Electronic Journal of Differential Equations, Vol. 2006(2006), No. 02, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

EXISTENCE OF LARGE SOLUTIONS FOR A SEMILINEAR ELLIPTIC PROBLEM VIA EXPLOSIVE SUB- SUPERSOLUTIONS

ZHIJUN ZHANG

Abstract. We consider the boundary blow-up nonlinear elliptic problems $\Delta u \pm \lambda|\nabla u|^q = k(x)g(u)$ in a bounded domain with boundary condition $u|_{\partial\Omega} = +\infty$, where $q \in [0,2]$ and $\lambda \geq 0$. Under suitable growth assumptions on k near the boundary and on g both at zero and at infinity, we show the existence of at least one solution in $C^2(\Omega)$. Our proof is based on the method of explosive sub-supersolutions, which permits positive weights $k(x)$ which are unbounded and/or oscillatory near the boundary. Also, we show the global optimal asymptotic behaviour of the solution in some special cases.

1. Introduction

The purpose of this paper is to investigate existence and global optimal asymptotic behaviour of solutions to the problems

$$\Delta u + \lambda|\nabla u|^q = k(x)g(u), \quad x \in \Omega, \quad u|_{\partial\Omega} = +\infty, \tag{1.1}$$

$$\Delta u - \lambda|\nabla u|^q = k(x)g(u), \quad x \in \Omega, \quad u|_{\partial\Omega} = +\infty, \tag{1.2}$$

where the boundary condition means $u(x) \to +\infty$ as $d(x) = \mathrm{dist}(x, \partial\Omega) \to 0$, Ω is a bounded domain with smooth boundary in $\mathbb{R}^N$ ($N \geq 1$), $q \in [0,2]$ and $\lambda \geq 0$. The solutions to the above problems are called 'large solutions' or 'explosive solutions'. Our assumptions on the function g are as follows:

- (G1) $g \in C^1([0,\infty))$ is non-decreasing on $[0,\infty)$, $g(s) \leq C_1 s^{p_1}$, for all $s \in (0,\infty)$ and $g(s) \geq C_2 s^{p_2}$ for large s, with $p_1 \geq p_2 > 1$ and C_1, C_2 are positive constants.
- (G2) $g \in C^1(\mathbb{R})$ is non-decreasing on $\mathbb{R}$, $g(s) \leq C_1 e^{p_1 s}$, for all $s \in \mathbb{R}$ and $g(s) \geq C_2 e^{p_2 s}$ for large $|s|$ with $p_1 \geq p_2 > 0$ and C_1, C_2 are positive constants.

We assume that $k \in C^{\alpha}_{\mathrm{loc}}(\Omega)$ for some $\alpha \in (0,1)$, is positive in Ω, and satisfies

- (K1) There exist constants C_1, C_2 such that $C_1(d(x))^{\gamma_2} \leq k(x) \leq C_2(d(x))^{\gamma_1}$, for all $x \in \Omega$ with $-2 < \gamma_1 \leq \gamma_2$.

When $\lambda = 0$, problems (1.1), (1.2) become

$$\Delta u = k(x)g(u), \quad x \in \Omega, \quad u|_{\partial\Omega} = +\infty. \tag{1.3}$$

2000 *Mathematics Subject Classification.* 35J60, 35B25, 35B50, 35R05.
Key words and phrases. Semilinear elliptic equations; explosive subsolutions; explosive superbsolutions; existence; global optimal asymptotic behaviour.

Submitted May 21, 2005. Published January 6, 2006.
Supported by grant 10071066 from the National Natural Science Foundation of China.

