same number of cards. "	3:91 Brittany: "Mh-hmm."
	3:92 Paige: " So, after you've taken away and \\ you've added \\ this, this would be what they both have. "	\\ Brittany: "So" (Possible initiation attempt) \\ Brittany: "Mh-hmm."
I (Guiding Procedures)		
3:93 Paige: " So? " R	F (Evaluation)	A
3:94 Brittany: "60 and 60."	3:95 Paige: " Mh-hmm. "	
3:96 Brittany: "Okay. So, it would be the answer?"	3:97 Paige: " Yeah, it would be 60. "	
3:98 Brittany: "{unintelligible} We will keep it like this."	3:99 Paige: " Yes, perfect. "	3:100 Brittany: "Okay."

Reese also had additional interactions in the task closure phase of two tasks in her pattern tiles task interview with Anita. Figure 13 illustrates the different flows of interactions that occurred in the eleven task closure phases. For the four task closures with peer-interviewer interactions beyond the task evaluation move act, the interviewers had additional initiation, response, and feedback exchange moves. For main task feedback, in addition to the eleven characteristic task evaluation move acts, there were two informative and three additional evaluation move acts. During peer-interviewer interaction sequences, interviewers had two directional move acts and one elicitation

move act as initiation exchange moves, as well as, one affirmation move act as a feedback exchange move. Interviewers responded with ten directional move acts during problem solver interaction sequences. Overall, interviewers used 28 out of 30 directional move acts, one support move act, and one elicitation move act in task closure phases.

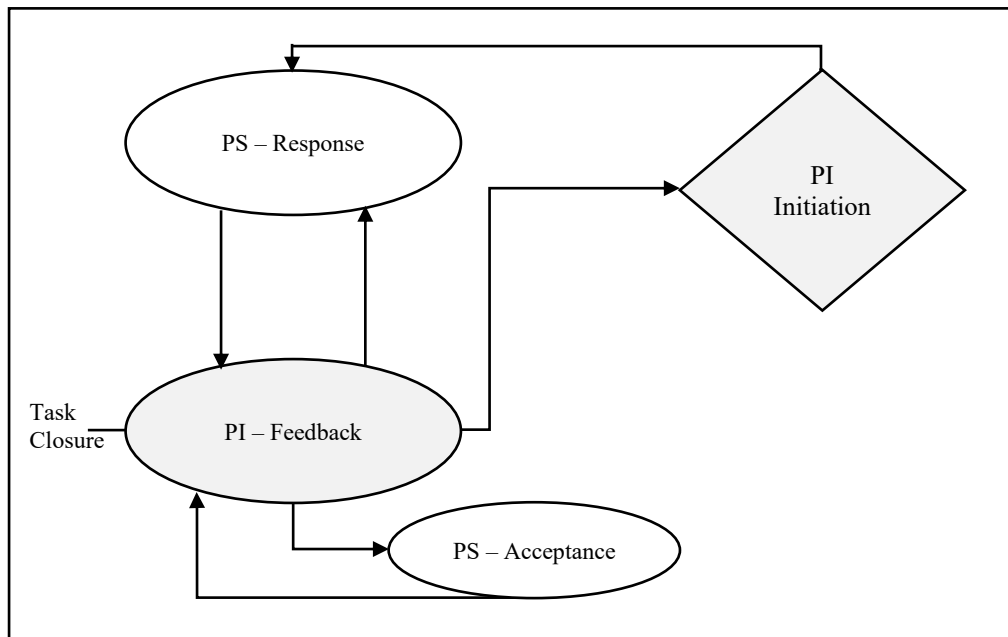


Figure 16. Task closure variation flowchart: Paige's sharing cards task closure.

Task reflection variations. When the interviewer or problem-solver reflected or asked questions about the problem-solver's work after the problem-solver had completed a solution, the interactions were considered the optional task reflection phase. Six of the nine interviews included task reflection phases. Task reflections occurred before the task closure, after the task closure, and in one case both before and after the task closure phase (see Table 10). Reese's pattern tiles interview with three tasks included additional task reflections after the third task reflection.

A task reflection could begin with initiated interaction sequences by either the interviewer or the problem-solver. Task reflection phases could be two moves, such as an

interviewer elicitation and problem-solver response or several interaction sequences by both the problem-solver and the peer-interviewer. Figure 17 displays a flow chart for the task reflection phases.

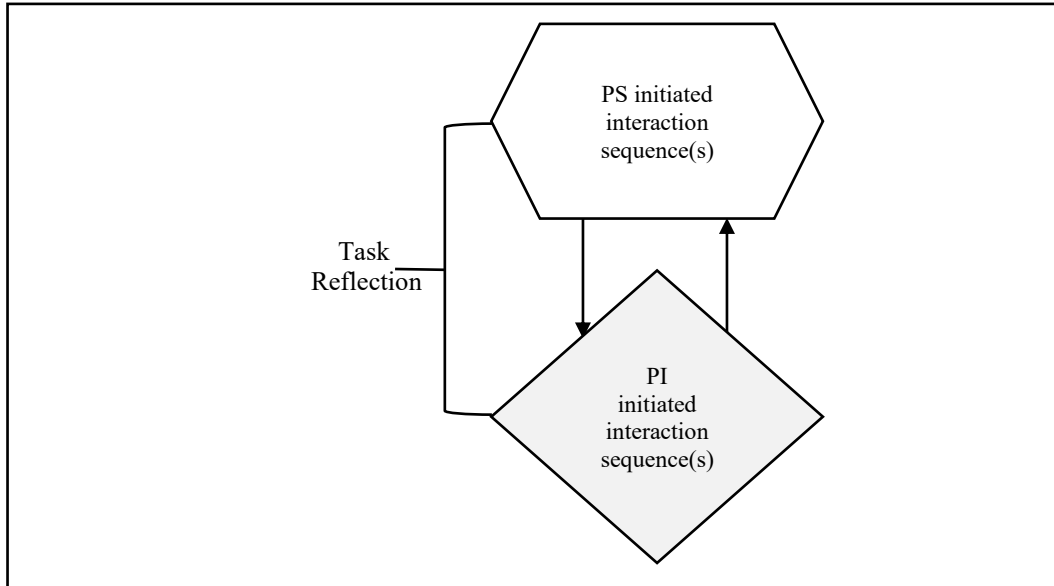


Figure 17. Task reflection interaction variations flowchart.

The interviewer used elicitations and directional moves in the initiation position with support and directional move acts in the feedback position. When the problem-solver initiated an interaction sequence, then the interviewer responded with directional or support move acts. In one interaction sequence, the interviewer added an acceptance exchange move.

Eight of the eleven tasks included task reflection phases. All the interviewers used elicitations in task reflections. Beth and Reese used directional and support move acts as well. I found direction move acts in the initiation, response, and feedback exchange moves. Support move acts were found in feedback, response, and acceptance exchange moves.

Interviewers used more types of elicitation moves during task reflection phases. Checking knowledge, guiding procedures, inquiring about procedures, and focusing on representation elicitation move acts that were used in task work were also used in task reflection. However, interviewers used four additional types of elicitations: (a) encouraging another strategy, (b) encouraging another solution, (c) probing thinking, and (d) encouraging adjustments. Examples of the four additional elicitation move acts used during the task reflection phases can be seen in examples of move acts in Table 8.

Beth's running times task interview in Table 7 lines 15 -18 provides an example of a task reflection phase. Beth used an elicitation to encourage the problem-solver to consider another solution strategy in task reflection. Beth's question was a closed question where an answer could be yes or no. In Beth's running times interview with Hoa, she elicited about another way to solve the task. Hoa gave a short answer that matched Beth's closed question. Beth continues by telling Hoa another way to solve the task with an informative move act, Hoa accepts the solution, and the interview ends.

Table 24 provides examples of directional and support moves acts during a problem-solver's initiated interaction sequences. After Anita worked on three tasks with Reese in the pattern tiles task interview, she initiated sequences about the tasks. Reese responds with affirmation and informative move acts. The interview ended with Reese's acceptance exchange move.

While task reflection phases were optional, they provided more interactions. Importantly, task reflection phases included a variety of elicitation move acts.

Table 24

Task Reflection Variation: Reese's Response and Acceptance Exchange Move Examples

Task Reflection			
I	R (Affirmation)		
1:180 Anita: "That's rough."	1:181 Reese: "Yeah."		
I	R (Affirmation/Informative)	F	
1:182 Anita: "That's hard."	1:183 Reese: "I know..." 1:183 Reese: "...and I was trying – I felt like I wasn't explaining it. Like I was trying to like get you to where like I wanted you to go \ towards. So, I was like I don't know how to like say it without telling you. Okay, \ this is what it will equal to."	\ Anita: "Yeah." \ Anita: {unintelligible}	
I	R (Informative)	F	A (Acknowledgment)
1:184 Anita: "No, it became easier like once I saw you using these [pattern block manipulatives]. But if I was just looking at this and trying to figure out."	1:185 Reese: "Yeah, I completely forgot – maybe that will help her out."	1:186 Anita: "Oh, okay."	1:187 Reese: "Yeah."

Summary of PST interviewer verbal interactions. Each PIRT interview interaction structure (see Figure 4) contained three essential task phases: task introduction, task work, and task closure. The optional task reflection was part of eight tasks. One interview had a shift to adjust the original task. The task introduction included a peer-interviewer directional move act, and the rest of the peer-interviewer moves followed-up on how the problem-solver responded to the task, including their written work, verbal work, and initiated interactions. Each interview was unique, given the

context of the mathematical task and the problem-solver's way of responding to the task. The common task move act was a peer-interviewer task evaluation.

I categorized the peer-interviewer interaction move acts into three categories: elicitation, directional, and support moves. The presence of four-part interaction sequences (IRFA) and problem-solver initiated interaction sequences show that PIRT interviews provided opportunities for balanced interactions between peer-interviewer and problem-solver.

The PIRT cycle was designed to encourage PSTs to explore their peer mathematical thinking, so I was particularly interested in peer-interviewer elicitations. Peer-interviewer elicitations were used the most in task work and task reflection phases. Peer-interviewer elicitations were about the interview process, procedural components of the task, probing thinking, building the problem-solver's mathematical thinking, and how to adjust a task to simplify it for students. Elicitations were closed (see Table 7, line 15), seeking a specific response (see Table 17 and Table 20), or open (see encouraging another solution and probing thinking examples in Table 8). Table 25 provides an overview of the elicitations in the interviews.

PST Peer-Interviewer Development of Verbal Interactions

I describe the development of verbal interactions for the participant PSTs according to their views shared in the post-PIRT participant interview and the findings of their interviewer interactions. The development of elicitation interactions was the shared theme in the post-PIRT participant interviews. Questions and prompts were terms that the PST participants used to describe elicitations. I present how each participant discussed their elicitation development.

Table 25

Overview of Peer-Interviewer Elicitation Moves

Elicitation Move	Task Phase	Task	Problem-solver Struggle	Elicitation Type	Amount	PST
Requesting permission	Task Work	Pattern Tiles	PS elicits about task	Closed	1	Reese
Offering manipulatives	Task Work	Pattern Tiles	PS elicits about task	Closed	1	Beth
Checking Knowledge	Task Work	Pattern Tiles	PS elicits about task	Closed	2	Reese
	Task Reflection			Specific Response	6	
				Specific Response	1	
		Sharing Cards	PS elicits about task	Closed	1	
Guiding Procedures	Task Work	Pattern Tiles	PS elicits about task	Specific Response	1	Reese
				Open	1	
					1	Beth
		Running Times	Possible – PI one elicitation	Specific Response	1	Paige
			Not evident		3	Reese
	Task Closure	Sharing Cards	PS elicits about task		2	
Inquiring about Procedures	Task Work	Sharing Cards	PS elicits about task	Closed	1	Paige
	Task Reflection		Not evident	Open	1	Beth
Focusing on Representations	Task Work	Sharing Cards	PS elicits about task	Specific Response	1	Reese
	Task Reflection	Pattern Tiles	Not evident	Closed	2	Paige
			PS elicits about task	Specific Response	2	Reese
Adjusting the Task	Shift to Adjust Task	Pattern Tiles	PS elicits about task	Closed	1	Reese
Encouraging another Strategy	Task Reflection	Running Times	Not evident	Closed	2	Reese
			Possible – PI informs once		1	Beth
Encouraging another Solution	Task Reflection	Pattern Tiles	PS elicits about task	Open	1	Reese
Probing Thinking	Task Reflection	Running Times	Not evident	Open	1	Reese
Encouraging Adjustments	Task Reflection	Running Times	Not evident	Closed	2	Reese

Paige's perception of her questioning development. Paige's teaching preference style at the beginning of the study was a showing teacher as being the closest to her style

and the best for children (type 1). During the post-PIRT interview, Paige identified her teaching preference as an affordance teacher being closest to her style and the best for children (type 2).

