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HARNACK INEQUALITY FOR (p, q)-LAPLACIAN EQUATIONS UNIFORMLY DEGENERATED IN A PART OF DOMAIN

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Abstract. We consider a (p, q)-Laplace equation with the exponent values p, q depending on the boundary which is divided into two parts by a hyperplane. Assuming that the equation is uniformly degenerate with respect to a small parameter in the part of domain where $q < p$, a special Harnack inequality is proved for non-negative solutions.

1. Statement of main result

We consider the elliptic equation

$$L_\varepsilon u = \operatorname{div}(\omega_\varepsilon(x)|\nabla u|^{p(x)-2}\nabla u) = 0 \tag{1.1}$$

in a domain $D \subset \mathbb{R}^n$, $n \geq 2$, with a positive weight $\omega_\varepsilon(x)$, and an exponent to be defined below. Assume that the domain is divided by the hyperplane $\Sigma = \{x : x_n = 0\}$ into two parts $D^{(1)} = D \cap \{x : x_n > 0\}$, $D^{(2)} = D \cap \{x : x_n < 0\}$, and that

$$\omega_\varepsilon(x) = \begin{cases} \varepsilon, & \text{if } x \in D^{(1)} \\ 1, & \text{if } x \in D^{(2)}, \end{cases} \quad \varepsilon \in (0, 1], \tag{1.2}$$

$$p(x) = \begin{cases} q, & \text{if } x \in D^{(1)} \\ p, & \text{if } x \in D^{(2)}, \end{cases} \quad 1 < q < p. \tag{1.3}$$

To define the solution of (1.1), we introduce a class of functions related to the exponent $p(x)$:

$$W_{\rm loc}(D) = \left\{u : u \in W^{1,1}_{\rm loc}(D), |\nabla u|^{p(x)} \in L^1_{\rm loc}(D)\right\}.$$

This set is a Sobolev space of functions locally summable in D together with their first order generalized derivatives.

By a solution of to (1.1), we mean a function $u \in W_{\rm loc}(D)$, which satisfies the integral identity

$$\int_D \omega_\varepsilon(x)|\nabla u|^{p(x)-2}\nabla u \cdot \nabla\varphi \, dx = 0 \tag{1.4}$$

for the test functions $\varphi \in C_0^\infty(D)$.

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For the exponent $p(\cdot)$, given by (1.3), the smooth functions are dense in $W_{\rm loc}(D)$ (see [13]). Therefore, finite functions from $W_{\rm loc}(D)$ can be considered as test functions in (1.4).

p-Laplace type equations with a variable nonlinearity exponent, $p(x)$, and variational problems with integrants satisfying non-standard coerciveness and growth conditions occur in the modeling of composite materials and electrorheological fluids whose characteristics depend on the considered electromagnetic field. In this work, we consider a model for the case of plane boundary between two different phases. Note that the problem is complicated by the degeneration, uniform in ε, in the domain $D^{(1)}$.

In each of the domains $D^{(i)}$, $i=1,2$, the regularity of the solution has been well studied before (see [10]). It was proved in [1] that for p defined by (1.3) and for every fixed $\varepsilon \in (0,1]$, every solution of the equation (1.1) in the arbitrary subdomain $D' \Subset D$ belongs to the space $C^\alpha(D')$ of Hölder functions in D'. The independence of the Hlder exponent α on ε in case $p=q$ has been established in [2, 9], and for our equation it was proved in [8].

Harnack inequality plays an important role in the qualitative theory of differential equations (see [12]): if $p(x)\equiv p$, then the following inequality holds for the solution u of the equation (1.1) which is non-negative in the ball $B_{4R}\subset D$:

$$\inf_{B_R} u \geq \gamma(n,p) \sup_{B_R} u. \tag{1.5}$$

In [5], it was shown that the classical inequality (1.5) does not hold for the solutions of the equation (1.1) if $\varepsilon = 1$ and $q<p$. This inequality is not satisfied in the balls B_R centered on the hyperplane Σ. To state the result obtained in [5], denote by B_R^- the set $\{x\in B_R : x_n < -R/2\}$. It was established in [5] that if u is a non-negative solution of the equation (1.1) in the ball $B_{8R}\subset D$ centered on the hyperplane Σ, then the following inequality holds in the concentric ball B_R of radius R:

$$\inf_{B_R} u + R \geq C(n,p,q) \sup_{B_R^-} u. \tag{1.6}$$

Along with the invalidity of classical Harnack inequality (1.5); it was proved in [5] that for large values of R the term R in (1.6) cannot be replaced by R^ν when $\nu<(p-q)/(p-1)$. Note that in case $p=q=2$ the Harnack inequality of the form (1.6) with no R has been first obtained in [3], and in [7] in case $q=p\neq 2$.