For $k(x) \equiv 1$ on Ω, $f(u) = e^u$ and $N = 2$, problem (1.3) was first considered by Bieberbach [21] in 1916. In this case, problem (1.3) plays an important role in the theory of Riemannian surfaces of constant negative curvatures and in the theory of automorphic functions. Rademacher [21], using the ideas of Bieberbach, showed that if Ω is a bounded domain in $\mathbb{R}^3$ with C^2 boundary, then problem (1.3) has a unique solution $u \in C^2(\Omega)$ such that $|u(x) + 2\ln d(x)|$ is bounded on Ω. In this case, this problem arises in the study of an electric potential in a glowing hollow metal body. For general increasing nonlinearities $f(u)$, $k(x) \equiv 1$ on Ω and a bounded smooth domain Ω, Keller [18] and Osserman [27] supplied a necessary and sufficient condition $\int^\infty 1/\sqrt{G(s)}\,ds < \infty$ where $G'(s) = g(s)$ for the existence of large solutions to problem (1.3). Later, Loewner and Nirenberg [23] showed that if $g(u) = u^{p_0}$ with $p_0 = (N+2)/(N-2)$, $N > 2$, then problem (1.3) has a unique positive solution u satisfying $\lim_{d(x)\to 0} u(x)(d(x))^{(N-2)/2} = (N(N-2)/4)^{(N-2)/4}$. In this case, the problem arises in the differential geometry. The asymptotic behaviour and uniqueness of solutions to (1.3) have been established in [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 22, 24, 25, 26, 29], where the uniqueness was derived through an analysis of the asymptotic behaviour of solutions near the boundary.

For $\lambda \neq 0$ and $k(x) \equiv 1$ on Ω, Bandle and Giarrusso [2] and Giarrusso [15, 16] established the asymptotic behaviour and uniqueness of solutions of (1.1) and (1.2). For more investigations of explosive problems for elliptic equations, we refer the reader to [5, 19, 20, 28, 30, 31, 32, 33].

Recently, the author [32] established an explosive sub-supersolution method for the existence of solutions to general elliptic problems with nonlinear gradient terms. Garcia-Melian [13] also established an explosive sub-supersolution method for the existence of solutions to (1.3). By constructing explosive subsolutions and explosive supersolutions, he showed the following results.

(I) If (K1) and (G1) are satisfied, then (1.3) has at least one positive solution $u \in C^2(\Omega)$ and satisfies
$$m[d(x)]^{-(2+\gamma_1)/(p_1-1)} \le u(x) \le M[d(x)]^{-(2+\gamma_2)/(p_2-1)}, \quad \forall x \in \Omega; \tag{1.4}$$
where m, M are positive constants with $m \le M$.

(II) If (K1) and (G2) are satisfied, then (1.3) has at least one solution $u \in C^2(\Omega)$ and satisfies
$$-m - (2+\gamma_1)/p_1 \ln d(x)) \le u(x) \le M - (2+\gamma_2)/p_2 \ln d(x), \quad \forall x \in \Omega. \tag{1.5}$$

In this paper, we extended the above results to problems (1.1) and (1.2). Let $w \in C^{2+\alpha}(\Omega) \bigcap C^1(\overline{\Omega})$ be the unique solution of the problem
$$-\Delta u = 1, \quad u > 0, \quad x \in \Omega, \quad u\big|_{\partial\Omega} = 0. \tag{1.6}$$

As is well known, $\nabla w(x) \neq 0$, for all $x \in \partial\Omega$ and $C_1 d(x) \le w(x) \le C_2 d(x)$, for all $x \in \Omega$, where C_1, C_2 are positive constants. Thus (K1) is equivalent to

(K2) $c_1(w(x))^{\gamma_2} \le k(x) \le c_2(w(x))^{\gamma_1}$, for $x \in \Omega$ with $-2 < \gamma_1 \le \gamma_2$.

For convenience in the following, we denote

$$|u|_\infty = \max_{x\in\bar\Omega}|u(x)|; \quad u \in C(\bar\Omega); \quad \beta_1 = \frac{2+\gamma_1}{p_1-1}; \quad \beta_2 = \frac{2+\gamma_2}{p_2-1};$$
$$c_0 = \min_{x\in\overline\Omega}[|\nabla w(x)|^2 + w(x)]; \quad C_0 = \max_{x\in\overline\Omega}[|\nabla w(x)|^2 + w(x)];$$
$$c_\beta = \min_{x\in\overline\Omega}[(1+\beta)|\nabla w(x)|^2 + w(x)]; \quad C_\beta = \max_{x\in\overline\Omega}[(1+\beta)|\nabla w(x)|^2 + w(x)]$$

for $\beta > 0$.

