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Resonance problems with respect to the Fučík spectrum of the p-Laplacian *

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Abstract

We solve resonance problems with respect to the Fučík spectrum of the p-Laplacian using variational methods.

1 Introduction

Consider the quasilinear elliptic boundary value problem

$$-\Delta_p u = a\,(u^+)^{p-1} - b\,(u^-)^{p-1} + f(x,u), \quad u \in W_0^{1,p}(\Omega) \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$, $n \geq 1$, $\Delta_p u = \operatorname{div}\big(|\nabla u|^{p-2}\,\nabla u\big)$ is the p-Laplacian, $1 < p < \infty$, $u^{\pm} = \max\{\pm u, 0\}$, and f is a Carathéodory function on $\Omega \times \mathbb{R}$ satisfying a growth condition

$$|f(x,t)| \leq V(x)^{p-q}\,|t|^{q-1} + W(x)^{p-1} \tag{1.2}$$

with $1 \leq q < p$ and V, $W \in L^p(\Omega)$. The set Σ_p of those points $(a,b) \in \mathbb{R}^2$ for which the asymptotic problem

$$-\Delta_p u = a\,(u^+)^{p-1} - b\,(u^-)^{p-1}, \quad u \in W_0^{1,p}(\Omega) \tag{1.3}$$

has a nontrivial solution is called the Fučík spectrum of the p-Laplacian on Ω. The nonresonance case for problem (1.1), $(a,b) \notin \Sigma_p$, was recently studied by Cuesta, de Figueiredo, and Gossez [4] and the author [23, 24]. The symmetric resonance case, $a = b \in \sigma(-\Delta_p)$, was considered by Drábek and Robinson [11]. The purpose of the present paper is to study the general resonance case $(a,b) \in \Sigma_p$.

The Fučík spectrum was introduced in the semilinear case, $p = 2$, by Dancer [6] and Fučík [12] who recognized its significance for the solvability of problems with jumping nonlinearities. In the semilinear ODE case $p = 2$, $n = 1$, Fučík [12] showed that Σ_2 consists of a sequence of hyperbolic like curves passing

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through the points (λ_l, λ_l), where $\{\lambda_l\}_{l\in\mathbb{N}}$ are the eigenvalues of $-\Delta$, with one or two curves going through each point. Drábek [10] has recently shown that Σ_p has this same general shape for all $p > 1$ in the ODE case.

In the PDE case, $n \geq 2$, much of the work to date on Σ_p has been for the semilinear case. It is now known that Σ_2 consists, at least locally, of curves emanating from the points (λ_l, λ_l) (see, e.g., [2, 5, 6, 9, 12, 13, 15, 16, 17, 20, 21]). Schechter [31] has shown that Σ_2 contains two continuous and strictly decreasing curves through (λ_l, λ_l), which may coincide, such that the points in the square $(\lambda_{l-1}, \lambda_{l+1})^2$ that are either below the lower curve or above the upper curve are not in Σ_2, while the points between them may or may not belong to Σ_2 when they do not coincide.

In the quasilinear PDE case, $p \neq 2$, $n \geq 2$, it is known that the first eigenvalue λ_1 of $-\Delta_p$ is positive, simple, and admits a positive eigenfunction φ_1 (see Lindqvist [19]). Hence Σ_p contains the two lines $\lambda_1 \times \mathbb{R}$ and $\mathbb{R} \times \lambda_1$. In addition, $\sigma(-\Delta_p)$ has an unbounded sequence of variational eigenvalues $\{\lambda_l\}$ satisfying a standard min-max characterization, and Σ_p contains the corresponding sequence of points $\{(\lambda_l, \lambda_l)\}$. A first nontrivial curve in Σ_p through (λ_2, λ_2) asymptotic to $\lambda_1 \times \mathbb{R}$ and $\mathbb{R} \times \lambda_1$ at infinity was recently constructed and variationally characterized by a mountain-pass procedure by Cuesta, de Figueiredo, and Gossez [4]. More recently, unbounded sequences of curves $\{C_l^{\pm}\}$ in Σ_p (analogous to the lower and upper curves of Schechter) have been constructed and variationally characterized by min-max procedures by Micheletti and Pistoia [22] for $p \geq 2$ and by the author [24] for all $p > 1$.

