

CASES OF TEACHER NOTICING TO POSITION STUDENTS IN
LINGUISTICALLY DIVERSE MIDDLE SCHOOL
MATHEMATICS CLASSROOMS

by

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LIST OF ABBREVIATIONS

Abbreviation	Description
CALLA	Cognitive Academic Language Learning Approach
CALP	Cognitive Academic Language Proficiency
CCSS	Common Core State Standards
ECLS-K	Early Childhood Longitudinal Study
EL	English Learner
ELL	English Language Learner
ESL	English as a Second Language
KT-MELL	Knowledge for Teaching Mathematics to English Language Learners
LEP	Limited English Proficiency
MDC	Mathematical Discourse Communities
MIELL	Mathematics Instruction for English Language Learners
MKT	Mathematics Knowledge for Teaching
MQI	Mathematics Quality of Instruction
NCLB	No Child Left Behind
NCTM	National Council of Teachers of Mathematics
SIOP	Sheltered Instruction Observation Protocol
TEDS-M	Teacher Education and Development Study in Mathematics

ABSTRACT

This study investigated how professional mathematics teacher noticing occurs in linguistically diverse middle school mathematics classrooms. The study illustrates how three middle school mathematics teachers are using this noticing, their beliefs, and various instructional strategies to position their students. The participants were chosen from the 34 teachers participating in a larger National Science Foundation funded study, CAREER: Mathematics Instruction for English Language Learners (MIELL). Data was collected by means of interviews and previously recorded classroom observations. The interviews and classroom observations were analyzed through various qualitative methods. The significance of this study lies in its contribution to mathematics education researchers interested in understanding how professional mathematics teacher noticing occurs in linguistically diverse classrooms and with emerging English Language Learners.

The participants in this study teach middle school mathematics in the same linguistically diverse Texas school district. They share similar language, educational, and work experiences. The three teachers believed that communication, inquiry, and accountability are the most important elements to foster in the students in their classrooms. The teacher demonstrated professional mathematics teacher noticing in their fostering of these classroom elements.

The components of noticing exhibited by the three participants varied in both effectiveness and purpose. Teachers employed noticing in whole class, partner, and group settings and this variety resulted in differences in how the students were positioned. Noticing that occurred during a whole class discussion served to position students whereas students designated as “teachers” during group work were also being positioned, but less by the noticing than the structure of the group. The teachers used numerous research-based instructional strategies with their students including revoicing, using language as a resource, and classroom mathematical discussions. Somewhat surprisingly the instructional strategies infrequently served to position students. This was due to several factors such as the frequency a particular strategy was used (revoicing) and how a strategy was used. Language as resource, for example, was primarily utilized by students for students.

Lastly, teacher noticing is thought to require a mathematics classroom where student engagement with mathematics is the norm. These teachers demonstrated noticing in classroom environments that both foster and smother student engagement. Classroom vignettes showed that noticing coupled with low student engagement was not dissimilar from the noticing occurring in a classroom with high student engagement.

Keywords: linguistically diverse classrooms, middle school mathematics, English language learners, professional mathematics teacher noticing, positioning, instructional strategies, teacher beliefs

I. INTRODUCTION

As of January 2015, the national debt of the United States is over \$18 trillion dollars and climbing (U.S. National Debt Clock: Real Time, n.d.). At the same time approximately 10,000 baby boomers are retiring each day (Kessler, 2014). With so many retirees leaving the work force who will be there to take their place, pay taxes to lower the debt, and care for a growing elderly population? The most recent United States census data shows that younger age groups have smaller percentages of non-Hispanic whites than older age groups. Because of this, the age ranges associated with child-bearing years include women from traditionally minority groups that are growing their families and creating a new generation where majority-minority will be the norm. In March 2014, for example, the Hispanic population in California was expected to become the majority (Cohn, 2014).

These majority minority populations together with new immigrants will combine to form the workforce of the future, charged with caring for the elderly and inheriting the national debt. Motel and Patten (2013) reported that, in 2011, 40.4 million immigrants were living in the United States. Mexico accounts for the largest percentage of immigrants at 29%, Central America and South America contribute an additional 8% and 7% respectively. Data also indicate that new immigrants tend to settle in just a handful of states, for example 10% of all immigrants in the United States live in Texas. In 2011, Texas was responsible for educating 832,000 students whose primary language was not English, trailing behind only the state of California with 1.1 million students (Flores, Batalova, & Fix, 2012).

Before these growing populations can reach the workforce they must first go through our school systems. Census data already shows that non-Hispanic whites only account for roughly half the students in our classrooms (Cohn, 2014). Immigrant children and the children of immigrants are coming to the classroom with their own culture and language. School age immigrant children that arrive in the United States may already have a solid educational foundation in another language or they may have had very little schooling. Educators have always been charged with teaching the students in their classrooms. Now more than ever those students are from increasingly diverse backgrounds and educators are expected to educate the workers of the future while keeping in mind federal legislation such as No Child Left Behind, the Common Core State Standards Initiative, the recommendations of professional organizations like the National Council of Teachers of Mathematics (NCTM), state and local mandates, and more.

The standards to which these students are expected to perform are demanding. Problem solving, communication, language, and technical skills are just some of what the United States citizenry will need to compete in the global economy (Partnership for 21st century learning, 2007). These skills are reflected in some of the aforementioned resources. For mathematics educators the Common Core State Standards demand that students be able to “construct viable arguments and critique the reasoning of other” as well as reason abstractly and make sense of problems (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The Common Core State Standards reiterate the process standards recommended by NCTM which include problem solving, reasoning and proof, communication, representation, and

connection. The communication standard expects students will “communicate their mathematical thinking coherently and clearly to peers, teachers, and others” and “use the language of mathematics to express mathematical ideas precisely” (NCTM, 2000, p. 60). This is just a portion of one of the five standards, all of which set equally lofty goals for mathematics teachers of all students to strive toward.

It stands to reason that minority, immigrant students (also referred to as 1.5 generation students) and second generation students from diverse cultural and linguistic backgrounds may have different needs from monolingual, native-born English speaking students in the mathematics classroom. They may require special attention or consideration when planning a lesson or different expectations with regards to language or the time it takes to acquire academic content. What can teachers do to ensure the academic success of these students in addition to their monolingual, native-born classmates, especially given the rigorous curriculum standards demanded of teachers and students? Research into the many questions concerning English Language Learners (ELLs) in the mathematics classroom has been pursued for a number of years and yielded many recommendations for classroom teachers to employ. Specific models or approaches to teaching ELLs have been developed over a number of years, programs such as the Sheltered Instruction Observation Protocol (SIOP) and Cognitive Academic Language Learning Approach (CALLA).

Due in part to the fact that Texas receives a large percentage of all U.S. immigrants, in 2009 the 81st Texas legislature called for “research to determine best practices in curriculum adjustments, instructional strategies, and professional development for teachers related to second dialects of English speakers” (Wilkinson et

al., 2011, p. 8). This research, according to Wilkinson et al., found that “the 2010 racial/ethnic composition of Texas schools was 48.6 % Hispanic (Latino), 33.3% White, 14% African American, 3.7% Asian/Pacific Islander, and 0.4% Native American. These percentages show that the majority of Texas students are members of racial/ethnic groups that are traditionally considered minorities. This is significant to the present study, in that minority group membership is associated with the use of language varieties that differ from standard English” (p. 8). “Another striking fact about the ELL population in Texas is that 85 percent of ELLs in grades K-5 and 59 percent of ELLs in grades 6-12 were born in the United States” (Flores, et al., 2012, p. 3). The importance of educating students is not in question, nor is the importance of educating ELLs, however these students are still falling behind their non-Hispanic peers in what has become known as the achievement gap (Wilkinson et al., 2011). In the 2012-2013 school year, the dropout rate for Hispanic students in Texas, at 8.2%, was more than double the rate for non-Hispanic white students at 3.5% (Texas Education Agency, 2014).

If we know minority, 1.5, and second generation students are important as workers of tomorrow, and we have research that helps us educate them with guidelines authored by education experts from the state and federal levels, why are we still faced with poor academic achievement from ELLs? What is happening with minority, 1.5, and second generation students at the classroom level? How are the teachers of ELLs interacting with their students?

One former ELL student remembers vividly his own interactions with an elementary mathematics teacher. José Valens¹ currently teaches eighth grade

¹ All names are pseudonyms.

mathematics at an urban middle school in central Texas. José is a personable forty-something. In the red t-shirt and jeans he wears he looks several years younger than this. He came to the United States as a child from Mexico and remembers sitting in an elementary school classroom, confused. “When I was a kid I was frustrated daily because I didn’t understand.” In fourth grade his teacher sat him at the back of the class because, “Mexicans can’t do math.” José recalled how this shaped his beliefs about mathematics until, as an adult, he decided to enroll at a community college and was required to take College Algebra. He realized he loved it and continued to take mathematics classes, eventually transferring to a university and graduating to become a mathematics teacher.

Research into teaching ELLs is not simply a recipe book of strategies and recommendations. It also catalogs, through qualitative and quantitative studies, questionable practices that have been used or are currently being used by educators with ELLs that are counterproductive to the goals of education. For example, José’s story is echoed by research conducted by Penfield in 1987 in which she surveyed a group of teachers to learn more about their belief about ELLs. Unsolicited, these teachers called out racial/ethnic groups and labeled them using pejorative terms. Like José’s teacher, they had acquired negative stereotypes about entire groups of people that influenced their teaching.

Some kind-hearted, but misguided teachers believe they are doing their ELLs a kindness by not including them in activities or asking them questions they might not understand (de Jong & Harper, 2005). Although these linguistically diverse children sit in the classroom, they never achieve membership as contributors or students. Other teachers, again trying to be helpful, will lower their expectations for ELLs. The work

ELLs are assigned is less challenging, held to lower standards, or not as rigorous (Pettit, 2011; Waxman & Padrón, 2002). Some teachers believe that math is the easiest thing to teach to ELLs because, since it is not Language Arts class, they don't really need to consider the language needs of students (de Jong & Harper, 2005; Penfield, 1987). Several states, such as California, and many central Texas school districts have banned the use of any language in the classroom other than English (García & Wei, 2014; Goldenberg, 2008). However, the idea that using a students' native language in the classroom and supporting their use of that language can be a benefit and a resource to the student in learning both English and mathematics has been shown in numerous published research studies (García & Wei, 2014; Sayer, 2013). Again, in an effort to help ELLs learn English and mathematics, an important resource is being denied to them.