Our main results are summarized in the following theorems.

Theorem 1.1. *Under assumptions (G1) and (K2), if $0 \le q < \min\{p_2, \frac{2p_2+\gamma_2}{p_2+\gamma_2+1}\}$, then problem (1.1) has at least one positive solution $u_\lambda \in C^2(\Omega)$ for each $\lambda \ge 0$, and satisfies*

$$m[w(x)]^{-(2+\gamma_1)/(p_1-1)} \le u_\lambda(x) \le M[w(x)]^{-(2+\gamma_2)/(p_2-1)}, \quad \forall x \in \Omega; \tag{1.7}$$

where m, M are positive constants with $m \le M$.

Theorem 1.2. *Under assumptions (G1) and (K2), if $1 < q \le \frac{2p_1+\gamma_1}{p_1+\gamma_1+1}$, then problem (1.2) has at least one positive solution $u_\lambda \in C^2(\Omega)$ for each $\lambda \ge 0$, and satisfies (1.7).*

Theorem 1.3. *Under assumptions (G2) and (K2), if $0 \le q \le 2$, then problem (1.1) has at least one solution $u_\lambda \in C^2(\Omega)$ for each $\lambda \ge 0$, and satisfies*

$$-m - (2+\gamma_1)/p_1 \ln w(x)) \le u_\lambda(x) \le M - (2+\gamma_2)/p_2 \ln w(x), \quad \forall x \in \Omega, \tag{1.8}$$

Theorem 1.4. *Assume (G2) and (K2).*

(I) *If $1 < q \le 2$, then problem (1.2) has at least one solution $u_\lambda \in C^2(\Omega)$ for each $\lambda \ge 0$, and satisfies*

$$-m - \beta \ln d(x) \le u(x) \le M - (2+\gamma_2)/p_2 \ln d(x), \quad \forall x \in \Omega; \tag{1.9}$$

where $\beta \in (0, (2+\gamma_2)/p_2)$ is small enough.

(II) *If $0 < q \le 1$, then problem (1.2) has at least one solution $u_\lambda \in C^2(\Omega)$ for each $\lambda \in [0, \lambda_0]$, and satisfies (1.8), where*

$$\lambda_0 = ((2+\gamma_1)/p_1)^{1-q} \frac{cc_0}{|w|_\infty^{2-q}|\nabla w|_\infty^q},$$

with $c \in (0,1)$.

The outline of this article is as follows. In section 2, we prove Theorems 1.1–1.4. In the final section, we give two examples.

2. Proofs of theorems

First we introduce an explosive sub - supersolution method. We consider the following general problem

$$-\Delta u = f(x, u, \nabla u), \quad x \in \Omega, \quad u|_{\partial\Omega} = +\infty, \tag{2.1}$$

where $f(x, s, \eta)$ satisfies the following conditions:

(F1) $f(x, s, \eta)$ is locally Hölder continuous in $\Omega \times I \times \mathbb{R}^N$ and continuously differentiable with respect to the variables s and η

(F2) There exists increasing function $h \in C^1([0,\infty), [0,\infty))$ such that

$$|f(x, s, \eta)| \le h(|s|)(1 + |\eta|^2), \quad \forall (x, s, \eta) \in \Omega \times I \times \mathbb{R}^N$$

(F3) f is nondecreasing in s for each $(x,\eta)\in\Omega\times\mathbb{R}^N$; where $I=[0,\infty)$ or $I=\mathbb{R}$.

Definition. A function $\underline{u}\in C^2(\Omega)$ is called an explosive subsolution of (2.1) if

$$\Delta\underline{u}\ge f(x,\underline{u},\nabla\underline{u}),\ x\in\Omega,\quad \underline{u}\big|_{\partial\Omega}=+\infty. \tag{2.2}$$

Definition. A function $\overline{u}\in C^2(\Omega)$ is called an explosive supersolution of (2.1) if

$$\Delta\overline{u}\le f(x,\overline{u},\nabla\overline{u}),\ x\in\Omega,\quad \overline{u}\big|_{\partial\Omega}=+\infty. \tag{2.3}$$

Lemma 2.1 ([32, Theorem 4.1]). *Suppose that* (2.1) *has an explosive supersolution* $\overline{u}$ *and an explosive subsolution* $\underline{u}$ *such that* $\underline{u}\le\overline{u}$ *on* Ω*, then* (2.1) *has at least one solution* $u\in C^2(\Omega)$ *satisfying* $\underline{u}\le u\le\bar{u}$ *on* Ω.