Paige described her interviewer style as “questioning and reaffirming.” Paige’s analysis of her interview style emphasized elicitations and support moves. Paige considered questioning in her preparation, along with building her mathematical knowledge of the task. Paige shares, “I have to look at them [the tasks] beforehand and think about ...what questions would help me to ...solve the problems and to remember them and then ask the person that I’m interviewing.” To ask questions, Paige noted that she had to be observant. She reflects, “I was really observant in what they [the problem-solvers] were doing ...”

Paige used five elicitation move acts over the three interviews. Paige’s third interview with two elicitations and the post-interview reflections provide insight into Paige’s perception of questioning. During task work, she used a closed question to inquire about Brittany’s procedures. This question was a clarification of a statement that Brittany had made. The interaction follows:

Brittany: “Oh, I know where I messed up.”

Paige: “What, did you skip a number?”

Brittany: “Right here at 45.”

Paige used a leading statement as an elicitation that Brittany answered with the task solution during the task closure.

In the interview reflections, both Brittany and Paige write about Paige’s questions. Brittany wrote, “[The] interviewer asked questions that helped me think broader.” The

problem-solver reflection form asked if there were moves of questions that the interviewer could use, and Brittany wrote, “None, [the] interviewer asked all the right questions.” Paige wrote on her reflection form, “I asked questions when she seemed stuck ...” Paige’s reflections indicate that she may have been thinking of questioning more than she put into practice. Paige and Brittany may also think of Paige’s directives as questions or consider that questions must have been part of the way that Brittany arrived the task solution.

Reese’s exploration of varied elicitation moves. Reese’s identified teaching preference style at the beginning of the study was an affordance teacher being closest to her style and the best for children (type 2). Her teaching preference style at the end of the study was the same.

Reese’s interviews showed the widest variety of elicitation move acts and types of elicitation move acts. Reese shared her views about her development as an interviewer in the post-PIRT:

In the beginning, ... try and explain it the way I know and kind of just give it to them. Like just show them the ways to do it ... in the last interview, I was more focused on like proving their thinking ... like just more getting their like understanding of the concepts, which then made it easier for me to explain it because I knew what exactly they didn’t understand.

Reese’s quote concentrates on the task work phase with a focus on the solution. Reese shares that her purpose in elicitations was to understand the problem-solver’s thinking so that she, as the interviewer, could then explain a task strategy better. Reese shows a shift in the purpose of her elicitations. The elicitation purpose is not just to

explore the thinking or to help support the problem-solver's thinking, but to assist herself as the interviewer to be able to explain the task.

Reese recalled using the following questions. "Can you show it to me visually?" "Can you use it with different numbers?" "Can you explain why you did it this way." These elicitation types were used during times of task reflection and illustrate some of the variety of questions that Reese remembered considering during the interviews. During the task reflections, Reese used elicitations to explore and build on her peer's mathematical thinking. Reese was the only participant that interacted with adjusting the task, encouraging another solution, probing thinking, and encouraging adjustments.

Beth's learning to prompt with questions in task reflections. Beth's teaching preference style was a mix of showing and affordance teaching (type 3) at the beginning of the study, and she did not change her preference by the end of the study. Beth saw a change in being able to prompt the problem-solvers by the third interview. Beth shares her views about her development as an interviewer in the post-PIRT:

Toward ... the last one, I was able to recognize right away like and prompt them with questions. Whereas in the beginning, we were like, oh, what do we do ... But, for sure, on the third one, I was able to prompt her. I think I asked her ... how come you came up with that reasoning? What could you do different[ly] if you explained it to somebody else?

Beth used four elicitation move acts. There is a shift in Beth's elicitations between the first and second interviews. In the first interview, Beth's elicitations were during task work, but in the second and third interviews, she initiated interaction sequences in the task reflection phases. Her second interview elicitation was encouraging another strategy

to solve the task, and her elicitation in the third interview was inquiring about procedures. The encouraging another strategies elicitation began with “Is there another way ...,” which is a closed question that brought a closed response, but the inquiring about procedures elicitation in the third interview was open beginning with “How did you ...,” which encouraged a more open response about the procedural reasoning.

Beth’s participant interview quotation shows that she remembered more question types than what she asked in the mathematical task-based interviews. Like Paige, Beth may have thought of more elicitations, than she put into practice.

Each PST identified elicitations as a significant component of their PIRT interaction development. Paige named her interviewing style as reaffirming and questioning. Beth added task reflection interaction sequences with elicitations. Both Paige and Beth indicate thinking about questioning more than they put into practice. Reese explored a variety of elicitations during the interviews. Reese identified her purpose in questioning as understanding the problem-solver’s thinking to explain the task better. In some task reflections, Reese interacted with elicitations to build on the problem-solver’s mathematical thinking.

Findings Synopsis

I analyzed the varied peer-interviewer’s interactions in the context of the interview interaction structure. A peer-interviewer interaction theme was that each task had a task evaluation move act. Dynamic variation of interaction patterns was the other theme. The interviewer’s interactions include initiations, responses, feedback, and acceptance exchange move in the categories of elicitations, support, and directional move acts. The emphasis of peer-interviewer interactions during task work focused on guiding

the problem-solver toward the solution and affirming the problem-solver. Peer-interviewer interactions during the task reflection phase included attempts to probe or build mathematical thinking. Peer interviews provided opportunities for some balanced discourse with acceptance exchange moves and problem-solver initiated interaction sequences.

In addition to identifying the interviewer interactions during PIRT interviews, I identified an elicitation development theme during interviewer interactions from the perspective of the PST participant. Each case study participant identified growth in questioning. Paige and Beth mention questioning that did not match the recorded interviews showing that they may have thought more about questioning than they put into practice. Beth added task reflection questioning in her second and third interviews. Reese interacted with a variety of elicitations and used some elicitation to build on the problem-solver's thinking during task reflection phases.

V. CONCLUSIONS AND RECOMMENDATIONS

Responsive teaching where teachers make in-the-moment decisions of how to respond to and build on student's mathematical thinking is an ambitious goal (Jacobs & Empson, 2016; Pierson, 2008; Schoenfeld, 2016). The difficulty is that teaching moves that are responsive to student thinking are challenging and often take years to develop (Jacobs et al., 2015). The question is how to begin the development of teaching moves for responsive teaching with PSTs in their mathematics education courses. As a starting point, I designed my study to explore PST's mathematical task-based interviewer interactions in an initial mathematics content course. I examined the peer-interviewer interactions of three PSTs as they occurred naturally during three cycles of PIRT activities. The PIRT interview and interaction preparation activities provided future teachers with some components of responsive teaching. Findings from my exploratory multiple case study identified a common task evaluation move act, dynamic variations of interactions in an interview structure of task phases, and common interest in questioning development.

Research Questions and Conclusions

The research questions that guided my study were:

1. In what ways did the PST peer-interviewers verbally interact with a peer problem-solver during three peer mathematical task-based interviews?
2. How did the PST's interviewer interactions during three peer task-based interviews develop over a semester?

I draw the following conclusions based on the findings reported in Chapter 4. First, I summarize my findings for the two research questions and then present my conclusion regarding PST development of interaction moves.

- The interviewer's interactions were in the context of task phases in an interview. Task introduction, work, and closure phases were a part of all mathematical tasks. Peer-interviewers evaluated each task during task closure.
- Three categories of interviewer interaction move acts were present. The categories included directional, support, and elicitation move acts.
- Aside from the essential task phases and the task evaluation move act, the PST interviewer's interactions varied dynamically within the context of the task and the problem-solver's interactions. Optional task phases included task reflection and shifting to another task. The peer-interviewer interaction variations included:
 - extending the problem-solver task work with feedback exchange moves.
 - allowing balanced interaction patterns, including acceptance exchange moves for optional four-part interaction sequences.
 - initiating additional sequences during task work, task closure, and task reflection phases with directional or elicitation move acts.
 - responding to problem-solver initiations during the task introduction, task work, task closure, and task reflection phases with support and directional move acts.
- The elicitations were present in some task work, shift to adjust the task, task closure, and task reflection phases. Most of the elicitations occurred during task work and task reflection phases. Task work elicitation move acts focused on

knowledge, procedures, and representations. Task reflection elicitations included questions about mathematical knowledge and task procedures, but also include elicitations about representations, probing thinking, and encouraging another strategy or solution.

- Each of the three PST participants showed an interest in developing elicitations. The PST participants reported that they thought about their ability to question peers during the interviews and shared about questions they asked during the interviews. Some of the questions that Paige and Beth remembered asking in the interviews were not part of the audio recordings. Each participant perceived growth in questioning another person during task-based interviews.

My analysis of PST's interviewer focused on identifying themes of beginning PST interview interactions and a view of how PST interviewer interactions develop.

Peer interviews include some balanced dialogue. One of the variations in the study was the balance of dialogue between the interviewer and problem-solver. Some interviewer, problem-solver, and mathematical task combinations produced more balanced interactions than others. Problem-solver initiated sequences and acceptance exchange moves added to interaction sequences show some balanced dialogue between the peer interview and problem-solver. One possible reason for more balanced interactions is that the interviewer and problem-solver were peers, which reduced the authority structure. The balanced interactions are noteworthy since the traditional classroom interaction patterns have the teacher talking about two-thirds of the time (Mehan, 1979; Cazden, 1988). While I consider peer interviews as a limitation to findings of how PST would interact with children, more research would help us determine if peer

approximations of practice (Grossman et al., 2009) could instill a more balanced interaction structure when working with another person on a mathematical task. A long-term study could examine if these peer interview approximations of practice would transfer to working with children.

A task solution focus and instrumental responsiveness. The typical pattern of the peer-interviewer interacting with an evaluation move act for each task showed a common expectation of the problem-solver completing a task and having an evaluation. Both the interviewer and problem-solver in the interviews often focused on help with the task when one of them perceived that this was necessary. Interviewers would offer directional moves or elicitations to help the problem-solver, and the problem-solver would also elicit information from the interviewer to help solve the task. Interviewers also sought to help the problem-solver's by providing support move acts to encourage the task work.

The common theme of a task evaluation for each task and task work elicitations that focused on the task work and procedures indicate a task solution focus during task work and task closure. Why were many peer interactions focused on the task solution and not on exploring their peer's relational mathematical thinking?

Jacobs and Empson's (2016) model of teaching responsive to children's mathematical thinking includes frameworks of researched children's solution strategies in the knowledge of children's mathematical thinking (see Carpenter et al., 2015). I compared the problem-solver's strategies to the peer-interviewers strategies in their mathematical task preparation assignments (see Table 4, Table 5, and Table 6). Table 26 shows a comparison of the peer-interviewer and problem-solver solution strategies with 5

of the eleven tasks showing different solution strategies. Directional move acts and a task solution focus happened in tasks with similar and different solution strategies. While knowledge of strategies is important, I considered additional PST dispositions.

