# ARITHMETICAL CONCEPTS AND UNDERSTANDINGS FOR THE JUNIOR HIGH SCHOOL LEVEL EDUCABLE MENTAL RETARDATE

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# CHAPTER I

# INTRODUCTION

## A. The Problem

Statement of the Problem. The purpose of this study is to present an orderly sequence of arithmetical concepts and understandings for the educable mentally retarded child on the junior high school level of achievement. Practical applications for enrichment of concepts, suggestions for methods and materials, and devices found especially helpful will be included. This curriculum will offer sufficient instructional material for a period of two or three years, depending on the child's rate of progress. It will not present a day-to-day lesson plan, but will indicate material appropriate for the designated level.

Importance of the Study. If the mentally retarded child is to succeed in arithmetic, the teacher must be willing to meet him on the level where she finds him, not the level where he ought to be according to his chronological age and/or his mental age level. As in the normal child, readiness must be present before a skill or concept can be mastered.

No phase of number work can be taken for granted. Each step must be preceded by an understanding of the concepts and the vocabulary involved. Each step in turn must be carefully built and mastered before a new one is introduced.

Many children with mental retardation lack the insight needed to select the appropriate mathematical process when faced with a problem solving situation. They also frequently lack the ability for transfer of learning from classroom lessons to life experiences. The relationship between any basic process and its real-life applications, and between instruments of measure and the purpose of measurement, must be clearly demonstrated.

Many educators believe that the intelligence of the mental retardate is different from the "normal" only in amount, and that his arithmetical ability, therefore, differs only in his rate of learning the same materials offered to average children. Consequently, arithmetical materials are presented to the mental retardate in much the same manner, and with the same emphasis as they are presented to average or "normal" children. The only adjustment made for the mental retardate is that the material designed for eight-year-old "normal" children will be employed for mentally retarded children with a mental age of eight years. This philosophy refuses to recognize that the individual needs of children differ. It also assumes the abilities of children are identical, or at least very similar, and that the mental retardate has no special abilities or disabilities in learning to use arithmetical concepts and skills as compared to "normal" children of the same mental age. Samuel A. Kirk states:

> This philosophy assumes (1) that materials which are designed for use with the normal children will meet the needs or future needs of all individuals whether superior, normal, or retarded in intellectual development, (2) that skills and concepts have been placed in order of difficulty; and (3) that factors essential to an individual's social and economic adjustment to the community are the easiest to learn and consequently taught first.[1]

Lorene Quay reports that there are qualitative and quantitative differences between the learning characteristics of the trainable retarded and the normal child. On the other hand, Quay states that the educable retardate may learn some tasks in the same fashion as normal children but may differ in the way in which they learn others. This implies that although some would require modification, programs that are designed for normal children could be used without modification for the educable mentally retarded.[2] Stolurow supports this notion.[3]

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[1]Samuel A. Kirk and G. Orville Johnson, Educating the Retarded Child (Boston: The Riverside Press, 1951), p. 49.

[2]Lorene Quay, "Academic Skills" In N. R. Ellis (Editor), Handbook of Mental Deficiency. (New York: McGraw-Hill Book Company, 1963), p. 664-666.

[3]L. M Stolurow, "Teaching Machines and Special Education." Educational and Psychological Measurement, 1960. 20:429-448.

Arithmetic by its very nature is abstract; hence there is great need for using many concrete examples and experiences to establish meaning for the mentally retarded child. Care must be taken, however, that the objects selected are interesting enough to gain the child's attention, but not so interesting that he loses sight of the lessons the objects are to convey.

Concepts must be presented in a variety of meaningful ways to help the child grasp the abstract facts of arithmetic. Once a skill is mastered, frequent short periods of review to help retain the knowledge gained are necessary. It is well to employ as many of the senses as possible in the presentation of the material.

> Retarded children frequently have individual strengths that may facilitate learning. Vision, hearing, and tactile sensation may be remarkably acute in many retarded children. Stimulation in these areas may be used to advantage. Many retarded children have relatively good memories and can learn through repetition.[4]

Methods advocated by many teachers include the use of meaningful materials, conscious assistance in transfer of training, repetition of concepts in a wide range of situations, verbal mediation of concepts, spaced rather than massed practice, over learning, and immediate reinforcement. Although many of these learning principles apply to normal children, all are of

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[4]Kirk and Johnson, op. cit., p. 50.

particularly important meaning in the education of the retarded and should be considered in programming specifically for them.

This curriculum broadens and develops skills and concepts which have been previously introduced. The material is based on the information the child has already learned, and has been selected on the basis of its predictive value in the adult life of the retardate.

In so teaching, the teacher should keep in mind at all times that utilization should be made of whatever number situations arise in the life of the child to give him additional practice in understanding and manipulating numbers in the basic skills.

> His arithmetic needs include (1) the development, understanding and use of an arithmetical vocabulary, (2) the development of number concepts and skills, (3) the development of the ability to apply number concepts, (4) the development of an understanding of various units of measure, and (5) the development of an understanding of fractional parts.[5]

The minimum essential concepts and skills to be developed in arithmetic may be listed as:

1. Number and number symbols
2. Counting, enumeration, notation
    a. Rote
    b. Rational

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[5] *Ibid.*, p. 51.

c. Hindu-Arabic

d. Roman (I to XII for clocks and chapters)

3. Addition and Subtraction

4. Multiplication and Division

5. Problems and problem solving (written with words rather than numerals)

6. Weights and measures

7. Common fractions

a. halves

b. thirds

c. fourths

8. Decimals and percents (limited to decimals in functional vocabulary and handling money)[6]

Some educators believe that special teaching techniques should be utilized with brain-injured retarded children. Strauss and Lehtinen proposes three principles for the education of brain-injured children:

(a) Reduce the space in which the learning situation takes place

(b) Control distracting stimuli

(c) Present information to be learned so that it receives the child's full attention.[7]

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[6] Curriculum Guide, Special Education, (Austin: Texas Education Agency, 1960), p. 95.

[7] A. A. Strauss and Laura E. Lehtinen, Psychopathology and Education of the Brain-Injured Child, Volume I. (New York: Grune and Stratton, 1947).

## B. Definitions of Terms Used

Special Education. According to the State Plan for Special Education published by the Texas Education Agency, February, 1965, Special Education is the provision of services additional to, or different from those provided in the regular school program by a systematic modification and adaptation of equipment, teaching materials, and methods to meet the needs of exceptional children.[8]

Exceptional Children. Exceptional children are those so different in mental and/or physical characteristics that special provisions must be made for them. Special Education programs are provided by law under the Minimum Foundation Program for children and youth who are partially sighted, blind, orthopedically handicapped, deaf, mentally retarded (both educable and trainable), and those requiring speech and hearing therapy.[9]

Mental Retardation. Mental Retardation can be defined in terms of functional characteristics, of significant impairments in intellectual functioning, and in the social adaptation of the individual. A recent definition of mental retardation,

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[8] State Plan for Special Education, (Austin: Texas Education Agency, February, 1965), p. 1.

[9] *Ibid.*, p. 1.

that proposed by the American Association on Mental Deficiency, is as follows:

> Mental Retardation refers to significant sub-average intellectual functioning which manifests itself during the developmental period and is characterized by inadequate adaptive behavior.[10]

The mentally retarded are generally classified in two groups for purposes of education: (a) the educable mentally retarded, who have I.Q. scores between 50 and 75, and can obtain functional literacy but will have learning difficulties in the regular grade; and (b) the trainable mentally retarded, who have I.Q. scores between 25 and 50 and cannot obtain functional literacy, but will be able to attain self-help skills, limited social skills, and minimal economic efficiency.[11]

In relation to the definition of the term "mental retardation," Dr. Gardner, Director of the Judge Baker Guidance Center in Boston, is of the opinion that definition has a bearing on diagnostic criteria and is therefore of practical consequence:

> There is general agreement that mental retardation is a syndrome which can be caused by many factors acting singly or in combination, and that the diagnosis is appropriate only when symptoms are present prior to the seventeenth year of life. Those who are closely tied to

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[10] R. F. Heber, A Manual on Terminology and Classification in Mental Retardation. American Journal on Mental Deficiency. (Revised Edition, 1961).

[11] Samuel A. Kirk, Educating Exceptional Children, (Boston: Houghton Mifflin Co., 1962), pp. 105-132.

> the behavioral sciences might define mental retardation primarily on terms of its functional characteristics. They would contend that retardation is a developmental impairment in adaptation which can result from psychological and socio-cultural factors, as well as from organic aberrations.[12]

Mentally Handicapped. Kirk and Johnson define "mentally handicapped," which in this paper will be referred to as "mentally retarded," as that term used for children who can be educated in special classes in the public schools.[13] Because of the complexity of diagnosis, there is confusion concerning these children and their educability.

Kirk and Johnson gave the following characteristics of the children termed "mentally handicapped" or "mentally retarded":

1. The mentally retarded child has some degree of educability in the social area, and with assistance may be expected to manage his own affairs with some prudence.

2. The mentally retarded child should have some degree of educability in the occupational areas so he must develop to the point where he can partially or totally earn a living.

3. The mentally retarded child is thought to have developmental retardation, that is to say that he is retarded at school age whether the retardation existed at birth or not.

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[12] George E. Gardner, Mental Retardation (American Medical Association: Chicago, 1964).

[13] Kirk and Johnson, op. cit., p. 10.

4. Also characteristic of the mentally retarded child is retardation obtaining at maturity; although this does not take into consideration the changes and advances such as more adequate medical, educational, and social training.

5. Since etiological factors often cannot be determined, the criterion of constitutional origin, may be theoretical in diagnosis.

6. The mentally retarded require a special curriculum for their social and occupational growth. This is one of the major criteria in the placement of the child in school. Children with I.Q 's of 50, 60, or 70, as determined by a battery of individual intelligence tests and psychological examinations, are usually likely candidates for classes for the mentally retarded.[14]

## C. Sources of Information

The information for this paper has been taken from professional classroom experiences of the author and from books, curriculum guides, and some personal interviews and experiences deemed applicable to the study.

## D. Limitations

This study was limited to a great extent to the child termed "educable mentally retarded" in the junior high school level of development (usually termed levels IV-V) of the Special Education program. Children in this group will be in the chronological age range of thirteen to possibly sixteen years, depending on their mental and social development.

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[14]Arnold Gesell and Catherine Amatruda, Developmental Diagnosis (New York: Harper and Row, Pub., 1964), p. 119.

# CHAPTER II

## INTRODUCING NUMBERS

Many of the arithmetical skills taught in the intermediate grades of the regular school are beyond the comprehension and abilities of the mentally retarded. Much of the material is of little use to them in practical life situations. The use and understanding of the basic arithmetical skills, however, are necessary in performing almost any number problem the individual might encounter.

It is up to the teacher to find the exact level at which each of the pupils is functioning in number sense, and to instruct him at that level. It is always a good idea to keep in mind the four stages in the development of real understanding of the abstract number. First, there is the concrete stage; second, the picture stage; third, the semi-concrete stage; and fourth, the abstract number stage. In cases of serious retardation in which the child has not yet formed the basic and fundamental concepts of the meaning of the abstract number, it is necessary to go back to the very beginning of arithmetic instruction and through patient effort and widely varied material to build up the elemental basis of understanding of the abstract number.

Primitive man kept a record of his flock of sheep by using pebbles. The set of pebbles matched the set of sheep in the flock. For each element or pebble in the set of pebbles there had to be one and only one sheep. Similarly, for each sheep in the set of sheep there had to be one and only one pebble in the set of pebbles. In that way there was a one-to-one correspondence between the two sets. The element of objects is a characteristic or property of the set.[15]

The owner of a flock of sheep did not know how to express the number of sheep he possessed except by indicating the collection of pebbles he had made. If the set of sheep and the set of pebbles matched, he could be sure no sheep were missing. Although the shepherd could tell whether the sheep were missing, he could not tell the number of sheep present or missing because he could not count. This concept of one-to-one correspondence must be thoroughly understood by the child before proceeding with number concepts.

## A. Number Concepts

The mentally retarded have need for arithmetical concepts from the time before they can handle simple number problems through adulthood. Consequently, every classroom activity in which a number situation arises should be made use

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[15]Foster E. Grossnickle and Leo J. Brueckner, Discovering Meanings in Elementary School Mathematics, (New York: Holt, Rinehart & Winston, 1963), p. 54.

of to add to his meaningful experiences and to further his grasp of the concepts.

> The mental retardate needs many meaningful experiences with numbers before he is ready to handle a simple problem. Every classroom activity in which a number situation arises can be made a part of his experiences. His ideas of number values should be systematically built up out of his immediate environment, and should be based upon objectives which he can handle.[16]

Addition, subtraction, multiplication, and division, are all specific, mechanical skills that can be acquired by all mentally retarded children to a greater or lesser degree. However, the child must know the method and reasoning behind each process.

> The child progresses by developing the concept of more and of less; then, when the concept is understood, meaningful practice material should be introduced, to facilitate its use. Finally, the materials should be presented in life-like situations.[17]

Concepts are formed by processes of comparisons, abstractions, and generalizations, and application of the correct name. For example, to form the concept of a bird, the child must compare a large number of various birds of different sizes, weights, breeds, colors, speeds and so on.

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[16] Curriculum Guide, Special Education, (Austin: Texas Education Agency, 1960), p. 95.

[17] Samuel A. Kirk and G. Orville Johnson, Educating The Retarded Child (Boston: The Riverside Press, 1951), p. 283.

> A concept is a device by means of which we can recall, condense, group, classify, and generalize all our past experiences. It enables us to use a "common" noun to refer to an illimitable number of objects for experiences without the arduous necessity of envisaging each object or experience.[18]

Although it is conceded to be difficult in the case of the mentally retarded child, the teacher should attempt to develop at least the basic conceptual tools that the child will need in adapting to the social sphere in which he will live.