For proving our main results, we use the above lemma; i.e., we construct an explosive supersolution $\overline{u}$ and an explosive subsolution $\underline{u}$ such that $\underline{u}\le\overline{u}$ on Ω.

Proof of Theorem 1.1. For $0\le q<\min\{p_2,\frac{2p_2+\gamma_2}{p_2+\gamma_2+1}\}$, let $\underline{u}=m(w(x))^{-\beta_1}$, where m is a positive constant satisfying $C_1C_2m^{p_1-1}\le\beta_1c_{\beta_1}$. Then

$$\begin{aligned}\Delta\underline{u}&=m\beta_1[(1+\beta_1)|\nabla w(x)|^2+w(x)](w(x))^{-2-\beta_1}\\&\ge m\beta_1c_{\beta_1}(w(x))^{-2-\beta_1}\\&\ge C_1C_2m^{p_1}(w(x))^{\gamma_1}(w(x))^{-\beta_1p_1}\\&\ge k(x)g(\underline{u}(x)),\quad x\in\Omega;\end{aligned}$$

i.e, $\underline{u}=m(w(x))^{-\beta_1}$ is an explosive subsolution of (1.1).

Let $\bar{u}=M(w(x))^{-\beta_2}$, where M is a positive constant satisfying

$$C_1C_2M^{p_2-1}\ge\beta_2C_{\beta_2}+\lambda M^{q-1}\beta_2{}^q|w|_\infty{}^{2+\beta_2-(1+\beta_2)q}|\nabla w|^q_\infty.$$

We see that

$$\Delta\bar{u}+\lambda|\nabla\bar{u}|^q\le C_1(w(x)^{\gamma_2}C_2\bar{u}^{p_2}\le k(x)g(\bar{u})+\lambda|\nabla\bar{u}|^q,\quad\forall x\in\Omega;$$

i.e., $\bar{u}=M(w(x))^{-\beta_2}$ is an explosive subsolution of (1.1), Clearly $M\ge m$, i.e., $\overline{u}\ge\underline{u}$ on Ω. Hence the desired conclusion follows by Lemma 2.1. □

Proof of Theorem 1.2. For $1<q\le\frac{2p_1+\gamma_1}{p_1+\gamma_1+1}$. Let $\overline{u}=M(w(x))^{-\beta_2}$, where M is a positive constant satisfying $M^{p_2-1}\ge\frac{\beta_2C_{\beta_2}}{C_1C_2}$. Then

$$\begin{aligned}\Delta\overline{u}&=M\beta_2[(1+\beta_2)|\nabla w(x)|^2+w(x)](w(x))^{-2-\beta_2}\\&\le C_1C_2M^{p_2}(w(x))^{\gamma_2}(w(x))^{-\beta_2p_2}\\&\le k(x)g(\bar{u}(x)),\quad x\in\Omega;\end{aligned}$$

i.e., $\overline{u}=M(w(x))^{-\beta_2}$ is an explosive supersolution of (1.2). Let $\underline{u}=m(w(x))^{-\beta_1}$, where m is a positive constant satisfying

$$C_1C_2m^{p_1-1}-\lambda m^{q-1}\beta_1^q|w|_\infty^{2+\beta_1-(1+\beta_1)q}|\nabla w|^q_\infty\le\beta_1c_{\beta_1}.$$

We see that

$$\Delta\underline{u}+\lambda|\nabla\underline{u}|^q\ge C_2(w(x)^{\gamma_1}C_1\underline{u}^{p_1}\ge k(x)g(\underline{u}),\quad\forall x\in\Omega;$$

i.e., $\underline{u}=m(w(x))^{-\beta_1}$ is an explosive subsolution of (1.2). Clearly $M\ge m$, i.e., $\overline{u}\ge\underline{u}$ on Ω. Hence the desired conclusion follows by Lemma 2.1. □