Table 26

Comparison of Problem-Solver and Peer-Interviewer Solution Strategies

Mathematical Task	Peer-Interviewer Participant	Problem-Solver's Solution Strategy	Alignment with PI's Strategy in the Mathematical Task Preparation Assignment
Pattern Tiles	Paige	A drawing of a hexagon and a trapezoid.	Different solution representation. Paige had a drawing of three trapezoids.
	Reese	Second easier task: The original hexagon and two trapezoids.	Different solution representation. Reese had a drawing of two hexagons and another representation of a hexagon, rhombus, triangle, and trapezoid.
		Original task: A drawing of a hexagon and three triangles.	Different solution representation. Reese had a drawing of three trapezoids, another representation of a hexagon and a trapezoid, and a final representation of a hexagon, rhombus, and a triangle.
		Third challenging task: A drawing of two rhombi. An additional drawing of a trapezoid, rhombus, and a triangle.	Same solution representation.
	Beth	A drawing of a hexagon and a trapezoid.	Different solution representation. Beth had a drawing of three trapezoids.
Running Times	Paige	Added with regrouping in base sixty.	Same strategy.
	Reese	Added with regrouping in base sixty.	Same strategy.
	Beth	Added with regrouping in base sixty.	Same strategy.
Sharing Cards	Paige	Beginnings of a proportion, then a completed ratio table.	Same as Paige's second strategy. Paige's first strategy was an algebraic equation.
	Reese	A ratio at the top, then a completed variation of a strip diagram.	A drawing variation of the same strategy.
	Beth	Multiplicative reasoning	Different strategy. Beth used a vertical ratio table strategy and then a double number line strategy.

As I considered this result of the study, I considered the possibility that a PST could interpret mathematical thinking differently from my perspective or the perspective of the preparation materials. An interpretation of understanding mathematics impacts one's interpretation of mathematical thinking. Skemp (1976/2006) used two phrases to emphasize differences in interpretations about understanding mathematics. He used the phrase *instrumental* understanding of mathematics to refer to knowing a rule or procedure and how to apply it to a mathematical task, in contrast to a relational understanding of mathematics to refer to “knowing both what to do and why” (p. 86). I understand a relational understanding of mathematics to include conceptual understanding that emphasizes relations or connections within mathematical concepts, as emphasized by Ma (1999) in PUFM.

If one's perception of mathematical understanding is knowing the procedures to get to the solution or instrumental understanding (Skemp, 1976/2006), then this person would consider that they understand the problem-solver's mathematical thinking when the problem-solver follows known procedures to complete a task. If a problem-solver completes a task using the same procedures that the interviewer used, then the problem-solver and interviewer have the same instrumental understanding. If an interviewer perceives that they understand the problem-solver's mathematical thinking, then there is no need to explore the problem-solver's thinking. If the problem-solver completes the task differently, but the interviewer can follow the procedures, then the interviewer instrumentally knows your mathematical thinking. If the problem-solver does not know the procedures to solve the task, then the problem-solver possibly does not have the mathematical thinking (or forgot the necessary mathematical thinking) for that task, and

the interviewer may feel the need to help you gain the instrumental mathematical understanding to complete the task.

Each PST responded to her peer's mathematical work in the interviews. The elicitation, directional, and support moves all corresponded to what the problem-solver was doing or saying. A PST could consider herself to be responsive to the problem-solver because she attended, interpreted, and responded based on her work even if it was with an instrumental understanding. Since the directional class of move acts consisted of traditional directive, informative, answers, and evaluation move acts focused on procedures and solutions, then directional move acts in this study could be classified as instrumentally responsive move acts. Note that directional move acts could be relational, but I did not observe relational move acts in this study. Elicitations that focused on the task procedures and solutions can classify as instrumentally responsive moves. Instrumentally responsive elicitation moves in this study included guiding procedures and inquiring about procedures.

PSTs display an interest in learning about questioning. The finding that there is a PST interest in learning about and practicing questioning in mathematics education courses is encouraging. More research is necessary to see if PST's interest in questioning existed before the course, was impacted by the instructor's ability to elicit in the course or was encouraged by the PIRT cycles. Although the reason for their interest in questioning is unclear, this finding shows that PST interest in questioning can be encouraged. Due to a common interest in questioning from the participant interviews, elicitations could be a good starting point for introducing PSTs to responsive interaction moves that include questioning as a significant component.

Task reflection phases to practice responsiveness. The finding that there were more types of elicitation move act type during task reflection than task work phases indicates that task reflection phases could be more a place to begin responsive interaction practice. Tasks not involving evident problem-solver struggle provided opportunities for elicitations during task reflection phases even when no interaction was present in the task work phase. To encourage PST interviewers to concentrate on responsive interactions and exploring mathematical relational thinking, PSTs could use tasks that may not initially encourage struggle, so that the focus was on the task reflection phase. PSTs could interview peers with number reasoning tasks similar to number talks (Humphreys & Parker, 2015) to see how peer PSTs calculate mentally, build number sense, and practice task reflection responsive interaction move acts.

Limitations

To explore PST interviewer interaction patterns, I limited my sample and worked with PSTs in the natural environment of a mathematics content course focused on problem-solving where interviews with children would not be required. The limitations of the study allowed me to concentrate on the PST interviewer interactions in PIRT mathematical task-based interviews.

Sample limitations. I limited my study to three PST interviewers to gain insight about the PST mathematical task-based interview interactions. Although small, the sample reflects three different perspectives of mathematics teaching practice. It was beyond the study's intent to measure the interviewer interactions as a result of the participants perspective of mathematics teaching practice, but I believe that selecting PST interviewers with different mathematics teaching perspectives contributed to the

variations of interviewer interactions. Mathematics teaching perspectives and dispositions may be a starting point to examine the types of interactions that PST's practice in different approximations of practice.

A diverse though limited sample size allowed me the opportunity to conduct discourse analysis on each of the interviews and develop a mathematical task-based interview interaction structure to analyze the interviewer interaction moves. The analysis helped me to document the task evaluation move acts for each task and the interaction variations.

The PSTs that participated in the PIRT interviews included elementary, middle school and all-grade PSTs, but the PST interviewers examined in my study were limited to elementary PSTs that were at the beginning of their mathematical education coursework. Further research examining PST peer-interviewer interactions with other groups of PSTs or with cultural groups of PSTs may give insight into other possible kinds of PST interviewer interactions. Different types of instructional practice in mathematics content courses, mathematical tasks, and PIRT preparation materials may also affect how interviewers interact with problem-solvers in mathematical task-based interviews.

Peer interview limitations. One could argue that elementary mathematics teaching interaction development should be done only with children. Interviewing peers is clearly different than interviewing children. The status of people involved in discourse impacts the interactions (Sinclair & Coulthard, 1975/1978). Thus, the interviewer's status impacts the interview interactions. PIRT is intended as preparatory practice for a PST before they work with a child. Peer mathematical task-based interviews could be a resource to use before child mathematical task-based interviews or lesson rehearsals in

other mathematics education courses. Further research could be done to see if students that used peer mathematical interviews in mathematics content courses are different in their interactions from those who did not use peer interviews.

Multiple simultaneous audio recordings. Audio recordings of the peer interviews happened simultaneously in one classroom. Many of the recordings were clear, but some of the recordings included interactions that had required additional concentration to transcribe. There were a few parts of interactions that were unintelligible due to the background noise. The multiple simultaneous recordings did limit the transcriptions a little and caused more time in editing transcriptions.

Implications

Implications of my study for the preparation of future teachers of mathematics include: (a) encouraging a shift from responsiveness to instrumental mathematics to responsiveness to relational mathematics and (b) leveraging PSTS' interest in developing elicitation interactions. PIRT provides one possible method to support this development.

Encouraging a shift from instrumental to mathematically relational responsiveness. This study on PST's interviewer interactions shows that PST may interact with a problem-solver in ways that are instrumentally responsive to the problem-solver's mathematical work. PSTs with an instrumental view of mathematical understanding may interpret preparation materials and mathematics teacher educators with a relational view of mathematical understanding differently than intended (Skemp 1976/2006). For PSTs that have an instrumental view of mathematics, mathematics education researchers and mathematics teacher educators should consider how to encourage a shift toward mathematically relational responsiveness. I use the phrase

mathematically relational responsiveness to mean responsiveness to relational mathematics involving responding to student's mathematical relational ideas and connections between students thinking and the mathematics discipline. Relational responsiveness could signify responsiveness in human relations instead of mathematical relations; thus, I use mathematically relational responsiveness to emphasize responsiveness to relational mathematics.

To encourage PSTs with a tendency to be responsive to instrumental mathematics to shift toward responsiveness to relational mathematics, I suggest that we consider the dispositions of PST's orientations and identities and that mathematics teacher educators concurrently model mathematically relational responsive teaching in mathematics content courses.

Addressing PST's orientations and identity. If PST's are observing responsive teaching models and learning about interaction moves and patterns with an instrumental understanding of mathematics (Skemp, 1976/2006) and calculational orientation (Thompson et al., 1994), then the PSTs are likely not to receive a relational understanding of responsive teaching practices. Any discipline could discuss responsive teaching practices, but I posit that what makes mathematics responsive teaching practices related explicitly to mathematics is a focus on the essence of a relational understanding of mathematics (Skemp, 1976/2006). The question to be addressed is: How can PSTs be encouraged to see mathematics from a relational perspective?

Do we hope that mathematical relational perspectives will be encouraged and developed from experiencing inquiry-based learning and preparing PSTs for ambitious teaching? Students with previous instrumental instruction in a content area may have

interference with relational instruction in the same content area even if they enjoy relational activities (Pesek & Kirschner, 2000). Pesek and Kirshner (2000) present three types of interferences that students may experience when learning content with instruction focused on a relational understanding of mathematics that they first learned with instruction focused on an instrumental understanding of mathematics. These interferences are part of the dispositions (Blömeke et al., 2015) of PSTs and hence the PST's mathematical identity (Aguirre, Mayfield-Ingram, & Martin, 2013).

For PSTs that learned grade school mathematics first with instrumental instruction, the challenge can be great to change the lens of instrumental mathematics understanding and appreciate relational instruction and mathematics understanding. Future studies can study PSTs perceptions along the spectrum of instrumental and relational understandings of mathematics with connections to PST's mathematical identities and how these dispositions impact approximations of practice.

PSTs cannot be expected to shift in perspectives if they do not realize that different perspectives exist. I propose that mathematics teacher educators purposefully discuss the different perspectives of instrumental versus relational understanding of mathematics with PSTs. PSTs may need time to analyze their mathematical identity, consider possible changes, and have the freedom to wrestle with possible interferences and changes to their dispositions. If mathematics teacher educators communicate the difference in instrumental and relational mathematics, PSTs may understand the purpose of inquiry-based mathematics content courses and the connection to reform-based mathematics teaching.

Modeling responsive teaching. The task evaluation was the only peer interaction move act that was the same in all the mathematical tasks. PSTs may expect the traditional IRE pattern in their mathematics content courses. Mathematics content course instructors have the opportunity to model a variety of follow-up responses as opposed to only evaluation. Wells' research (1993) showed that initiation-response-follow-up patterns could be used effectively in different classroom situations. Mathematics content course instructors can take advantage of studying their practice to increase responsive teaching interactions.

PSTs as students may expect IRE patterns in mathematics content courses (Mehan, 1979). Instructors of mathematics content courses need to be sensitive to how some PSTs may feel uncomfortable with non-IRE patterns. Introducing and explaining different classroom interaction patterns before modeling the pattern can prepare PSTs for differences in classroom expectations and how this connects to learning mathematics. Research on how mathematics teacher educator interactions in mathematics content courses impact PST's affective and cognitive areas could provide tools for improving mathematics content courses.