It is therefore of prime importance, in teaching, to use objects, representations, illustrations, demonstrations, movies, film strips, activity projects, units, and real school room experiences, rather than words, symbols, descriptions, explanations, rules, principles, and abstractions. Concrete activities hold their interest and attention because the children can understand them, whereas interest and attention flag quickly in the face of incomprehensible abstractions.

> An assortment of supplementary browsing materials belong in every schoolroom. For number concepts, many different kinds of scales, rules, containers, for liquid measure (these can be used with sand, in a deep tray), old clocks, thermometers, boxes and jars of various

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[18] J. E. Wallace Wallin, Education of Mentally Handicapped Children (New York: Harper & Brothers, 1955), p. 189.

> dimensions, sticks, checkers, poker chips, and other materials offer opportunities for matching, sorting, arranging, estimating, comparing and measuring.[19]

Many times number concepts are developed as they are with a normal child except that the rate is slower for some mentally retarded children, and again much structuring is done. Concrete and graphic materials are emphasized for a longer period. For example, Hortense Barry follows this method:

> Large, then smaller, wooden blocks are used initially. The written symbols, the verbal names, and the concepts are taught. The child handles the blocks, learns to count them, to match them to the written symbols, to match the written symbols to groups of blocks and other objects. Individual bead boards are supplied, and the child is encouraged to touch and move each bead when counting, to prevent skipping or random counting. Some children cannot count unless each block or bead is placed in their hands.[20]

When the transfer from objects to configurations is made, it might be necessary to circle a group of dots, to tie them together, before some children can perceive them as a whole. Wheat says the child must learn to see numbers as units made up of smaller groups and as essential parts of larger groups without breaking them into ones. He must study

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[19] Elizabeth E. Freidus, "The Needs of Teachers for Specialized Information on Number Concepts," The Teacher of Brain-injured Children, William Cruickshank, Editor, (Syracuse University Press, 1966), p. 124.

[20] Hortense Barry, The Young Aphasic Child (Washington: Alexander Graham Bell Association for the Deaf, Inc., 1961), p. 58.

groups of up to ten. The child learns to identify, select, and reproduce a group of given size. The child learns about groups by comparing each with others, and breaking them down into their component parts, and by counting.[21]

Complete understanding of what is being done should come before practice; otherwise, practice could be definitely harmful. However, this is not always possible with the mentally retarded child. Many teachers take the child from a very inadequate understanding of the meaning of numbers into the area of drill in the fundamentals of addition and subtraction, which the child must then learn by rote, attaching little meaning to what he has learned. They expect the child to know numbers without having been taught the basic concepts of number. Adults are sometimes so accustomed to the fact of the fourness of the word "four" and the symbol "4" that they sometimes forget that there is absolutely nothing inherent in the work or symbol to denote such a concept. It is only through careful planning by the teacher and varied experiences on the childs part that he arrives at that fundamental concept.

The printed forms of the number words are introduced early and the child learns to match the symbol, the printed

[21]Harry Grove Wheat, <u>How to Teach Arithmetic</u> (New York: Row, Peterson & Company, 1961), p. 53.

form, and the configuration. A wide variety of activities is used in this training. Simple addition and subtraction are started as soon as the child has a basic understanding of number.

More than is common in normal children, the mentally retarded individual needs help in focusing on his task.[22] The teacher can do this by removing extraneous material from the tables or chalkboard. For example, a heavy outline can be made on a frame around the numbers or the area being studied.

> It is important to remember that during all training with the child, the atmosphere of the classroom remains structured rather than permissive. The children continue to need routines and patterning. They are calmer and happier in an organized situation where the teacher sets the goal.[23]

## B. Counting

The arithmetical activities should be developed through the discovery method, using social applications. Only when the child relates his knowledge of numbers to his general activities will the arithmetical facts become useful to him. There is a certain sequential development of understanding, processes, and ideas that cannot be left to chance alone. Counting is the foundation for all processes in arithmetic. It

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[22] Abraham Feingold, *Teaching Arithmetic to Slow Learners and Retarded*, (New York: The John Day Company, 1965), p. 14.

[23] Barry, *op. cit.*, p. 50.

is complex and closely associated with the meanings expressed by numbers. The child must learn counting and the meanings of number names. He must learn that each number has a fixed position in the counting series. This is called the "ordinal" number. The ordinal number stands for a single item in a series and points out its numerical position as second, third, fourth, and so on. Teachers do not have to call numbers by these seemingly complicated terms. They can be distinguished by asking "which one" or "how many." Each number in a counting series means one more than the preceding number. This concept can be illustrated by using the number line. The teacher must emphasize the importance of building a meaningful vocabulary as the child learns numbers. Although some of the terms will be difficult for the retardate to remember, many of the words will be "picked up" as time goes on and they are used regularly. The term numerals indicates number names.

The work in arithmetic begins with the adjustment to concrete situations at the level of the child's comprehension.

> In the case of the mental defective, his level of comprehension will be so low that the child's first work may be the counting of objects in small groups, as having two cents and discovering how much he would have if someone gave him two more. If a child cannot solve a simple problem like this without counting the pennies, then he needs to count them. The child who has no concept of two, three, four, and other simple numbers will count as the very small child does, beginning each time with "one." That is, he will find that he has two cents by counting "one, two." If he is given two more and asked how many

he has, he will count "one, two, three, four" instead of simply adding "three, four" to the two he has just counted.[24]

Counting is the foundation for all processes in arithmetic. The child must learn counting and the meanings of number names. These names express two meanings: one designates our idea of the size of the group or the number of ones in a group; the other designates the place of the number in the series.[25]

The real usefulness of a child's being able to count by rote, no matter how high, is a questionable matter. Nevertheless, the fact of his being able to do so shows his interest in numbers. It is a grave error to continue rote counting as an end in itself. Its only apparent usefulness is that it teaches the child the number names and teaches him the sequence in which the number names belong. Though rote counting is almost universal among younger children, it is a pity that some teachers fall back on this feeble and incompetent means of beginning arithmetic. Such a practice should be replaced by very extensive foundational work in which the child has ample experience with numbers as represented by concrete

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[24] Grace Fernald, Remedial Techniques in Basic School Subjects (New York: McGraw-Hill Book Co., 1943), p. 266.

[25] Lucy Lynde Rosenquist, Young Children Learn to Use Arithmetic (Boston: Ginn and Company, 1949), p. 114, 116, 122.

objects, pictures, and then semi-concrete material which would lead to a far more real understanding of arithmetic throughout all levels.

The teacher should understand the child's learning characteristics in the development of arithmetical concepts and utilize principles which take these characteristics into consideration. The motives must be concrete and specific, and the learning units must be short and closely related to his immediate environment and past experiences. Also, it must be remembered that the child interprets what he meets in terms of specific rather than generalized experiences.[26]

### C. Place Value

The concept of place value is ordinarily a difficult one for the retarded child to grasp; therefore, a multifaceted approach is usually necessary. Variations should, however, correlate with other phases of the arithmetic instruction.

The place value of a digit in a whole number is its position in reference to other digits in the number. The positions of digits are numbered beginning at the right. Thus in 243, three express that number of ones, four expresses four tens, and two expresses two hundred. Each digit contributes

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[26]Karl G. Garrison, The Psychology of Exceptional Children (New York: The Ronald Press Co., 1950), p. 159.

to the number to which it belongs. The value of the digit depends on its position in the number.

The development of the concept of tens and ones and of place value is first introduced. Rote counting to 20 should be developed previously as essential readiness. The teacher writes the numbers one through nine in a vertical column on the chalkboard. She then writes the number ten, placing the zero directly under the single digits. She then stimulates the children to tell her that there are two figures.

The term "place" is introduced by illustrating that this number has two places. Ten objects are then used; apples, cookies, or anything that can be joined visibly by a binding factor such as a plate or transparent container. The children count the ten objects and are told that this is a group, a group of ten. After identifying the group of ten, a few scattered objects are then identified by the teacher as ones, loose ones, not enough to make ten. Large tongue depressors are bundled by the children to make ten.

<u>Teen Numbers</u>. The children are then guided in counting objects, beginning with ten (recognized as a group). The children watch as the teacher writes the number on the board and shows how, for example, 13 means one ten and three ones and that any number which is more than ten is a two-place number. Practice exercises are done with the teacher.

Arithmetic terms which should be mastered here in reading are number words one through twenty, and zero.[27]

Variations in Presentation. Bundles of sticks may be used, and the presentation given previously can be reviewed with emphasis now shifting to the analysis of teen numbers. That is, the teacher selects a number and asks the child to tell how many tens and how many ones it has. A few periods throughout the day might be devoted to counting a dime and pennies to 20 cents, introducing substitution of another dime for the second ten pennies. After rote counting to 100 has been taught, the child can be asked to show various numbers in terms of dimes and pennies, tens and ones.

Zero. Beginning with a vertical column of single digit numerals, the teacher re-emphasizes the meaning of ten and the teen numbers as two place numbers; the importance of placing ones under ones and tens under tens is also introduced. The concept that zero is a place holder, that it keeps the 1 (or other numeral representing tens) in its proper place and indicates a numerical value is stressed.

Hundreds Concept. The teacher directs the children in fastening tongue depressors or ice cream sticks into bundles of tens. The children then count the sticks by tens until

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[27] Curriculum Guide, Special Education, *op. cit.*, p. 95.

they reach one hundred. The teacher then goes to the chalkboard and asks, "How much is one ten?" The response "10" is recorded on the chalkboard. Using the bundles to illustrate the question, the teacher continues, asking "How many are two tens?" and so on, placing the response in a vertical column. When the symbol 100 is written, pupils are guided to observe that now three digits are necessary; one hundred is a three-place number.

Hundreds, tens, ones. Practice in analyzing three-place numbers might take the form of a guessing game, the teachers leading with, "I am thinking of a number that is three hundreds, five tens and two ones. What is it?" After the skill is well established the children can take turns, the one giving the correct response providing the next example. Numbers should also be presented for the children to break into hundreds, tens and ones.

Enriching the concept. Counting of dimes to a dollar and presenting the dollar as ten dimes, ten tens or 100 cents could advisedly follow. A two dollar bill might be presented as being 20 dimes or 20 tens or 200 cents or a five dollar bill as being 50 dimes, 50 tens or 500 cents.

Numbers to 100. A review of reading, writing, and serial order of numbers from 1 to 100 precedes the introduction of reading and writing number names to one hundred. Practice in comparing numbers might also be included in the review.

<u>Numbers to 200</u>. Recognition and serial order of the numbers 1 to 100 are reviewed as readiness for extension of the skill to 200. The numbers 100 to 200 are introduced a decade at a time through reading of the number, writing of the symbol, and placement in serial order. The fact that these are three-place numbers is brought out; the children are stimulated to observe and discover this for themselves. Review of the tens and ones concept introduction to the concept of hundreds appropriately follows.

<u>Numbers to 1,000</u>. The review is now extended to reading, writing, and serial order of numbers to 200. Reading and writing of numbers to 1,000 follows with some practice in comparing non-consecutive numbers. The number name "thousand" is introduced. Development of the concepts of hundreds and thousand will appropriately follow.

<u>Thousand</u>. Since the concept of thousand is rather removed from the realm of the child's personal conceptual experience, introduction is made by way of an example which the teacher tailors to the child's interests. She might have it on the cost of a piece of equipment, or an article being installed in a familiar building. Any other type of situation in which the thousand concept might be involved (thousand people at a picnic) would be suitable. The child builds the concept of thousand by building ones to tens, tens to hundreds,

and so on. The ten bundles of one hundred each are counted by hundreds and the count is recorded on the chalkboard in a vertical column.

When the thousand sticks have been combined and recorded, children are guided to observe that writing 1,000 involves another, a fourth place value. The sticks are unbundled and piled loosely to give the children an idea of the bulk involved. Teacher-directed practice in reading, writing, and interpreting four-place numbers follows. The relationship between dollars and hundreds, ten dollars and a thousand is also pointed out.

<u>House Numbers, Telephone Numbers, Room Numbers</u>. Enrichment of the skill of reading larger numbers includes reading house and telephone numbers, room numbers, and also the numbers on license plates. Zero is read as 0; four place numbers are grouped in reading. 1822 is eighteen twenty-two.

<u>Roman Numerals</u>. Introduction to reading and writing Roman numerals to twelve is made as an aspect of time-telling, namely that of reading Roman numerals on a clock face.[28] The teacher presents two large clock faces, real or pictured, one having Arabic, the other Roman numbers. Children are challenged to read the Roman clock face with aid of comparison

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[28]Robert E. Eicholz and Emerson Martin, <u>Elementary Mathematics Series</u>. (Massachusetts: Addison-Wesley Pub., Inc., 1963), p. 81-82.

with the Arabic. The underlying system of Roman numeration is explained and reading and writing of Roman numeration is practiced for mastery. The teacher also points out to the class other uses for the numerals, as chapter numbers in books and date markings.

Developing an understanding of numeration is an important part of the arithmetical program. There are a variety of devices which can be used to formulate and illustrate the concepts underlying the decimal system.

1. Tongue depressors, pencils, or sticks can be used separately as counters or tied together to form bundles of 10 or more.

2. The abacus or counting frame may be obtained in many forms and sizes from various distributors. An inexpensive open-ended abacus can be constructed without much difficulty, using objects such as nails and poker chips. The well-supplied classroom should have a small abacus (to 3 places) for each child and a large one for the teacher.

3. Place-value charts are easily made using colored sticks and a manila folder with 3 attached "pockets" (for the 1's, 10's, and 100's columns). A similar illustrative device can be constructed by nailing 3 tin cans to a board and using tongue depressors or sticks as counters.

4. Pegboards in sizes of 10" by 10" and 100" by 100" can be used to show two and three place numbers.

5. The number line is an important device which can be used to illustrate many concepts. Several textbook companies and commercial firms produce inexpensive paper lines which can be placed above the chalkboard in the front of the room. Numeration, the basic facts of computation, the concept of order of numbers, inequalities and fractions are just a few of the topics which can be illustrated and reinforced on the number line. A tape measure can also be used effectively as

the number line. It has the advantage of being flexible, and it can be easily stored.