Proof of Theorem 1.3. For $0 \le q \le 2$. Let $\underline{u} = -m - (2+\gamma_1)/p_1 \ln w(x)$, where m is a positive constant satisfying

$$e^{-mp_1} \le \frac{(2+\gamma_1)c_0}{C_1C_2p_1}.$$

Then

$$\begin{aligned}\Delta\underline{u} &= (2+\gamma_1)/p_1(|\nabla w(x)|^2 + w(x))(w(x))^{-2}\\ &\ge C_2(w(x))^{\gamma_1}C_1e^{-mp_1}(w(x))^{-2-\gamma_1}\\ &\ge k(x)g(\bar{u}(x)), \quad \forall x \in \Omega;\end{aligned}$$

i.e.,, $\underline{u} = -m - (2+\gamma_1)/p_1 \ln w(x)$ is an explosive subsolution of (1.1). Let $\bar{u} = M - (2+\gamma_2)/p_2 \ln w(x)$, where M is a positive constant satisfying

$$C_1C_2e^{Mp_2} \ge C_0(2+\gamma_2)/p_2 + \lambda((2+\gamma_2)/p_2)^q|w|_\infty^{2-q}|\nabla w|_\infty^q.$$

Then

$$\Delta\bar{u} \le k(x)g(\underline{u}(x)) + \lambda|\nabla\bar{u}|^q, \quad \forall x \in \Omega;$$

i.e., $\bar{u} = M - (2+\gamma_2)/p_2 \ln w(x)$ is an explosive superbsolution of (1.1). Clearly, $\overline{u} \ge \underline{u}$ on Ω. Hence the desired conclusion follows by Lemma 2.1. □

Proof of Theorem 1.4. For $0 < q \le 2$. Let $\overline{u} = M - (2+\gamma_2)/p_2 \ln w(x)$, where M is a positive constant satisfying

$$C_1C_2e^{Mp_2} \ge C_0(2+\gamma_2)/p_2.$$

Then

$$\begin{aligned}\Delta\overline{u} &= (|\nabla w(x)|^2 + w(x))(2+\gamma_2)/p_2(w(x))^{-2}\\ &\le C_1(w(x))^{\gamma_2}C_2e^{Mp_2}(w(x))^{-2-\gamma_2}\\ &\le k(x)g(\bar{u}(x)), \quad \forall x \in \Omega;\end{aligned}$$

i.e, $\overline{u} = M - (2+\gamma_2)/p_2 \ln w(x)$ is an explosive supersolution of (1.2).

We need to construct an explosive subpersolution of (1.2).

Case (I) $1 < q \le 2$. Let $\underline{u} = -m - \beta \ln w(x)$, where $\beta \in (0, (2+\gamma_1)/p_1)$ is small enough such that

$$c_0\beta/2 \ge \lambda\beta^q|w|_\infty^{2-q}|\nabla w|_\infty^q$$

and m is a positive constant satisfying

$$c_0\beta/2 \ge C_1C_2e^{-mp_1}|w|_\infty^{2+\gamma_1-p_1\beta}.$$

Then

$$\begin{aligned}\Delta\underline{u} &\ge \beta(|\nabla w(x)|^2 + w(x))(w(x))^{-2}\\ &\ge C_1(w(x))^{\gamma_1}C_2e^{-mp_1}(w(x))^{-p_2\beta} + \lambda\beta^q w^{-q}|\nabla w|^q\\ &\ge k(x)g(\underline{u}(x)), \quad \forall x \in \Omega;\end{aligned}$$

i.e., $\overline{u} = -M - \beta \ln w(x)$ is an explosive subsolution of (1.2). Clearly, $\overline{u} \ge \underline{u}$ on Ω.