Statement of the Problem

If someone is interested in ELLs and their learning of mathematics, it is only natural to first ask, what is happening in their linguistically diverse classrooms? More specifically, what is the teacher, charged with educating these diverse students, doing to provide instruction for all her students, ELLs and native speakers? According to mathematics education research there are numerous instructional strategies recommended for ELLs, however they are all predicated on a reform model. By reform model we mean those educational ideals such as cooperative learning, engaging problem-solving, and a de-emphasis of direct teaching promoted by NCTM and others. Research further warns that although reform ideas have been around for decades they have not necessarily found their way into many classrooms (Kennedy, 2005). What does that mean for our ELLs and

their linguistically diverse classroom? Can the recommended instructional strategies be found in today's classroom?

When considering the available literature about ELLs and linguistically diverse classrooms we see that far more mathematics education research connected to ELLs has taken place at the K-5 level. Why? There are services available to younger students such as bilingual classrooms that are less likely to occur in secondary schools. Since younger students are learning key foundational mathematics this is a crucial time to reach these ELLs. These facts entice researchers and paint linguistically diverse elementary school mathematics classrooms, and rightly so, as a desirable unit of study. Many people believe that by the time students get to middle and high school they will already be capable English speakers and so the need for research is diminished. In fact, some students who begin to learn English in elementary school may still be classified as ELLs well into high school. Additionally, many older students enter the school system and struggle as much as their younger counterparts. There is no reason not to explore linguistically diverse secondary school mathematics classrooms if there is any possibility that researchers can find ways to increase learning opportunities for ELLs.

Additionally, research confirms that the beliefs a teacher holds about mathematics and mathematical teaching influence the decisions and choices she makes in her classroom (Donaghue, 2003; Mustani, Celedón-Pattichis, & Marshall, 2009; Thompson, 1984). However, merely observing a classroom will not necessarily inform a researcher about what those beliefs are because even though a teacher possesses beliefs, this does not mean they are observable in her classroom. Beliefs that are espoused are not necessarily those that are seen in action (Fang, 1996; Philipp, 2007; Šapkova, 2014).

Teachers can hold different beliefs for different students (Penfield, 1987). So knowing a teacher's beliefs is not an automatic window into understanding her classroom routine. Research however, indicates the importance teacher's beliefs play in the success of her ELLs.

Professional mathematics teacher noticing has been a focus in mathematics education research in recent years. Professional mathematics teacher noticing can be thought of as a three part process of attending to a student's strategy, considering the student's understanding, and then taking the appropriate next steps. Noticing is an important part of what teachers choose to do in the classroom. There is very little research investigating professional mathematics teacher noticing, in-service middle school teachers, and ELLs. So it is reasonable to wonder what this noticing might look like inside the classroom.

Purpose of the Study

The purpose of this study is to gain an understanding of how middle school mathematics teachers in linguistically diverse classrooms use research-based instructional strategies, their beliefs, their positioning, and professional mathematics teacher noticing to position students in the classroom through a qualitative case study. Middle schools were chosen because considerably less research has been devoted to post elementary, linguistically diverse classrooms. Three middle school mathematics teachers from a south Texas school district will be observed in their classrooms three times over the course of a school year. They will also be interviewed in order to better understand the beliefs they hold concerning mathematics and the teaching of mathematics. The theoretical framework for this study is positioning theory which considers all social interactions as

opportunities for participants to be positioned in various ways as contributors or not, valuable or not. It is anticipated that teachers can be observed positioning their students both negatively and positively through their choices of instructional strategies, their beliefs, and their noticing. This positioning can impact a student's sense of belonging in the community of learners, i.e. the classroom, and therefore their sense of self. The data collected will be analyzed using qualitative methods in an attempt to better understand how these teachers' choices position their students. Evidence of professional mathematics teacher noticing will also be noted in order to better understand what this might look like in a linguistically diverse classroom with as many as 30% of students classified as ELLs.

Significance of the Study

This research will add to the body of knowledge concerning secondary school mathematics classrooms and ELLs by describing and interpreting what is happening in linguistically diverse classrooms. Unlike quantitative studies, this research will describe what teaching looks like through rich descriptions and personal accounts.

This research will also begin a conversation for future researchers connecting Jacobs, Lamb, and Philipp's (2010) definition of professional mathematics teacher noticing of student thinking to teaching ELLs. They define professional mathematics teacher noticing "as a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172).

Research Questions

The study will address the following questions:

1) What do teachers of linguistically diverse classrooms believe concerning mathematics, mathematics teaching, and their linguistically diverse students of mathematics? How do teachers position themselves as teachers? What about these beliefs and positioning can be witnessed in the classroom?

2) What does professional mathematics teacher noticing look like in a linguistically diverse middle school mathematics classroom?

3) How do teachers use of research-based instructional strategies, their beliefs, and professional mathematics teacher noticing position students in the classroom?

4) How are teachers in linguistically diverse mathematics classrooms using invitation moves to promote student engagement?

Definitions of Terms

Terminology used in this document and their usage is provided. Additionally, some terms provided are defined here simply because the field does not agree on a standard definition and therefore the definition adopted for this research is being identified.

Beliefs – Although this can mean different things to different people this dissertation uses Philipp's (2007) definition which is as follows,

Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action.

Beliefs, unlike knowledge, may be held with varying degrees of conviction and

are not consensual. Beliefs are more cognitive than emotions and attitudes. (p. 259)

Bilingual – a person that is “knowing and using two autonomous languages”
(García & Wei, 2014)

English Language Learner (ELL) – student who may speak a language different from English outside of school or “...any child for whom standard academic English is not his or her first language. Generally these children have some kind of relationship to a language other than English” (Houk, 2005, p. 7). This can also be referred to in the literature as English Learners (EL), Limited English Proficient student (LEP), English as a Second Language student (ESL), or Emerging Bilingual.

Latina/o – Individual whose family was originally from a Spanish speaking country in North America, Central America, South America, or the Caribbean. This can also be found in the literature as *Hispanic*.

Linguistically diverse – classrooms containing a mixture of monolingual English speaking students and students classified as English Language Learners that may speak a language other than English outside of school

Instructional Strategies- techniques and pedagogical choices teachers employ in their classrooms

Professional mathematics teacher noticing- “as a set of three interrelated skills: attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings” (Jacobs et al., 2010, p. 172).

Delimitations

The participants in this study will be middle school mathematics teachers from a large south Texas public school district. The participant teachers in this study are also participants in a larger NSF funded research study, CAREER: Mathematics Instructional for English Language Learners (MIELL). Most participant teachers in this study are bilingual, with different degrees of proficiency in English and Spanish (Mejía Colindres, 2015). Moreover, around 30% of middle school students in this district are ELLs; the rest are mostly bilingual English proficient (Mejía Colindres, 2015). As Yoon (2008) mentioned, studying urban classrooms with higher percentages of ELLs may yield different results than suburban classrooms with just a few ELLs. This will be considered when the data and reported findings are analyzed.

Summary

Mathematics education studies reveal that while there are instructional strategies recommended for teacher of ELLs, these strategies and these studies are based on reform mathematics. Research also finds that reform mathematics is not widespread in our schools (Kennedy, 2005). While research touts the importance of teacher's beliefs on her classroom teaching, these beliefs may or may not be felt in the classroom itself (Fang, 1996; Philipp, 2007). Recently, mathematics education research has focused on the value of professional mathematics teacher noticing in our classrooms and as an important area of study. However, research connecting professional mathematics teacher noticing, and secondary school teachers of ELLs is lacking. Because teacher noticing of students' is so important in the classroom and is presumably influenced by teachers choices and beliefs,

this study can also reveal how middle school mathematics teachers are able to position their ELLs.

These disconnects can only be answered at the source, our classrooms. Three middle school mathematics teachers in a south Texas public school district will be the focus of my study in order to better understand the instructional choices they make, their beliefs, and what professional mathematics teacher noticing looks like for them. From the positioning that teachers take with their students through their use of noticing and their instructional choices they may be able to take students from a different culture and a different language and make them feel integral, valuable contributors in a community of learners. The sense of inclusion and value these ELLs experience in class may encourage them personally and academically to finally close the achievement gap.

II. LITERATURE REVIEW

Harklau (2000) said that the actions of teachers of ELLs, “not only serve to teach language, but also serve to shape our students’ attitude toward schooling and their very sense of self” (p. 64). This is a powerful statement that points to the importance of teachers’ classroom teaching and their ELLs. This chapter contains the theoretical framework for this research and the accompanying review of the literature. The review considers research-based instructional strategies recommended for teaching ELLs given the mathematics classroom reform movement begun by NCTM in the 1980’s and the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NCTM, 2000). The review also considers professional mathematics teacher noticing as an important component of what teachers should do in the classroom. Teachers’ beliefs about mathematics instruction and ELLs will also be considered to include the ways they influence and inform classroom practice.

Theoretical Framework

Positioning theory is the theoretical framework for this research because it is pervasive in the mathematics classroom and also in the beliefs and stories of the teachers. Merely contemplating what teachers do in the classroom can be an exhausting task. There is constant activity that the teacher must manage, described by Sherin and Star (2011) as, the “blooming, buzzing confusion of classroom events” (p. 73). Researching such a dynamic, unpredictable space requires a theoretical framework that focuses on the moment (Wagner & Herbel-Eisenmann, 2009). Positioning theory, defined by Harré and van Langenhove (1999a), is “the study of local moral orders as ever-shifting patterns of

mutual and contestable rights and obligations of speaking and acting” (p. 1). They continue on to frame positioning theory as, “one possible conceptual apparatus that allows for social constructionist theorizing based on a dynamic analysis of conversations and discourse” (1999a, p. 2). What else is a classroom, we hope, but conversations and conversation-like activities?

In the context of teaching mathematics to ELLs, Khisty and Chval (2002) utilize Gee’s definition of *Discourse*. Gee (1996) considers *Discourses* as “ways of being in the world, or forms of life which integrate words, acts, values, beliefs, attitudes and social identities, as well as gestures, glances, body positions and clothes” (p. 127). A Discourse, singular, would include ways of talking, listening, reading, writing, and acting. “Discourses create ‘social positions’ (perspectives) from which people are ‘invited’ (summoned) to speak, listen, act, read and write, think, feel, believe...” (p. 128). Turner, Dominguez, Maldonado, and Empson (2013b) studied ELLs’ participation in mathematical discussions or Discourse. They saw that ELLs can struggle to engage in these conversations with their native English speaking peers. In order to better understand how the benefits of mathematical discussion can be made accessible to ELLs, the researchers organized and ran a ten week after-school mathematics club for fourth and fifth graders. In their research of this after-school mathematics club they used,

discursive positioning to include sociolinguistic moves by teachers and students, as well as other elements of the classroom discursive environment that function to position students in particular roles relative to tasks and one another, such as the availability of tools and resources or the development of classroom norms.