A number line from 1 to 20, made on clear pliofilm using a 9-inch interval, can be taped to the floor. The child who starts at 0 and moves 5 places easily visualizes how much farther he has moved than his friend who moves only 3 places. This gives meaning to the relative value of numbers. Two children may start at 0 and move 3 places; 1 child continues to move 4 additional places, finding he is now 3 + 4 places from 0, and that he is standing on the numeral 7. They discover that 3 + 4, or 7, name the same place. This method might be the forerunner of using the traditional number line as described previously.

6. An equation bar is a manipulative device which can be helpful in reinforcing the concept that every number can be named in various ways. Such a bar can be made by using a 1-inch square bar of wood, 42 inches long, placing cup hooks at 2-inch intervals, beginning with 1 in the exact center and counting in each direction to 10. The consecutive numerals should be painted above each hook. A small screw eye on top in the center allows the bar to swing freely when suspended. (A little sanding may be necessary to get good balance). Using plastic clothespins, a child finds that 8 on one side balances 5 and 3 on the other. The 8 may be replaced by clipping pins on 7 and 1. Again the child discovers that there are many names for each number. The equation bar is also useful when beginning multiplication. The child finds that 3 pins clipped together on the numeral 2 are perfectly balanced by 1 pin on 6. To work with products larger than 10, such as 3 times 8 equals 24, 3 pins are clipped on 8 and the expanded form of 24 is used . . . 20 + 4 would be 2 pins on 10 and 1 on 4. Supplement these devices with the well-known pocket chart and cards to build equations.

7. Add-a-bead frame with removable beads gives the opportunity for experimentation with equivalent sets and repeated addition to build readiness for multiplication and division. The child uses 5 sets plus 1 set of 5 each to find how many are 6 sets of 4. Other aids which might prove helpful to the child are counters, place value charts and place value diagrams.

## D. Number Relationships

The teacher should provide specified times in classes at this level for the development of the fundamental number skills. During these times the teacher will probably find it necessary to supplement the activity-teaching situations with practice in the various skills so that all the children may become familiar with their use. To be of the greatest value, the practice must be meaningful to the child. He must know why the practice is included, how it will help him, and what its values are. Experiences preceding the practice can demonstrate the need as well as teaching the child the basic principles underlying the processes. It should be remembered that the learning of number skills through practice is not an end in itself. It becomes the means to an end by saving the child time through short-cutting and improving accuracy.

> The formation of various relationship concepts begin long before a child reaches school-age, while he is learning to understand bigger, smaller, above-below, in front of-behind, nearer-farther, heavier-lighter, before-after, sooner-later, and more-less.[29]

Children are constantly discovering relationships, confirming one discovery with another, using one sense to elaborate upon what another has learned. Clarity in relationship concepts is basic to understanding number concepts.

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[29] Freidus, _op_. _cit_., p. 117.

> Some children approach school learning without having established this foundation strongly during their school years. They will need much opportunity to solve many kinds of problems involving quantitative comparison--opportunities planned to fill gaps left by experiences that are insufficient in number or inadequate in awareness or clarity of meaning.[30]

Comparison of Large Numbers. After the child has mastered the ability to group objects into sets and compare individual objects to each other, he is ready to begin comparing sets. In comparing large numbers, children are taught to scrutinize the thousand place first, then the hundreds, then the tens and finally the ones. Comparison of numbers less than a hundred will have more practical value to the child and should be stressed most.

Estimative Ability. The ability to make sensible rough estimates requires practice. Opportunities should be provided for the child to make a judgment on comparative quantities. He might be asked to compare the ages of siblings or of classmates or to judge about how old a small child or an obviously elderly person might be. A skill in judgment might be attained also by estimating on the basis of comparative amounts of time required to get somewhere, (to the store, and so on). The child might be asked which is the farther of two places grossly different in distance but with which the child is well

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[30] Ibid.

acquainted. He might be asked which would probably cost more money, a small toy or a radio. He might be asked to show with his hands the length of a foot. Descriptive terms of relative size which are essential to the understanding of arithmetic in everyday life are as follows: big, bigger, biggest, small, smaller, smallest, large, larger, largest, long, longer, longest, little, short, shorter, shortest.[31]

Estimative ability is developed through frequent challenges to estimate various heights, lengths, and weights. Whenever possible, the estimate should be verified by measuring, and other objects of the same height, length or weight should be pointed out to the child to strengthen the basis of his estimates. Estimating time to the hour occasionally serves to help sharpen the child's time sense.

Estimating Problem Answers. Estimation should be carried into problem solving when the child is challenged to estimate what his answer will be. This device often provides the teacher with an insight into whether the child really understands the problem and whether he can select the appropriate process for getting the solution.

Terms which should be mastered are as follows:

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[31] Curriculum Guide, loc. cit.

| Relative Quantity | | Descriptive Terms | |
|---|---|---|---|
| many | the same as | big | largest |
| enough | fewer than | bigger | long |
| fewer | less than | biggest | longer |
| for each | more than | small | longest |
| more | | smaller | little |
| as many as | | smallest | short |
| less | | large | shorter |
| | | larger | shortest[32] |

[32] *Ibid.*

# CHAPTER III

## BASIC PROCESSES

The minimum everyday demands of arithmetic in the adult life of the retardate are relatively few, but they are important. Many of them involve the use of money and of making change. The chief problems relate to household expenses and are concerned with such items as groceries, fruit, meat, milk, clothing, dry goods, house furnishings, fuel, electricity, light, gas, rent, or taxes. Which of several items would it be better to buy? How can the household budget be managed? These are typical questions that face the young mental retardate almost daily. Other problems involve the figuring of wage rates per hour, week, or month, the use of time schedules, and common weights and measures.

> Whatever number situations arise in the life of the child or in the life of his family could be considered good content in arithmetic, provided the child's mental level is high enough to enable him to cope with them.[33]

Through such experiences the child can develop a concept of number values and relations. He should learn in

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[33]Elise H. Martens, Mental Retardation (New York: Holt, Rinehart & Winston, 1961), p. 246.

doing things that the "and" relationships means additional; and "difference" "how much more," "lost" or "gave" means subtraction.

> It is not sufficient for the educable mentally handicapped child to know how to add. He must also acquire the concept of addition so that he may independently differentiate when to add from any one of a number of arithmetical behaviors available to him. Devoid of concepts, arithmetic becomes nothing more than exercises; numbers and reading become word calling.[34]

The retarded child is usually able to achieve at a high grade level in the more mechanical computational skills (basic processes) than in arithmetical reasoning (problem solving). Kirk feels that this condition may be a reflection of the methods of teaching, since it is easier to drill on computational problems than to develop insight into quantitative concepts.[35] However, in a study by Dunn, it was found that the retarded and normal subjects did not differ significantly in arithmetical computation. When the results of arithmetic reasoning tests were scrutinized, Dunn noted that the normal subjects surpassed their retarded counterparts.[36]

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[34] Ibid., p. 225.

[35] Samuel A. Kirk, Educating Exceptional Children (Boston: Houghton Mifflin Co., 1962), p. 127.

[36] Lloyd M. Dunn, "A Comparison of the Reading Processes of Mental Retardates and Normal Boys of the Same Mental Age Level" The Exceptional Child, Philip Trapp, Philip Himelstein, Editors, (New York: Appleton-Century-Crofts, Inc., 1962), p. 119.

R. J Capobianco, Associate Professor, Educational Psychology, University of Minnesota, in studying the school curriculum for the mental retardate, points up the important fact that if based strictly on mental age, the curriculum in itself is inadequate.[37] He feels that it is important to identify other factors and show their interaction within the learning process before an effective curriculum can be established.

> Education, in developing a curriculum, must take into consideration many factors if it is to be designed to meet the needs of individual children. Among these factors, or primary importance, are the potentialities and characteristics of special children enrolled within the total school environment.[38]

Some of the differences which have been found in the process of arithmetic suggest that retarded children are inferior to average children of the same mental age in the following areas:

(1) solving arithmetic problems presented verbally
(2) establishing mature habits such as eliminating "counting on fingers"
(3) decreasing careless mistakes in reading and
(4) understanding the abstract terms of mathematics, space, time, and quantity.[39]

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[37] R. J. Capobianco, "Reasoning Methods and Reasoning Ability in Mentally Retarded and Normal Children," The Exceptional Child (New York: Appleton-Century-Crofts, Inc., 1962), p. 212.

[38] Ibid., p. 213.

[39] Kirk, op. cit., p. 261.

## A. Addition

The basic processes of arithmetic-addition, subtraction, multiplication, and division--are processes of grouping and regrouping. Children need many grouping experiences. As soon as the child fully and completely understands the abstract number, he quite naturally starts putting numbers together and taking them apart. These two processes, in the final analysis, constitute the whole arithmetic field. The child must be made to see this over-all scheme with each new computational operation which is taken up to show the relation between the various processes.

It is up to the teacher to find out the exact achievement level at which each of the pupils is competent, and to instruct each of them at that level. It is a good idea to keep always in mind the four stages in the development of real understanding of the abstract number. First there is the concrete stage; second, the picture stage; third, the semi-concrete stage; and fourth, the abstract number stage. In cases of serious retardation in which the child has not yet formed the basic and fundamental concepts of the meaning of the abstract number, it is necessary to go back to the very beginning of arithmetic instruction and through patient effort and greatly varied materials to build up the elemental basis of understanding of the abstract number.

Buckingham defines addition as a process of putting together or regrouping.[40] It is not a process of getting more. The sum is equal to the combined addends. In addition one may have equal sized or unequal sized groups. The sum is not affected by the order of the addends. That is, the addends may be interchanged without changing the sum, according to the Law of Commutation:

| | | | | |
|---|---|---|---|---|
| 3 + 4 = 7 or | 0 | 00 | 00 | 00 |
| 4 + 3 = 7 | 0 | 00 | 00 | 00 |
| | 4 | 1 + 3 | 2 + 2 | 3 + 1 |

Only those numbers which represent subjects that have a common characteristic can be added together. Only digits in like places may be added. In addition one begins with the ones column, then moves to tens, and so on.

0 + 00 = 000          1 + 1 = 2

Common Characteristics

# + 0 = ?          1 + $ = ?

Uncommon Characteristics

Correct terminology should be stressed with the retarded child as with the normal. Words with exact meanings are needed to acquire vocabulary for the expression of arithmetical ideas.

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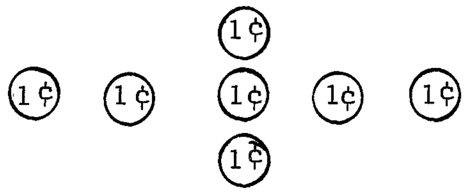
[40]B. R. Buckingham, Elementary Arithmetic: Its Meaning and Practice (Boston: Ginn and Co., 1947), Chapters VI, VII.

In addition, the child must learn the words "sum" and "addends." These terms, if started early in the arithmetical program should be no more difficult to the child than other simplified words. In like manner, the child should learn the terms used in the other fundamental processes.

Where practice exercises are designated to develop the understanding of the facts through the use of pictured objects, the use of rubber stamps is often to be preferred instead of directing the child to draw pictures. It is easier for the child to concentrate on counting as he stamps than on counting as he draws. Additional advantage is that incidentally the one-to-one correspondence is reviewed, one stamped picture for each number counted.

Column Addition of Single Digits. Introduction to column addition can be made through use of a simple sales slip interpreted in the form of objects, then in picture, finally in symbols. Children are directed to add down, adding the first number and the next, with the second number being added to the following number, until the children are adding the column with ease. The teacher might present the sales slip as the record of a trip where a candy bar for one cent, an ice cream cone for five cents, and a stick of gum for one cent were purchased. The pennies required to pay for each item are arranged on the table and the child counts them as "1 and 5 are 6; 6 and 1 are 7; the bill is 7 cents." The pennies are next drawn on the chalkboard, counted again, and pictures are replaced by number symbols. Sums to ten are introduced.




| | |
|---|---|
| 1 | 1 and 5 are 6 |
| 5 | 6 and 1 are 7 |
| 1 | |
| 7¢ | The sum is 7 ¢ |

It should be emphasized to the child that the difference in order of the addends makes no difference. The sum will remain the same.

$$5 + 1 = 6$$
$$1 + 5 = 6$$

The Law of Commutation

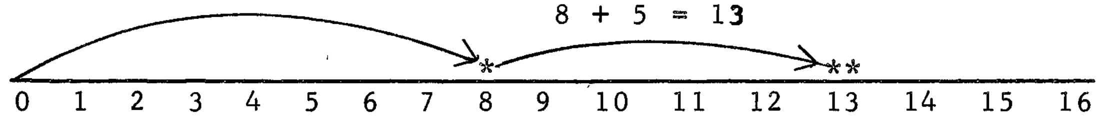
The addends may be grouped in any order without affecting the sum:

Add: 7, 4 → 10; 3 — 10, 4, 14 or 7, 4, 3 → 7 — 7, 7, 14 or 7, 4 → 11; 3 — 11, 3, 14

Associative Property

The use of the "number line" will be helpful in teaching the retardate addition. A number line can be purcahsed commercially or teacher constructed.

8 + 5 = 13



The following steps are outlined for teaching the use of the number line in addition:

(1) Place finger on one addend (*)
(2) Count from finger the number of spaces indicated by the other addend. (**)
(3) Where you stop is the sum.

Much practice should be given the child in the correct use of the number line.

Use of the tape recorder for independent practice work is usually an effective method of sustaining the child's interest. The teacher records the set of facts being drilled, using some type of mnemonic device--a jingle, simple melody or pronounced beat--and providing opportunity for the child to supply responses. Variations of the related facts presentation might include adding 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 to the other numbers to make sums through 10.