Different mathematics practices as classroom routines can encourage students to share their mathematical thinking and provide instructors an opportunity and framework to practice responsiveness to students' relational mathematical reasoning. Number talks (Humphreys & Parker, 2015) are a routine that supports number sense for number and operations in mathematics content courses, as well as responsiveness to student's relational mathematical reasoning. PSTs can participate in number talks as students, watch videos of number talks, (see <https://www.teachingchannel.org/video/3rd-grade->

number-talks) and prepare a number talk for a group of people. Some additional mathematical classroom routines that encourage discussions about relational mathematics include estimation tasks (see <http://www.estimation180.com/>), determining which one doesn't belong (see <http://wodb.ca/index.html>), and visual patterns (see <http://www.visualpatterns.org/>).

Mathematics content course instructors can explore the use of math routines that encourage student's communication of relational mathematics in mathematics content courses. Research on math routines with PSTs can focus on the impact these routines have on PST's perceptions of teaching interactions.

Leveraging PST's interest in questioning. The finding that PST participants were interested in the development of questioning is encouraging. Mathematics teacher educators can leverage PSTs' interest in questioning by actively teaching PSTs about different interaction patterns and move acts. Interaction instruction can complement instruction on PST's orientations and identities. PSTs can consider how interaction patterns and move acts impact perspectives on understanding mathematics and student's mathematical identities.

Developing interaction patterns. Mathematics teacher educators have the opportunity to present interaction patterns to PSTs. The Initiation-Response-Evaluation pattern (Mehan, 1979) can be presented to PSTs and contrasted to other Initiation-Response-Follow-up interaction patterns (Wells, 1993). PSTs can analyze one-on-one and classroom teacher-student(s) interaction patterns from videos or written transcripts. Mathematics teacher educators can teach PSTs the difference between funneling and focusing interaction patterns (Wood, 1998) and explore different types of classroom

interactions. PSTs may also benefit from learning about levels of discourse in a mathematics classroom (Hufferd-Ackles, Fuson, and Sherin, 2004). PSTs can analyze the types of mathematics classroom discourse environments that they experienced and their own goals for mathematics classroom discourse.

Further research can study how PSTs respond to different activities involving mathematics teaching interaction patterns in mathematics teaching. We need more research about the development of PST's perspective of interaction patterns and how to develop a repertoire of interaction patterns. PSTs may also benefit from learning about levels of discourse in a mathematics classroom.

Developing interaction moves. The theme of dynamic variation in the peer-interviewer's interaction move acts, and the finding that the PST participants are interested in elicitation move acts shows promise for PSTs practicing different interaction moves. We are at an exciting time in mathematics education because researchers are developing materials for interaction practice based on their research (e.g., Chapin, O'Conner, & Anderson, 2013; Parrish, 2014; Humphreys & Parker, 2015; Smith & Stein, 2018).

Mathematics teacher educators can present different interaction models to PSTs during their teacher preparation. "Math talk" is a phrase being used to present different moves in the classroom that encourage students to share, listen, and justify thinking. One resource that includes video and transcript examples of the math talk moves is a book entitled – Talk Moves: A Teacher's Guide for Using Classroom Discussion in Math (Chapin et al., 2013).

Milewski and Strickland (2016) present a framework based on research work with teachers that uses a common language for responding to students. Milewski and Strickland's (2016) work built on the language of teachers and linguistic research to form clusters of moves. Since the teacher's language built the cluster of moves, PSTs may respond to this language. Research can investigate how PST's respond to this framework of interaction moves.

I used Jacobs et al.'s (2015) work on a framework of teaching moves in PIRT. The framework of supporting and extending moves organizes moves into categories related to the problem-solving process. The framework directly builds on concepts of responding to children's mathematical thinking. The children's mathematical thinking that these researchers refer to are frameworks of how children think of mathematical tasks. The framework of supporting and extending moves would complement courses that taught researched frameworks of children's mathematical thinking.

Recommendations

Both mathematics education researchers and mathematics teacher educators can work together to develop additional ways that PSTs can be encouraged to actively consider their mathematical identities, including their perceptions of mathematics understanding and mathematics teaching. PSTs leave university mathematics departments and teacher preparation programs to teach in school districts where school administrators, parents, and students have a spectrum of perceptions of mathematical understanding. Our PSTs will decide how to teach and influence each community where they work, live, and participate socially. I propose inviting PSTs to consider their mathematical identities and directly present how differences in perceptions of mathematics understanding have a role

in the mathematical identities they developed. I propose recommendations for research and practice.

Research recommendations. This study is a step towards exploring the ways that beginning PSTs interact as peer-interviewers with a mathematical task. Some specific research questions arising from this study are:

- Does some balanced dialogue in peer mathematical task-based interviews transfer to balanced dialogue in child mathematical task-based interviews?
- How does combining instruction on researched frameworks of children's mathematical thinking impact mathematically relational responsiveness in PSTs?
- In what ways do PSTs develop relational responsiveness during task reflection phases?
- In what ways do PSTs develop relational responsiveness during task work phases when the problem-solver struggles?

There are still questions about how PST dispositions, including affective and cognitive domains; professional noticing; and the practice of responsive interactions can develop together. Research can test responsive teaching interactions as a continuum of competency to develop trajectories of competence across the continuum and develop effective strategies for the development of responsive interactions. In what ways does the development of any one area of the competence continuum impact other areas? How can we develop mathematics education programs that develop PSTs holistically and prepare them for continual growth?

Practice recommendations. To assist PSTs with the development of responsive interactions, university mathematics education leaders and instructors can work together

on a plan for addressing PSTs orientations and responsive interaction development across the competency continuum (Blömke et al., 2015). In some universities, mathematics education courses for PSTs are in both education and mathematics departments. In this case, mathematics education leaders in different departments would need to face the challenge of planning together how to impact responsive interactions.

Mathematics education courses concentrating on the mathematical knowledge that PSTs need should focus on relational mathematical knowledge. Neubrand (2018) posit: “I would say that a significant part of teachers’ professional knowledge should be devoted to building up specific [relational] mathematical knowledge, not sheer instrumental knowledge, but in direct relation to the educational issues a teacher is faced with during teaching” (p. 610). Yet, Pesek and Kirshner (2000) found that while some students who learned mathematical content first instrumentally and later enjoyed relational mathematics activities in the same mathematical content area could have attitudinal interference. These same students claimed that they learned more from the instruction based on instrumental understanding than the relational instruction, and the students showed cognitive and metacognitive interference with conceptually understanding the same topic. So relational mathematical instruction for PSTs may not be enough for all PSTs to change their perceptions about understanding mathematics. PSTs need additional instructions about mathematics orientations and identities.

In addition to teacher preparation that addresses interpretations toward mathematics and relational mathematical knowledge for teaching, content can relate to teaching children mathematics, including noticing and practice (see Neubrand, 2018 quote above). One may argue that interaction development belongs in a methods course.

I understand the concern that mathematics content courses are already full of content.

Mathematics content course instructors may be trying to make sure future teachers know the mathematics content required to teach or prepare PSTs to pass a mathematics content test for teachers.

Although the PIRT activities did take time from a class session, peer mathematical task-based interviews in this study were a value-added component to the class. In this study, the mathematical tasks came from the curriculum and complemented the classroom content. Paige shared in a post-PIRT interview that the mathematical task preparation and interviews allowed her to concentrate on the mathematical ideas in a task. She thought the PIRT activities impacted her learning of mathematics and helped her do better on formal assessments. She comments:

I was more familiar with it [the mathematics] at that point because I had spent so much time going in detail and it makes me be able to answer you know quiz and test questions better because like now I know the thinking behind everything and now I know like if you take the same problem and you change the numbers or you kind of like tweak it a little bit, I still know you know everything about it. I know like why, how, how to define it, things like that. I feel like it [PIRT activities] did. I feel like it helped my understanding of everything.

Peer approximations of practice in mathematics content courses can be part of a plan to prepare for other approximations of practices such as lesson rehearsals (Lampert et al., 2013), mathematical task-based interviews with children (Philipp et al., 2007), and mathematics conferencing (Munson, 2016). Since responsive teaching moves take time

to develop, a mathematics education plan throughout teacher preparation courses could be developed and studied in different institutions.

We should consider the timing of the blend of content and teaching practice. Just as instrumental instruction before relational instruction can cause interferences (Peseck & Kirschner, 2000), our decisions about when to address identities and perceptions of mathematical understanding in preparation could be significant. Future research studies can study the impact of inviting PSTs to consider these areas beginning in the first mathematics education course. Philipp et al. (2019) propose that PSTs can learn from researched frameworks of children's mathematical thinking while they are learning mathematics content so that the PSTs can view mathematics through the lens of children's mathematical thinking. Connecting PST's interest in children to children's relational thinking of mathematics has been effective with some PSTs while the PSTs were studying in a mathematics content course (Philipp et al., 2007).

I propose university mathematics education leaders and instructors consider Philipp et al.'s (2019) recommendation to include frameworks of children's mathematical thinking at the beginning of PSTs mathematical education coursework. I propose blending Philipp et al.'s (2002) circles of caring model with the PIRT framework (see Figure 2) that was informed by Philipp et al.'s (2002) model. Connecting to PST's interest in children to engage PSTs with mathematics and mathematical teaching practices by way of children's mathematical thinking and teaching children can guide universities in developing a plan to address PSTs orientations and responsive interaction development across mathematics content and methods courses.

Jacobs and Empson's (2016) model of responsive teaching that is responsive to children's mathematical thinking complements the adapted circles of caring framework and aligns with this study's dynamic competency continuum informed by Blömke et al., 2015. The relational mathematical understandings that are part of children's solutions strategy frameworks are essential for teaching that is responsive to children's mathematical thinking.

For PST development of mathematically relational responsiveness when interacting with others working on mathematical tasks, I recommend the following considerations for PSTs using the dynamic competency continuum of dispositions, situation-specific skills, and performance. Mathematics teacher educators can include instruction on mathematical orientation and identity, along with interaction knowledge to complement the knowledge of children's mathematical thinking, pedagogical content knowledge, and mathematics content knowledge to develop dispositions. Noticing of children's mathematical thinking can develop using examples of children's mathematical thinking (Jacobs, 2010) to develop situation-specific skills. Approximation of practices such as PIRT leading to child task-based interviews (Ginsburg, 1997) and lesson rehearsals (Lampert et al., 2013) leading to classroom teaching can complement the learning of interaction moves to elicit, support, and extend children's mathematical thinking responsively.

I hope this study encourages mathematics teacher educators and mathematics education researchers to consider PST mathematical task-based interview interactions and how PST's perceptions of mathematics impact their mathematical task-based interview interactions. Moreover, I hope that this study may pave the way for further

investigations of PIRT and other ways to better prepare future teachers of mathematics for the challenges they will encounter in the classrooms.

APPENDIX SECTION

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APPENDIX A: Interview and Interaction Preparation Assignments

Interview and Responsive Interaction Preparation 1 Task-Based Interview Introduction and Questioning

Name _____

Task-Based Interview Introduction

Teachers and mathematics education researchers use task-based interviews with children to explore children's mathematical ways of reasoning or thinking. The interview situation consists of a mathematical task, an interviewer (teacher or researcher), and a student who shares his thinking while solving a task.

Read the attached article "Assessing for Learning" by Ed Lavinowicz for an introduction to the interview method. Note: The article mentions some pages of questions that are unavailable. We will consider another questioning framework.

1. Based on your reading, what do you think is the goal of conducting an interview with a child using a mathematical task? Why do you think this is the goal?
2. Based on the reading, what does a teacher need to *begin* (get started) interviewing children with mathematical tasks?
3. What similarities and/or differences do you see between interviewing a child and teaching a child a mathematical task?

Interview and Responsive Interaction Preparation 1
Task-Based Interview Introduction and Questioning

4. The article mentions four aspects (establishing an atmosphere of acceptance, productive questioning, waiting and listening, and coaxing and encouraging) of the interview method. Which area do you think would be the easiest for you to develop as a future teacher? Why?

5. Of the four aspects of interviewing mentioned in the article, which area do you think would be the most difficult to develop as a future teacher? Why?

Read the attached Pose Purposeful Questions with Question Type Framework.

1. What question type would you like to develop as a goal in learning to explore and support another person's mathematical thinking?