In this work, we establish the Harnack inequality of the form (1.6) with a constant C independent of ε. Our main result is the following theorem.

Theorem 1.1. *If* (1.2) *and* (1.3) *hold, and u is a non-negative solution of* (1.1) *in the ball $B_{8R}\subset D$ centered on the hyperplane Σ, then the inequality* (1.6) *holds in the concentric ball B_R of radius R with the constant C depending only on n, p q.*

The proof is based on the modified technique of Mozer [11], developed in [4, 6], where the domains $D^{(1)}$ and $D^{(2)}$ play different roles.

The assertion of Theorem 1.1 Also holds for the equation

$$\operatorname{div}\Big(\omega_\varepsilon(x)|\nabla u|^{p(x)-2} a\nabla u\Big) = 0,$$

where α is a measurable uniformly positive definite matrix. Besides, the constant in (1.6), it will additionally depend on ellipticity constants of this matrix.

2. Proof of main result

Below B_R will denote an open ball centered on $\Sigma\cap D$, so that $B_{4R}\subset D$, $B_R^{(i)}=D^{(i)}\cap B_R$, $i=1,2$, u is a non-negative solution of the equation (1.1) and $w=u+R$. Here $|E|$ is n-dimensional Lebesgue measure of the measurable set $E\subset\mathbb{R}^n$, and

$$\fint_E f\,dx=\frac{1}{|E|}\int_E f\,dx.$$

Let us first establish auxiliary estimates for the solutions. Taking $\varphi=w^\beta\eta^p$, as a test function in the integral identity (1.4) with $\beta<1-p$, $\eta\in C_0^\infty(B_{3R})$ and $0\le\eta\le1$ by (1.2) we have

$$\int_{B_{3R}^{(2)}}|\nabla w|^p w^{\beta-1}\eta^p\,dx\le C\Big(\int_{B_{3R}^{(2)}}w^{\beta+p-1}|\nabla\eta|^p\,dx+\int_{B_{3R}^{(1)}}w^{\beta+q-1}|\nabla\eta|^q\,dx\Big).\tag{2.1}$$

Below $\tilde f$ will denote a continuation of a function from $D^{(2)}$ to $D^{(1)}$ even with respect to the hyperplane Σ. Let

$$G_R=B_{3R}^{(1)}\cap\{x:w(x)<\tilde w(x)\}\tag{2.2}$$

and, assuming $G_R\ne\emptyset$,

$$\varphi(x)=\begin{cases}(w^\gamma(x)-\tilde w^\gamma(x))\eta^q(x) & \text{in } G_R\\ 0 & \text{in } B_{3R}\setminus G_R,\end{cases}$$

as a test function in (1.4), with the constant $\gamma<1-q$ to be defined later. Then we obtain (see (1.2))

$$\begin{aligned}&|\gamma|\int_{G_R}|\nabla w|^q u^{\gamma-1}\eta^q\,dx\\&\le|\gamma|\int_{G_R}|\nabla w|^{q-1}|\nabla\tilde w|\tilde w^{\gamma-1}\eta^q\,dx+q\int_{G_R}|\nabla w|^{q-1}|\nabla\eta|\tilde w^\gamma\eta^{q-1}\,dx\\&\quad+q\int_{G_R}|\nabla w|^{q-1}|\nabla\eta|w^\gamma\eta^{q-1}\,dx.\end{aligned}\tag{2.3}$$

Let us estimate the integrands on the right-hand side of (2.3) by using Young's inequality, definition of G_R and relation $\gamma<0$. We have

$$\begin{aligned}|\nabla w|^{q-1}|\nabla\tilde w|\tilde u^{\gamma-1}\eta^q&\le\varepsilon_1|\nabla w|^q\tilde w^{\gamma-1}\eta^q+C(\varepsilon_1,q)|\nabla\tilde w|^q\tilde w^{\gamma-1}\eta^q\\&\le\varepsilon_1|\nabla w|^q u^{\gamma-1}\eta^q+C(\varepsilon_1,q)|\nabla\tilde w|^q\tilde w^{\gamma-1}\eta^q,\end{aligned}\tag{2.4}$$