(2013b, p. 200)

The four researchers analyzed problem solving situations that were generated during the mathematics club and found that the students were positioned, by the facilitator, to take on roles as problem solvers. Moreover, when the facilitator modeled positioning moves, the students began to emulate and use positioning moves themselves. For example, when a teacher asks a student to clarify or explain his thinking, this lets the student and his classmates know that the student is the authority in the matter and what he has to say has value.

Imagine a traditional classroom where the teacher leads a whole-class lecture. She has positioned herself as the authority of what can be said and learned. Simultaneously she may also be positioning her students as passive receptacles of information with little or nothing to contribute. In a classroom across town, the teacher facilitates her students in robust discussions about mathematical concepts. She encourages students to make conjectures and justify their reasoning. The students are active learners. By supporting their ideas and encouraging them, this teacher has put her students in a position where their contributions are valued. Classmates are able to view each other as successful collaborators. As Turner et al. (2013b) stated “By asking other students to intellectually engage with an ELL’s contribution, the teacher is in fact realigning students to challenge patterns of participation that do not recognize ELL’s mathematical ideas as important contributions” (p. 225).

The ways a teacher was herself positioned as a student may be responsible for the beliefs about mathematics and mathematics teaching that she later holds. McVee, Baldassarre, and Bailey (2004), like Khisty and Chval (2002), employ Gee’s (1996) idea of Discourse when they discussed the reality of pre-service and in-service teachers that

participated in a research study concerning teachers' beliefs about literacy and culture as, "the positions they take up in their discourse. Positioning theory thus provides an analytic framework for analyzing teacher discourse, that is, the saying [writing]-doing-being-valuing-believing combinations produced by teachers' written and discussion based responses..." (p. 3). Yoon (2008) researched middle school English teachers of ELLs and found that the background and beliefs teachers held led them to position themselves as either teachers of students, teachers of regular education students, or teachers of a single subject. This positioning influenced the choices they made in the classroom. These choices positioned ELLs either positively or negatively as reported by the students themselves. One ELL, described by his ESL teacher as "active" and "funny", is described by his regular English teacher as "quiet," and "shy" (p. 510). His regular English teacher admits not making accommodations for his ELLs and it is reflected in how he positions the students. He rarely engages them or tries to include them in the class activities and his ELLs concede feeling uncomfortable speaking in his class.

Research-Based Instructional Strategies

In 1989, NCTM published their first edition of guidelines for mathematics education, titled *Curriculum and Education Standards for School Mathematics* (NCTM, 1989). The principles and standards being recommended by NCTM in the 1990s were a departure from what came to be called a traditional education model. NCTM called for a more constructivist learning environment where students create their own understanding and are provided opportunities to grapple with mathematical ideas. Teachers become facilitators providing activities that allow their students to discover and uncover meaning. This is a contrast from rote memorization of mathematical facts and procedures derived

from teacher lecture. These principles and standards were revised, updated, and republished again in 2000 (NCTM, 2000).

Approximately fifteen years that have passed since the second reincarnation of NCTM's principles and standards and mathematics education researchers have contributed a large number of publications that advance these principles and standards through their findings. Particular to mathematics education for ELLs, researchers have recommended numerous instructional strategies and techniques that foster the academic success of this population. Moschkovich (2012) stressed the importance of research-based teaching practices. Waxman and Padrón (2002) similarly consider research, "the best criteria we have for determining effective practices in education" (p. 9). These strategies are defined and addressed below. It is worth noting that these strategies propositioned by the research are a departure from the erroneous belief that teaching strategies that work for native English speakers, or in de Jong and Harper's (2005) words, "just good teaching", are going to reach all students regardless of language ability (p. 102) (Coggins, Kravin, Coates, & Carroll, 2007; Driscoll, Heck, & Malzahn, 2012; Waxman & Padrón, 2002). What follows is a review of proposed instructional techniques which includes revoicing, heteroglossia - a three-fold strategy which encompasses language as resource, multidiscursivity and multivoicedness, gestures, scaffolding, multiple representations, and fostering classroom mathematical discussions. We would hope to find these research recommended strategies in use, in whole or in part, employed by teachers in linguistically diverse classrooms.

Revoicing. Revoicing is an instructional technique that has received a lot of attention in the mathematics education literature. It is defined by Ramirez and Celedón-

Pattichis (2012) as, “restating earlier comments to scaffold students’ understanding and position them as contributors of knowledge” (p. 34). They define revoicing in the context of a teacher revoicing a student’s comments, although certainly another student could be called upon to revoice a classmate’s observation. In keeping with the theoretical framework of this research, the technique of revoicing allows teachers to position their students as members of the classroom with important contributions, in fact specific cases of using revoicing to position has been studied by Enyedy et al. (2008). This is in contrast to a traditional class where the teacher is viewed as the keeper of knowledge and the student’s passive vessels waiting to receive that knowledge. Dominguez and Adams (2013) classify revoicing as a validation for students, a way of saying, “I, the teacher, am noticing what you are noticing” (p.40).

The roots of revoicing in the literature are traced to O’Connor and Michaels (1993) who state the three possibilities for revoicing in the classroom as used to, position students in differing alignments with propositions and allow them to claim or disclaim ownership of their position; share reformulations in ways that credit students with teachers’ warranted inferences; scaffold and recast problem-solution strategies of non-native-language students. (p. 318)

O’Connor and Michaels (1993) offer several examples of revoicing in the classroom, one in particular involves a discussion about creating concentrations of lemonade in which students Paulina, Sarita, Allison, Steven, and their teacher Lynne are participating. Steven’s comments appear below as he discusses his classmates’ ideas, but is then interrupted by his teacher, Lynne:

Steven: ... um, but if she kept her um sugar and used that and then took her things of ten to twenty-two and just picked another number like halfway like Allison said and then just made that her concentrate...

Lynne: So then, you don't agree with Sarita that if she picks a number halfway between that that's not really making her first concentrate either (p. 322)

Here, Lynne the teacher is restating Steven's position in opposition to classmate Sarita, allowing him to agree, disagree, and generally continue the conversation.

Moschkovich (1999a) recommends revoicing as a tool to keep students' conversation mathematical and to facilitate student participation. Fernandes, Anhalt, and Civil (2009) also commend revoicing as a way to encourage ELLs communication. Herbel-

Eisenmann and Schleppegrell (2008) noted a middle school mathematics teacher's use of revoicing to focus her students on the relationship instead of the procedures found in the mathematics of their lesson. Khisty and Chval (2002) studied teacher and classroom discourse and revoicing and found in the transcript of an elementary school teacher's lesson on geometry that an elementary school teacher they studied revoiced student responses and took the opportunity to rephrase those responses with the proper mathematical terminology involving geometry.

Interestingly, there is some hesitation in the literature surrounding the use of revoicing as a technique with ELLs. Enyedy et al. (2008) discuss revoicing in a linguistically diverse high school classroom in which they noted that student contributions made in Spanish were typically revoiced in English. This may be of greater benefit to native speakers of English who can better understand and learn from the revoicing. However, Enyedy et al. (2008) continues on to say that,

The value of revoicing is not just in the way the teacher spruces up the mathematical content of student utterances, but lies in its potential to contribute to the development of students' identities as mathematic scholars. We speculate that the way revoicing is used to position students as people who are successfully engaging in mathematical roles and activities increases the likelihood of affiliation and engagement. (p. 158)

The teacher's role in validating a student's home language during classroom discourse is something that will be addressed in more detail in the coming section titled, Language as Resource.

Heteroglossia. Busch (2014) conducted a study of multilingual elementary school students in Vienna, Austria and specifically focused on how the students used their linguistic resources. She defines heteroglossia, using Bhaktin's framework, as acknowledging different languages as a resource, encouraging students to bring their interests into the classroom, and a process where learning takes place in a give and take with teacher and student assuming different roles. Busch (2014) notes how the heteroglossia pedagogical approach encourages learners to use their prior knowledge and language abilities. The assuming of different roles necessitates the teacher using positioning to classify herself and her students as learners or teachers depending on the circumstances. The three overlapping pieces of Busch's definition, which are expanded on below, are all recommended instructional strategies for teaching ELLs. In Busch's study, multi-grade, multilingual elementary students author and illustrate their own short books. The students are seen writing in their native language and then translating with the help of dictionaries, classmates, and family members into German. The teacher helps as

well to create the finished version. Over the years, students have created a library of these books which are a testament to the school's encouragement of students' linguistic resources.

Language as resource. Historically, a priority for ELLs in the classroom was to become proficient English speakers. To that end, the speaking of their native language was discouraged or prohibited. It was even recommended to parents that their child's native language not be spoken in the home. There is a generation of adults in this country that are monolingual as a result of their bilingual immigrant parents choosing not to speak a second language in the home. Parents believed their children's lives would be easier in the United States if they spoke English as native speakers from birth. This goal was achieved at the expense of depriving their own children of an opportunity at bilingualism or multilingualism.

The more things change the more they stay the same. The number of non-English speakers in the United States has increased 32% since 2000 (Camarota & Zeigler, 2014). Therefore, it is a greater priority than ever in the United States, given this increased population, to nurture ELLs into proficient and capable English speakers, however there are still state or district level policies in place that prohibit the use of students' primary languages in the classroom (Goldenberg, 2008). Goldenberg believes that such policies are not based on the research and, "make educators' jobs more difficult" (p. 43). Campbell, Adams, and Davis (2007) conclude their summary of the state of affairs in linguistically diverse classrooms by observing that problems identified years ago have not measurably changed, namely Nieto's (2000) observation that the majority of ELLs will spend their day in a monolingual English classroom. Conversely, Goldenberg's

(2008) review of the research in teaching ELLs shows that teaching students to read in their native language actually promotes achievement in reading English.

Khisty and Chval (2002) shared a classroom excerpt from an elementary school mathematics class in which the teacher connects the English word quadrilateral, with cuadro, which is one way to express the word square in Spanish (square can also be known as cuadrado). The teacher is able to connect a new concept to her students' prior knowledge by using their bilingualism as a resource.

The recent research into the use of a native language with ELLs is overwhelmingly affirmative. Turner et al. (2013b) discuss in detail a bilingual transcript from an after school math program for elementary school students. Their results show that the teacher's use of English and Spanish positions her students as all being contributing members of the group, moreover Spanish speaking students are aware that either language is available to them. The teacher, "elevates Spanish as an intellectual resource, which is equivalent to elevating an integral part of many Latino/a students' identities" (p. 215).