Other forms of drill might include pantomime games, card games with the object of gaining all four parts of a fact family, or matching of jig-saw type pieces with one piece having the fact, the other piece the answer.

<u>Adding Two-place Numbers</u>. Review of the tens and ones concept and adding of two-place numbers may be accomplished by using the same form as previously used. Addition of even tens is presented first as a bridge from adding single digits to adding two-place numbers. Dimes or stick bundles

of tens are used to illustrate that two and one are three; two tens and one ten are three tens; 20 (two tens) and 10 (one ten) are 30 (three tens). Much practice should be given, and possibly a period of germination, before the addition of other two-place numbers is introduced. After much teacher direction stressing the need to start with the ones column, some independent work is assigned.

```
Add:  10 = 1 ten                    10 = 1 (10)
      10 = 1 ten                    10 = 1 (10)
      ----------                    -----------
           2 tens or 20                  2 (10) or 20
```

Morton and Spitzer illustrate addition of higher decade numbers as follows. This method definitely shows relationships in the number system.

```
Add:  12 This is 1 ten + 2 units
      17 This is 1 ten + 7 units
      -------------------------
                2 tens + 9 units or 29

Add:  27 This is 20 + 7 or 2 tens plus 7 units
      28 This is 20 + 8 or 2 tens plus 8 units
      ------------------------------------
                40 + 15 or 4 tens plus 15 units

          (15 units = 1 ten plus 5 units)
```

Rewritten it becomes 1 tens plus 5 units which is 55.[41]

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[41]Robert Lee Morton, Teaching Arithmetic in the Elementary School, (New York: Silver-Burdett Co., 1938), Vol. II, pp. 59-60.

Herbert F. Spitzer, The Teaching of Arithmetic, (Boston: Houghton Mifflin Co., 1948), p. 149.

Add: 17 and 28

$$
\begin{array}{rcl}
17 & = & 1(10) + 7(1) \\
\underline{28} & = & \underline{2(10) + 8(1)} \\
 & & 3(10) + 15(1) \\
 & \text{or} & 3(10) + 10(1) + 5(1) \\
 & & 3(10) + 1(10) + 5(1) \\
 & & 4(10) + 5(1) \\
 & & 45
\end{array}
$$

To further illustrate:

Add: 345 and 266

$$
\begin{array}{rcl}
345 & = & \text{3 hundreds + 4 tens + 5 units} \\
\underline{266} & = & \underline{\text{2 hundreds + 6 tens + 6 units}} \\
 & & \text{5 hundreds + 10 tens + 11 units}
\end{array}
$$

11 units is equal to 1 ten + 1 unit, so the last line may be written in this way:

5 hundreds + 11 tens + 1 unit

11 tens is equal to 1 hundred + 1 ten. The sum reduces to 6 hundred + 2 tens + 1 unit = 611.

In an attempt to determined practice effects and to measure the amount of transfer of teaching to a similar task, Tizard and Loos found that the mentally retarded subject exhibited successive gains with practice on subsequent tasks. They observed that transfer of training had taken place. Specifically studying the sequence of behavior exhibited, they identified several stages; general response, perseveration behavior, trial and error, and general orientation to the various stimuli offered them.[42]

The addition of tens and units follows the addition of tens and tens. The same procedure is used as before.

$$\begin{array}{lrcl} \text{Add:} & 21 & = & 2(10) + 1(1) \\ & \underline{1} & = & \underline{\phantom{2(10) + }7(1)} \\ & & & 2(10) + 8(1) = 28 \end{array}$$

Adding Three-place Numbers. The concept of tens, ones, and hundreds is reviewed briefly. Adding of three place numbers is then introduced with emphasis being given to the correct use of the terms "ones," "tens," and "hundreds."

$$\begin{array}{lrcl} \text{Add:} & 115 & = & 1(100) + 1(10) + 5(1) \\ & \underline{113} & = & \underline{1(100) + 1(10) + 3(1)} \\ & & & 2(100) + 2(10) + 8(1) = 228 \end{array}$$

Column Addition of Hundreds. Working with three place addends is introduced, and when facility has been developed, four addends are used. Mixed columns of four addends with two and three places are practiced. Finally columns of four addends with one, two, and three places are introduced.

$$\begin{array}{lrcl} \text{Add:} & 1{,}245 & = & 1(1{,}000) + 2(100) + 4(10) + 5(1) \\ & \underline{123} & = & \underline{\phantom{1(1{,}000) + }1(100) + 2(10) + 3(1)} \\ & & & 1(1{,}000) + 3(100) + 6(10) + 8(1) = 1{,}368 \end{array}$$

---

[42] J. Tizard and F. M. Loos, "The Learning of a Special Relations Test by Adult Imbeciles," American Journal on Mental Deficiency, 1954, 59:85-90.

Adding Dollars and Cents. Further practice in adding three-place numbers is provided in addition of dollars and cents. The importance of placing cents signs correctly is emphasized. Two dollars and cents addends are used, next three, then four, all written with the decimal point. Also, dollars and cents are added using two, then three, then four addends.

$$
\begin{array}{lrcl}
\text{Add:} & \$2.25 & = & 2(\text{dollars} + 2(\text{dimes}) + 5(\text{cents}) \\
 & \underline{1.21} & = & \underline{1(\text{dollar}) + 2(\text{dimes}) + 1(\text{cent})} \\
 & & & 3(\text{dollars}) + 4(\text{dimes}) + 6(\text{cents}) = \$3.46
\end{array}
$$

In the development of the concept "regroup" in addition, the idea must first be introduced intuitively. Following this intuitive introduction, a three-step process is presented for solving the exercises. The children find and write the sum of the numbers in the ones place; then, below the first sum, they write the sum of the numbers in the tens place; finally, the child finds the sum of these two numbers. This process, as a final step leading to the more formal procedure of addition involving "regrouping" helps children to discover and understand the ideas involved in doing problems in the adult "trick method."

Add: 28 + 64

$$
\begin{array}{rl}
28 & \\
\underline{64} & \\
12 & 8 \text{ units and } 4 \text{ units} = 12 \text{ units} \\
\underline{80} & 2 \text{ tens and } 6 \text{ tens} = 8 \text{ tens} \\
92 &
\end{array}
$$

Now investigate the mathematics of a simple exercise of "regrouping."

$$
\text{Add:}\quad \begin{array}{r} 28 \\ \underline{64} \\ 92 \end{array}
\qquad\qquad
\begin{array}{rcl}
28 & = & 2(10) + 8(1) \\
\underline{64} & = & \underline{6(10) + 4(1)} \\
 & & 8(10) + 12(1) \\
 & & 8(10) + 10(1) + 2(1) \\
 & & 8(10) + 1(10) + 2(1) \\
 & & \underline{9}(10) + 2(1) \\
 & 92 &
\end{array}
$$

<u>Vocabulary</u>. Written addition vocabulary terms taught on this level are sum, plus, addition, addend, pairs, plus sign, regroup, and group.

In judging the child's progress two things must be considered: (1) Has the child mastered the regrouping process? "Are the children able to work the exercise involving regrouping?) (2) Are the ideas involved thoroughly understood?

## B. Subtraction

Subtraction is the opposite process of addition. It is not a process of getting less. It is the inverse of addition, the process of "taking apart groups." The minuend is equal to the sum of the subtrahend and remainder. Again, as in addition, only digits in like places may be subtracted. In subtraction one begins at the right with the ones column, then moves left to tens, then to hundreds, and so on. Only those numbers can be subtracted which represent objects that have a common characteristic.

Many times teachers lack understanding of the types of subtraction. If the teacher uses only the "take-away" situation or decomposition, she often wonders why the child has trouble knowing when to subtract when the problem calls for "how many more," or "how many needed." Other subtraction terms used are group, regroup, subtract, subtrahend, minuend, difference, "how many are left," "how many less," and "the rest are" (find the other number). The term minus is introduced.[43]

Change-making is also introduced as a subtraction process. Many practice periods in solving of simple change-making problems are held, and the fact is emphasized that this is another method of doing something already learned as a counting process.

Counting cubes, sticks, or other objects that will not distract the child with their intrinsic interest (as small toys) are used for introducing the concepts as an illustration of the problem presented orally by the teacher. As the child meets the teacher's challenge to solve the problem, his solution is illustrated with the cubes. This illustration is transferred in picture form to the chalkboard. Substitution

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[43]Esther J. Swenson, "Arithmetic for Preschool and Primary Grade Children," <u>The Fiftieth Yearbook of the National Society for the Study of Education</u> (Chicago: The University of Chicago Press, 1951), Part II, pp. 67-195.

of number symbol for pictured object follows. Practice in reading problems using symbols and signs is given. Both vertical and horizontal forms are used.

The child then receives opportunity to manipulate the objects as he uses the problem solving terms given previously. When the single fact is mastered, its related subtraction fact is introduced. When the latter is mastered, the relation between the two is demonstrated, and the process is followed with further repetition.

The relation between the subtraction and the addition process is demonstrated with real objects, then pictured, and practice for mastery follows. The child should be made to see that as 4 + 3 = 7, so also 7 - 4 = 3.

$$4 \underset{\text{Subtract (3)}}{\overset{\text{Add (3)}}{\rightleftarrows}} 7$$

In arithmetic, we may add the number 3 to the number 4, thus performing an operation of addition to get the result of 7. This process implies the "existence" of an inverse operation (involving the same number 3) which we are to apply to 7 in order to get back to the 4 with which we began. This we call subtraction.[44]

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[44] Robert L. Swain, Understanding Arithmetic (New York: Holt, Rinehart, & Winston, 1960), p. 59.

> After understandings have been developed, practice must be used to make permanent what has been learned and to increase the ease with which it can be used. This means we must move slowly in introducing new concepts and that meaningful repetition must continue until mastery is assured.[45]

Repetition should be of three types: written and individual; oral with the teacher, fellow-pupil, or using the tape recorder; and mental, as in teacher-directed activity. Games of a competitive nature leave value as stimulation to greater speed in using facts, but should not be attempted until all the competitors are fairly sure of the material and the competition is a probable means of achieving success.

The number line might be used to start the children in subtraction. Listed are the steps involved in subtraction on the number line:

Subt:

$$\begin{array}{rl} 13 & \text{(minuend)} \\ \underline{5} & \text{(subtrahend)} \\ 8 & \text{(difference)} \end{array}$$

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| | | | | | * | | | | | | | | * | | | |

(1) Place the finger on the minuend.
(2) Place another finger on the subtrahend.
(3) Count from the subtrahend to the minuend for the difference.

(If counting to the right the difference is positive; if counting to the left, the difference is negative.)

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[45] Joyce Benbrook and Cecile Foerster, Contributions in Arithmetic (Austin: The Steck Company, 1954), p. 42.

Subtracting Tens and Ones. Subtracting two-place numbers without having to regroup is introduced after addition of two place numbers has been mastered, and the tens and ones concept has been thoroughly reviewed. Even tens are used at first:

Subt: 40 = four tens<br>
      10 = one ten<br>
      three tens or 30

Subt: 17 = 1 ten and seven units<br>
      6 = six units<br>
      1 ten and one unit = 11

In subtracting higher decade numbers, Spitzer uses the following procedure:[46]

Subt: 46 = 4 tens + 6 ones = 3 tens + 16 ones<br>
      18 = 1 ten + 8 ones = 1 ten + 8 ones<br>
      2 tens + 8 ones

Subtracting Three-place Numbers. An introduction to subtracting three-place numbers is preceded by review of the tens and hundreds concept and of adding three-place numbers. Even hundreds then hundreds and even tens and finally hundreds, tens and ones are subtracted; dollars and cents are subtraced as an extension of the skill. Emphasis is placed on keeping digits correctly placed.

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[46]Spitzer, *op. cit.* p. 156.

Subt: Even hundreds

$$\begin{array}{rcl} 300 & = & \text{three hundred} \\ \underline{100} & = & \underline{\text{one hundred}} \\ & & \text{two hundred} \end{array} \qquad \begin{array}{rcl} 300 & = & 3(100) \\ \underline{100} & = & \underline{1(100)} \\ & & 2(100) \text{ or } 200 \end{array}$$

Subt: Hundreds and even tens

$$\begin{array}{rcl} 540 & = & \text{five hundred + four tens} \\ \underline{220} & = & \underline{\text{two hundred + two tens}} \\ & & \text{three hundred + two tens} \end{array}$$

$$\begin{array}{rcl} 540 & = & 5(100) + 4(10) \\ \underline{220} & = & \underline{2(100) + 2(10)} \\ & & 3(100) + 2(10) \text{ or } 320 \end{array}$$

Subt: Hundreds, tens and ones

$$\begin{array}{rcl} 247 & = & \text{two hundred + four tens + seven units} \\ \underline{123} & = & \underline{\text{one hundred + two tens + three units}} \\ & & \text{one hundred + two tens + four units} \end{array}$$

$$\begin{array}{rcl} 247 & = & 2(100) + 4(10) + 7(1) \\ \underline{123} & = & \underline{1(100) + 2(10) + 3(1)} \\ & & 1(100) + 2(10) + 4(1) = 124 \end{array}$$

Subt: Dollars and cents

$$\begin{array}{rcl} \$6.42 & = & 6(\text{dollars}) + 4(\text{dimes}) + 2(\text{cents}) \\ \underline{3.21} & = & \underline{3(\text{dollars}) + 2(\text{dimes}) + 1(\text{cent})} \\ & & 3(\text{dollars}) + 2(\text{dimes}) + 1(\text{cent}) = \$3.21 \end{array}$$

The process of "regrouping" in subtraction is next taught following a thorough review of the tens and ones concept. As a preliminary step exercises are given in which the child is to solve such equations as 80 + 5 = 70 + ?

$$\begin{array}{lrl}
\text{Subt:} & 34 - 6 = & 3(10 + 4(1) \\
 & & \underline{\qquad\quad - 6(1)} \\
 & & 2(10) + 1(10) + 4(1) \\
 & & 2(10) + 10(1) + 4(1) \\
 & & 2(10) + 14(1) \\
 & & \underline{\qquad\qquad - \ 6(1)} \\
 & & 2(10) + \ 8(1) = 28
\end{array}$$

Regrouping or Decomposition Method

In previous subtraction problems, ones were subtraced from ones, which is impossible in this case since 6 is greater than 4.