Let us also mention that some Morse theoretical aspects of the Fučík spectrum have been studied in Dancer [7], Dancer and Perera [8], Perera and Schechter [25, 26, 27, 28, 29, 30], and Li, Perera, and Su [18].

Denote by N the set of nontrivial solutions of (1.3), and set

$$F(x,t) := \int_0^t f(x,s)\, ds, \quad H(x,t) := p\, F(x,t) - t f(x,t). \tag{1.4}$$

The main result of this paper is:

Theorem 1.1. *The problem* (1.1) *has a solution if*

(i). $(a,b) \in C_l^+$ *and* $\int_\Omega H(x,u_j) \to +\infty$, *or*

(ii). $(a,b) \in C_l^-$ *and* $\int_\Omega H(x,u_j) \to -\infty$

for every sequence (u_j) *in* $W_0^{1,p}(\Omega)$ *such that* $\|u_j\| \to \infty$ *and* $u_j/\|u_j\|$ *converges to some element of* N.

As is usually the case in resonance problems, the main difficulty here is the lack of compactness of the associated variational functional. We will overcome this difficulty by constructing a sequence of approximating nonresonance problems, finding approximate solutions for them using linking and min-max type arguments, and passing to the limit. But first we give some corollaries. In what follows, (u_j) is as in the theorem, i.e., $\rho_j := \|u_j\| \to \infty$ and $v_j := u_j/\rho_j \to v \in N$.

First we give simple pointwise assumptions on H that imply the limits in the theorem.

Corollary 1.2. *Problem* (1.1) *has a solution in the following cases:*

(i). $(a,b) \in C_l^+$, $H(x,t) \to +\infty$ *a.e. as* $|t| \to \infty$, *and* $H(x,t) \geq -C(x)$,

(ii). $(a,b) \in C_l^-$, $H(x,t) \to -\infty$ *a.e. as* $|t| \to \infty$, *and* $H(x,t) \leq C(x)$

for some $C \in L^1(\Omega)$.

Note that this corollary makes no reference to N.

Proof. If (i) holds, then $H(x,u_j(x)) = H(x,\rho_j\, v_j(x)) \to +\infty$ for a.e. x such that $v(x) \neq 0$ and $H(x,u_j(x)) \geq -C(x)$, so

$$\int_\Omega H(x,u_j) \geq \int_{v\neq 0} H(x,u_j) - \int_{v=0} C(x) \to +\infty \tag{1.5}$$

by Fatou's lemma. Similarly, $\int_\Omega H(x,u_j) \to -\infty$ if (ii) holds. □

Note that the above argument goes through as long as the limits in (i) and (ii) hold on subsets of $\{x \in \Omega : v(x) \neq 0\}$ with positive measure. Now, taking $w = v^+$ in

$$\int_\Omega |\nabla v|^{p-2}\, \nabla v \cdot \nabla w = \int_\Omega \Big[a\,(v^+)^{p-1} - b\,(v^-)^{p-1}\Big] w \tag{1.6}$$

gives

$$\begin{aligned} \|v^+\|^p &= \int_{\Omega_+} a\,(v^+)^p \leq a\,\|v^+\|_{p^*}^p\, \mu(\Omega_+)^{p/n} \\ &\leq a\,S^{-1}\,\|v^+\|^p\,\mu(\Omega_+)^{p/n} \end{aligned} \tag{1.7}$$

where $\Omega_+ = \{x \in \Omega : v(x) > 0\}$, $p^* = np/(n-p)$ is the critical Sobolev exponent, S is the best constant for the embedding $W_0^{1,\,p}(\Omega) \hookrightarrow L^{p^*}(\Omega)$, and μ is the Lebesgue measure in $\mathbb{R}^n$, so

$$\mu(\Omega_+) \geq \left(\frac{S}{a}\right)^{n/p}. \tag{1.8}$$

A similar argument shows that

$$\mu(\Omega_-) \geq \left(\frac{S}{b}\right)^{n/p} \tag{1.9}$$

where $\Omega_- = \{x \in \Omega : v(x) < 0\}$, and hence

$$\mu\Big(\,\{x \in \Omega : v(x) = 0\}\,\Big) \leq \mu(\Omega) - S^{n/p}\,\Big(a^{-n/p} + b^{-n/p}\Big). \tag{1.10}$$