2. In what ways do you think you could practice questioning this type of question in an upcoming peer task interview?

You may read about clinical interviews and watch examples of interviews at <http://www.learnnc.org/lp/editions/pemath/1.0>.

Another set of videos with classroom instruction and then an interview related to place value can be watched at https://www.youtube.com/watch?v=_ofQ_WnQiZ4 and <https://www.youtube.com/watch?v=1puQxclB2aw>.

Pose Purposeful Questions

“Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships” (NCTM, 2014, p. 35)

Questions Types Framework

Question Type	Description	Examples
<i>Gathering Information</i>	Questions asked encouraging students to recall facts, definitions, and procedures.	What place value does “2” have in “28”? When you write an equation, what does the equal sign tell you?
<i>Probing Thinking</i>	Questions asked encouraging students to explain, elaborate, or clarify their thinking.	Can you do this task out loud? How did you figure that out? Can you show me how you did it? Can you explain your idea? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans Task?
<i>Making the Mathematics Visible</i>	Questions asked encouraging students to discuss mathematical structures and make connections.	How do the blocks that you counted relate to the number (number greater than ten) you wrote? How does that array relate to multiplication and division?
<i>Encouraging Reflection and Justification</i>	Questions asked encouraging students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How do you know that you can do that? Can you always do that? How do you know that the sum of two odd numbers will always be even?

Adapted from:

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author. (pages 35-37)

Some probing questions taken from:

Ginsburg, H. P. (1997). *Entering the child’s mind*. New York: Cambridge University Press.

Interview and Responsive Interaction Preparation 2
Elicit and Use Student Thinking to Support and Extend Learning.

Future Teacher Number or Name _____

Elicit and Use Student Thinking to Support and Extend Learning

Mathematics education researchers studied teachers using interviews with story problems to develop a list of teacher's moves that support and extend a child's mathematical thinking. This interview and responsive interaction focuses on teacher moves in interviews that support and extend learning.

Read the attached article "Making the Most of Story Problems" by Victoria Jacobs and Rebecca Ambrose that provides a list of questions used in child task based interviews to elicit and use children's thinking that support and extend the children's mathematical thinking. An updated list of questions is also attached.

1. The article provides a rich toolbox of resources worthy of some time to process all the information. Take some time to look over the article. What is your overall impression or initial reactions to the article? Summarize your reaction in one to three sentences.

2. What teacher moves would you like to concentrate on developing for teacher's moves before a correct answer is given?

3. What teacher moves would you like to concentrate on developing for teacher's moves after a correct answer is given?

Interview and Responsive Interaction Preparation 2
Elicit and Use Student Thinking to Support and Extend Learning.

4. Review the question type that you chose to develop in the pose purposeful questions framework (#1 in Pose Purposeful Question – Problem Set 1 Assignment due January 30th). What was that question type? In what ways is that question type similar or different from the teacher moves that you chose in the last two questions?

5. Think back on the peer interview or responding to classmate's work done in classroom activities. Have you been able to practice the question type that you selected previously or other teacher moves? Why or why not?

6. In order to prepare for the next peer interview, what category or teacher moves or questions would you like to work on? It may be helpful to consider both areas of before and after a correct answer is given since each interview situation is unique. After answering this question, prepare a personalized guide of example questions that you would like to work on to take have with you during the next peer interview.

Interview and Responsive Interaction Preparation 3
Supporting Productive Struggle

Future Teacher Name _____

Consider the Rain Barrel problem:

Suppose we have a 48 gallon rain barrel containing 24 gallons of water and a 5 gallon water jug containing 3 gallons of water. Which container is said to be fuller? If we drain a gallon of water from each container, does this change your answer about which container is fuller?

Source: McCabe, Warshauer, and Warshauer (2009)

1. Compare the 48 gallon rain barrel containing 24 gallons of water with the 5 gallon water jug containing 3 gallons of water. In detailed sentences, tell which container is fuller and why.
2. Now imagine draining a gallon of water from each container. In detailed sentences, tell which container is fuller and why. Is your answer different than in number one? Explain in full sentences why your answers are the same or why your answers are different.

Read the attached article “Strategies for Productive Struggle” by Hiroko K. Warshauer. This article shares two classroom episodes of the Rain Barrel problem that you completed above. Think about your perspective of mathematics and productive struggle.

3. After reading the article, take some time to reflect on the idea of “productive struggle” in mathematics. What is your overall impression or reactions to the article? Summarize your reaction in two to four full sentences.

4. Consider Ms. George's approach to Drew. Explain in what ways did Ms. George's response catch your attention (or what did you notice) and why?

Interview and Responsive Interaction Preparation 3
Supporting Productive Struggle

5. Think back over one of the classroom interviews or activities where you or a classmate struggled with the mathematics. (If you cannot think of any struggles in your current course, think back to another mathematical struggle you have observed.) In the episode that you chose, was the struggle productive in deepening your or your classmate's mathematical understanding?
 - a. Tell in full sentences about the struggle with the mathematics.
 - b. Explain whether the goal during the struggle was focused on the answer and/or the mathematical understanding.
 - c. Explain in full sentences, why the struggle was productive in deepening mathematical understanding or why the struggle was not productive in deepening mathematical understanding.

Interview and Responsive Interaction Preparation 3
Supporting Productive Struggle

6. Consider productive struggle as you prepare for another peer interview in class and teaching mathematics in the future. If there are student struggles, explain how you can specifically prepare to use the struggle as an opportunity to deepen student understanding.

APPENDIX B: Pattern Tiles Mathematical Task Preparation Assignments

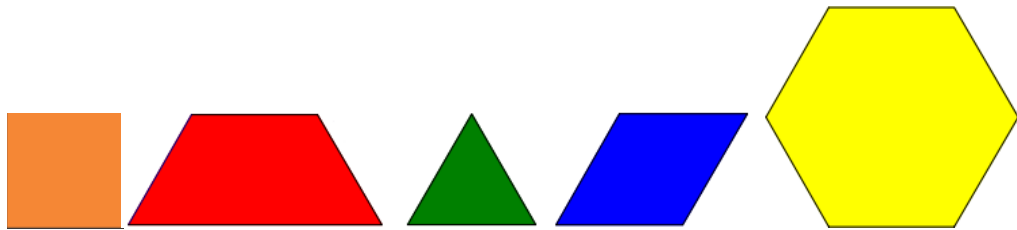
Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: _____

You may handwrite, type, or do both for this assignment. For handwritten work, please use pen. We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday.

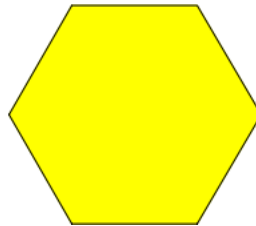
This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

This assignment uses pattern tiles (blocks). Here are visual representations of the pattern tiles.



1. Work through the following mathematical task. Use more paper if needed.

The hexagon pattern tile is $\frac{2}{3}$ of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design.



Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: _____

2. Describe in detail with full sentences the way that you went about solving the task. You may include drawings.
3. Describe in full sentences how your answer in part 1 relates to the original problem.
4. Think of another way that a student may work on solving this problem and draw this other way of working on the task.

Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: _____

5. Explain in sentence form what is a key concept about fractions that is necessary in order to work on this task productively.

6. List any potential difficulties a student may have in completing this task.

7. What is a question that you could ask if a student has difficulty beginning this task?

8. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{1}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.

Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: _____

9. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{3}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.

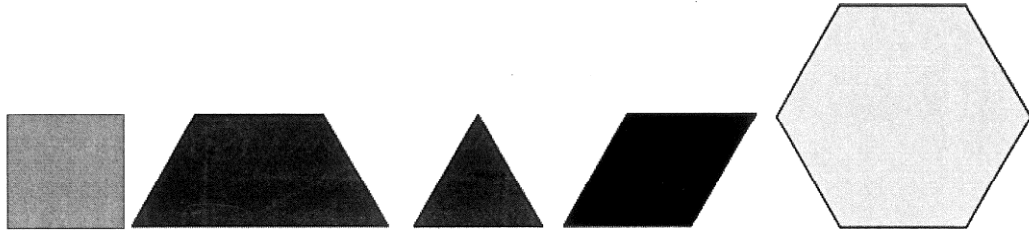
10. Explain in complete sentences how each of the task adaptations in part 8 and 9 change the difficulty of the task in part 1.

Peer Interviewer Name: _ Paige

You may handwrite, type, or do both for this assignment. For handwritten work, please use pen. We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday.

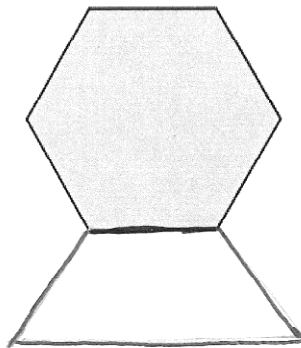
This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

This assignment uses pattern tiles (blocks). Here are visual representations of the pattern tiles.



1. Work through the following mathematical task. Use more paper if needed.

The hexagon pattern tile is $\frac{2}{3}$ of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design.



Peer Interviewer Name: 22 Paige

2. Describe in detail with full sentences the way that you went about solving the task. You may include drawings.

I split the hexagon in half to identify that $\frac{1}{2}$ the hexagon = $\frac{1}{3}$ of the pattern tile design. By adding one trapezoid, I completed the pattern tile design.

3. Describe in full sentences how your answer in part 1 relates to the original problem.

The design that I drew met the requirements for part one by using the hexagon given and adding a trapezoid to the bottom. If you split the hexagon that I used into two trapezoids then there would be three total which makes up the whole pattern tile design that is being described.

4. Think of another way that a student may work on solving this problem and draw this other way of working on the task.

Students could possibly work to solve the surface area of the shape and then divide that by two. After dividing the surface area by two to get $\frac{1}{3}$, the students could then multiply that answer by 3 to get the total surface area and work to find a shape that would compliment the dimensions of the matching area.

Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: 22

Paige

5. Explain in sentence form what is a key concept about fractions that is necessary in order to work on this task productively.

It is important to know how to split things into fractions and know how many of the fractions can be put together to make a whole number.

6. List any potential difficulties a student may have in completing this task.

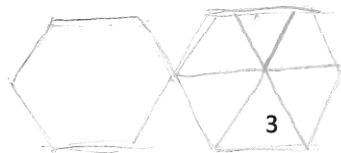
Students may not be able to work proficiently with fractions and if they are not aware of how to match $\frac{2}{3}$ to $\frac{1}{3}$ then they will struggle to find a pattern that matches the directions given.

7. What is a question that you could ask if a student has difficulty beginning this task?

If this hexagon takes up 2 parts out of the total 3 parts, how many more parts do we need to make a whole piece?

8. Consider and work through the following adaptation to the task.

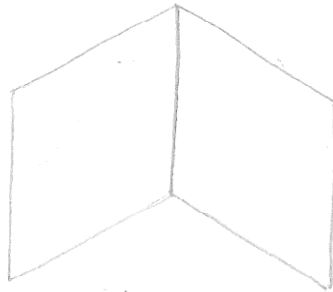
The hexagon pattern tile is $\frac{1}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.



Peer Interviewer Name: 22 Paige

9. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{3}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.



10. Explain in complete sentences how each of the task adaptations in part 8 and 9 change the difficulty of the task in part 1.

In part 8, I believe the difficulty is lessened by the fact that it is easier for students to identify half of something rather than $\frac{2}{3}$ of something. In part 9, the difficulty increases because students have to indicate that the hexagon pattern tile makes up more than the total pattern design, which might be a hard concept to visualize.

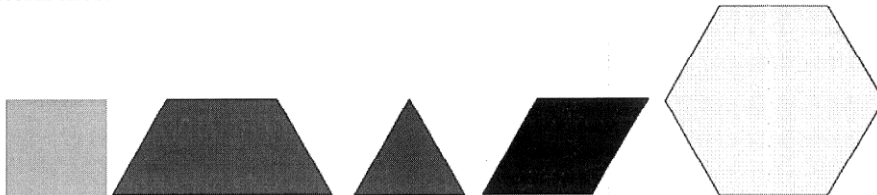
Peer Interviewer Name: **Reese**

4

You may handwrite, type, or do both for this assignment. For handwritten work, please use pen. We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday.