Campbell et al. (2007) stress the value of reflecting on language and finding ways to incorporate it into models of mathematics teaching. Pettit (2011) reviewed the literature of teachers' beliefs about ELLs and found that teachers will be more effective when they encourage their ELLs to speak their native language at home and at school. Students should not be penalized for using their native language as necessary and Turner, Dominguez, Empson, and Maldonado (2013a) found that, "research with latino/a bilinguals has shown that students draw upon informal, everyday language, metaphors,

gestures, and code-switching between Spanish and English as resources to articulate their ideas” (p. 351). Celedón-Pattichis (2008) in her research describes the ways in which Mrs. Brown, a bilingual teacher of elementary ELLs, uses students’ existing language abilities to make connections with mathematics. Sayer’s (2013) work with bilingual teachers and elementary school students shows the valuable role of language in students’ sense-making processes.

Specific types of classroom language inclusion receive a significant amount of attention in the literature. Translanguaging is defined by Sorto, Mejía Colindres, and Wilson (2014) as a form of revoicing, more specifically the interpretation by a student, as opposed to a translation, of what another student has said in their native language. This use of the two languages is meant to, “deepen the understanding of the subject matter” (p. 74). This is in keeping with Sayer’s (2013) example of a classroom that reads a story in one language, but discusses the story in a second language. Code switching in Sayer’s (2013) research of ELL elementary school students in San Antonio, Texas is the mixing or blending of both Spanish and English words in classroom conversation. Code switching may be done by students or the teacher. Campbell et al. (2007) in developing a framework for teacher reflection concluded that, “students learning English as a second language should have the opportunity to learn mathematics in classrooms in which they can negotiate meaning as the dialogue moves freely between their primary language and English” (p. 25).

Multidiscursivity. Multidiscursivity is the term Busch (2014) uses to mean the inclusion of students’ culture and prior experiences in the classroom. The idea that native language has a place in a mathematics classroom seems to go hand in hand with the belief

that a students' interests and cultural background also belong in the classroom. Although as with language, cultural understanding has not always enjoyed a place in American classrooms. According to Gay (2002), "students have been expected to divorce themselves from their cultures and learn according to European American cultural norms. This places them in double jeopardy – having to master the academic tasks while functioning under cultural conditions unnatural to them" (p. 114). McVee et al. (2004) paint a classic picture of a teacher who, through seeking equality for her students, strives towards her goal of teaching in a color-blind classroom. This mentality simultaneously serves to ignore the richness and value of classroom diversity such as Busch (2014) described witnessing in an Austrian elementary school.

A teacher observed during Yoon's (2008) study of middle school English classrooms would ask her ELLs to share their culture and traditions with classmates. "She believed that prompting English-speaking peers' understanding about other cultures was a way to help ELLs be a part of the community" (p. 506). Instead of conforming these students to the traditional American classroom experience she celebrated and valued their cultural differences.

The existing research touts the value of culturally responsive teaching practices. Goldenberg (2008) found that students' comprehension was improved when they were exposed to culturally relevant materials. Campbell et al. (2007) list students' culture and prior experiences along with language as important factors to consider in formulating a model for mathematics teaching. Celedón-Pattichis (2008) states NCTM's position statement for ELLs as including mathematics curriculum with, "connections to the cultural heritage of students" (p. 60). The middle school mathematics teachers Celedón-

Pattichis (2008) observed were seen to access students' identities and experiences to further classroom aims and she concludes by stressing the importance research places on doing just that. Moschkovich (1999b) states that some important considerations that can guide the design of mathematics instruction for Latina/o students are to, "honor the diversity of Latino students' experiences and know the students and their experiences" (p. 9).

Waxman and Padrón (2002) include cultural responsiveness in their list of the five best instructional strategies for ELLs. Although the strategy goes by many names, and likely as many definitions, Waxman and Padrón use Darder's (1993) definition of culturally responsive instruction as one that, "focuses on the students' needs and culture and tries to create conditions that support the empowerment of students" (p. 10). Darder (1993) compared and contrasted Latina/o educators and white educators in her discussion of teacher culture and stressed the importance of a classroom that empowers and values cultural differences for Latina/o students. Gay (2002) states that culturally responsive teaching, "is based on the assumption that when academic knowledge and skills are situated within the lived experiences and frames of reference of students, they are more personally meaningful, have higher interest appeal, and are learned more easily and thoroughly" (p. 106).

Gay distills culturally responsive teaching down to five elements which include, "developing a knowledge base about cultural diversity, including ethnic and cultural diversity content in the curriculum, demonstrating caring and building learning communities, communicating with ethnically diverse students, and responding to ethnic diversity in the delivery of instruction" (p. 106). She later adds, "Too many teachers and

teacher educators think that their subject (particularly math and science) and cultural diversity are incompatible, or that combining them is too much of a conceptual or substantive stretch for their subjects to maintain disciplinary integrity. This is simply not true” (p. 107).

Mustani et al. (2009) present problems students can grasp culturally. Dominguez and Adams (2013) view, “bilingualism and everyday experiences as cognitive resources for learning mathematics” (p.38). Gay (2002) advocates for teachers learning to multiculturalize subjects, mathematics in particular. Ramirez and Celedón-Pattichis (2012) and de Jong and Harper (2005) stress cultural understanding, using culturally relevant tasks and cultural acceptance in their extensive list of essential teacher actions for ELLs.

Multivoicedness. Busch’s (2014) last tenet of heteroglossia is also known as multivoicedness and is that of a give and take learning environment between teacher and student or between students. Multivoicedness is an opportunity for teacher and student to be repositioned as learner and educator. Students have the opportunity to be seen by their teacher and their peers as contributors of knowledge. Gay (2002) calls for resource sharing between students and teachers and creating a community of learners. She cites numerous sources validating the benefits of learning communities for Latina/o students, “the reciprocity involved in students working with each other and with teachers as partners to improve their achievement” (p. 108). Ramirez and Celedón-Pattichis (2012) also recommend Willey’s idea of a creating a mathematics discourse community (MDC), “in which teachers interact with students and students interact with their peers to develop knowledge of the mathematics while using the language of mathematics” (p. 20).

An example of multivoicedness familiar to many secondary teachers is the activity commonly called a jigsaw where students in groups learn different aspects of a new concept, then reorganize the groups so that each new group has one expert in each aspect. Each expert in turn teaches their peers about what they have already learned. In this way the roles of teacher and student are constantly changing.

Fernandes et al. (2009) remarked, “Regularly occurring cognitive interactions between students and the teacher have the potential to develop students’ mathematical thinking and confidence. This is especially the case for Latino ELLs because they develop both their mathematical reasoning and their English language in math through such interaction” (p. 168). Turner et al. (2013a) shared Mercer’s definition of a shared communicative space as “the dynamic, reflexive maintenance of a purposeful, shared consciousness by a teacher and learner, focused on the task at hand and dedicated to the objective of learning” (p. 351). O’Connor and Michaels (1993) pointed to Erickson’s research into classroom discourse, noting that teacher and student were constructing lessons jointly. Teachers and student assuming multiple roles in the learning environment is the essence of multivoicedness.

Gestures. Gestures are defined by Shein (2012) as “any actions or movements made by the teacher or students that pertain to the mathematical tasks at hand, excluding gestures that do not appear to directly contribute to the communication of mathematical ideas (i.e., hair tossing, body adjustment, or facial expression)” (p. 185). Gestures are something that most people exhibit unconsciously, but they play a special role as a learning tool for both teachers and ELLs. Gesturing is included by researchers as another useful component of communication available for ELLs (Fernandes et al., 2009; Ramirez

& Celedón-Pattichis, 2012; Shein, 2012; Turner et al., 2013a; Turner et al., 2013b).

Fernandes and McLeman (2012) classify gesturing as a significant tool for ELLs still learning academic English. Shein (2012) believes the study of gestures and ELLs serves to illuminate their participation and dismiss the deficit view that says ELLs cannot participate because they do not speak the language.

Moschkovich (1999b) witnessed teachers and students using gestures in a geometry lesson and recommends the practice of using gestures to foster language development. Gestures can clarify the words a teacher uses to convey meaning to students. Fernandes and McLeman (2012) also witnessed the use of gestures in a geometry lesson where gestures served to clarify what students meant when they said the words area and perimeter. Fernandes and McLeman continue on to describe a task based interview between a sixth grade ELL and a pre-service teacher. Their encounter, a vignette demonstrating professional mathematics teacher noticing, shows the value of attending to the gestures that students make to support their verbiage. The research encourages teachers to notice student use of gestures as a means to assess student understanding as well as encourage students to incorporate gesturing into their collection of tools as language continues to develop (Fernandes & McLeman, 2012; Sorto et al., 2014).

Scaffolding. The term scaffolding is used somewhat ambiguously in the literature, especially when applied to ELLs, but is a research-based recommended strategy for teachers of ELLs (Ramirez & Celedón-Pattichis, 2012). Fernandes et al. (2009) use Gibbon's definition of scaffolding and describe it as, "the step-by-step process of building students' ability to complete tasks on their own" (p. 164). The process of

scaffolding for ELLs can incorporate other strategies like gesturing, hands-on learning activities, and visuals (Fernandes et al., 2009). Scaffolding then is an umbrella term for what teachers do that may incorporate several other strategies. Scaffolding may be used in various ways by a teacher in her practice. She can scaffold culturally, linguistically, or otherwise.

Gay (2002) uses the phrase cultural scaffolding to mean teachers using their students, “own cultures and experiences to expand their intellectual horizons and academic achievement” (p. 109). Aguirre and Bunch (2012) use the term linguistic scaffolding. O’Connor and Michaels (1993), in their article on revoicing, describe how a teacher scaffolds an ELL through a discussion of an arm-balance scale. Walqui and van Lier (2010) discuss scaffolding at length in their book where they describe it as a spontaneous action in the classroom that can only work if it serves to support student autonomy. Therefore, when designing supports for English Language Learners in mathematics instruction, teachers are advised to heed Walqui and van Lier’s (2010) advice: “amplify, don’t simplify” (p. 38).

For example, a teacher heeding the above advice, may devise an activity for all her students involving solving two-step equations. Her ELLs may be better served with different or more numerous examples, extra time, or a glossary of terms, but not a simpler set of equations to solve. Walqui and van Lier (2010) state that a task would not be considered simplified “if the goal – the knowledge or skill to be attained – remains the same with the task simply broken down into more manageable subactivities so that the learner’s progress is scaffolded by judicious guidance only as needed” (p. 38).