Thus,

Corollary 1.3. *Problem* (1.1) *has a solution in the following cases:*

(i). $(a,b) \in C_l^+$, $H(x,t) \to +\infty$ *in* Ω' *as* $|t| \to \infty$, *and* $H(x,t) \geq -C(x)$,

(ii). $(a,b) \in C_l^-$, $H(x,t) \to -\infty$ *in* Ω' *as* $|t| \to \infty$, *and* $H(x,t) \leq C(x)$

for some $\Omega' \subset \Omega$ *with* $\mu(\Omega') > \mu(\Omega) - S^{n/p}\left(a^{-n/p} + b^{-n/p}\right)$ *and* $C \in L^1(\Omega)$.

Next note that

$$\begin{aligned} \underline{H}_+(x)\,(v^+(x))^q + \underline{H}_-(x)\,(v^-(x))^q &\leq \liminf \frac{H(x,u_j(x))}{\rho_j^q} \\ \leq \limsup \frac{H(x,u_j(x))}{\rho_j^q} &\leq \overline{H}_+(x)\,(v^+(x))^q + \overline{H}_-(x)\,(v^-(x))^q \end{aligned} \tag{1.11}$$

where

$$\underline{H}_\pm(x) = \liminf_{t\to\pm\infty} \frac{H(x,t)}{|t|^q}, \quad \overline{H}_\pm(x) = \limsup_{t\to\pm\infty} \frac{H(x,t)}{|t|^q}. \tag{1.12}$$

Moreover,

$$\frac{|H(x,u_j(x))|}{\rho_j^q} \leq (p+q)\,V(x)^{p-q}\,|v_j(x)|^q + \frac{(p+1)\,W(x)^{p-1}\,|v_j(x)|}{\rho_j^{q-1}} \tag{1.13}$$

by (1.2), so it follows that

$$\begin{aligned} \int_\Omega \underline{H}_+(v^+)^q + \underline{H}_-(v^-)^q &\leq \liminf \frac{\int_\Omega H(x,u_j)}{\rho_j^q} \\ \leq \limsup \frac{\int_\Omega H(x,u_j)}{\rho_j^q} &\leq \int_\Omega \overline{H}_+(v^+)^q + \overline{H}_-(v^-)^q. \end{aligned} \tag{1.14}$$

Thus we have

Corollary 1.4. *Problem* (1.1) *has a solution in the following cases:*

(i). $(a,b) \in C_l^+$ *and* $\int_\Omega \underline{H}_+(v^+)^q + \underline{H}_-(v^-)^q > 0 \quad \forall v \in N$,

(ii). $(a,b) \in C_l^-$ *and* $\int_\Omega \overline{H}_+(v^+)^q + \overline{H}_-(v^-)^q < 0 \quad \forall v \in N$.

Finally we note that if

$$\frac{tf(x,t)}{|t|^q} \to f_\pm(x) \quad \text{a.e. as } t \to \pm\infty, \tag{1.15}$$

then

$$\frac{F(x,t)}{|t|^q} = \frac{1}{|t|^q}\int_0^t \left[\frac{sf(x,s)}{|s|^q} - f_\pm(x)\right]|s|^{q-2}s\,ds + \frac{f_\pm(x)}{q} \to \frac{f_\pm(x)}{q} \tag{1.16}$$

and hence

$$\frac{H(x,t)}{|t|^q} \to \left(\frac{p}{q} - 1\right) f_\pm(x), \tag{1.17}$$

so Corollary 1.4 implies

Corollary 1.5. *Problem* (1.1) *has a solution in the following cases:*

(i). $(a,b) \in C_l^+$ *and* $\int_\Omega f_+(v^+)^q + f_-(v^-)^q > 0 \quad \forall v \in N$,

(ii). $(a,b) \in C_l^-$ *and* $\int_\Omega f_+(v^+)^q + f_-(v^-)^q < 0 \quad \forall v \in N$.