$$\begin{aligned}|\nabla w|^{q-1}|\nabla\eta|\tilde w^\gamma\eta^{q-1}&\le|\nabla w|^{q-1}|\nabla\eta|w^\gamma\eta^{q-1}\\&\le\varepsilon_2|\nabla w|^q w^{\gamma-1}\eta^q+C(\varepsilon_2,q)w^{\gamma+q-1}|\nabla\eta|^q,\end{aligned}\tag{2.5}$$

$$|\nabla w|^{q-1}|\nabla\eta|w^\gamma\eta^{q-1}\le\varepsilon_3|\nabla w|^q w^{\gamma-1}\eta^q+C(\varepsilon_3,q)w^{\gamma+q-1}|\nabla\eta|^q.\tag{2.6}$$

Considering the relations (2.4)–(2.6) in (2.3), by a proper choice of ε_1, ε_2 and ε_3, we have

$$\int_{G_R}|\nabla w|^q w^{\gamma-1}\eta^q\,dx\le C(q)\Big(\int_{G_R}|\nabla\tilde w|^q\tilde w^{\gamma-1}\eta^q\,dx+\int_{G_R}w^{\gamma+q-1}|\nabla\eta|^q\,dx\Big).\tag{2.7}$$

Introduce the constant γ as

$$\gamma=\beta+p-q.\tag{2.8}$$

Then
$$|\nabla\tilde{w}|^q\tilde{w}^{\gamma-1}\eta^q = |\nabla\tilde{w}|^q\tilde{w}^{(\beta-1)q/p}\tilde{w}^{(\beta-1)(p-q)/p+p-q}\eta^q,$$
and, by Young's inequality,
$$|\nabla\tilde{w}|^q\tilde{w}^{\gamma-1}\eta^q \le R^{p-q}|\nabla\tilde{w}|^p\tilde{w}^{\beta-1}\eta^p + R^{-q}\tilde{w}^{\beta+p-1}. \tag{2.9}$$
Now we can rewrite the inequality (2.7) as
$$\begin{aligned}\int_{G_R}|\nabla w|^q u^{\gamma-1}\eta^q\,dx \le C(q)\Big(R^{p-q}\int_{G_R}|\nabla\tilde{w}|^p\tilde{w}^{\beta-1}\eta^p\,dx\\ + R^{-q}\int_{G_R}\tilde{w}^{\beta+p-1}\,dx + \int_{G_R}w^{\beta+p-1}|\nabla\eta|^q\,dx\Big).\end{aligned} \tag{2.10}$$
Let
$$v(x) = \begin{cases} w(x), & \text{if } x\in D^{(2)}\\ \min\,(w(x),\tilde{w}(x)), & \text{if } x\in D^{(1)}\,. \end{cases}$$
Note that (2.10) implies
$$\begin{aligned}&\int_{B_{3R}^{(1)}}|\nabla v|^q v^{\gamma-1}\eta^q\,dx\\ &\le C(q)\Big(R^{p-q}\int_{B_{3R}^{(1)}}|\nabla\tilde{w}|^p\tilde{w}^{\beta-1}\eta^p\,dx + R^{-q}\int_{B_{3R}^{(1)}}\tilde{w}^{\beta+p-1}\,dx\\ &\quad+\int_{B_{3R}^{(1)}}v^{\beta+p-1}|\nabla\eta|^q\,dx\Big).\end{aligned} \tag{2.11}$$
To prove the theorem, it suffices to add the integral
$$\int_{B_{3R}^{(1)}\setminus G_R}|\nabla\tilde{w}|^q\tilde{w}^{\gamma-1}\eta^q\,dx$$
to both sides of the inequality (2.10) and then use (2.9) on the right-hand side.

Using the definition of the function v, we rewrite (2.1) as follows:
$$\int_{B_{3R}^{(2)}}|\nabla w|^p w^{\beta-1}\eta^p\,dx \le C\Big(\int_{B_{3R}^{(2)}}w^{\beta+p-1}|\nabla\eta|^p\,dx+\int_{B_{3R}^{(1)}}v^{\beta+q-1}|\nabla\eta|^q\,dx\Big). \tag{2.12}$$
Hence, from (2.11) and the properties of even continuation of a function we obtain
$$\begin{aligned}\int_{B_{3R}^{(1)}}|\nabla v|^q v^{\gamma-1}\eta^q\,dx \le C(q)\Big(\int_{B_{3R}^{(2)}}w^{\beta+p-1}(R^{-q}+R^{p-q}|\nabla\eta|^p)\,dx\\ + \int_{B_{3R}^{(1)}}(v^{\beta+p-1}+v^{\beta+q-1}R^{p-q})|\nabla\eta|^q\,dx\Big).\end{aligned} \tag{2.13}$$