Multiple representations. NCTM, in 2000, published their ten principles and ten standards for K-12 mathematics education. Of the ten standards five are considered content standards and five are process standards. One of these process standards is titled Connections and included in that standard was the idea that representations matter. In the words of NCTM,

The term representation refers both to a process and to product-in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself...Some forms of representation-such as diagrams, graphical displays, and symbolic expressions-have long been part of school mathematics. Unfortunately, these representations and others have often been taught and learned as if they were ends in themselves. Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling. (p. 67)

The importance of representing mathematical ideas in different ways is also stressed in the Common Core State Standards beginning in seventh grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

The research stresses the importance of using multiple representations in the teaching of ELLs both as a strategy for teachers and as a tool for the students (Aguirre &

Bunch, 2012; Campbell et al., 2007; Fernandes et al., 2009; Goldenberg, 2008; Sorto et al., 2014).

Aguirre and Bunch (2012) in their research discussed a video clip from a classroom and commented, “By paying close attention to the representing demands of the lesson, the teacher enabled Miguel to successfully navigate the various representation demands and be a mathematical resource for his classmates” (p. 191). Notice that when the teacher in the video considers multiple representations it can enable her to position students as not only successful learners, but also as resources for peers.

Classroom mathematical discussions. The current mathematics education research advocates discussion and social interaction in classrooms (NCTM, 2000; Wagner & Herbel-Eisenmann, 2009; Waxman & Padrón, 2002). Ramirez and Celedón-Pattichis (2012) said,

As NCTM (2008) states, ‘It is important for all students, but especially critical for ELLs, to have opportunities to speak, write, read, and listen in mathematics classes, with teachers providing appropriate support and encouragement.’ Having ELLs share their mathematical thinking positions them as *competent problem solvers*, and thus as *contributors of mathematical knowledge*, and places them on a trajectory for increased participation in the learning process. (p. 20)

Ramirez and Celedón-Pattichis (2012) also cited the Application of Common Core State Standards for English Language Learners when they said, “Regular and active participation in the classroom—not only reading and listening but also discussing, explaining, writing, representing, and presenting—is critical to the success of ELLs in mathematics” (p. 20).

The topic of classroom discussion is not a new one. Already we have seen numerous instances where the research encourages discussion between ELLs, their classmates, and teacher. Some of the recommended instructional strategies we have already seen require a discussion space to function. Revoicing, using language as a resource, and arguably the remaining strategies all depend on a classroom environment that encourages the positioning of ELLs as contributors. Moschkovich (1999a) offered classroom vignettes where a teacher is able to support students through their discussion using techniques like revoicing as well as noticing a student's strategy and building on that foundation. A portion of one of these vignettes is included below in which the teacher and students are discussing rectangles.

Teacher: ... Anybody else have a different idea that they could tell me about rectangles?

Student: Another different [unclear] parallelogram.

Teacher: It's called a parallelogram, can you say that word?

Students: Parallelogram.

Teacher: What were you going to say, Betsy?

Betsy: Also a parallelogram it calls a rectangle.

Teacher: A Parallelogram is also a rectangle? They can be both?

Betsy: Yeah.

Teacher: Wow, very interesting. Can you convince me that they can be both?

Betsy: Because a rectangle has four sides and a parallelogram has four sides.

Teacher: [unclear]

Eric: [unclear] a parallelogram

Teacher: You want to borrow one? [a tangram piece] I really want to remind you that you really have to listen while your classmate is talking...

Eric: Because these side [runs his fingers along the widths of the rectangle] will never meet even though they get bigger, and these sides [runs his fingers along the lengths of the rectangle] will never meet even though they get bigger. And these sides [picks up a square] will never meet [runs his hand along two parallel side] and these sides will never meet. [runs his hand along the other two parallel sides]

Teacher: When you say get bigger you mean if we kept going with the line? [gestures to the right with his hand]

Eric: Yeah.

Teacher: Very interesting.

During this lesson, the teacher employed some important strategies to orchestrate and support students' mathematical talk. In general, he established and maintained norms for discussions, asking students to listen to other students, to agree or disagree, to explain why they believed something and to convince the teacher of their statements. The teacher also used gestures and objects, such as the cardboard geometric shapes, to clarify what he meant. (Moschkovich, 1999a, p. 14)

The reform mathematics that NCTM promotes does not expect students to passively sit, perhaps listening, or perhaps not. Students are now expected to contribute to

classroom activities such as discussions and presentations (Moschkovich, 1999b; NCTM, 2000). But what exactly is a mathematics classroom discussion? The word discussion is not particularly unusual, but it can have various specific meanings in mathematics education research. Moschkovich (1999a) cited Pirie's definition of a mathematical discussion from Pirie's research into peer discussion at a time when it received minimal attention. Pirie defined mathematical discussion as "purposeful talk on a mathematical subject in which there are genuine pupil contributions and interactions" (1991, p. 143). As an early foray into the study of peer mathematical discussions, she admittedly uncovered more questions than answers. Ballenger (1997) and Warren and Rosebery (1995), after studying Haitian middle school science students, found that profound, thoughtful scientific conversations took place even though students were using their native language and/or speaking in a less formal, conversational way.

Both Moschkovich (1999a), and Khisty and Chval (2002) referenced Gee's understanding of the term Discourses when referring to discussion. Moschkovich (1999a) reminds us Gee's (1996) definition of Discourses includes actions such as gestures, glances, body positions, and clothes. Gesturing, one of the research recommended instructional strategies for working with ELLs, may not generate noise, but would be an integral part of a mathematical discussion according to Gee via Moschkovich.

Waxman and Padrón (2002) used the term instructional conversation in lieu of mathematical discussion, but the idea is the same. This term comes from a study by Gallimore and Goldenberg (1992) begun in the early 1970's, the goal of which was to develop a reading program for native Hawaiian student in primary grades. Many years after the fact, they set out to define the term, instructional conversation (IC), in

cooperation with teachers and the following ten elements emerged from their collaboration.

1. Thematic focus for the discussion
2. Activating, using, or providing background knowledge and relevant schemata
3. Direct teaching, as necessary
4. Promoting more complex language and expression by students
5. Promoting bases for statements or positions
6. Minimizing “known-answer” questions in the course of the discussion
7. Teacher responsivity to student contributions
8. Connected discourse, with multiple and interactive turns on the same topic
9. A challenging, but nonthreatening atmosphere
10. General participation, including self-selected turns

(Gallimore & Goldenberg, 1992, p. 208)

Again, we see other instructional strategies reflected in this description of instructional conversation. Using students’ prior knowledge would, for ELLs, naturally include utilizing students’ language, culture, and prior experiences. Responding to and using students’ contributions are a paraphrasing of professional mathematics teacher noticing if the contributions are used to interpret a student’s understanding.

Pimm (1987) points out that there is a difference between spoken and written mathematics and notes that the desire for more discussion in classrooms serves to point out the inequity between the two. Pimm (1987) states that “many teachers do not see the

value or even the possibility of discussion in mathematics as a consequence of the view of mathematics which they hold” (p. 47). Teachers who, for example, view mathematics as a collection of right and wrong answers that require little consensus may not be inclined to promote classroom discussions. Pimm explains at length what he calls the mathematics register, the specialized language that we use when conversing about mathematics. He believes that only once the register has been developed can meaning be made available in the language of mathematics. This speaking mathematically was witnessed by Khisty and Chval (2002) in reference to their case study participant, Mrs. Martinez, who is successful because she actively engages her students in “problem solving and collaboration, oral and written communication and justification and independent thinking” (p. 157).

Ramirez and Celedón-Pattichis (2012) combined the terms mathematics and discourse and define it “as communication that centers on making meaning of mathematical concepts” (p. 20). They emphasized the importance of Mathematical Discourse Communities (MDC) as a way to support an ELL’s acquisition of academic language. A MDC was briefly mentioned when introducing multivoicedness.

Ivory, Chaparro, and Ball (1999) speak to how reform mathematics appears in classrooms when they say “it is entirely consistent with constructivism that if students are asked to discuss, are listened to, and are required to explain or defend their thinking, they will actually begin to discuss, explain, defend, and think better” (p. 116). Students being listened to and held responsible for defending their thinking are being positioned by their teachers as valued and accountable for their learning. Moschkovich (1999b) on comparing reform and traditional mathematics classrooms said,

In reform-oriented mathematics classrooms latinos are no longer grappling mainly with acquiring technical vocabulary, developing comprehension skills to read and understand mathematics textbooks, or solving standard word problems. Instead students are now expected to participate in both verbal and written mathematical discourse practices such as explaining solution processes, describing conjectures, proving conclusions and presenting arguments. (p. 6)

Cummins (2000) finds it valuable to separate ELLs' language proficiency into its conversational and academic aspects. This cognitive academic language proficiency (CALP) can be thought of as the language specific to mathematics and necessary for success in the discipline. Recall, this is also known as the mathematics register (Fernandes, 2012; Pimm, 1987). If it can be agreed that the acquisition of academic language is important, then Ramirez and Celedón-Pattichis (2012) warn of the pitfall of focusing on the vocabulary of mathematics without the proper context and discussion to promote meaning and understanding. Rather, educators should focus on student acquisition of academic development through context and meaningful discussions (Khisty & Chval, 2002). Mustani et al. (2009) presented a case study of a first grade teacher who realized that academic language in a reform class needs to be learned in a particular way. This teacher connected and recognized the importance of talking and learning. Khisty (1995) studied elementary school classrooms with a significant number of Latina/o ELLs led by bilingual teachers. In teacher 1's classroom student talk is dominated by choral response and pro forma repetition. In teacher 2's class the students carry on active discussions centered on problem solving challenges. Khisty contrasts these teachers and notes how "talk or the absence of talk results in either drawing

students closer to mathematics or alienating them from it” (p. 290). Mustani et al. (2009) quoted Khisty’s 1995 research when stating, “Discussing the characteristics of teacher discourse as an avenue for Latina/o students’ mathematical learning explained that, ‘Talk... is the critical vehicle by which an individual internalizes meanings’” (p. 37).

With all this talk about talking, it seems logical to fear that discussion can marginalize ELLs that are expected to, but can’t fully contribute (Turner et al., 2013b). While there is a lack of research into mathematics classroom discussions, ELLs, and positioning at the secondary school level, however there is related research available from primary grades and other subject areas. Yoon (2008) also saw in her case study of middle school English teachers that even in interactive student-center classes, ELLs can find themselves outside the learning. Turner et al. (2013b) recommend positioning as the way to draw students into the conversation. Students positioned as contributors will believe themselves to be contributors, not apathetic observers. Khisty and Viego (1999), in their case study of Mrs. Martinez, a fifth grade mathematics teacher of ELLs, pointed out that mathematics is now considered a socially constructed entity with its own cultural life. They believed learning mathematics means acquiring this culture and that students must “actively make meaning of its tools through the use of those tools” (p. 73). They describe Mrs. Martinez using rich discussion and persistent questioning that allows her students access to the culture of mathematics. Mrs. Martinez’s students know they are members of the learning community and responsible contributors because she has positioned them as such.