2 Preliminaries on the Fučík Spectrum

As in Cuesta, de Figueiredo, and Gossez [4], the points in Σ_p on the line parallel to the diagonal $a = b$ and passing through $(s, 0)$ are of the form $(s + c_+, c_+)$ (resp. $(c_-, s + c_-)$) with $c_\pm$ a critical value of

$$J_s^\pm(u) = \int_\Omega |\nabla u|^p - s\,(u^\pm)^p, \quad u \in S = \Big\{u \in W_0^{1,p}(\Omega) : \|u\|_p = 1\Big\} \tag{2.1}$$

and $J_s^\pm$ satisfies the Palais-Smale compactness condition. Since Σ_p is clearly symmetric with respect to the diagonal, we may assume that $s \geq 0$. In particular, the eigenvalues of $-\Delta_p$ on $W_0^{1,p}(\Omega)$ correspond to the critical values of the even functional $J = J_0^\pm$. As observed in Drábek and Robinson [11], we can define an unbounded sequence of critical values of J by

$$\lambda_l := \inf_{A \in \mathcal{F}_l} \max_{u \in A} J(u), \quad l \in \mathbb{N} \tag{2.2}$$

where

$$\mathcal{F}_l = \big\{A \subset S : \text{there is a continuous odd surjection } h : S^{l-1} \to A\big\} \tag{2.3}$$

and S^{l-1} is the unit sphere in $\mathbb{R}^l$, although it is not known whether this gives a complete list of eigenvalues.

Suppose that $l \geq 2$ is such that $\lambda_l > \lambda_{l-1}$ and let $0 < \varepsilon < \lambda_l - \lambda_{l-1}$ be given. By (2.2), there is an $A^{l-2} \in \mathcal{F}_{l-1}$ such that

$$\max_{u \in A^{l-2}} J(u) < \lambda_{l-1} + \varepsilon. \tag{2.4}$$

Let $h_{l-2} : S^{l-2} \to A^{l-2}$ be any continuous odd surjection and let

$$\begin{aligned}\mathcal{F}_l^+ = \big\{&A_+ \subset S : \text{there is a continuous surjection } h : S_+^{l-1} \to A_+ \\ &\text{such that } h|_{S^{l-2}} = h_{l-2}\big\}\end{aligned} \tag{2.5}$$

where S_+^{l-1} is the upper hemisphere of S^{l-1} with boundary S^{l-2}. Then $\mathcal{F}_l^+$ is a homotopy-stable family of compact subsets of S with closed boundary A^{l-2}, i.e,

(i). every set $A_+ \in \mathcal{F}_l^+$ contains A^{l-2},

(ii). for any set $A_+ \in \mathcal{F}_l^+$ and any $\eta \in C([0,1] \times S; S)$ satisfying $\eta(t,u) = u$ for all $(t,u) \in (\{0\} \times S) \cup ([0,1] \times A^{l-2})$ we have that $\eta(\{1\} \times A) \in \mathcal{F}_l^+$.

For $s \in I_l^\varepsilon := [0, \lambda_l - \lambda_{l-1} - \varepsilon]$, set

$$c_l^\pm(s) := \inf_{A_+ \in \mathcal{F}_l^+} \max_{u \in A_+} J_s^\pm(u). \tag{2.6}$$

If $c_l^\pm(s) < \lambda_l - s$, taking $A_+ \in \mathcal{F}_l^+$ with

$$\max_{u \in A_+} J_s^\pm(u) < \lambda_l - s \tag{2.7}$$

and setting $A = A_+ \cup (-A_+)$ we get a set in $\mathcal{F}_l$ for which

$$\max_{u \in A} J(u) = \max_{u \in A_+} J(u) \le \max_{u \in A_+} J_s^\pm(u) + s < \lambda_l, \tag{2.8}$$

a contradiction. Thus

$$c_l^\pm(s) \ge \lambda_l - s \ge \lambda_{l-1} + \varepsilon > \max_{u \in A^{l-2}} J(u) \ge \max_{u \in A^{l-2}} J_s^\pm(u), \tag{2.9}$$

and it follows from Theorem 3.2 of Ghoussoub [14] that $c_l^\pm(s)$ is a critical value of $J_s^\pm$. Hence

$$C_l^\pm := \left\{ (s + c_l^\pm(s), c_l^\pm(s)) : s \in I_l^\varepsilon \right\} \cup \left\{ (c_l^\pm(s), s + c_l^\pm(s)) : s \in I_l^\varepsilon \right\} \subset \Sigma_p. \tag{2.10}$$

Note that (2.9) implies $c_l^\pm(0) \ge \lambda_l \to \infty$.