Let us estimate from below the integrand on the left-hand side of (2.13) using the inequality (2.9) with $\tilde{w}$ replaced by w. Also, note that $v^q \le R^{q-p}v^p$ as $w \ge R$. Taking into account this relation, we can rewrite (2.12) and (2.13) as
$$\begin{aligned}&\int_{B_{3R}^{(2)}}|\nabla w|^q w^{\gamma-1}\eta^q\,dx\\ &\le C\Big(R^{p-q}\int_{B_{3R}^{(2)}}w^{\beta+p-1}|\nabla\eta|^p\,dx + \int_{B_{3R}^{(1)}}v^{\beta+p-1}|\nabla\eta|^q\,dx\Big)\end{aligned} \tag{2.14}$$

and

$$
\begin{aligned}
&\int_{B_{3R}^{(1)}} |\nabla v|^q v^{\gamma-1}\eta^q\,dx \\
&\le C(q)\Big(\int_{B_{3R}^{(2)}} w^{\beta+p-1}(R^{-q}+R^{p-q}|\nabla\eta|^p)\,dx + \int_{B_{3R}^{(1)}} v^{\beta+p-1}|\nabla\eta|^q\,dx\Big)
\end{aligned}
\tag{2.15}
$$

respectively. Summing both sides of the inequalities (2.14) and (2.15), and using again the definition of the function v, we obtain

$$
\begin{aligned}
&\int_{B_{3R}} |\nabla v|^q v^{\gamma-1}\eta^q\,dx \\
&\le C(q)\Big(\int_{B_{3R}} v^{\beta+p-1}(R^{-q}+R^{p-q}|\nabla\eta|^p)\,dx + \int_{B_{3R}} v^{\beta+p-1}|\nabla\eta|^q\,dx\Big)
\end{aligned}
$$

Hence, from the choice of γ (see (2.8) and by the Sobolev embedding theorem, we conclude that

$$
\begin{aligned}
&\Big(\fint_{B_{3R}} v^{k(\beta+p-1)}\eta^k\,dx\Big)^{1/k} \\
&\le C(n,p,q)|\beta|^q\Big(\fint_{B_{3R}} v^{\beta+p-1}(1+R^p|\nabla\eta|^p+R^q|\nabla\eta|^q)\,dx\Big),
\end{aligned}
\tag{2.16}
$$

where $k=n/(n-1)$. Iterating the relation (2.16)) by Mozer method, we arrive at the following conclusion.

Lemma 2.1. *For every $q_0>0$, we have*

$$
\inf_{B_R} v(x) \ge C(n,p,q,q_0)\Big(\fint_{B_{2R}} v^{-q_0}(x)\,dx\Big)^{-1/q_0}. \tag{2.17}
$$

As $w\ge v$, (2.17) implies

$$
\inf_{B_R} w(x) \ge C(n,p,q,q_0)\Big(\fint_{B_{2R}} v^{-q_0}(x)\,dx\Big)^{-1/q_0}. \tag{2.18}
$$

Lemma 2.2. *For every ball $B_{2r}\subset B_{3R}$ centered in B_{3R}, it holds*

$$
\int_{B_r} |\nabla \ln v|^q\,dx \le C r^{n-q}, \tag{2.19}
$$

where the constant C does not depend on u, R and r.

Proof. As before, it is assumed below that $B_r^{(i)}=D^{(i)}\cap B_r$ $i=1,2$. Consider a cutting function $\eta\in C_0^\infty(B_{3R})$, such that $\eta\equiv 1$ in B_r, $|\nabla\eta|\le Cr^{-1}$. Assuming $\varphi= w^{1-p}\eta^p$ in the integral identity (1.4), by simple calculation with the consideration of (1.2) we obtain

$$
\int_{B_{2r}^{(2)}} |\nabla \ln w|^p\eta^p\,dx \le C\Big(\int_{B_{2r}^{(2)}} |\nabla\eta|^p\,dx + \int_{B_{2r}^{(1)}} w^{q-p}|\nabla\eta|^q\,dx\Big).
$$

Or, from $w^{q-p}\le R^{q-p}$,

$$
\int_{B_{2r}^{(2)}} |\nabla \ln w|^p\eta^p\,dx \le C\big(r^{n-p}+R^{q-p}r^{n-q}\big)\le Cr^{n-p}.
$$

Thus,

$$
\int_{B_{2r}^{(2)}} |\nabla \ln w|^q\eta^q\,dx \le C r^{n-q} \tag{2.20}
$$

which proves (2.19) in the case $B_r \subset D^{(2)}$.