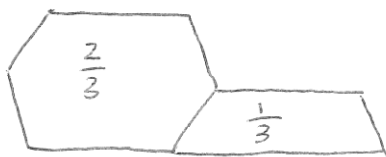
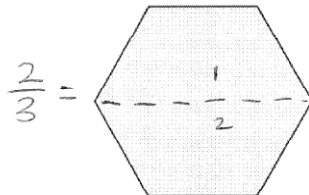
This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

This assignment uses pattern tiles (blocks). Here are visual representations of the pattern tiles.



1. Work through the following mathematical task. Use more paper if needed.

The hexagon pattern tile is $\frac{2}{3}$ of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design.



Peer Interviewer Name: Reese

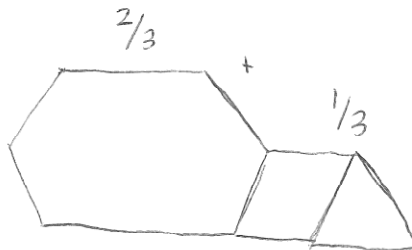
2. Describe in detail with full sentences the way that you went about solving the task. You may include drawings.

I started by dividing the hexagon into 2 since it represented $\frac{2}{3}$. I then proceeded to look for the shape that was the same size as each of the $\frac{1}{3}$ that I already had. Choosing the shape allowed me to find the last $\frac{1}{3}$ of the tile and then put the whole design together.

3. Describe in full sentences how your answer in part 1 relates to the original problem.

The full $\frac{2}{3} + \frac{1}{3}$ of the shapes equals the total area of the pattern tile design.

4. Think of another way that a student may work on solving this problem and draw this other way of working on the task.



Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: Reese

5. Explain in sentence form what is a key concept about fractions that is necessary in order to work on this task productively.

A key concept in a fraction is that the denominator is split into equal parts represented by the numerator.

6. List any potential difficulties a student may have in completing this task.

A student might not understand what the fraction represents.

7. What is a question that you could ask if a student has difficulty beginning this task?

Do you understand what the numbers on a fraction means?

8. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{1}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.



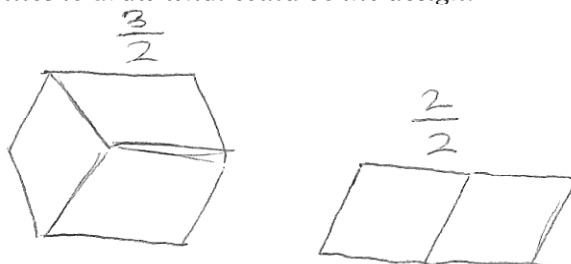
or



Peer Interviewer Name: Reese

9. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{3}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.



10. Explain in complete sentences how each of the task adaptations in part 8 and 9 change the difficulty of the task in part 1.

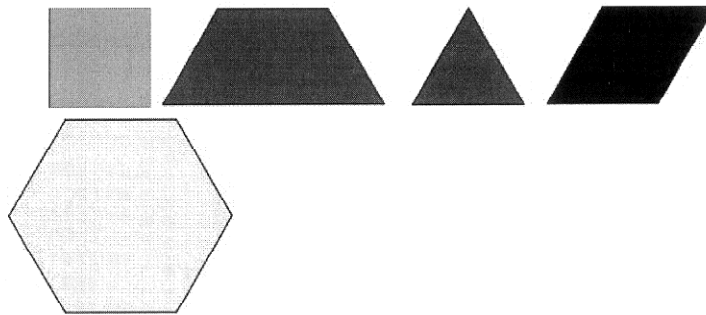
Each task adaptation changes in difficulty due to the fractions that the hexagon tile represents change to more difficult fractions, such as improper fractions.

Peer Interviewer Name: Beth

You may handwrite, type, or do both for this assignment. For handwritten work, please use pen. We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday.

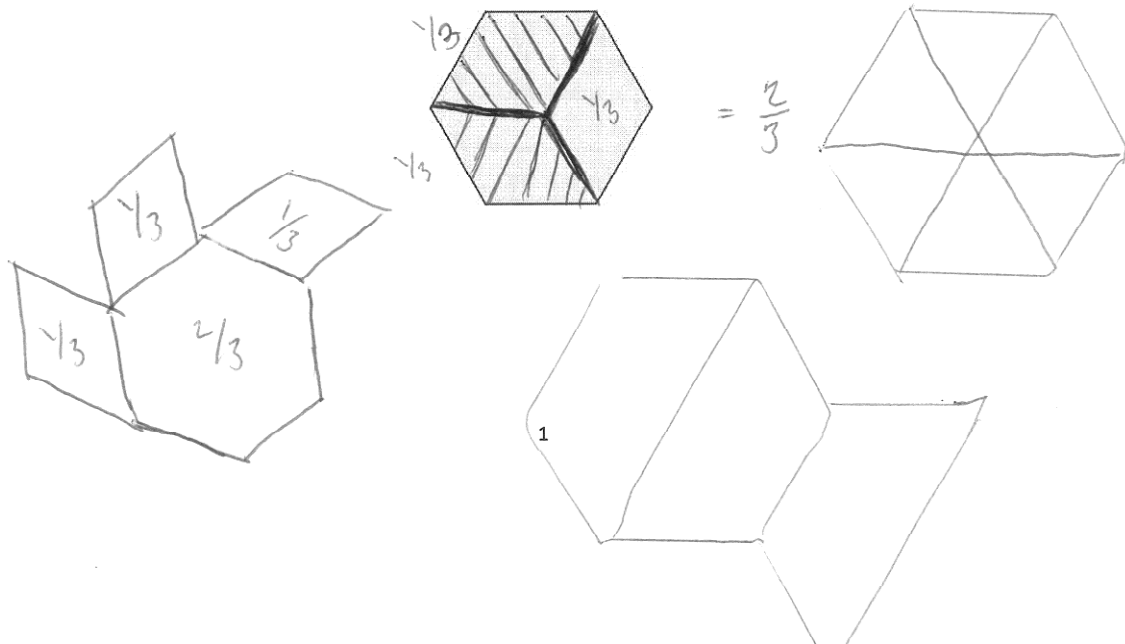
This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

This assignment uses pattern tiles (blocks). Here are visual representations of the pattern tiles.



1. Work through the following mathematical task. Use more paper if needed.

The hexagon pattern tile is $\frac{2}{3}$ of the area of a pattern tile design. Use the pattern tile shapes above to draw what could be the design.



Peer Interviewer Name: Beth

2. Describe in detail with full sentences the way that you went about solving the task. You may include drawings.

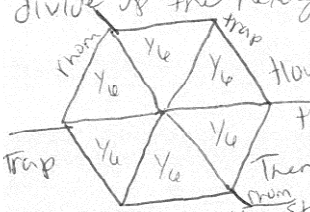
In my head first I fit all of the above shapes into the hexagon first. Then I, with the process of elimination, got rid of the one that would be bigger than $\frac{2}{3}$ or that would not fit more than 2. (square & trapezoid). Then I realized the triangle and rhombus would fit perfectly. later concluding that the triangle was too many ($\frac{1}{6}$) and that the rhombus would be just right for the $\frac{2}{3}$ in the hexagon.

3. Describe in full sentences how your answer in part 1 relates to the original problem.

It relates to the original problem because I was able to find and solve what the question was asking for. However in the student situation they may have not been able to solve it in their head - so with that I could have shown an example of how I went about the problem with each step.

4. Think of another way that a student may work on solving this problem and draw this other way of working on the task.

One way a student could see the problem would be to first divide up the hexagon into the nameable shapes first (triangles). However they would have noticed that it wasn't the $\frac{2}{3}$ they were looking for. (Unless they converted the fraction ex.) Then with a little guidance, or persistence from the student or teacher - the student would then try and solve the problem again. However if they noticed they made 2 other shapes (trapezoid and rhombus) they could do and work with the process of elimination by eliminating the square out of the solution with that the trapezoid would be only $\frac{1}{2}$ and would be too much for the $\frac{2}{3}$ so that leaves us with the rhombus again. (Finding and finding the solution of $\frac{2}{3}$ shapes and organizing it this way



Peer Interviewer Name: 8

5. Explain in sentence form **Beth** key concept about fractions that is necessary in order to task productively.

The key concept is to know value and placement. Without these two things these kids won't be able to visualize the problem in the 1st place.

6. List any potential difficulties a student may have in completing this task.

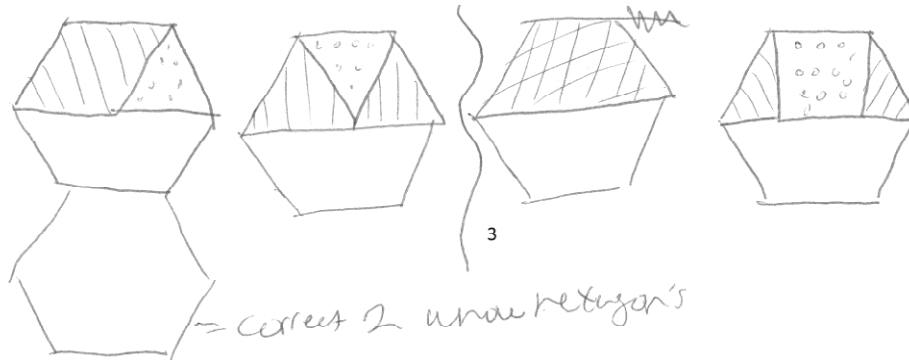
- They could right away freak out and block a way(s) to solve the problem.
- They could panic if there wasn't a visualization up to know what the shapes.
- They could have trouble picturing in their mind how the shapes would fit.
- Since no practice shapes were provided.
- They could be having a hard time listening and focusing in on what you had asked them to do. The list goes on and on, but that doesn't mean it's the child's fault. They could have a learning disability.

7. What is a question that you could ask if a student has difficulty beginning this task?

I would say something along the lines of - "Well how could we change this if you are stuck," or "what do these two shapes create and make?" etc... It really depends on the child's learning ability on the task itself.

8. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{1}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.



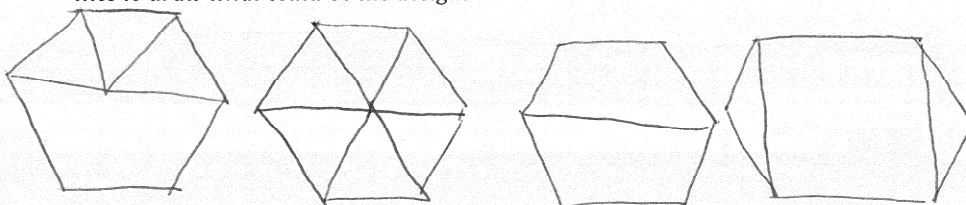
Preparing a Mathematical Task: Pattern Tiles

Peer Interviewer Name: Beth

Correct $\Rightarrow$

9. Consider and work through the following adaptation to the task.

The hexagon pattern tile is $\frac{3}{2}$ of the area of a pattern tile design. Use pattern tiles to draw what could be the design.



The design could be none of the above ... the measurements and fractions are off, therefore they are all wrong.

10. Explain in complete sentences how each of the task adaptations in part 8 and 9 change the difficulty of the task in part 1.

In part A the level of difficulty was on easy (for my own personal experience), however in part 9 the difficulty level was high. I was unable to solve anyway to create $\frac{3}{2}$ in a hexagon. However it all has to do with how familiar we are and how comfortable with fractions. Still to this day it takes me a little bit longer to solve equations with fractions because I'm a visual learner. Just like my kids, I myself too get stumped sometimes. In part A the kids are able to create and play with the images. In part B the kids will have a hard time grasping the concept of what it is asking in a visual formation.