3 Proof of Theorem 1.1

As is well-known, solutions of (1.2) are the critical points of

$$\Phi(u) = \int_\Omega |\nabla u|^p - a\,(u^+)^p - b\,(u^-)^p - p\,F(x,u), \quad u \in W_0^{1,p}(\Omega). \tag{3.1}$$

We only consider (i) as the proof for (ii) is similar. Let $(a,b) = (s + c_l^+(s), c_l^+(s))$, $s \ge 0$ and

$$\Phi_j(u) = \Phi(u) + \frac{1}{j} \int_\Omega |u|^p = \int_\Omega |\nabla u|^p - s\,(u^+)^p - \left(c_l^+(s) - \frac{1}{j} \right) |u|^p - p\,F(x,u). \tag{3.2}$$

First we show that, for sufficiently large j, there is a $u_j \in W_0^{1,p}(\Omega)$ such that

$$\|u_j\|\, \|\Phi_j'(u_j)\| \to 0, \quad \inf \Phi_j(u_j) > -\infty. \tag{3.3}$$

Let ε and A^{l-2} be as in Section 2. By (2.9),

$$\max_{u \in A^{l-2}} J_s^\pm(u) \le c_l^\pm(s) - \frac{2}{j} \tag{3.4}$$

for sufficiently large j. For such j, $u \in A^{l-2}$, and $R > 0$,

$$\begin{aligned}\Phi_j(Ru) &= R^p \left[J_s^+(u) - \left(c_l^+(s) - \frac{1}{j}\right)\right] - \int_\Omega p\, F(x, Ru) \\ &\le -\frac{R^p}{j} + p \left(\|V\|_p^{p-q} R^q + \|W\|_p^{p-1} R\right) \end{aligned} \tag{3.5}$$

by (1.2), so

$$\max_{u \in A^{l-2}} \Phi_j(Ru) \to -\infty \quad \text{as } R \to \infty. \tag{3.6}$$

Next let

$$F = \left\{u \in W_0^{1,p}(\Omega) : J_s^+(u) \ge c_l^+(s)\, \|u\|_p^p\right\}. \tag{3.7}$$

For $u \in F$,

$$\Phi_j(u) \ge \frac{\|u\|_p^p}{j} - p \left(\|V\|_p^{p-q} \|u\|_p^q + \|W\|_p^{p-1} \|u\|_p\right), \tag{3.8}$$

so

$$\inf_{u \in F} \Phi_j(u) \ge C := \min_{r \ge 0} \left[\frac{r^p}{j} - p \left(\|V\|_p^{p-q} r^q + \|W\|_p^{p-1} r\right)\right] > -\infty. \tag{3.9}$$

Now use (3.6) to fix $R > 0$ so large that

$$\max \Phi_j(B) < C \tag{3.10}$$

where $B = \left\{Ru : u \in A^{l-2}\right\}$.

Next consider the homotopy-stable family of compact subsets of X with boundary B given by

$$\begin{aligned}\mathcal{F} = \big\{A \subset X : &\text{ there is a continuous surjection } h : S_+^{l-1} \to A \\ &\text{such that } h|_{S^{l-2}} = R\, h_{l-2}\big\}\end{aligned} \tag{3.11}$$

where h_{l-2} is as in Section 2. We claim that the set F is dual to the class $\mathcal{F}$, i.e.,

$$F \cap B = \emptyset, \quad F \cap A \ne \emptyset \quad \forall A \in \mathcal{F}. \tag{3.12}$$

It is clear from (3.9) and (3.10) that $F \cap B = \emptyset$. Let $A \in \mathcal{F}$. If $0 \in A$, then we are done. Otherwise, denoting by π the radial projection onto S, $\pi(A) \in \mathcal{F}_l^+$ and hence

$$\max_{u \in \pi(A)} J_s^+(u) \ge c_l^+(s), \tag{3.13}$$

so $F \cap \pi(A) \ne \emptyset$. But this implies $F \cap A \ne \emptyset$.