Now let $B_r \cap D^{(1)} \neq \emptyset$. To prove the similar estimate in $B_r^{(1)}$ we first assume that the set G_R defined by (2.2) is not empty and consider

$$\varphi(x) = \begin{cases} (w^{1-q}(x) - \tilde{w}^{1-q}(x))\eta^q(x) & \text{in } G_R \\ 0 & \text{in } B_{3R} \setminus G_R. \end{cases}$$

as a test function in (1.4). Then it is not difficult to see that

$$\begin{aligned} &\int_{G_R} |\nabla \ln w|^q \eta^q \, dx \\ &\le \int_{G_R} |\nabla w|^{q-1} |\nabla \ln \tilde{w}| \tilde{w}^{1-q} \eta^q \, dx \\ &\quad + q \int_{G_R} |\nabla w|^{q-1} |\nabla \eta| \tilde{w}^{1-q} \eta^{q-1} \, dx + q \int_{G_R} |\nabla w|^{q-1} |\nabla \eta| w^{1-q} \eta^{q-1} \, dx. \end{aligned}$$

As $w(x) \le \tilde{w}(x)$ in G_R, then, by Young's inequality, we obtain

$$\int_{G_R} |\nabla \ln w|^q \eta^q \, dx \le C\Big(\int_{G_R} |\nabla \ln \tilde{w}|^q \eta^q \, dx + \int_{G_R} |\nabla \eta|^q \, dx\Big). \tag{2.21}$$

First consider the case where the center of the ball B_r is located in $\overline{D}^{(2)}$. Then, by (2.20),

$$\int_{B_{2r}^{(1)}} |\nabla \ln \tilde{w}|^q \eta^q \, dx \le C r^{n-q}. \tag{2.22}$$

Summing (2.21) and (2.22), we have

$$\int_{B_{2r}^{(1)}} |\nabla \ln v|^q \eta^q \, dx \le C\Big(\int_{B_{2r}^{(1)}} |\nabla \ln \tilde{w}|^q \eta^q \, dx + r^{n-q}\Big) \le C r^{n-q}. \tag{2.23}$$

Hence, by (2.20), we obtain the inequality

$$\int_{B_{2r}} |\nabla \ln v|^q \eta^q \, dx \le C r^{n-q}, \tag{2.24}$$

which implies (2.19).

If the set G_R is empty, then $v(x) = \tilde{w}(x)$ in $B_{3R}^{(1)}$, and (2.19)follows from (2.20). Now consider the case where $G_R \neq \emptyset$ and the center of the ball B_r is located in $D^{(1)}$. Denote by $\hat{B}_r$ the image of the ball B_r under mirror reflection with respect to the hyperplane Σ. By inequality (2.20), for the ball $\hat{B}_{2r}$ we have

$$\int_{\hat{B}_{2r}^{(2)}} |\nabla \ln w|^q \eta^q \, dx = \int_{B_{2r}^{(1)}} |\nabla \ln \tilde{w}|^q \eta^q \, dx \le C r^{n-q}. \tag{2.25}$$

As above, summing (2.21) and (2.25), we obtain again (2.23), which, combined with (2.20), leads to the inequality (2.24), which in turn implies (2.19). If the set G_R is empty, then (2.19) follows from (2.20) and (2.25). The proof is complete. $\square$

John-Nirenberg lemma is a corollary of (2.19): there exist the positive constants q_0 and C, independent of u and R such that

$$\Big(⨍_{B_{2R}} v^{-q_0}(x) \, dx\Big)^{-1/q_0} \ge C\Big(⨍_{B_{2R}} v^{q_0}(x) \, dx\Big)^{1/q_0}. \tag{2.26}$$

Proof of Theorem 1.1. As before, let $w = u + R$. Using (2.18) and (2.26), we obtain

$$\inf_{B_R} u(x) \geq C\Big(\fint_{B_{2R}} v^q(x)\,dx\Big)^{1/q} \geq C \inf_{B_R^-} w(x).$$

Then (1.6) follows from the classical Harnack inequality for the solutions of the equation (1.1) in the domain $D^{(2)}$, which states $\inf_{B_R^-} w(x) \geq c \sup_{B_R^-} w(x)$. Theorem 1 is proved. $\square$

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