Case (II) $0 < q \le 1$ and $\lambda \in [0, \lambda_0]$. Let $\underline{u} = -m - (2+\gamma_1)/p_1 \ln w(x)$, where m is a positive constant satisfying

$$(1-c)c_0(2+\gamma_1)/p_1 \ge C_1C_2e^{-m}.$$

Since $\lambda \in [0, \lambda_0]$, 1.e.,

$$cc_0(2+\gamma_1)/p_1 \ge \lambda((2+\gamma_1)/p_1)^q|w|_\infty^{2-q}|\nabla w|_\infty^q;$$

we see that

$$\begin{aligned}\Delta \underline{u} &\geq \beta(|\nabla w(x)|^2 + w(x))(w(x))^{-2}\\ &\geq C_1(w(x))^{\gamma_1} C_2 e^{-m}(w(x))^{-p_2\beta} + \lambda\beta^q w^{-q}|\nabla w|^q\\ &\geq k(x)g(\underline{u}(x)), \quad \forall x \in \Omega;\end{aligned}$$

i.e., $\underline{u} = -m - (2+\gamma_1)/p_1 \ln w(x)$ is an explosive subsolution of (1.2). Hence the desired conclusion follows by Lemma 2.1. □

3. Examples

As an applications of Lemma 2.1, we give the following two examples.

Example 3.1. Consider the problem

$$\Delta u = C_0(R^2 - r^2)^\sigma u^p, \quad u > 0, \quad x \in B_R, \quad u\big|_{\partial B_R} = +\infty, \tag{3.1}$$

where $p > 1$, C_0 is a positive constant, $-2 < \sigma$ and $B_R = \{x \in \mathbb{R}^N : ||x|| < R\}$ with $N \geq 2$.

Note that in the case of $\Omega = B_R$, $w(x) = w(r) = (R^2 - r^2)/2N$. We see that $\overline{u} = M(R^2 - r^2)^{-(2+\sigma)/(p-1)}$ is an explosive supersolution to (3.1) and $\underline{u} = m(R^2 - r^2)^{-(2+\sigma)/(p-1)}$ is an explosive subsolution to (3.1), where

$$C_0 M^{p-1} = \frac{2(2+\sigma)R^2}{p-1} \max\big\{N, \frac{2(p+\sigma+1)}{p-1}\big\};$$

and

$$C_0 m^{p-1} = \frac{2(2+\sigma)R^2}{p-1} \min\big\{N, \frac{2(p+\sigma+1)}{p-1}\big\}.$$

Thus (3.1) has at least one positive solution u satisfying

$$m(R^2 - r^2)^{-(2+\sigma)/(p-1)} \leq u(x) \leq M(R^2 - r^2)^{-(2+\sigma)/(p-1)}.$$

In particular, if $N = \frac{2(p+\sigma+1)}{p-1}$, i.e., $p = \frac{N+2(\sigma+1)}{N-2}$ with $N \geq 3$, $M = m$, then $u = m(R^2 - r^2)^{-(2+\sigma)/(p-1)}$ is an exact solution to (3.1).

We remark that $u(x)$ is radially symmetric, as shown in [4].

Example 3.2. Consider the problem

$$\Delta u = C_0(R^2 - r^2)^\sigma e^u, \quad x \in B_R, \quad u\big|_{\partial B_R} = +\infty, \tag{3.2}$$

where C_0 is a positive constant, $-2 < \sigma$ and $B_R = \{x \in \mathbb{R}^N : \|x\| < R\}$ with $N \geq 2$.

We see that $\overline{u} = M - (2+\sigma)\ln(R^2 - r^2)$ is an explosive supersolution to (3.2) and $\underline{u} = m - (2+\sigma)\ln(R^2 - r^2)$ is an explosive subsolution to (3.2). Where

$$C_0 e^M = 2N(2+\sigma)R^2 \quad \text{and} \quad C_0 e^m = 4(2+\sigma)R^2.$$

Thus (3.2) has at least one solution u satisfying

$$m \leq u(x) + (2+\sigma)\ln(R^2 - r^2) \leq M.$$

In particular, if $N = 2$, $M = m$, then $u = m - (2+\sigma)\ln(R^2 - r^2)$ is an exact solution to (3.2).

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Zhijun Zhang

Department of Mathematics and Informational Science, Yantai University, Yantai, Shandong, 264005, China

E-mail address: zhangzj@ytu.edu.cn