APPENDIX C: Running Times Mathematical Task Preparation Assignments

Preparing a Mathematical Task: Running Times

Future Teacher Number or Name: _____

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, March 6th. Please use pen while completing.

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

1. Work through the following mathematical task. Use more paper if desired.

Ruth runs around a lake two times. The first time takes 1 hour, 43 minutes, and 38 seconds. The second time takes 1 hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes, and seconds.

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings.

Preparing a Mathematical Task: Running Times

3. Review the addition and subtraction problem types from your textbook (pp. 94-95). What problem type is in number one?
4. Now consider what sub-type (result unknown, change unknown, start unknown) is the task in number one?
5. Think of another way that a student may work on solving this problem and explain in full sentences this other way of working on the task.

Preparing a Mathematical Task: Running Times

6. List any potential difficulties a student may have in completing this task.
7. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

Future Teacher Number or Name: Paige (22)

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, March 6th. Please use pen while completing.

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

1. Work through the following mathematical task. Use more paper if desired.

Ruth runs around a lake two times. The first time takes 1 hour, 43 minutes, and 38 seconds. The second time takes 1 hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes, and seconds.

3 hours, 32 minutes, 7 seconds

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings.

I added the hours, minutes, and seconds separately. Adding the hours first was the easiest because it is simply $1+1=2$ hours. Adding the minutes together was slightly more difficult because I had to recognize that one-hour was 60 minutes and be able to make that conversion. $43 \text{ minutes} + 48 \text{ minutes} = 91 \text{ minutes}$ OR one hour and 31 minutes. After reaching that solution I added the hour to the two hours I already had equaling 3 hours and then the remaining 31 minutes. Lastly, I added the seconds and had to recognize that $60 \text{ seconds} = 1 \text{ minute}$ and be able to make that conversion. $38 \text{ seconds} + 29 \text{ seconds} = 67 \text{ seconds}$ OR one minute and 7 seconds. I added the one minute onto the 31 minutes equaling 32 minutes and then the remaining time was 7 seconds.

- 1 hour	43 minutes	38 seconds
- 1	48	29
-	91	67
2	- 60	- 60
+ 1	1	
3 hours	+ 31 minutes	7 seconds
	32 minutes	

3. Review the addition and subtraction problem types from your textbook (pp. 94-95). What problem type is in number one?

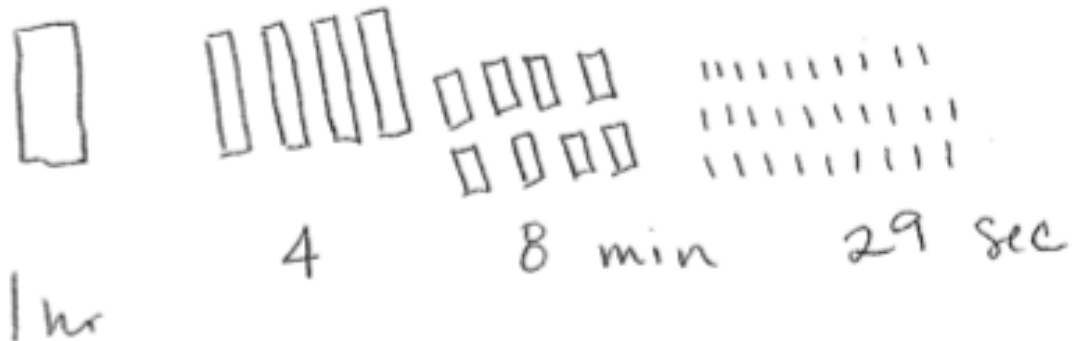
Put together

4. Now consider what sub-type (result unknown, change unknown, start unknown) is the task in number one?

Total unknown

5. Think of another way that a student may work on solving this problem and explain in full sentences this other way of working on the task.

Another way a student could solve this problem would be to use visuals to represent each amount of time.



6. List any potential difficulties a student may have in completing this task.

Students will have difficulty with this task if they are unable to make time conversions: seconds to minutes and minutes to hours. The students must be aware of how many seconds make up a minute and how many minutes make up an hour. If they are unable to do this then the problem will be difficult for them to complete.

7. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

You could make the task easier but avoiding time conversions. If the numbers were 1 hour, 30 minutes and 10 seconds for the first lap and 2 hours and 10 minutes and 12 seconds for the second lap the student wouldn't have to worry about converting minutes to hours or seconds to minutes.

Future Teacher Number or Name: Reese

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, March 6th. Please use pen while completing.

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

1. Work through the following mathematical task. Use more paper if desired.

Ruth runs around a lake two times. The first time takes 1 hour, 43 minutes, and 38 seconds. The second time takes 1 hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes, and seconds.

$$\begin{array}{r}
 1 \text{ hour} + 43 \text{ minutes} + 38 \text{ seconds} \\
 + 1 \text{ hour} + 48 \text{ minutes} + 29 \text{ seconds} \\
 \hline
 2 \text{ hours} \quad 91 \text{ minutes} \quad 67 \text{ seconds} \\
 + (1) \text{ hour} \quad + (1) \text{ minute} \quad \swarrow \\
 \quad \quad \quad 92 \text{ minutes} \quad \downarrow \\
 \hline
 3 \text{ hours} \quad 32 \text{ minutes} \quad 7 \text{ seconds}
 \end{array}$$

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings.

By sitting the problem up in a basic addition algorithm I added starting from the seconds and carried over when the number needed to be regrouped into the next place value. (when it reached our 60) after regrouping and adding all of the variables up it gave me the total time of the 2 laps.

1

b/c 60 sec = 1 min
60 min = 1 hour

3. Review the addition and subtraction problem types from your textbook (pp. 94-95). What problem type is in number one?

Put together / take apart problem

4. Now consider what sub-type (result unknown, change unknown, start unknown) is the task in number one?

Total unknown

5. Think of another way that a student may work on solving this problem and explain in full sentences this other way of working on the task.

The student may look at the problem differently and set up the problem like a Take from problem with the start unknown. calculating it by the total lap - 1st lap = 2nd lap or use a different method all together, such as making a base-ten math drawing. (but not use 10) as the base (using 20)

6. List any potential difficulties a student may have in completing this task.

A difficulty that a student might face would be grouping/regrouping in 60 for the time instead of 10s like normally. Realizing that seconds and minutes only go up to 60 before it reaches the next place value could be difficult for some students.

7. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

By keeping the same wording but changing the numbers to a smaller number that won't add up over 60 to effect the next place value is a way to make this task a little bit easier for the student.

Future Teacher Number or Name: Beth

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, March 6th. Please use pen while completing.

1 min = 60 sec

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

1 hr = 60 min

$$\begin{array}{r} \text{hr} \quad \text{min} \quad \text{sec} \\ 1 = 1.45.38 \\ 2 = 1.48.29 \\ \hline \text{tot. } 2.91.67 \\ \text{+1} \quad \text{+1} \end{array}$$

1. Work through the following mathematical task. Use more paper if desired.

Ruth runs around a lake two times. The first time takes 1 hour, 43 minutes, and 38 seconds. The second time takes 1 hour, 48 minutes, and 29 seconds. What is Ruth's total time for the two laps? Give the answer in hours, minutes, and seconds.

$$\begin{array}{r} \text{hr} \quad \text{min} \quad \text{sec} \\ 1 \quad 43 \quad 38 \\ +1 \quad 48 \quad 29 \\ \hline 2 \quad 91 \quad 67 \\ -60 \quad -60 \\ \hline 3 \quad 32 \quad 7 \end{array}$$

3 hours, 32 minutes, 7 seconds

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings.

I first added up all the seconds, ~~the~~ minutes and hours up individually in a group or at 100 as the total. Knowing that the units of time is out of 60, I subtracted the measurements of time to be out of 60 now. So with that my 67 seconds soon became 7 seconds - now with an extra minute to the 91 minutes now making it 92 minutes. I carried my addition over to the minutes calculation. Since time is consistent with all 3 calculations of hours, minutes and seconds. My minutes now with 92 minutes will soon become 32 minutes. Because 92 - 60 = 32. With that 1000 carried over the 60 minutes to the hour place making 3 hours soon become 3. The work I did/shown is above in the first question.

$$\begin{array}{l} \text{min} = 60 \text{ sec} \\ \text{hr} = 60 \text{ min} \\ 8 \end{array}$$

$$\begin{array}{r} 1 \quad 43 \quad 38 \\ 1 \quad 48 \quad 29 \\ \hline 2 \quad 91 \quad 67 \\ -60 \quad -60 \\ \hline 3 \quad 32 \quad 7 \end{array}$$

- either
- put together & both addends unknown
 - add to & result unknown
 - take from & result unknown

Beth

Preparing a Mathematical Task: Running Times

3. Review the addition and subtraction problem types from your textbook (pp. 94-95). What problem type is in number one?

~~add to problems~~

→ add to problems

4. Now consider what sub-type (result unknown, change unknown, start unknown) is the task in number one?

~~result unknown~~

→ result unknown

5. Think of another way that a student may work on solving this problem and explain in full sentences this other way of working on the task.

→ ~~add to~~ take from

→ result unknown

With this problem all you would have to do is change the wording of the question and reuse the answer choices you have. You would say it took Beth _____ long to run around the park 2 times, how long did she run around the park both times?

6. List any potential difficulties a student may have in completing this Beth

- changing the time conversion to 60 seconds.
- understanding time
- adding the total $\#$ $\rightarrow$ then subtracting the placement value of 60 from that $\#$.
- they may not know 60 seconds is 1 minute & that 60 minutes is 1 hour etc.

Conversion from addition to a time value

- Carrying over additional values over

7. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

- yes you could keep the wording the same ✓
- yes changing the value of the $\#$ to 60 59 & under that way the kids won't get confused with the addition of the times values. That way they won't be subtracting the placement value over.
 ↑ ↑
 confused value

ex:

1 hour	20 minutes	5 seconds
+ 2 hours	30 minutes	6 seconds
= 3 hours 50 minutes 11 seconds		

APPENDIX D: Sharing Cards Mathematical Task Preparation Assignments

Preparing a Mathematical Task: Sharing Cards

Name: _____

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, April 24th. Please use pen while completing.

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

1. Work through the following mathematical task. Use more paper if desired.

The ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?

Preparing a Mathematical Task: Sharing Cards

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings. Consider what ratios you used and if you used any proportions. Consider if you used multiplication, division, and/or simple logical reasoning in solving the task.

Preparing a Mathematical Task: Sharing Cards

3. Review the techniques of strip diagrams, ratio tables, and double number lines for solving proportion problems in textbook section 7.2 (pages 284-288). Think of another way from what you did in number one, using one of the techniques from section 7.2. Work the task this other way.
4. Describe in detail with full sentences the reasoning that you used to solve the task in number three. You may include drawings. Consider what ratios you used and if you used any proportions. Consider how you used multiplication and/or division in number three.

5. What difficulties could a student have in solving this mathematical task.

6. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

22
Name:

Paige

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, April 24th. Please use pen while completing.

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

I. Work through the following mathematical task. Use more paper if desired.

The ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?

answer



Samuel and Robert both have 60 cards.

$$\begin{array}{rcl} 5(x) - 15 & = & 3(x) + 15 \\ + 15 & & + 15 \end{array}$$

$$\begin{array}{rcl} 5(x) & = & 3(x) + 30 \\ - 3(x) & - & 3(x) \end{array}$$

$$\begin{array}{rcl} 2x & = & 30 \\ \hline 2 & & 2 \end{array}$$

$$x = 15$$

$$5(15) - 15 = 3(15) + 15$$

$$75 - 15 = 45 + 15$$

$$60 = 60 \text{ (same number of cards)}$$

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings. Consider what ratios you used and if you used any proportions. Consider if you used multiplication, division, and/or simple logical reasoning in solving the task.