Now it follows from a deformation argument of Cerami [3] that there is a u_j such that

$$\|u_j\|\, \|\Phi_j'(u_j)\| \to 0, \quad |\Phi_j(u_j) - c_j| \to 0 \tag{3.14}$$

where

$$c_j := \inf_{A \in \mathcal{F}} \max_{u \in A} \Phi_j(u) \ge C, \tag{3.15}$$

from which (3.3) follows.

We complete the proof by showing that a subsequence of (u_j) converges to a solution of (1.1). It is easy to see that this is the case if (u_j) is bounded, so suppose that $\rho_j := \|u_j\| \to \infty$. Setting $v_j := u_j/\rho_j$ and passing to a subsequence, we may assume that $v_j \to v$ weakly in $W_0^{1,p}(\Omega)$, strongly in $L^p(\Omega)$, and a.e. in Ω. Then

$$\begin{aligned}\int_\Omega |\nabla v_j|^{p-2}\,\nabla v_j\cdot\nabla(v_j-v) = \frac{\big(\Phi_j'(u_j), v_j-v\big)}{p\,\rho_j^{p-1}} + \int_\Omega \bigg[\Big(a-\frac{1}{j}\Big)(v_j^+)^{p-1} \\ - \Big(b-\frac{1}{j}\Big)(v_j^-)^{p-1} + \frac{f(x,u_j)}{\rho_j^{p-1}}\bigg](v_j-v) \to 0, \quad (3.16)\end{aligned}$$

and we deduce that $v_j \to v$ strongly in $W_0^{1,p}(\Omega)$ (see, e.g., Browder [1]). In particular, $\|v\| = 1$, so $v \neq 0$. Moreover, for each $w \in W_0^{1,p}(\Omega)$, passing to the limit in

$$\begin{aligned}\frac{\big(\Phi_j'(u_j), w\big)}{p\,\rho_j^{p-1}} = \int_\Omega |\nabla v_j|^{p-2}\,\nabla v_j\cdot\nabla w - \bigg[\Big(a-\frac{1}{j}\Big)(v_j^+)^{p-1} \\ - \Big(b-\frac{1}{j}\Big)(v_j^-)^{p-1} + \frac{f(x,u_j)}{\rho_j^{p-1}}\bigg]w \qquad (3.17)\end{aligned}$$

gives

$$\int_\Omega |\nabla v|^{p-2}\,\nabla v\cdot\nabla w - \Big[a\,(v^+)^{p-1} - b\,(v^-)^{p-1}\Big]w = 0, \qquad (3.18)$$

so $v \in N$. Thus,

$$\frac{\big(\Phi_j'(u_j), u_j\big)}{p} - \Phi_j(u_j) = \int_\Omega H(x,u_j) \to +\infty, \qquad (3.19)$$

contradicting (3.3).

References

[1] F. Browder. Nonlinear eigenvalue problems and group invariance. In *Functional Analysis and Related Fields (Proc. Conf. for M. Stone, Univ. of Chicago, Chicago, Ill., 1968)*, pages 1–58. Springer, New York, 1970.

[2] N. Các. On nontrivial solutions of a Dirichlet problem whose jumping nonlinearity crosses a multiple eigenvalue. *J. Differential Equations*, 80(2):379–404, 1989.

[3] G. Cerami. An existence criterion for the critical points on unbounded manifolds. *Istit. Lombardo Accad. Sci. Lett. Rend. A*, 112(2):332–336 (1979), 1978.

[4] M. Cuesta, D. de Figueiredo, and J.-P. Gossez. The beginning of the Fučík spectrum for the p-Laplacian. *J. Differential Equations*, 159(1):212–238, 1999.

[5] M. Cuesta and J.-P. Gossez. A variational approach to nonresonance with respect to the Fučik spectrum. *Nonlinear Anal.*, 19(5):487–500, 1992.

[6] E. N. Dancer. On the Dirichlet problem for weakly non-linear elliptic partial differential equations. *Proc. Roy. Soc. Edinburgh Sect. A*, 76(4):283–300, 1976/77. Corrigendum: *Proc. Roy. Soc. Edinburgh Sect. A*, 89(1-2):15, 1981.

[7] E. N. Dancer. Remarks on jumping nonlinearities. In *Topics in nonlinear analysis*, pages 101–116. Birkhäuser, Basel, 1999.