To further counteract the difficulties ELLs may have in contributing to discussions, the characteristics of a MDC incorporate the recommended instructional

strategies we have already seen, such as the aspects of heteroglossia and multiple representations (Ramirez & Celedón-Pattichis, 2012). Garcia and Gonzalez (1995), in their discussion of curricula issues for linguistically diverse students, said that,

Effective curricula provide abundant and diverse opportunities for speaking, listening, reading, and writing, along with scaffolding to help guide students through the learning process. Further, effective schools for diverse students encourage them to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts. (p. 424)

So it is not enough to be a reform, student-centered classroom, but the instructional strategies critical to ELL success must be in place.

The recommended research-based instructional strategies for teaching ELLs discussed above include revoicing, heteroglossia, gestures, discussion, and scaffolding. All of these strategies are geared towards giving ELLs an avenue into classroom citizenship by letting teachers position students as valuable. When a teacher or a peer revoices an ELL's words they are communicating, "I value what you said." Heteroglossia enables teachers to show ELLs that their language and cultural are available learning tools; the students and these tools are positioned as valuable. Encouraging ELLs in classroom discussions, multiple representations, gestures, and scaffolding are ways that teachers can position ELLs that lack academic language skills as contributing and learning members of the classroom community. The bond unifying these instructional strategies is that they set up ELLs for success by positioning them as classmates. These instructional strategies are summarized below in Table 1.

Table 1. Research-based Instructional Strategies Recommended for Use with ELLs.

Type	Description	Citation
Revoicing	restating earlier comments to scaffold students' understanding and position them as contributors of knowledge	Ramirez & Celedón-Pattichis, 2012, p. 34
Heteroglossia		
Language as Resource	students draw upon informal, everyday language, metaphors, code-switching, and translanguaging between Spanish and English as resources to articulate their ideas	Sayer, 2013; Turner, Dominguez, Empson, & Maldonado, 2013a
Multidiscursivity	stressing cultural understanding, using culturally relevant tasks and fostering cultural acceptance in the classroom	de Jong & Harper, 2005; Ramirez & Celedón-Pattichis, 2012
Multivoicedness	a community of learners where teachers and students are working and learning together	Gay, 2002
Gestures	any actions or movements made by the teacher or students that pertain to the mathematical tasks at hand	Shein, 2012
Scaffolding	the step-by-step process of building students' ability to complete tasks on their own	Fernandes, Anhalt, & Civil, 2009, p. 164
Multiple Representations	[e.g. diagrams, graphical displays, and symbolic expressions are] essential elements in supporting students' understanding of mathematical concepts and relationships	NCTM, 2000
Classroom Mathematical Discussion	to have opportunities to speak, write, read, and listen in mathematics classes... Having ELLs share their mathematical thinking	Ramirez & Celedón-Pattichis, 2012, p. 20

Student Engagement

An investigation into the conducted research concerning the topic of student engagement returns a wealth of information, the vast majority of which occurred within the last fifteen years. This is likely a result of the standards published by NCTM (2000)

that call for a middle school mathematics classroom replete with students solving challenging problems, understanding mathematical ideas, making conjectures, abstracting, and generalizing in a supportive environment. During these fifteen years, student engagement has been studied in detail and from many approaches. The literature available offers different concepts that can be used to understand, foster, and discuss student engagement. These facets of student engagement, as it may relate to K-12 mathematics and ELLs, will be examined in this section.

Student engagement matters, particularly for Latina/o learners. Moller, Stearns, Mickelson, Bottia, and Banerjee (2014) ask the question, “Is academic engagement the panacea for achievement in mathematics across racial/ethnic groups?” in the title of their research paper. Their quantitative, longitudinal study examines “the effects of teachers’ perceived organizational culture of schools on mathematics achievement by students’ engagement and race/ethnicity” (p. 3). Organizational culture is examined using a Collective Pedagogical Teacher Culture. “A Collective Pedagogical Teacher Culture has two distinct components 1) the presence of strong professional community and 2) a norm of collaboration among teachers where students’ needs are centralized” (p. 5). Moller et al. measured student achievement in the elementary grades using data from the 1998 Early Childhood Longitudinal Study (ECLS-K). Moller et al. found that “engaged Latina/o students who spend their elementary years in schools where teachers collaborate experience 7 points greater growth on the mathematics achievement test by fifth grade, on average” (p. 21). Disengaged Latina/o students had lower achievement regardless of collaboration. Their conclusion showed a relationship between collaborative teaching environments and Latina/o student achievement. Schools where teachers are encouraged

and able to work together to meet students' needs will raise the mathematics achievement for engaged Latina/o children. Moller et al. (2014) do not address how to increase and maintain student engagement, however, Matos (2015) looked at Latina/o student engagement in college and made some recommendations for K-12 teachers and administrators. These include rejecting deficit beliefs surrounding Latina/o families and the value they place on education. She would also like to see K-12 schools create strong partnerships with Latina/o parents and do more to utilize the Spanish language as a resource in the classroom.

Azevedo, diSessa, and Sherin (2012) define student engagement as students' investment in and effort directed towards learning. This is the definition that Nyman's (2015) research into student engagement builds on. Nyman approached the idea of student engagement from the teacher's perspective instead of the students' or researchers'. Thus, she used a kind of teacher noticing. In Nyman's study, Swedish middle school mathematics teachers were videotaped teaching in their classrooms. These teachers were then asked to choose instances from the videos that exemplified student engagement. The results yielded six indicators of engagement: verbalizing thinking, concentration, gestures, asking/answering questions, enhancing ideas, and justifying arguments. These indicators were seen to take place during whole class interactions, group interactions with the teacher, and in individual student/teacher interactions. Nyman concluded that some aspects of student engagement, that is evidence of active listening, can be perceived by teachers, but not by researchers.

Franke et al. (2015) looked at the teacher's role in student engagement in elementary school mathematics classrooms. To facilitate this they collected video of

teachers and their classrooms from kindergarten through fifth grade. They coded the videos for instances of student engagement and found that teachers used six different invitation moves to foster student engagement. These six invitation moves were explain someone else's solution, discuss differences between solutions, make a suggestion to another student about his or her idea, connect their ideas to other students' ideas, create a solution together with other students, and use a solution that was shared by another student. Franke et al. found that teachers also used three follow up moves to support the students' engagement with mathematical ideas. These support moves were probing, scaffolding, and positioning.

Edwards (2015) believes that developing students' participation during the middle school grades is one of the most important goals that educators can have. She looks at student participation from an active learning framework. Collins and O'Brien (2003) define active learning as,

The process of having students engage in some activity that forces them to reflect upon ideas and upon how they are using those ideas. Requiring students to regularly assess their own degree of understanding and skill at handling concepts or problems in a particular discipline. The attainment of knowledge by participating or contributing. The process of keeping students mentally, and often physically, active in their learning through activities that involve them in gathering information, thinking, and problem solving.

Edwards looks at classroom activities as potentially falling into three categories: intellectual, social, and physical. Intellectual activities may include problem solving, summarizing, or creating concept maps. Whole class and small class discussions or group

work are considered social activities. Experiments, projects, games, model building, and using manipulatives are examples of physically active learning. Some activities may fall into multiple categories.

Marshman and Brown (2014) support teachers' use of collective argumentation to scaffold student engagement in the middle school mathematics classroom. Collective argumentation is a four step sequential process that includes represent individually, compare cooperatively, explain, justify, and agree collaboratively, and validate communally. The first step, represent individually, gives students a chance to study a mathematical task by themselves and consider their solution or their strategy to finding a solution. In step two, compare cooperatively, the students compare their ideas in a small group of their peers. During the third step, explain, justify, and agree collaboratively, the small groups will work together and craft a single solution. The group members are charged with working together to assist members who are unsure of the mathematics. In the final step, validate communally, the class comes together and each group reports their solution. The student presenters give details of the steps used and the thinking behind the solution. Students in the audience are encouraged to engage in mathematical discussion with the presenters about their work.

Hand, Kirtley, and Matassa (2015) believe there is value in student engagement. They view it as a tool that teachers can use to better understand their students' needs. Teachers have a history of bemoaning unmotivated students and searching for new and better ways to turn them into motivated students. Hand et al. (2015) believe that classifying students as unmotivated is blame wrongly placed. If a student is unmotivated in class it is because the teacher has not organized the classroom in such a way as to

engage the student. Hence, classroom engagement is a teacher's responsibility and a measure for how well the mathematics classroom is meeting students' needs. They consider engaging instead of motivating a strategy for narrowing participation gaps in linguistically diverse mathematics classrooms.

Student engagement is an important component of the kinds of mathematics classrooms that NCTM, CCSS, and mathematics educators promote. Research has shown its value to all students' achievement in mathematics, including Latina/o students and other minorities. Researchers have looked at the relationship between teachers and student engagement using a variety of strategies which share commonalities that encourage student communication and interaction with mathematical ideas.

Professional Mathematics Teacher Noticing

Teacher noticing is currently undergoing a resurgence of interest in the mathematics education research community. According to Erickson (2011) it has its roots in the beginnings of the twentieth century when the child study movement encouraged teachers to closely watch their students. But watching what exactly? Dewey (1904) juxtaposed a student's inner attention versus their external attention when considering a teacher's responsibilities;

The inner attention is the giving of the mind without reserve or qualification to the subject in hand. To be able to keep track of this mental play, to recognize the signs of its presence, or absence to know how it is initiated and maintained, how to test it by results attained, and to test *apparent* results by it, is the supreme mark and criterion of a teacher...External attention, on the other hand, is that given to the book or teacher as an independent object. It is

manifested in certain conventional postures and physical attitudes rather than in the movement of thought. Children acquire great dexterity in exhibiting in conventional and expected ways the *form* of attention to school work, while reserving the inner play for their own thoughts image and emotions for subjects that are more important to them, but quite irrelevant. (p. 148-149)

Dewey was popular at the turn of the twentieth century, but his words predicted the current learning environment for many ELLs. As long as their external attention is acceptable, the ELLs will not be asked questions, be expected to participate, or be held to high standards by the teacher who believes she is doing them a service (Pettit, 2011). But what if their appearance and behavior, Dewey's external attention, is not acceptable? In 1987 Penfield conducted research into teachers' beliefs about teaching ELLs where such Hispanic ELLs were labeled as wild, disruptive, and time consuming. Penfield's study will be addressed in more detail later in this chapter.