<u>Place Variations in Minuend and Subtrahend</u>. After each step has been mastered separately, the teacher again presents a combination of problems having three-place minuends and one-place subtrahends; three-place minuends and two-place subtrahends, as well as other one- and two-place combinations.

$$\begin{array}{lrcr}
\text{Subt:} & 17 & = & 1(10) + 7(1) \\
 & 3 & = & \underline{\qquad\qquad 3(1)} \\
 & & & 1(10) + 4(1) \ = 14
\end{array}$$

$$\begin{array}{lrcr}
\text{Subt:} & 183 & = & 1(100) + 8(10) + 3(1) \\
 & \underline{21} & = & \underline{\qquad\qquad 2(10) + 1(1)} \\
 & & & 1(100) + 6(10) + 2(1) \ = 162
\end{array}$$

$$\begin{array}{lrcr}
\text{Subt:} & 187 & = & 1(100) + 8(10) + 7(1) \\
 & 6 & = & \underline{\qquad\qquad\qquad\qquad 6(1)} \\
 & & & 1(100) + 8(10) + 1(1) \ = 181
\end{array}$$

$$\begin{array}{lrcr}
\text{Subt:} & 432 & = & 4(100) + 3(10) + 2(1) \\
 & \underline{121} & = & \underline{1(100) + 2(10) + 1(1)} \\
 & & & 3(100) + 1(10) + 1(1) \ = 311
\end{array}$$

## C. Multiplication

Multiplication is a short way of adding groups to form a whole group. It is simply a short way of adding equal groups. Multiplication may be a specialized case of addition, one in which all the addends are of equal size. For example, 6 + 6 + 6 + 6 = 24, while 6 + 4 + 5 + 9 cannot be transformed to multiplication. If the numbers in the multiplication fact are interchanged the product is not changed. This rule is the law of commutation. Thus, 3 x 7 = 21 and 7 x 3 = 21.

Names for the basic items in a multiplication in terms of the already known process of addition are as follows: the multiplicand is the typical addend; the multiplier is the number of times the typical addend appears, and the product is the result of the operation corresponding to the sum. Thus, our 6 + 6 + 6 + 6 = 24 transferred to multiplication would be:

$$\begin{array}{lrl} \text{Mult:} & 6 & \text{(multiplicand)} \\ & \underline{4} & \text{(multiplier)} \\ & 24 & \text{(product)} \end{array}$$ [47]

It should be noted that in the corresponding addition problem, the principles of likeness guarantee agreement between the addends and the sum (if the addend are "people"; the sum will have to be "people" also).

[47] Francis J. Mueller, Arithmetic, Its Structure and Concepts (New Jersey: Prentice-Hall, Inc., 1956), p. 69.

Although many retardates do fairly well on simple, concrete problems such as measuring, weighing, and counting objects, provided the conditions are not changed, they are unable to envison abstract numerical concepts. The retarded are most competent in addition and least competent in fractions and division. Many learn to repeat the multiplication tables glibly, although they may be unable to apply the multiplication process in solving concrete problems.[48]

In the study of multiplication, it seems unlikely that any teacher would fail to show that multiplication is merely a short way to add the same number two or more times. The many modern textbooks and workbooks available at present offer an abundance of helpful plans in showing this relationship. One of the most effective is the use of dots:

| | | |
|---|---|---|
| . . 2 | . . 2 | . . 2 |
| . . 2 | . . 2 | . . 2 |
| 2 x 2 = 4 | . . 3 | . . 2 |
| | 3 x 2 = 6 | . . 2 |
| | | 4 x 2 = 8 |

The relation between multiplication and division is easily shown when the multiplication and division combinations are taught simultaneously. For example, it is easy for the child to see that if 4 x 8 = 32, then 32 ÷ 8 = 4. In other

[48] Benbrook and Foerster, op. cit., p. 44.

words, if four eights make thirty-two, then, when asked how many eights there are in thirty-two, the child usually sees immediately that there are four.

The multiplication tables can play a useful part in learning arithmetic facts.[49] After multiplication has been illustrated through problems, if the nature of the facts has been illustrated through the use of objects and marks, and the child understands the process, it is sometimes advantageous to have the multiplication facts in table form.

> Any organization that makes for easier learning should be used. If the child understands the process first, studying the facts in serial order as in a table is not harmful.[50]

Spitzer illustrates multiplication as follows:[51]

$$
\begin{array}{r}
3 \text{ tens} + 1 \text{ one} \\
\underline{2 \text{ tens} + 1 \text{ one}} \\
3 \text{ tens} + 1 \text{ one} \\
6 \text{ hundreds} + 2 \text{ tens} \qquad\qquad\qquad \\
\hline
6 \text{ hundreds} + 5 \text{ tens} + 1 \text{ one} = 651
\end{array}
$$

In beginning the recognition of multiplication groups concepts, small groups might be used:

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[49] Spitzer, _op. cit._, p. 187.

[50] _Ibid._, p. 188.

[51] _Ibid._, p. 377.

```
X X X   X X X   X X X   How many groups of X's?
 X X     X X     X X    How many in each group?
                        How many X's in all three
                          groups together?
```

Equal products that arise in the application of the commutative law can be illustrated with objects and diagrams. To illustrate that four 3's give a product equal to three 4's, use the following diagram:

| Diagram of four 3's (4 rows with 3 in each row) | Diagram of three 4's (3 rows with 4 in each row) |
|---|---|
| . . .<br>. . .<br>. . .<br>Total 12 | . . . .<br>. . . .<br>. . . .<br>. . . .<br>Total 12 |

The product of the factors is unaffected by the combination by which the factors are grouped:

$$4 \times 2 \times 3 \times 5 = 4(2 \times 3 \times 5) = 4 \times 30 = 120$$
$$= (4 \times 2 \times 3)5 = 24 \times 5 = 120$$

```
Mult:  17                 4 (people)
        4 (people)       17
                         --
                         28          The Law of Commutation[52]
                          4
                         --
                         68 (people)
```

By developing and using the distributive law of multiplication, the following product is obtained:

---

[52] Mueller, op. cit., p. 86.

4 x 9 becomes 4(3 + 6) or 12 + 24 = 36

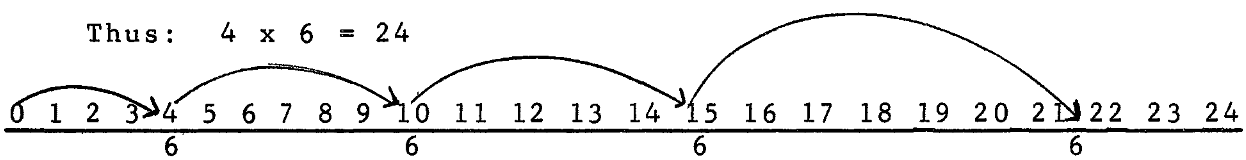
or

4 x 9 becomes 4(2 + 2 + 5) or 8 + 8 + 20 = 36

Distributive Law of Multiplication

<u>Multiplication and the Number Line</u>. Multiplication is a specialized case of addition, one in which all the addends are of equal size. On the number line, this involves increments of equal size. For instance, 4 x 6 means ultimately the sum of four sixes.[53]

Thus: 4 x 6 = 24




<u>Practice Variation</u>: Since interest is a key factor in making practice effective, variation should be introduced from time to time. Use of the tape recorder for teacher-directed practice requiring a written response is advocated. Card games for small group practice can be designed to follow the rules for Old Maid or Spoof. The use of multiplication records is frequently very helpful.

<u>Lattice Method of Multiplication</u>. To multiply 24 x 683, the 2 square by 3 square lattice is used. The three digits of the multiplication are arranged in order from left to right

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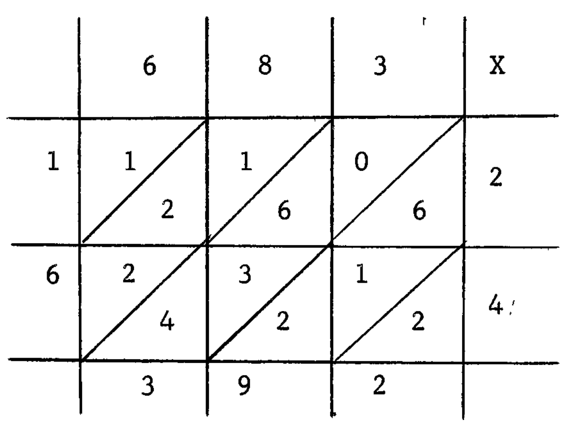
[53]<u>Ibid</u>., p. 87.

at the top of the rectangle, while the digits of the multiplier are arranged in sequence from top to bottom on its right-hand edge.

Each square of the lattice receives the product of its particular column digit and row digit, with the unit value written below the diagonal and the tens digit (when present) above the diagonal. Starting at the top right, combinations of 2 x 3 = 6 is written in its coordinating square below the diagonal. Then going to the left in this top row to the second square, 2 x 8 = 16. The six is written below the diagonal and so on until all squares (or all combinations) have been filled. The ultimate product is found by starting at the lowest right-hand square and adding diagonally, writing the digit when necessary into the next diagonal. Thus we see 24 x 683 = 16,392.




Russian Peasant Method: Divide one factor by 2, multiply the other factor by 2 and disregard the remainder--continue dividing the columns by 2 and multiplying the other by 2 until the last number in the column being divided is one. Next, mark out the even numbers in the column being divided by 2 and also the corresponding numbers in the column being multiplied by 2. Add the remainder in the column being multiplied for the factor.

| 146 x 12 | |
|---|---|
| ~~146~~ | ~~12~~ |
| ~~72~~ | ~~24~~ |
| ~~36~~ | ~~48~~ |
| ~~18~~ | ~~96~~ |
| 9 | 192 |
| ~~4~~ | ~~384~~ |
| ~~2~~ | ~~768~~ |
| 1 | 1536 |
| | 1728 |

## D. Division

Since division is ordinarily the most difficult of the four basic processes for the retarded child to grasp, the teacher must prepare the child for the concepts through many informal problem solving situations long before the process is formally introduced. Using objects, real and pictured, the children are challenged to figure out how many cars will be needed to transport them in groups of four or five on an experience trip, how many times a simple recipe must be increased if taken one time it will serve four or how many cookies which child should receive to share the supply equally. Problems of both measurement and partition types are used in the informal readiness.

> Research over a long period of time has brought us to the recognition that the technical aspects of arithmetic

can be taught better if they are related to the social activities of daily life.[54]

Division is a faster process of taking apart a large single group into two or more equal-sized groups. The number of items before and after division is the same.[55]

Division is introduced formally in a problem solving situation of the measurement type. For example the classroom store might provide the setting.[56] Candy is on sale in our store today for two cents apiece. How many pieces can you buy with ten cents? Pennies are then deducted in groups of two until the division is complete. The teacher asks, "How many 2's are there in 10? The number of 2's in 10 can be found by using pictures as shown:

oo/oo/oo/oo/oo

Number Line. The number line may be used to illustrate the idea of division as successive subtraction of equal groups, which is measurement division.

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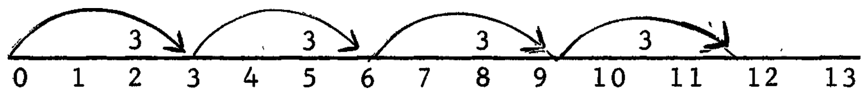
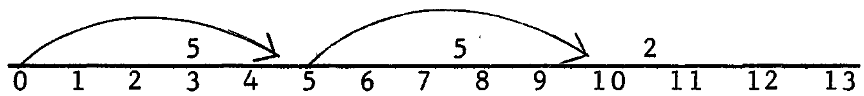
[54] Leo J. Brueckner, The Three R's Plus, (Minnesota: University of Minnesota Press, 1961), p. 155.

[55] Swenson, op. cit., p. 66.

[56] Francis W. Rahn, "Learning Elementary Arithmetic By Means of the General Store," Journal of Exceptional Children, March, 1960, pp. 15-20.




12 ÷ 4 = 3  Because we can subtract four groups of 3




12 ÷ 5 = 2  Remainder 2, we can subtract 2 groups of 5 and have 2 left

This is incomplete division

Initial instruction with the algorithm should stress two principles, the distributive property and division or successive subtraction. For example, in finding $2\overline{)84}$ the child should see the problem as finding $2\overline{)80}$ and $2\overline{)4}$. The dividend is expressed as 80 + 4 and the divisor distributes over the tens. The first step should be:

$$\begin{array}{r} 2\overline{)84} \\ \underline{80} \\ 4 \end{array}$$

The child's thinking is that "Forty 2's is 80, minus forty 2's leaves 4". The second step becomes:

$$\begin{array}{r} 2 \\ 40 \\ 2\overline{)84} \\ \underline{80} \\ 4 \\ \underline{4} \\ 0 \end{array}$$

The child thinks, "Two 2's in 4, take away 4.[57]

Incomplete division should be introduced early; the existence of a remainder helps to develop insight into what is happening as the division is performed--that is, successive subtraction of the divisor. When that which remains is less than the divisor we can no longer subtract the divisor.[58]

To divide 698 by 3 we think of 698 as 600 + 90 + 8.

```
      2
     30
    200
3 ) 698
    600
    ---
     98
     90
     --
      8
      6
      -
      2
```

Another way of illustrating the same concept easily in small numbers is this:

```
    1 + 1 + 1 = 3
8 ) 24
     8
    --
    16
     8
    --
     8
     8
     -
     0
```

---

[57] J. Houston Banks, Learning and Teaching Arithmetic (Boston: Allyn & Bacon, Inc., 1960), p. 216.