[8] E. N. Dancer and K. Perera. Some remarks on the Fučík spectrum of the p-Laplacian and critical groups. *J. Math. Anal. Appl.*, 254(1):164–177, 2001.

[9] D. de Figueiredo and J.-P. Gossez. On the first curve of the Fučik spectrum of an elliptic operator. *Differential Integral Equations*, 7(5-6):1285–1302, 1994.

[10] P. Drábek. *Solvability and bifurcations of nonlinear equations*. Longman Scientific & Technical, Harlow, 1992.

[11] P. Drábek and S. Robinson. Resonance problems for the p-Laplacian. *J. Funct. Anal.*, 169(1):189–200, 1999.

[12] S. Fučík. Boundary value problems with jumping nonlinearities. *Časopis Pěst. Mat.*, 101(1):69–87, 1976.

[13] T. Gallouët and O. Kavian. Résultats d'existence et de non-existence pour certains problèmes demi-linéaires à l'infini. *Ann. Fac. Sci. Toulouse Math. (5)*, 3(3-4):201–246 (1982), 1981.

[14] N. Ghoussoub. *Duality and perturbation methods in critical point theory*. Cambridge University Press, Cambridge, 1993.

[15] A. Lazer. Introduction to multiplicity theory for boundary value problems with asymmetric nonlinearities. In *Partial differential equations (Rio de Janeiro, 1986)*, pages 137–165. Springer, Berlin, 1988.

[16] A. Lazer and P. McKenna. Critical point theory and boundary value problems with nonlinearities crossing multiple eigenvalues. *Comm. Partial Differential Equations*, 10(2):107–150, 1985.

[17] A. Lazer and P. McKenna. Critical point theory and boundary value problems with nonlinearities crossing multiple eigenvalues. II. *Comm. Partial Differential Equations*, 11(15):1653–1676, 1986.

[18] S. J. Li, K. Perera, and J. B. Su. On the role played by the Fučík spectrum in the determination of critical groups in elliptic problems where the asymptotic limits may not exist. *Nonlinear Anal.*, 49(5):603–611, 2002.

[19] P. Lindqvist. On the equation $\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda |u|^{p-2}u = 0$. *Proc. Amer. Math. Soc.*, 109(1):157–164, 1990. Addendum: *Proc. Amer. Math. Soc.*, 116(2):583–584, 1992.

[20] C. Magalhães. Semilinear elliptic problem with crossing of multiple eigenvalues. *Comm. Partial Differential Equations*, 15(9):1265–1292, 1990.

[21] C. Margulies and W. Margulies. An example of the Fučik spectrum. *Nonlinear Anal.*, 29(12):1373–1378, 1997.

[22] A. Micheletti and A. Pistoia. On the Fučík spectrum for the p-Laplacian. *Differential Integral Equations*, 14(7):867–882, 2001.

[23] K. Perera. An existence result for a class of quasilinear elliptic boundary value problems with jumping nonlinearities. to appear in Topol. Methods Nonlinear Anal.

[24] K. Perera. On the Fučík spectrum of the p-Laplacian. preprint.

[25] K. Perera and M. Schechter. Computation of critical groups in Fučík resonance problems. preprint.

[26] K. Perera and M. Schechter. Type (II) regions between curves of the Fučík spectrum and critical groups. *Topol. Methods Nonlinear Anal.*, 12(2):227–243, 1998.

[27] K. Perera and M. Schechter. A generalization of the Amann-Zehnder theorem to nonresonance problems with jumping nonlinearities. *NoDEA Nonlinear Differential Equations Appl.*, 7(4):361–367, 2000.

[28] K. Perera and M. Schechter. Multiple nontrivial solutions of elliptic semilinear equations. *Topol. Methods Nonlinear Anal.*, 16(1):1–15, 2000.

[29] K. Perera and M. Schechter. Double resonance problems with respect to the Fučík spectrum. *Indiana Univ. Math. J.*, 50(4), 2001.

[30] K. Perera and M. Schechter. The Fučík spectrum and critical groups. *Proc. Amer. Math. Soc.*, 129(8):2301–2308, 2001.

[31] M. Schechter. The Fučík spectrum. *Indiana Univ. Math. J.*, 43(4):1139–1157, 1994.

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