Dewey's idea of noticing the inner attention can still be found in the literature. When discussing the importance of multiple representations NCTM (2000) said, "Moreover, the term [representations] applies to processes and products that are observable externally as well as to those that occur "internally", in the minds of people doing mathematics" (p. 67). Noticing can mean different things to different people as a quick glance of the recent literature will attest. However, an often cited definition of professional noticing of children's mathematical thinking comes from Jacobs et al. (2010), "as a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's

understandings” (p. 172). This is commonly referred to as professional mathematics teacher noticing, but based on Dewey’s remarks it is a cornerstone of effective teaching.

As previously stated, this is a growing field of interest to mathematics education researchers. A number of researchers are interested in the question of whether noticing can be taught to pre-service and in-service K-12 mathematics teachers; the agreed upon answer is yes (Schoenfeld, 2011; Star & Strickland, 2008). If the research tells us that noticing can be taught, then how can that best be accomplished? A popular response to this question involves the use of videos to capture in the moment noticing (or lack thereof) and provide reflective opportunities for participating pre-service and in-service teachers (Erickson, 2011; Kazemi et al., 2011; Mason, 2011; Miller, 2011; Sherin, Russ, & Colestock, 2011b; Sherin & van Es, 2005; Star, Lynch, & Perova, 2011; Star & Strickland, 2008).

In the preface to their book on professional mathematics teacher noticing, *Mathematics Teacher Noticing: Seeing through Teachers’ Eyes*, Jacobs, Philipp, and Sherin (2011b) state that, “noticing is a critical component of mathematics teaching expertise and thus a better understanding of noticing could become a tool for improving mathematics teaching and learning” (p. xxv). Schoenfeld (2011) agrees that noticing is valuable and can lead to improved teaching practice in mathematics. Researchers consider noticing valuable because the three prongs of professional mathematics teacher noticing taken together are an important aspect of effective teaching (Miller, 2011; Sherin et al., 2011b). Mason (2011) believes that the discipline of noticing can be used in the research of various aspects of mathematics teaching including contributing, “to our appreciation of intricacies of learning and teaching mathematics” (p. 35).

Sherin, Jacobs, and Philipp (2011a) in the introduction to their book, point to researchers of noticing as asking the- “primal questions of teaching: Where do teachers look, what do they see, and what sense do they make of what they see” (p. 3)? Noticing focuses on the teacher’s interaction with the classroom and their response to a child’s thinking (Sherin et al., 2011a; Jacobs, Lamb, Philipp, & Schappelle, 2011a). This is reinforced by Jacobs et al. (2011a) who call on noticing as a way to, “understand how teachers make sense of complex classrooms” (p. 98).

Fernandes (2012) studied pre-service teachers, their noticing, and their understanding of ELLs through task-based interviews as part of a larger intervention. Fernandes aimed to educate pre-service teachers in the needs of ELLs and improve their abilities to notice. He found that the pre-service teachers were able to better understand ELLs after conducting the interviews and employing a noticing framework. What about in-service teachers of ELL populations? What does professional mathematics teacher noticing look like in their classrooms? A review of the literature of ELLs and professional teacher noticing uncovers no additional research, beyond Fernandes’ short article, into what noticing looks like in these classrooms.

The researchers previously cited, who study the field of professional mathematics teacher noticing, consider it an important aspect of successful teaching. They believe it is important when considering the basic questions of what teachers are doing in the classroom and how they respond to their students. We can easily find the statistics about the number of ELLs in our classrooms and the difficulties they must overcome. There are many things we know about ELLs in our classrooms, but we don’t know how professional mathematics teacher noticing looks in these classrooms. What do teachers of

ELLs notice? Do they notice differently than other teachers? Are there different levels of noticing? How does the presence of ELLs and the challenges that they pose color teacher noticing? What do the teachers themselves have to say about their practice and their noticing? Professional mathematics teacher noticing is not a natural or inherent skill, perhaps teachers of ELLs are not engaging in any kind of noticeable noticing (Ball, 2011; Jacobs et al., 2010).

Teachers' Beliefs Affect Instruction

Macnab and Payne (2003) in their study of Scottish primary school pre-service teachers said, "The beliefs and attitudes of teachers- cultural, ideological and personal are significant determinants of the way they view their role as educators" (p. 55). This adds to Richardson's (1996) understanding that beliefs affect what teachers believe as their purpose in teaching, influence the ways they think about their subject matter, the choices they make in their teachings, and how they approach professional development. This is especially important as research into ELLs often calls for professional development as a way to improve education for this population (Penfield, 1987; Pettit, 2011). This is why the question of what teachers are doing must consider the beliefs the teachers hold.

Researchers unanimously support the connection of teachers and their beliefs to classroom practices (Donaghue, 2003; Fang, 1996; Pettit, 2011; Philipp, 2007; Šapkova 2014; Thompson, 1984).

McVee et al. (2004) give evidence that teachers' beliefs influence and can be identified in their classroom discourse. McVee et al. used positioning theory to study teachers' beliefs about literacy and culture and found that, "the reality of teachers, in this case their perceptions of culture as related to literacy instruction, is revealed in the

positions they take up in their discourse” (p. 283). Mustani et al. (2009) said that, “beliefs affect practice, and they play an important role in how teachers interpret and implement curriculum” (p. 30).

But what are beliefs? Philipp (2007) authored the chapter on teachers’ beliefs and affect for the *Second Handbook of Research on Mathematics Teaching and Learning*. He defines beliefs as follows,

Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one’s view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes. (p. 259)

Once a definition for beliefs can be agreed upon, although this has yet to happen, it might seem the hard work is done. Teachers can be interviewed or observed and their beliefs will be clear; this is unfortunately not the truth. The research also repeatedly warns that teachers often say one thing, but do another when it comes to beliefs. Donaghue (2003) labels these disparate states as espoused theory and theory in action. Philipp (2007) and Fang (1996), in her review of the available literature on teacher beliefs and practices, use the terms consistency and inconsistency when describing these two theories in practice. Philipp (2007) cites Raymond whose own study of teacher

beliefs concluded that a teacher's inconsistencies were explained by, "time constraints, scarcity of resources, concern over standardized tests, and students' behavior" (p. 272).

Šapkova (2014) studied the relationship between student achievement and teachers' beliefs in Latvia; in particular the connection between teacher beliefs and classroom achievement. Šapkova acknowledges the importance of beliefs in the classroom and credits the earlier research into teachers' thought processes of Clark and Peterson. Clark and Peterson's (1986) model of teacher thought and action gives equal importance to teachers' thought processes and their actions in the classroom. Šapkova's (2014) study found teachers' stated beliefs were inconsistent with their classroom practices. Her data analysis compared teachers' espoused theory and their theory in action with student achievement and showed that teachers who espoused more traditional beliefs about teaching mathematics had lower student achievement scores. However, teachers who reported using more traditional techniques in their classroom had higher test scores. It seems these inconsistent teachers, who espouse reform style beliefs about mathematics, but tended to more traditional teaching methods, had the best student achievement results. Šapkova's (2014) results were supported by her literature review, specifically from the findings in the IEA Teacher Education and Development Study in Mathematics (TEDS-M) authored by Tatto, et al. (2012) who corroborated that students are more successful when their teacher believes that mathematics is a process of inquiry. Although there is research that supports the use of more traditional practices in the classroom, Šapkova (2014) noted that there is no proof for the claim that teachers holding traditional beliefs translates to positive student achievement.

Philipp (2007) reports that researchers who have studied teachers' beliefs are doing so in the reform spirit even though they may not say so. Philipp (2007) also cautions that although the reform movement has made a large impact in mathematics education research literature it has not enacted an equal amount of change in America's classrooms. So when researchers debate and study teachers' beliefs with a reform ideal in mind, they may well have witnessed teachers with more traditional beliefs and classroom practices.

Teachers' Beliefs About ELLs

While there is less research on teachers' beliefs dedicated specifically to their beliefs about ELLs, what can be found is still noteworthy. Penfield published her groundbreaking study of teachers' beliefs about ELLs in 1987. She surveyed 162 regular classroom teachers of ELLs in New Jersey as part of a professional development session. Of these teachers, 137 taught K-8 and the remainder taught grades 9-12. Penfield's goal was to qualitatively study and uncover these teachers' beliefs and assumptions about their ELLs as well as learn more about how these teachers felt about their ESL teacher colleagues. Penfield ultimately recommended that teachers require more training in order to successfully educate ELLs. Along the way she noted that teachers believed mathematics, a universal language, was an easier class to teach to ELLs than other subjects. This erroneous belief has been re-corroborated in other, more recent articles (Aguirre & Bunch, 2012; Fernandes, 2012). Penfield cites Fillmore's (1986) research of elementary age Chinese and Hispanic ELLs. Fillmore (1986) noted that many ELLs she witnessed are being taught basic mechanical skills and not a deeper understanding of the mathematics, which in her view did not qualify as a real education. The teachers surveyed

by Penfield felt that the traditional teacher-centered classroom does not work for ELLs. Penfield references numerous articles and recognized that, “teachers must learn new ways for improving classroom dynamics” (p. 30). This idea, witnessed by teachers in the classroom and justified by Penfield’s research, points to the NCTM reforms that came after her 1987 publication as beneficial for ELLs.

An interesting outcome of Penfield’s survey was that although it elicited no race-specific information, they were offered spontaneously by teachers. All comments made by Penfield’s participants towards Hispanic students were negative. The teachers characterized their Hispanic students as- lazy, wild, and difficult to discipline. Teachers complained that Hispanic students took extra time to discipline and needed to be kept busy. It is no surprise that one teacher, when discussing eighth grade students, was quoted as remarking, “This age level seems to put down those from other countries- especially Hispanic students” (p. 33). According to the comments from other teachers these Hispanic students are lazy discipline problems and were likely positioned as such by their teachers. Would anyone be surprised to learn that their fellow students also perceive and position the Hispanic students similarly? When comparing Hispanic students to Asian students, Penfield’s survey data yielded the following,

Hispanic students (mostly Mexican Americans), on the other hand, responded to irrelevant curriculum material by losing interest. These same students did very well when teachers presented well-organized instruction rich in content and emphasized comprehension and student participation in instructional activities. In short, the nature of the curriculum and the way it is presented may be the

explanatory factor for the classroom disjuncture which these teachers' comments reflect. (p. 32)

Penfield's analysis lends support to the reform mathematics movement and the instructional techniques discussed in this chapter as benefitting ELLs. Teachers also specifically reported using student collaboration and incorporating ELLs' cultural experiences and multidiscursivity into the classroom.