[58] Swenson, *op. cit.*, p. 67.

Or it can be illustrated by using larger numbers and subtracting the divisor ten or one hundred tens at a time, thus:

```
      100 + 100 + 5 = 205
24)4926
   2400
   2526
   2400
    126
    120
      6
```

Still another way to illustrate this concept is as follows:

```
8)24  1
   8
  16  1
   8
   8  1
   8
   0  3
```

Signs and Terms: The sign $\overline{)\quad\quad}$ is introduced along with the concept of division; the alternate division sign ÷ is taught when the horizontal form of the division tables is presented. Terms to be mastered in oral and written form are "divide," "quotient," "dividend," "divisor" and "remainder."

<u>Two-figure Dividend and Quotient</u>. Two-place dividends, both digits of which contain the divisor evenly, are introduced in a problem solving content which is illustrated with counting sticks. For example the teacher might challenge the group to find how 48 cookies should be equally placed on

two plates. Using sticks, bundles, and loose sticks to illustrate, the children are guided to find that 4 tens divided by 2 are 2 tens; 8 ones divided by 2 are 4 ones; 2 tens and 4 ones are 24. Written record made by the teacher on the chalkboard is this:

$$\begin{array}{rl}
 & \underline{2 \text{ tens, } 4 \text{ ones}} = 24 \\
2) & 4 \text{ tens, } 8 \text{ ones} \\
 & \underline{4 \text{ tens} \phantom{, 8 \text{ ones}}} \\
 & \phantom{4 \text{ tens, }} 8 \text{ ones} \\
 & \phantom{4 \text{ tens, }} \underline{8 \text{ ones}}
\end{array}$$

The teacher explains the process fully, then tells the children that this problem can be written using numbers only. For example:

$$\begin{array}{rl}
 & 4 \\
 & \underline{20} = 24 \\
2) & 48 \\
 & \underline{40} \\
 & 8 \\
 & \underline{8} \\
 & 0
\end{array}$$

48 means 4 tens; 4 tens divided by 2 are 2 tens or 20; 2 times 4 is 8; 20 and 4 are 24. This form is used until the teacher is certain that the child has a thorough grasp of the reasoning processes of division.

Three-figure Dividend and Quotient. The three-figure dividend resulting in a three-figure quotient is next presented according to the pattern given for the table of 2's.

```
      3
     40
    200  = 243
 2)486
    400
     86
     80
      6
      6
      0
```

<u>Dollars and Cents</u>. Extension of the drill of using a three-figure dividend and resulting in a three-figure quotient is next presented. The division table of 4's is presented according to the pattern given for the table of 2's.

<u>Averages</u>. Conceptual background and conversational use of the term "average" as usual or ordinary is introduced on this level, but no formal problem solving is done concerning the concept. Batting averages of baseball players, average height and weight, average rates of speed and the like are discussed, as well as the method of gaining these figures through column addition and division.

The use of the newly acquired tool over and over again in many and varied situations makes it a permanent part of the child's knowledge. The mentally retarded pupil is, of all children, most in need of such repetition if he is to acquire a given skill. Short practice periods have a place in arithmetic as in other fundamental processes and can be, in practically every case, based upon the content of the unit of experience. One may, for instance add 3 cupfuls of fruit juice and 2 cupfuls of water to make a pitcher of fruit punch, make a garden path 3 feet wide and 8 feet long, or divide a day's wages into halves.

> Through countless applications the attainment of skill in the fundamental processes can become for the child an interesting, purposeful activity rather than a meaningless process.[59]

[59] Martens, *op. cit.*, p. 248.

# CHAPTER IV

## PROBLEM SOLVING

The major objectives of programs provided for educable mental retardates emphasize the development of satisfactory social adjustment and relationships physical competencies and desirable health habits, the appropriate use of leisure time, the acceptance of home responsibilities, and the attainment of vocational proficiency.[60] Therefore, it will be in these particular areas of experience that the teacher will find arithmetical problems for use in the classroom. Every opportunity should be given the child to practice his number skills and to use and expand his number knowledge in the above areas, because they are the areas in which the need for competence will arise. Such practice will familiarize the child with their use in real-life situations. Kirk and Johnson support this notion with the following assertions:

> Rather than the verbal or theoretical substitutes presented in the typical arithmetic text, problems the children are encountering in their work situations should be used. In school situations, the boys will have use for arithmetical concepts and skills in conjunction with their manual training, home mechanics, and in the study of occupations. Similarly, the girls will have need for them in cooking, sewing, homemaking, and in the study of

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[60] Samuel A Kirk and G Orville Johnson, Educating the Retarded Child (Boston: The Riverside Press, 1951), p. 49.

> occupations and vocations in their areas of interest. By including these skills as an integral part of their work we are depending less upon the child's ability to transfer his knowledge and skills from school to make the transfer by explaining the relationships and associations, and training their ability to make generalizations in the future.[61]

*One-step Problems*. These problem types are introduced in the context of situations that the child is currently encountering. As the child's maturity increases situations presented focus more and more on situations that will need to be met in adult life, as keeping records of expenditures, totaling grocery bills, restaurant checks, clothing expenses, and other bills. Data for formulating these problems can be obtained from price lists, newspaper ads, catalogues and menus.

*Problems in Handling Money*. Much practice, using real money, is usually necessary before the retarded child is able to find correct amounts of change with accuracy and reasonable speed. Specific directions should also be given in careful handling of money, in honesty, and in regard to where reliable advice can be obtained. Concepts of thrift, wise spending, investing, and saving are introduced. The purpose and recording of a bank account are also explained.

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[61] *Texas State Plan for Special Education*, (Austin: Texas Education Agency, February, 1965), p. 1.

Two-step Problems. Problems of a more complex nature are introduced when an appropriate occasion arises to provide a situation having significance to the child. The fact that there are two questions involved must be clearly explained. An example might be a classroom party. Our cake will cost $ .89 and we need 15 bottles of soda water at $ .05 a bottle. How much will the soda water cost? What will be our total bill?

Converting Measures. Introduction of each new measurement concept and further development of those previously presented should be followed by practice in solving problems requiring the converting of large measures to smaller measures, and also converting small units of measure to larger ones. Equivalence of various units of measure is emphasized as a purchasing skill. (How much can be saved through buying by the gallon?)

Purchasing in Groups. The significance of 2 for a dime, 2 for a quarter, and the like, is illustrated in purchasing situations. The reversal of this skill, finding the cost of one article when the price is listed at 2 for, or 3 for, is also taught.

Interpretation of Facts. Interpretation of the problem situations, aided by vocabulary clues, is indispensable for independent problem solving; therefore, frequent practice periods should be devoted to discussing problems, problem solving techniques, vocabulary, and checking for the correct solution. The fact that there may be more than one way to arrive at a correct solution should naturally become evident to the child during the course of the discussions.

## A. Addition

Social situations which the child encounters offer rich sources of material for arithmetic lessons. When to use which process, so frequently a puzzle to the retarded child, is best illustrated in a problem solving situation which has personal meaning for him. At times the situation might call for a solution requiring a skill the child does not yet possess. As a teacher-directed activity, emphasis is placed on the usefulness of skill in arithmetic in meeting such situations. Few retarded children can solve problems of any degree of complexity. They do not know which process to use, or they lose the trail of the problem and so get the process confused.[62]

Vocabulary clues are stressed and some simple speculation is stimulated as to the reasonableness of the proposed answer. Particularly challenging are those problems in which the facts presented have the same label. Clues presented

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[62] J. E. Wallace Wallin, Education of Mentally Handicapped Children (New York: Harper & Brothers, 1955), p. 189.

include: How many in all? How many have both? How many altogether? What is the total amount, number or cost? How many in both groups? (Unequal groups having the same label.) Terms developed are: plus, sum, and total.

## B. Subtraction

Social situations which personally affect the child are chosen as contexts for problem solving in subtraction. Besides solving real-life problems and teacher formulated problems, practice should also be given to the child in formulating problems of his own.

The number work in the schoolroom should be thoroughly functionalized and socialized, by means of meaningful experiences provided in the classrooms. The teacher should plan to supply day by day experiences that will develop concepts of numerical relationships of temporal and spatial relationships, and of various units of measure, and provide abundant practice in solving the kinds of problems that the child has to wrestle with.

Problems should center on variations of how many are left, how much change is received, exact differences in comparisons, how many or how much more is needed. Problems should also provide practice in finding the missing number or other part. (Mother made ham sandwiches and egg sandwiches. She made 24 sandwiches; 16 are ham, how many egg sandwiches did she make?)

## C. Multiplication

Problems having equal groups with the same label frequently prove confusing. Extended practice in interpreting presented facts might be necessary before the child can independently and accurately ascertain when to add, when to multiply and when to divide. Vocabulary clues might be how much or how many in each? (of equal groups); if one item costs contains or requires a certain amount, how much or how many would 2, 6, etc., cost, contain, or require? The meaning of doubling and tripling is introduced as well as "a" or "an" (as in so much an ounce, etc.), "each" and "apiece."

## D. Division

Problem solving practice in division should include both measurement and partitive types. Sharing problems are presented both as dividing into equal parts or shares and as finding fractional parts (1/2, 1/3, 1/4). Children should also be challenged to find the price of one article when the total of several like articles is known.

Attention is given to problems requiring the label of the answer to be grouped when other numbers in the problem share a different label. (Four children can work together at a table for art class; since we have 16 children in our class, how many tables must we have?)

Lessons in problem solving must be held very frequently. The teacher should have a specific, clear objective for each, a specific vocabulary term, and one certain type of problem.

> The child's first goal is to gain insight into the meaning of mathematical concepts--quantitative relationships, symbols, terms, processes and measure. His second goal is to employ mathematical processes with precision in meaningful situations until he has achieved manipulative mastery.[63]

The problems and problem types are introduced in the context of situations the child is currently encountering. In helping the retarded child to improve his ability to solve verbal problems, it is most helpful to devote some time each week to problem analysis and to the improvement of arithmetic reading.

A method which may be helpful in developing ability to solve verbal problems is, showing the child a picture involving some number operation, such as a little girl picking flowers, which shows seven in her hand and two yet to be picked. The child is asked to tell the story of the picture. After some practice the child will probably end his story with the equation seven flowers plus two flowers equal nine flowers. A second method is to allow the child to make up an oral, verbal problem for some of his computational drill

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[63] Wallin, op. cit., p. 339.

work based upon his own experiences, real or imaginary. After a few short periods of this type, the child seems to develop more understanding of what is expected of him when he is given verbal problems to solve.

An important factor which should be kept in mind in helping the retarded child to improve his ability to solve verbal problems is that all objects used in the problem should be familiar to the child. For example, it is better to use the expression "bundles or sticks" rather than "faggots" or "horses" rather than "llamas."

Another point to be remembered is that all unnecessary details should be excluded. It used to be the practice to include many irrelevant details with the idea that if the child understood the problem, he would be able to pick out the important facts and reject the unimportant details. However nothing could be more useless and farther from the real life situations with which the child comes in contact.

There are many methods that have been used in helping the child to solve verbal problems. Some are good, and some can be improved upon. No perfect method has yet been found. The verbal problem is still the greatest failing of most classroom teachers, and it is usually their greatest concern in the teaching of arithmetic.

CHAPTER V

# MEASUREMENT, FRACTIONS, AND DECIMALS

## A. Measurement

The understanding of units of measurement is a much-needed concept for the mentally retarded child. In practice this understanding includes mathematical vocabulary, the number skills, and the ability to apply them. For the solution of certain types of problems a mere understanding of the relationships may be sufficient; whereas for others the individual may need to use one or more of the fundamental arithmetical skills.

The pupils themselves are able to transfer their skill and knowledge to actual use by actively participating in situations in the classroom similar to the ones they will face outside of school in adult life.[64] Many arithmetic problems can be taken from newspapers and catalogues, such as the price of a turkey at Thanksgiving and the price of Christmas toys; the meanings of dozen and half dozen are

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[64] Samuel A. Kirk, Educating Exceptional Children. (Boston: Houghton Mifflin Co., 1962), pp. 105-132.

shown from an empty egg carton, and the meaning of a ruler and measurements may also be demonstrated.[65]

The child should acquire a working vocabulary of the arithmetical terms he will encounter in the everyday social and economic worlds. Since the whole program of the education of the mentally retarded is based on the premise that getting along with his fellows socially and economically is the prime consideration in his education, this vocabulary must be limited to those terms used in daily living at his state of development. Such a vocabulary might include the following, which was extracted from Kirk and Johnson:

(1) Terms relating to size, such as big, small, huge, and tiny.

(2) Terms relating to length and distance such as inch, foot, near, far, long, and short.

(3) Terms relating to other types of measurement such as penny, nickel, dime, quarter, half-dollar, dollar, dozen, ounce, quart, cup, and pound.

(4) Terms relating to amount, such as more, less, increase, decrease, some, none, apiece, double, twice, each, enough, both, many, pair, part, and half.

(5) Terms relating to locations, such as up, down, above, below, right, left, under, over, at the beginning, at the end, and between.

(6) Terms relating to time, such as early, late, soon, on time, day, week, months, and year.

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[65] Curriculum Guide, Special Education, (Austin: Texas Education Agency, 1960), p. 95.

(7) Terms relating to comparisons, such as larger, smaller, longer, shorter, higher, lower, heavier, and lighter.

(8) Terms commonly used in commercial practice such as bill, cost, rent, lease, buy, earn, sell, change, expense, spend, charge, price, worth, lay-away, time-payments, and due.[66]

A little more concise vocabulary list as found in the Curriculum Guide is as follows:[67]

| | | |
|---|---|---|
| empty | one-fourth | half-pint |
| full | halves | cup |
| half | quart | inch |
| whole | pint | foot |

In addition to the teaching of the meanings of these words and the related concepts, the commonly used abbreviations should also be included.[68] These abbreviations might be ft., in., yd., qt., and so forth.