I created an equation using the proportions and solved for 'x' in the equation.

$$5x-15=3x+15$$

I used this particular equation to demonstrate the ratio of Samuel to Robert's cards, which is 5 to 3. Because Samuel gives Robert 15 of his cards I needed to demonstrate 15 being subtracted from Samuel's side and added to Robert's. The 5 and the 3 are present in the equation to ensure that the ratio remains constant and true and the 15 is present to demonstrate the transferring of the cards.

- ## Ratio table

		Ratio table				
(samuel)	x	5	15 15	15 30	45	75
(Robert)	y	3	9	18	27	45
		-15	0	15	30	60
		+15	24	33	42	60
difference:			24	18	12	0

- answer

* I adjusted the ratio table to be shorter to save space and time

5. What difficulties could a student have in solving this mathematical task?

A student might not know the resources they can use to solve this. For example: an equation or a ratio table. If the student is not familiar with the multiple ways that can be used to find the solution then they will struggle to determine the answer.

6. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

Creating a problem that requires a lower ratio would make it easier to solve and more adaptable for younger students. Instead of having to go all the way to 75:45 to solve the problem lower the numbers. You wouldn't be able to keep the same wording for the problem because 75:45 is the only ratio that works for this specific problem so the problem structure and wording would need to be changed as well.

4
Name: Reese

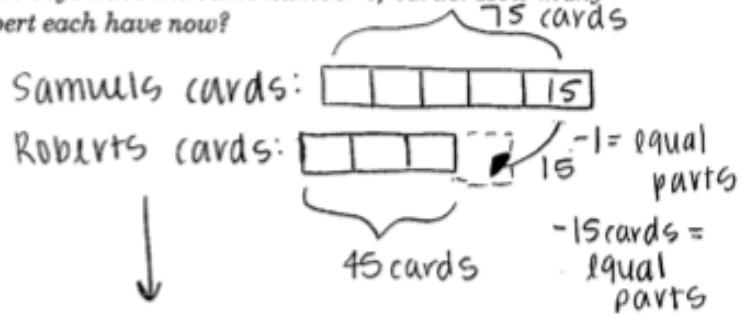
We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, April 24th. Please use pen while completing.

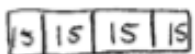
This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

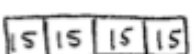
1. Work through the following mathematical task. Use more paper if desired.

The ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?

5:3



Samuel's cards:  = 60 cards

Robert's cards:  = 60 cards

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings. Consider what ratios you used and if you used any proportions. Consider if you used multiplication, division, and/or simple logical reasoning in solving the task.

Using the ratio 5 to 3, I set up a strip diagram to visually see the 5 parts of Samuel's cards and the 3 parts of Robert's cards. If giving Robert 15 cards make their collections equal and giving 1 part of Samuel's 5 parts to Robert makes their parts equal then you can see that each part represents 15 cards. Now that the 1 part was given to Robert they both have 4 equal parts and since we know each part is worth 15 cards I multiplied 4×15 showing that Robert and Samuel both have 60 cards each now.

Reese

3. Review the techniques of strip diagrams, ratio tables, and double number lines for solving proportion problems in textbook section 7.2 (pages 284-288). Think of another way from what you did in number one, using one of the techniques from section 7.2. Work the task this other way.

ratio table →

$$5:3 \rightarrow 4:4$$

$$5-1=4$$

$$3+1=4$$

1 part = 15 cards

x	5	15	30	45	75
y	3	9	18	27	45

They both
have 60
cards

after moving 15 →

0	15	30	45
20	35	40	60

difference 14 18 22

4. Describe in detail with full sentences the reasoning that you used to solve the task in number three. You may include drawings. Consider what ratios you used and if you used any proportions. Consider how you used multiplication and/or division in number three.

I wrote a ratio table using the original ratio of 5 to 3 and the total number of parts which is $5+3=8$. With this ratio table I found out that each part is 15 cards and to make the ratio represent equal amounts Samuel has to have 4 parts and Robert has 4 parts making the ratio 4:4. Now that the ratio is 4:4 you can multiply by 15 showing that each person has 60 cards with a total of 120 cards out of the 8 parts total as well.

- also showing through the x and y ratio table and count up by 5 in the x and add 3 in the y category to keep equivalent ratios then show the same table w/ the 15 removed which visually shows the difference.

Samuel's cards	5	?	$5 \cdot 15$	75
Robert's cards	3	?	$3 \cdot 15$	45
total # cards	8	$\cdot 15$	120	$8 \cdot 15$ 120



Samuel's cards	4	→ ?	$4 \cdot 15$	60
Robert's cards	4	→ ?	$4 \cdot 15$	60
total cards	8	$\cdot 15$	120	$8 \cdot 15$ 120

0	15	30	45
20	35	40	60

difference 14 18 22

5. What difficulties could a student have in solving this mathematical task.

A student could have difficulties in how losing 15 cards affects the 5:3 ratio as well as knowing that a ratio that is equal (4:4) means the parts represent the same number or are equal.

6. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

I think this task would be made easier by using easier numbers, such as even numbers to use for the ratio. Yes you could keep the same wording just use simpler numbers in the problem.

8

Name: Beth

We will be working with this in class for the peer task interviews, so please bring the assignment to class on Monday, April 24th. Please use pen while completing.

This assignment is to prepare the mathematical task below in part 1 to give a classmate in a peer interview situation.

1. Work through the following mathematical task. Use more paper if desired.

The ratio of Samuel's cards to Robert's cards is 5 to 3. After Samuel gives Robert 15 of his cards, both boys have the same number of cards. How many cards do Samuel and Robert each have now?

Samuel
Robert

50	30
45	25
40	20
35	15
30	10
25	5
20	0
15	
10	
5	
0	

at least
need table in order
to subtract & gain

needs even number (even)
to be divided by 5

now only 3 possible choices

50-15=35 X not going to work because other
if it way to much.

75-15=60 ✓
45+15=60 ✓

100-15=85 X other number will be too much

- The answer is 60 cards.
- kept the same ratio 5-3
- Samuel had 75 cards while Robert had 45
- now = both have 60 cards

$$\begin{aligned}\text{Samuel} &= 75 - 15 = 60 \\ \text{Robert} &= 45 + 15 = 60\end{aligned}$$

8

2. Describe in detail with full sentences the reasoning that you used to solve the task. You may include drawings. Consider what ratios you used and if you used any proportions. Consider if you used multiplication, division, and/or simple logical reasoning in solving the task.

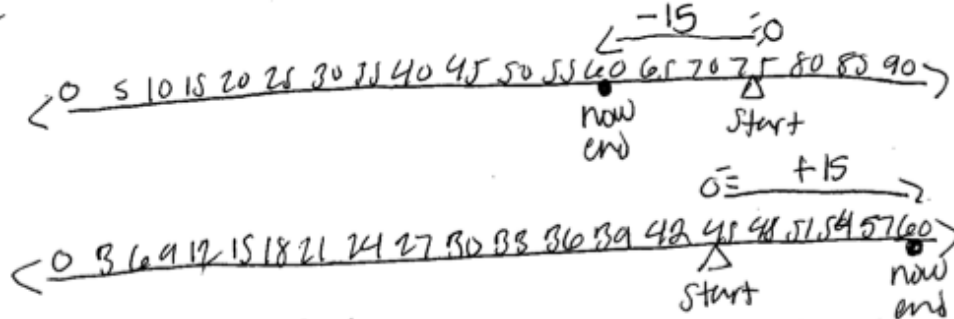
I used the table from the previous page to solve my problem.

I kept the same ratio 5:3 therefore I went up by 5 and 3 each time. The numbers to ratio were constant. I used simple logic and order of operations to solve this problem. I knew some problems wouldn't work so I kept going up from there. I knew that Samuel's number of cards needed to be doubled (30) in order to produce/give away 15 to Robert. Then I realized that the number I was looking for had to be divisible by 5 in the 3 column. So that eliminated a lot of answers. I was then left with three choices so I did the math and found out that Samuel had 75 cards and Robert had 45 both with the 5-3 ratio still. Therefore the answer to both Samuel and Robert is 60.

8

3. Review the techniques of strip diagrams, ratio tables, and double number lines for solving proportion problems in textbook section 7.2 (pages 284-288). Think of another way from what you did in number one, using one of the techniques from section 7.2. Work the task this other way.

double #
line



It was not like the
car diagram

$$75 - 15 = 60$$

$$45 + 15 = 60$$

4. Describe in detail with full sentences the reasoning that you used to solve the task in number three. You may include drawings. Consider what ratios you used and if you used any proportions. Consider how you used multiplication and/or division in number three.

I thought of the double number line experiment (drawing) at the car model we did in class. If I'm adding $45 + 15 = 60$ then I am facing to the right. If I am subtracting $75 - 15 = 60$ then I am facing to the left. (drawing above)
 I used the same ratio 5 to 3 = 75 to 45.
 I did not use multiplication or division.

8'

5. What difficulties could a student have in solving this mathematical task.

- They could stop and not go as far down the number line - like I did my first try. I gave up too easily.
- They could not realise you need to subtract one and then add to the other.
- They may not understand ratios

6. What would make this task easier? Could you keep the same wording of the task, but change some numbers so that the level of difficulty is lowered?

I would change it to where you start higher in the ratio.
 So S to B would be equivalent to 50 and 80.
 That way the long process of finding the answer
 would be shortened. You could keep the same wording.

APPENDIX E: Reflection Forms

Pattern Tiles

Problem Solver Reflection

Problem Solver Number: _____

Peer Interviewer Number: _____

- *In what ways did the peer interviewer elicit and support my mathematical thinking? Address that following questions in your answer.*
 - *What questions did the peer interviewer ask that caused you to think more deeply about the mathematics?*
 - *What questions did the peer interviewer ask that encouraged me to explain my mathematical thinking?*

- *What understanding about fractions was needed to work on the task(s)?*

- *What do you think may have been helpful to elicit or support your mathematical thinking in the task that you worked on?*

Peer Interviewer Reflection

Peer Interviewer Number_____

- 196

Running Times **Problem Solver Reflection**

Running Times **Problem Solver Reflection**

Problem Solver Number: _____ **Peer Interviewer Number:** _____

Problem Solver Number: _____ **Peer Interviewer Number:** _____

- *Name at least one strength that the interviewer had in the interview.*

- *What did the peer interviewer ask or say that encouraged me to explain my mathematical thinking?*

- *What did the peer interviewer ask or say that helped me to think more deeply about the mathematics?*

- *What moves or questions may have been helpful to elicit or support your mathematical thinking in the task that you worked on?*

Running Times

Peer Interviewer Reflection

Problem Solver Number: _____

Peer Interviewer Number _____

- *In what ways did the problem solver's work on the task connect (or not connect) to the mathematical ideas of regrouping for addition or subtraction?*

- *Think about the ways that you responded to the problem solver's mathematical work and/or ideas. Did you use any of the question types or categories that you planned to practice in this interview? Why or why not?*

- *Name at least one strength that you had as an interviewer today.*

- *Name an aspect of interviewing that you would like to continue to develop or improve.*

Sharing Cards

Problem Solver Reflection

Problem Solver Number: _____

Peer Interviewer Number: _____

- *Name at least one strength that the interviewer had in the interview.*

- *What did the peer interviewer ask or say that encouraged me to explain my mathematical thinking?*

- *What did the peer interviewer ask or say that helped me to think more deeply about the mathematics (especially ratios and/or proportions)?*

- *What moves or questions may have been helpful to elicit or support your mathematical thinking in the task that you worked on?*

Sharing Cards

Peer Interviewer Reflection

Problem Solver Number: _____

Peer Interviewer Number _____

- *In what ways did the problem solver's mathematical thinking with the task connect (or not connect) to the mathematical ideas or techniques used with ratios and/or proportions? (Was algebra, strip diagrams, ratio tables, and/or double number lines used? Did the methods to solve the problem show an understanding of ratios and/or proportions?)*

- *In what ways, did you explore the problem's mathematical reasoning?*

- *Name at least one strength that you had as an interviewer today.*

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