> Isolated facts are important, and their learning by a rote memory process is often necessary. Remembering your telephone numbers and street address are important items although they cannot be expected to produce any major change in over-all responses. In like manner many basic pieces of information must be committed to memory and a rote memory process must be used.[69]

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[66] Samuel A. Kirk and G. Orville Johnson, Educating the Retarded Child (Boston: The Riverside Press, 1951), p. 282.

[67] Curriculum Guide, op. cit., p. 96.

[68] Kirk and Johnson, op. cit., p. 283.

[69] Newell C. Kephart, The Slow Learner in the Classroom (Ohio: Charles E. Merrill, 1965), p. 67.

_Size_. Discussions of the interrelationship between size, bulk, weight, capacity and age, as well as the absence of this relationship in some instances, will be helpful in developing the child's ability to make reasonable quantitative estimates. Some practice should also be given in estimating comparative sizes of pictured objects. For example: Is our school as big as the Empire State Building? Rather grossly different objects should be chosen.

_Form_. Practice in using correct names of geometric forms--such as the circle, square, rectangle, triangle, and diamond can be correlated with practice in drawing lines with a ruler.[70] Recognition of these forms in a variety of positions is introduced, followed by teacher-directed discussion of where these forms are found in one's environment.

_Position_. Positional terms should be reviewed specifically a few times each year to prevent their falling into disuse. They should be incorporated whenever possible into directions for written work. Such terms might be as follows:

| | | |
|---|---|---|
| left | higher | after |
| right | lower | next |
| beside | highest | last[71] |
| before | lowest | |

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[70]Curriculum Guide, _op_. _cit_., p. 96.

[71]_Ibid_.

Left and Right. Left and right as positional terms (the tree on the left side of the building) and as directional terms (go two blocks to the right, then turn left), are reviewed as practice in giving directions. Places with which the child is well acquainted are chosen and turns are taken in giving imagined strangers exact directions for getting there. This is a learned activity. Kephart is of the opinion that:

> It is only by experimenting with the two sides of the body and their relationship to each other that we come to distinguish between the two systems . . .[72]

Ordinals. Oral and written use of the ordinals through thirty-first is developed through daily calendar discussions: the ordinal position of letters in words correlates well with dictionary study, telephone book study, and alphabetizing. The concept of "middle of a group" is extended to groups of nine and eleven. Finding the middle of a square and rectangle having sides with even numbers of inches, by finding half the inches on a side, is presented as a teacher-directed activity. This activity serves as an introduction to the concept, not as a skill to be mastered at this level.

Linear Measure. Practice in making rough estimates and comparisons of the height of persons precedes formal

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[72] Kephart, *op. cit.*, p. 43.

introduction to measuring height in feet and inches. The child should be able to tell his own height with a fair amount of accuracy. He ought to have a reasonable concept of average adult height and of the height of familiar rooms.

> The importance of space is particularly obvious in numbers or arithmetic. Arithmetic, as Strauss and Lehtinen have indicated, is a visual-spatial problem. Stern has devised a method of teaching arithmetic based in large part on the spatial concepts involved. Mathematics deals with groups of objects and the characteristics of groups and grouping phenomena. If the child has not developed an adequate space world, he will have difficulty in dealing with grouping phenomena, since groups can only exist in space.[73]

Yardstick. The yardstick, its reading and use are introduced in a problem-solving situation. A situation formulated by the teacher to demonstrate the use of the ruler is at times inadequate. Children are guided to find that a yard is three feet long by placing a three-foot-ruler on it. Some time should be devoted to becoming familiar with the markings on the yardstick, to reading these markings, and to discovering the practical uses of the yardstick. The concept that a yard is 36 inches long receives emphasis.

Measuring. Another period of time is devoted to measuring with the yardstick, emphasis being placed on matching the end of the yardstick with the edges of the object measured. Objects selected should be exactly a yard, less than two feet,

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[73] *Ibid.*, pp. 93-94.

less than three feet, or more than a yard. The concept that a yard is three feet is once more emphasized. Drawing lines to specified lengths, stated in inches, is next developed.

Feet and Inches. Converting inches to a statement of feet and inches is introduced and practiced in reading markings, measuring objects and drawings of lines stated in feet and inches. The use of feet and inches in measuring height is also presented.

One-fourth Inch. Review of 1/4 of a whole precedes introduction to measuring in 1/4 inches. Reading and writing of 1/4, 3/4, and the substitution of 1/2 for 2/4, are also reviewed. One-fourth and three-fourths inch are introduced in the same manner as the whole inch, namely by reading of the ruler markings, measuring objects and drawing lines to a specified length.

Terms of Comparison. Oral and written knowledge is developed concerning the terms tall and short, and their comparative and superlative forms, as well as the terms foot, feet, inch, inches, and ruler. Acquaintance with the appearance and use of the tape measure is also reviewed.[74]

Abbreviations. The abbreviations in., ft., yd., are introduced for use in reading and writing terms. These are including the terms already specified above.

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[74] Ibid., pp. 199-202.

Length. The concept of length is refined through problem-solving measurement situations that require use of the terms mentioned in Terms of Comparison. Kephart discusses the problem of teaching these terms to the child in the Slow Learner in the Classroom:[75]

> Many children who can adequately produce the vertical and horizontal lines aloud will be found to have difficulty orienting these two lines to each other in the proper relationship. Thus, the horizontal line may be placed too high or too low on the vertical axis or the horizontal line may be too long either on the left or, more frequently, on the right. These deviations indicate that the child is having difficulty with the problem of the relationships between the parts of this figure and particularly with the problem of bisecting a line. In certain cases, where one arm is drawn markedly longer than its counterpart, he may also be having difficulty stopping.[76]

Kephart discusses the long line in comparison with the short. The difference between two unequal lines are attacked in three areas: kinesthetic, tactual, and visual. When the child has become aware of "longer" and "shorter," he can move on the concept of "equal," which is neither longer nor shorter.

Width. Practice in finding which is the length and which is the width of clearly rectangular objects is used for

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[75] Ibid., p. 203.

[76] Ibid.

development of the terms wide, narrow, wider, widest, too wide, not wide enough, and too narrow.

Depth. Conversational knowledge of the concept of depth in the third dimension, (not in these exact words), is developed through examples drawn from the child's experiences. Depth of containers, particularly of liquids, is discussed. Terms: deep, deeper, and not as deep, and shallow are used.

Distance. The term "blocks" as a measure of distance is introduced by having children describe in terms of blocks the distance to various well-known, nearby places. Comparison of blocks and mile, and of both of these with feet, is presented conversationally. The abbreviation mi. is presented as a reading and writing term. The representation of distance as shown on maps is introduced, and comparisons of distances thus pictured are made. The term "miles an hour" is developed; conversational acquaintance with the meaning of speed limit signs along roads is introduced.

Time. The clock is an instrument which translates time into changes in space. The hand moves around the face of the clock, and from an observation of the extent of this movement, we deduce the passage of time which has elapsed.

Review of telling time on the hour should take place daily in an informal way, and as a formal instruction subject approximately once a month. The term "o'clock" should be mastered in oral and written use.

Next, mastery of telling time on the half-hour is sought. Brief reviews of telling time on the hour and of half of a whole circle, using the vertical division, preface the lesson. Hands of the clock are introduced as the hour hand and the minute hand, and the concept of an hour as being composed of 60 minutes is presented.

The position of the minute hand and its indication of half-past the hour are developed before the movement of the hour hand is discovered. An educational clock with movable hands serves best at this stage of instruction. A real clock with large face will illustrate clearly the amount of movement of the hour hand and its synchronization with the minute hand. Correlation should be made throughout the instruction between time recorded on the clock and the child's daily schedule.

After the concept of one-fourth of a whole has been presented, a brief introduction to telling time on the quarter hour is given. A large clock face with no hands is drawn on the chalkboard. The teacher divides the circle vertically and rapidly reviews half of a whole, then goes on to divide the circle horizontally into fourths. The teacher reminds the child that the terms "one-fourth" and "quarter" are synonymous, then draws the hour hand to point to a quarter past twelve and the minute hand to the three. After the position of the minute hand has been thoroughly discussed, it is erased and drawn to the nine. The hour hand is adjusted and the explanation of "quarter to" is made, with emphasis being placed on the number toward which the hour hand is moving.

A real clock is used then to illustrate the synchronization of the two hands, and the space between the two numbers on the picture clock on the chalkboard is divided to show the reason for the terms quarter to the hour and quarter past the hour.

The term "quarter past the hour" is used for introducing the concept since the child knows the term half "past the hour." The term "quarter after the hour" can be substituted for "quarter past the hour."

Time-telling skills are extended to the five-minute interval on this level. Review of telling time on the hour, half-hour and quarter-hour; of reading and writing o'clock, half-past the hour, quarter to the hour, quarter after the hour, and the colon form for the hour and half hour are reviewed, as well as counting by 5's to 55.

After telling time to the five minute interval has been introduced on a large instructional clock, a real clock with a large, easily read dial should be used so that the synchronization of the hand will be apparent to the child. The child first learns to read the clock at various settings, to place the hands of an instructional clock (later of a real clock) to a specified time, then to read and write time when given in literal form (nine twenty-five) and in colon form (9:25). Specific instruction is given to present the identity of half-past the hour and :30; quarter after the hour and :15; quarter to the hour and :45. Television and radio guides and simple bus schedules might profitably be used for teaching reading of time stated in colon form.

Setting a Clock. The skill of setting a clock, its purpose and technique, correlate well with time-telling lessons. Practice should be given in setting wrist watches, alarm clocks and electric wall clocks. Social studies or guidance lessons on punctuality would correlate well and provide motivation to the introduction to the alarm clock. Robinson and Robinson state:

> Even such a simple act as going to the movies is much more feasible and pleasant if one can learn from the newspaper the time at which the feature begins, take along enough money for a ticket and popcorn, and recognize the President of the United States when he appears on the newsreel.[77]

Alarm Clock. Reading of the alarm setting is introduced (actual setting of the alarm is deferred for the time being), using different types of examples. Practice should also be given in winding watches and clocks, and in the use

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[77] Halbert B. Robinson and Nancy M. Robinson, The Mentally Retarded Child (New York: McGraw-Hill Book Company, 1965), p. 471.

of stop-watches, sundials, oven and egg-timers. Review of Roman numerals as found on the clock face should be developed to reading time to the five-minute interval on such a clock.

Time Measurement. Acquaintance with the second hand and concept of seconds are introduced. The facts that there are sixty minutes in an hour, and 24 hours in a day, are taught for mastery as well as the abbreviations min., hr., da., and so on. The technique of finding the number of minutes in a given interval of time is introduced with an instructional clock, and the interval is found by counting. Relative duration of varying lengths of time is refined to the use of five-minute intervals. (Which is a longer period, 1:10 to 1:15 or 1:00 to 1:30?)

A.M. and P.M. The terms noon and midnight are reviewed and the terms A.M. and P.M. are introduced in the context of activity carried on at various hours of the day. Simple plane, bus and railroad schedules, and advertisements of entertainments including mention of the performance time, radio and television guides, and theater programs can be used to enrich the concept. Names and sequence of the days of the week and their relation to today, tomorrow, and yesterday are brought to mastery by daily mention during calendar discussion. The terms "day after tomorrow," "day before yesterday," "week ago" and "week from today" are introduced during practice in identifying those days on the calendar. Counting weeks using ordinal terms and mastery of the concept of a week of seven days are developed gradually during the course of several months. Dated bulletin board items also help to sharpen the child's awareness of the measure of passage of time. Mastery in reading the names of the days of the week is also sought.

Week. Calendar study offers a concrete aid to the child in understanding the passage of time. Daily discussions on this level include counting how many weeks until birthdays or holidays, and ordinal naming of the week (first week of December, etc.).

Weeks in a Year. Terms such as "a week ago," or "three weeks from now" are used. A New Year bulletin board with twelve pages of the calendar displayed provides the material for discovering that there are 52 weeks in a year, or 365 days in a year. The use of terms "day before yesterday," and "day after tomorrow" is also continued.

Month. The sequence of the months and their placement in the seasons are frequently reviewed as an item of calendar discussion. A short form of writing the date (8-2-66) is practiced by dating each written assignment. This practice also aids the teacher in filing the child's work to ascertain his progress. Practice in reading dates, especially birth dates, should be provided. Introduction to the number of days in the various months is also made.

Abbreviations. Reading and writing abbreviations for the names of the months and of the days of the week are introduced for mastery at this level. The regular written word should also be mastered if at all possible.

Season. Passage of the seasons is discussed in terms of the months in each, the usual temperature and weather, activities appropriate to each, and holidays occuring during their intervals. This is not considered a problem in Texas; however there is a weather change at certain times of the year. The names of holidays occurring in each season might help distinguish one from the other to the child.

Year. The composition of the year in seasons, months, weeks and days is practiced by short daily period calendar discussions. Questions are varied, emphasizing each in turn until mastered. The term "leap year" is developed to include recognition of the extra day and the variations of the number of days in February.

Age. The child should know how old he is, the date of his birthday, and how old he will be at his next birthday. The concept of age is based on his own age. Guided discussions should be held to help him master the concepts of "older" and "younger" in reference to himself. He should have a reasonable idea of the ages of his siblings.

Speed. Distinctions between various speeds and their relation to the terms "R.P.M." and "miles an hour" should be fairly apparent to the child. The usual speed of various types of transportation is included. The astronauts would be an excellent example of speed. Skill in reading a speedometer is extended to exact miles per hour. A bicycle speedometer might be used for instructional purposes.

Temperatures. Televised or broadcast weather reports can provide much material for group discussion and clarification of such terms as "increasing cloudiness," "fair," "clearing," "clear," "humid," and "freezing." Discussions should encourage the child's use of the terms conversationally in describing a day in an interesting and accurate manner.

Freezing Point. The terms "freezing point at 32 degrees above freezing," "below freezing," "zero temperature," "above zero," "below zero," "mercury," and "room temperature" are introduced and developed. Daily weather discussion includes reading the outdoor thermometer which is fastened outside the classroom window within easy view of the children. Reading to the exact degree is expected.

Thermostat. In development of the term "room temperature," recognition of the appearance and use of the thermostat is explained in a very simplified manner. Much practice in reading thermostat settings is provided.

Money. Money concepts developed on this level include careful spending, checking one's change, simple ideas about budgeting and saving. Use of a classroom store where real money is earned, saved, spent and exchanged for articles that may be kept, supplemented by purchasing experience in a real store, will provide meaningful situations where change-making for amounts to a dollar will be practiced as a counting process. (Count from the cost of the article to the amount given the clerk.)

In playing store it is helpful to use forms which can be run off on the duplicator for children to make grocery lists and calculate costs. Use cans that have been opened from the bottom. Have the children practice writing checks to pay the total bill and use advertisements of "weekly specials."[78]

Making change for amounts for more than a dollar is practiced as subtraction progresses. Children are instructed to count change received after a purchase has been made.

Writing Dollars and Cents. Writing dollars and cents with the decimal point and dollar sign is practiced for mastery, first when no zeros occur in the tens place then when zeros do occur. Writing cents only, using a decimal point, is introduced. Begin with no zeros in the tens place.

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[78] Curriculum Guide, *op. cit.*, p. 110.

*Applications*. Meaning of the term "sale" and problems in finding out how much is saved are introduced. Some discussion should be held to introduce principles of wise buying. The children should learn that the cheapest in price is not always the most economical selection, and about buying in the off-season, being sure of the integrity of the merchant, and so on. Terms "bargain" and "price" are introduced; and "cheap," "expensive," "cost," "two for (group price)" are further developed. Acceptable behavior in stores is also taught, such as not squeezing fruit or handling articles that one does not intend to buy. Proper response to service offered and courtesy in declining unwanted articles are important points. Budgeting of weekly allowance introduces the idea of planned spending and is of great importance.

Play money may be substituted for real money for a time, but it is suggested that real money be used as a culminating activity. The use of play money for the mentally retarded child is discouraged. The use of actual coins is more meaningful.[79]

*Fares, Menus, Sales Slips*. Social situations should be structured occasionally, either as experience trips or as classroom dramatizations, to furnish experience in selecting items from a simple menu and figuring the total cost, and

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[79] *Curriculum Guide*, *op*. *cit*., p. 102.

also in correct payment of fares on buses, in purchasing tickets, and in reading sales slips. Mastery of these skills is slowly developed over a long period of time and can be correlated with various skills involving the basic processes and with social studies projects.

Weight. The child on this level should be able to state his weight with reasonable accuracy and to read a bathroom scale to the exact pound. Weights of objects are also discussed and compared in terms of "heavy" and "light" and their comparative and superlative forms. Gain and loss are presented for conversational use.

Half and Quarter Pound. Reading of the household scale to the half and quarter pound is practiced, but mastery of the latter is not expected on this level. Items commonly bought by the whole, half and quarter pound are discussed and weighed. The fact that there are sixteen ounces in a pound, and the abbreviations oz. and lb. are introduced and developed for mastery.

Ton. The concept of ton, its weight as 2,000 lbs., examples of things bought by the ton, and the abbreviation T. are introduced in a situation that has personal meaning for the children. The description of parts of a building the child can observe being constructed offers a good example.

Liquid Measure. A review of liquid measure concepts presented previously is developed, with many opportunities for the children to manipulate the liquid measure containers. The concept of "gallon" is introduced and developed in a social problem solving situation (beverages for a class party or the like), and then its relationship with quart, pint, and cup is developed. A quart is a quarter of a gallon, and 16 cup servings to 1 gallon are demonstrated with liquids. The terms "half-full" and "half-empty" are reviewed, and "one-fourth" and "three-fourths full" are introduced.

Abbreviations. The abbreviations ga., qt., pt., and c. as reading and writing terms are introduced. Liquid measure concepts are enriched by discussion and handling of things ordinarily purchased by the pint, quart, and gallon.

Dry Measure. The bushel basket, peck sack, fruit crate, and pint berry boxes are presented for recognition of the container and its usual contents; also they are satisfactory for conversational use as terms of measure.

Actual experience in situations where the measure is used in real life gives the measurement concepts practical value for the child. Including a recipe being increased for a classroom baking or cooking project or trips to the grocery store, where articles are purchased with real money and kept, are valuable experiences for the retarded child who often lacks the experiential background to make arithmetic meaningful.

B. Fractions

An understanding and use of simple fractional parts such as 1/4, 1/3, 1/2, and 3/4, is needed by mentally retarded children in everyday living. While a few of the higher mentally retarded group may be able to apply addition, subtraction, multiplication, and division facts to fractions, the need does not arise very often in the life of the mental retardate.

Half. The concept of half of a whole is reviewed by extending it to half-past the hour, half-inch, half-dollar, half-pound. Recognition of objects divided into halves horizontally, vertically, or diagonally, and methods of dividing objects in these ways are practiced. Review is also established for reading and writing half, one-half and 1/2.

One-third. One-third of a whole is introduced as a sharing situation with an object being divided so each of three children receive an equal part. Recognition of objects and forms divided horizontally, vertically or in Y is followed by practice in dividing objects, then pictured objects and forms, into thirds. Reading and writing one-third and 1/3, two-thirds and 2/3, are also introduced for the written record of the sharing activity.

One-fourth. One-fourth of a whole is introduced in the same manner as one-third. The term "quarter" is presented as synonymous with "one-fourth." Recognition of objects divided into fourths is established with the vertical, horizontal, and corner to corner diagonal and cross-cut division. Reading and writing of one-fourth and 1/4, three-fourths and 3/4 are presented. Telling time on the quarter hour, and the quarter of a dollar, are practiced based on the concept of one-fourth.

Three-quarters. Introduction is made of the quarter and three-quarters inch, and the quarter and three-quarter pound. "Three-quarters" is presented as synonymous with "three-fourths." Less emphasis is placed on 3/4 and 2/3 than on 1/4 and 1/3, but particularly in demonstrating with liquids, 2/3 and 3/4 are stressed.

Parts of a Group. Half of a group of 2 through 12 objects is reviewed and its relation to division emphasized. One-third or a group of 3, 6, 9 and 12, one-fourth of a group of 4, 8 and 12, and one-fifth of a group of 5 and 10 are introduced through use of objects. Their relationship to dividing by 3, 4 and 5 is stressed.

Half of Eight. The idea of half of eight is presented as a problem in sharing structured by the teacher, who challenges the child to find the solution. The problem is first worked out with representative objects (blocks or sticks), then by drawing the division of pictured objects, and finally by using the written form to record the activity; half of eight is four; four is 1/2 of eight.

Half of Ten. Half of ten is introduced in the same manner as half of eight. The nickel as being equivalent to half a dime, and the fingers on one hand as representing half of all one's fingers are brought out as illustrations.

Half of Twelve. Half of twelve is developed according to the plan for half of eight. The concept of dozen is reviewed and half dozen is introduced. Discussion is held concerning things that are ordinarily bought by the dozen and half-dozen. Six inches is introduced as being half of a foot.

Fractional parts as used in increasing and decreasing recipes are shown on charts which may be used successfully in the classroom. Recognition of parts of a whole can be illustrated by teacher-made cards showing circles, squares, triangles, diamonds, and rectangles divided into various fractional parts, one or more of the parts being colored to illustrate the fractions. Cards naming the fractions in numerals, and a second set naming them in words are made and children match the three types of cards. Commercial card sets might also be used.

## C. Decimals

The decimal point, called "cents point," is practiced in writing dollars and cents. Emphasis is placed on keeping it in the correct position. Reading and writing of dollars and cents is first introduced, then sums are written in dollars and cents with zero in the tens place, and finally written in cents only using the decimal point. This skill is merely introduced on this level, but the substitution of the cents point for the cents sign is carefully emphasized.

# CHAPTER VI

## SUMMARY AND CONCLUSIONS

This guide has sought to fill the gap that presently exists in the area of arithmetic curriculum for the educable mental retardate on the junior high school level of achievement. At present educators use the Special Education Curriculum Guide written by the Texas Education Agency and last published in 1960. This is a general outline of informational material which is to be taught within the educational years of the retardate. Since this guide is so general in nature, many interpretations can be derived, and the chance of poor curriculum organization in a single school district is obvious. However, it is virtually impossible for the Texas Education Agency to write a complete Curriculum guide for every area of special education on every level of development.

This guide presents an orderly sequence of arithmetical concepts and understandings, many of which are based on the same factors presented in the Texas Curriculum Guide, but which are much more specific in nature. Practical suggestions for enrichments of concepts, suggestions for methods and materials, and devices found especially helpful are included.

This guide is not intended to be a complete innovation to the area of special education, but rather a sequential curriculum which can be followed by the teachers of educable mental retardates in order to assure some consistency in the program, and to present a complete arithmetical curriculum for the child. A new approach can always be beneficial to a teacher of long-standing, and a flexible guide can relieve much tension and unsureness in the adjustment of the embarking teacher.

The material in this paper needs to be modified to suit the individual child. Some would take years to master the contents of this paper while others would do it in less time. Those who cannot write would have to postpone or eliminate the sections involving such skills. A lesson-by-lesson division of this paper would therefore not be practical. The exercises which are discussed in each unit of work are only suggestive. The teacher is expected to add to these exercises and to vary them.

Teachers should learn to start on the child's level of understanding and to accept his immature ways of thinking and not impose on him the adult methods of arithmetic. They must lead the child to work gradually toward more mature methods. Arithmetic is a developmental process, and mathematical understandings grow slowly. Uniform procedures in the teaching of arithmetic should be established throughout the school system.

This uniformity is very important and necessary if confusion and waste of time on the part of the child is to be avoided. It is essential for understanding and for development of interest in mathematics.

Teachers need to plan experiences that will provide specifically for what they want the child to learn. They must guide the child to discover relationships for himself. This systematic teaching must start in the primary levels and continue throughout his school career. Teachers must take advantage of incidental situations and correlate them with arithmetic whenever possible.

Teachers must lead the child gradually from concrete situations to abstract situations. Often when the child enters the school room, his attention is taken away from numbers and quantities and centered on symbols.

Drill has a place in an arithmetic program. It should come after understanding. Without understanding, drill sets up an impossible task for the learner. It causes the child to stay on an immature level in handling numbers. It works against the very nature of arithmetic as system of related ideas.

Success in mathematics throughout a school system depends upon the quality of work done in the primary levels. A well-informed, enthusiastic teacher can do much for her pupils.

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Intended to be used as a textbook in upper division and graduate courses on mental retardation. An introductory text in specified professional training programs designed to prepare students in mental retardation.

Rosenquist, Lucy Lynde. Young Children Learn to Use Arithmetic. Boston: Ginn and Company, 1949. 175 pp.

Deals with the principles, suggested program of the skills and understandings to be learned by children in kindergarten and grades I and II. Material is based on practical application and the psychology of learning.

Rothstein, Jerome H. Mental Retardation. New York: Holt, Rinehart & Winston, 1961. 38 pp.

Discusses various considerations in the teaching of retarded children. Emphasizes home-school relationship, developmental aspects, some skill areas, group work and school-community relationships.

Spitzer, Herbert F. The Teaching of Arithmetic. Boston: Houghton Mifflin Co., 1940. 397 pp.

Presents some of the teaching procedures given to the improvement of the understanding of arithmetic with emphasis being placed on "seeing through" a mathematical fact or process, or perceiving its simplicity and purpose.

State Plan for Special Education. Austin: Texas Education Agency, February, 1965. 53 pp.

Administrative guide and state plan for special education. Provides information for local school administrators, relative to policies for the initiation, organizing, and operation of special education classes in Texas.

Stolurow, L. M. "Teaching Machines and Special Education." Educational and Psychological Measurement. 1960. 20:429 19 pp.

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Strauss, A. A., and Laura E. Lehtinen. Psychopathology and Education of the Brain-Injured Child. New York: Grune and Stratton, 1947. 270 pp.

Identification and description of certain problems associated with brain-injury, plus principles and teaching of instruction in arithmetic fundamentals, reading and writing.

Swain, Robert L. Understanding Arithmetic. New York: Holt, Rinehart & Winston, 1960. 277 pp.

Orderly and systematic way of teaching and learning about the number of things--quantities, amounts, sizes--to elementary school children.

Swenson, Esther J. "Arithmetic for Pre-school and Primary Grade Children," The Fiftieth Yearbook of the National Society for the Study of Education. Chicago: The University of Chicago Press, 1951. 202 pp.

The requirements of the number system which the teacher must observe and teach and which the pupils must recognize and respect are discussed in detail.

Tizard, J., and F. M. Loos. "The Learning of a Special Relations Test by Adult Imbeciles." <u>American Journal on Mental Deficiency</u>. 1954. 59:85-90. 6 pp.
The learning of a complex problem by adult imbeciles and normal individuals.

Wallin, J. E. Wallace. <u>Education of Mentally Handicapped Children</u>. New York: Harper & Brothers, 1955. 394 pp.
Discusses the history and care of handicapped children in general and the theories of mental deficiency and education. A description of instructional procedures with the mental retardate; curriculum and objects of special classes.

Wheat, Harry Grove. <u>How to Teach Arithmetic</u>. New York: Row, Peterson, & Co., 1961. 438 pp.
Describes the work of teachers and pupils, and what teachers must do to set up, stimulate, and guide the learning activities of pupils. It tells what pupils must learn, what the order of their learning must be, and what they must do to learn. It is written for teachers of arithmetic and students who are preparing to become teachers.

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