Pettit (2011) more recently used a qualitative open coding system to review the literature of teachers' beliefs about ELLs and concluded that there are five beliefs teachers can hold to more effectively teach their ELLs.

This new set of beliefs include (a) high expectations for ELLs, (b) accepting responsibility for ELLs, (c) encouraging native language use both at home and in the classroom, (d) an awareness of the time it takes ELLs to learn academic English, and (e) a desire for professional development in relation to ELLs when needed. (p. 144)

Pettit's article was published twenty-four years after Penfield's and yet she is still calling for professional development for teachers about ELLs and echoes Penfield's sentiment that teachers need to take responsibility for educating the ELLs in their classroom and not expecting the school specialist or the English department to take care of it.

Karabenick and Noda (2004), after surveying nearly an entire Midwestern school district's teachers, found evidence that teachers who believed themselves capable of effectively teaching ELLs translated to higher student performance. This may explain part of the reason why so many researchers call for additional professional development

of teachers of ELLs. If teachers feel more confident in their abilities as teachers of ELLs, they will be positioning themselves differently which will transfer to positive changes in the classroom. Teachers who believe they are capable and competent will position themselves as classrooms leaders and teach accordingly.

In developing a framework to allow teachers to identify factors that influence the way ELLs learn, Campbell et al. (2007) noticed that oftentimes teachers are unaware of how these factors influence the decisions they make in their classroom instruction. Campbell et al. (2007) proposed that these teachers' beliefs were shaped by their experiences which did not include considering or reflecting on the cultural and linguistic backgrounds of their students when crafting instruction.

When considering what teachers of ELLs are doing in their classrooms, we look to the research literature on the subject. The research cites several instructional strategies, namely, revoicing, heteroglossia, multiple representations, gestures, and discussion. The theoretical framework for this review, positioning theory, was chosen as the lens for studying what teachers are doing and saying in their classrooms because that saying and doing is about more than just adding and subtracting. That saying and doing has the potential to situate ELLs as capable learners. The recommended research-based instructional strategies allow teachers to position their student as successful. But this is only a part of what influences what teachers are doing. The research also says that a teacher's beliefs will impact the decisions she makes in her classroom practice and how she positions her students. A teacher's own experiences have positioned her in ways that have shaped her beliefs about mathematics and the teaching of mathematics. Interestingly, a teacher's beliefs may or may not be discernible in her practice for various

reasons discussed. Are the beliefs of teachers of ELLs more or less likely to be consistent or inconsistent with their classroom practices? Although literature tells us what some teachers of ELLs believe and what they all should believe (Pettit, 2011), there is no information on whether their beliefs are consistent or inconsistent with their instructional choices and practices.

Putting It All Together

The research gives evidence that *professional mathematics teacher noticing*, - “... attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings”, is important for learning and the learner (Jacobs et al., 2010, p. 172). The importance of noticing is evidenced by the amount of research dedicated to the teaching of this particular skill to pre-service teachers. The ability to effectively apply this kind of noticing in the classroom brings educators closer to providing the high quality instruction touted by mathematics education researchers and reform organizations such as NCTM (Jacobs et al., 2010). It would seem impossible to witness a teacher “interpreting children’s understandings” (Jacobs et al., 2010) without employing a brain scan or at the very least, as some researchers have done, equip the teacher with a camera that captures her every action that she must later account for. The actions a teacher undertakes in noticing that are, well, noticeable are what a teacher will do to, attend to children’s strategies and deciding how to respond on the basis of children’s understandings. They can use different types of formative and summative assessments, but they will also use various instructional strategies. Instructional strategies are the tools a teacher has at her disposal when she

needs to elicit a particular type of knowledge about her students, from her students, or make meaningful some piece of knowledge to her students.

So we have teachers' noticing which equates to attending to their students' strategies and deciding how to respond based on their students' understandings, at least as far as can be observed. Teacher noticing can be accomplished with the use of instructional strategies and in the case of ELLs, particular instructional strategies shown to effectively work with this population. These are worth observing in a secondary mathematics classroom. ELLs' learning of mathematics using a standards-based curricula is an area of research that has been neglected at the secondary level, not receiving the attention of the comparable Language Arts (Celedón-Pattichis, 2008). Shein (2012) also calls for more study of ELLs in the era of reform mathematics, claiming that the discussion based learning that reforms demand may serve to further marginalize the ELL population. Recall that Turner et al. (2013b) considered the positioning of elementary ELLs in mathematics discussions by forming an after-school mathematics club where they controlled the curriculum and facilitated the discussion. Although that may be an important step in the research considering positioning and ELLs, it is not representative of what ELLs and their teachers will encounter in the classroom. Additionally, Turner et al. (2013b) called for "more study of instructional practices that support equitable participation" (p. 200). They discuss revoicing as an instructional strategy that can position students as it credits students with their ideas and puts them in the role of problem solver in front of their peers. However, the review of the literature revealed several additional instructional strategies recommended for use with ELLs. As found in Table 1, they were using language as resource, multidiscursivity, multivoicedness,

gestures, scaffolding, multiple representations, and fostering classroom mathematical discussions. One claim of this study is that the reason these strategies are recommended for ELLs and the reason they have been found to be successful is that they can be used to position ELLs positively in the classroom. Students are positioned by their teacher, and also peers, as problem solvers, authorities, and worthwhile members of the community of learners. In the long run how ELLs are positioned will influence how they view themselves (Turner, et al., 2013b).

The preceding sentence can be explored, in part, through the investigation of teachers' beliefs. Recall Yoon's (2008) case study of middle school English teachers of ELLs found that the beliefs teachers held about themselves and students were evidenced in the ways they positioned their students. Additionally, it feels natural to ask in what ways were today's teachers positioned as learners. How has that influenced their beliefs and the choices they make for their own classrooms? If positioning is critical to the classroom experience of an ELL and teachers' beliefs are an important part of understanding that classroom, then the positioning their teachers experienced may also be of value to uncover. An understanding of how a teacher was herself positioned may also be an important step in unpacking the teacher's relationship with Pettit's (2011) five beliefs that a teacher in a linguistically diverse classroom needs to possess in order to be effective.

This new set of beliefs include (a) high expectations for ELLs, (b) accepting responsibility for ELLs, (c) encouraging native language use both at home and in the classroom, (d) an awareness of the time it takes ELLs to learn academic

English, and (e) a desire for professional development in relation to ELLs when needed. (p. 144)

III. METHODOLOGY

Classrooms in the United States are becoming more linguistically diverse, making the classroom education a challenge for teachers and administrators. Review of the literature indicates a number of best practices when teaching English Language Learners (ELLs) such as revoicing, using language as a resource, and scaffolding. However, even after twenty plus years of research into the education of ELLs, many are still lamenting the lack of teacher education with regards to teaching ELLs (Penfield, 1987; Pettit, 2011). Therefore, it is a question whether educators are even utilizing these research-based best practices in the classroom. Recent mathematics education research touts the value of professional mathematics teacher noticing in the classroom (Ball, 2011; Jacobs et al., 2011b; Schoenfeld, 2011), but there is little to no research available that addresses what that even looks like in an ELLs' classroom. Finally, research emphasizes the importance of teacher beliefs about mathematics, themselves, and their students with regard to the choices teachers make in their classroom as well as how they position their ELLs and effectively educate these students. Positioning considers the interaction between people, in this case teacher and students, and the positioning of ELLs as powerful members of a learning community can contribute to their success in the classroom (Yoon, 2008).

The purpose of this study is to gain an understanding of professional mathematics teacher noticing and positioning in a linguistically diverse middle school mathematics classroom. Specifically, the study investigates the role of teacher beliefs, teacher positioning, instructional strategies, and teachers' use of professional mathematics teacher noticing and how these can position their students.

Research Questions

The study will address the following questions:

- 1) What do teachers of linguistically diverse mathematics classrooms believe about mathematics, mathematics teaching, and their linguistically diverse students of mathematics? How do teachers position themselves as teachers? What aspects of these beliefs and positioning can be witnessed in the classroom?
- 2) What does professional mathematics teacher noticing look like in a linguistically diverse middle school mathematics classroom?
- 3) How do teachers' use of research-based instructional strategies, their beliefs, and professional mathematics teacher noticing position students in the classroom?
- 4) How are teachers in linguistically diverse mathematics classrooms using invitation moves to promote student engagement?

The design of this study is a case study analysis consisting of three middle school mathematics teachers working in linguistically diverse classrooms. A case study approach is appropriate for this type of research as it is recommended for investigating phenomenon within a real-life context (Yin, 2003). An in depth study situated in a real-life context that develops understanding of the phenomenon in question is the purpose of a case study and the reason it is the best choice for this particular research (Creswell, 2013).

Population and Sampling

The participants for this study were purposefully sampled from the middle school mathematics teachers currently participating in the CAREER: Mathematics Instruction for English Language Learners (MIELL) project. MIELL is a National Science

Foundation funded five year research study. The main goal of this project is to empirically estimate whether and which classroom factors contribute to mathematics gains of English Language Learners in Texas schools. The emphasis is on mathematical knowledge for teaching (MKT), knowledge of students as English Language Learners, and the mathematical quality of instruction (MQI) in middle school classrooms.

There are thirty-three middle school mathematics teachers from a south Texas school district currently associated with MIELL. The teacher participants of the MIELL study represent classrooms from every middle school in the district. During the 2013-2014 school year these teachers were videotaped three times. That year, the district's student population was classified as 99% Hispanic, 96% economically disadvantaged, 65% at risk, 31% limited English proficient, and 3% migrant. The mathematical knowledge for teaching (MKT) of each participant in the MIELL project was assessed during the 2013-2014 school year. The mathematical quality of their instruction (MQI) was quantified during the 2014-2015 school year utilizing the three videotaped lessons. Additionally, the knowledge for teaching mathematics to English Language Learners (KT-MELL) of the participant teachers was assessed (Wilson, 2013) as well as their Spanish language proficiency (Mejía Colindres, 2015) via a web survey.

These various scores were used, similar to the methods of Hill et al. (2008), to choose three participants for this study. The participants were chosen based on their scores on the MKT, MQI, and KT-MELL assessments. The overall MQI lesson scores were used as well as component scores that would reflect a more student-centered classroom. This was done to maximize instances of teacher noticing that could be explored. This enabled the research to create the most illuminating case studies possible

with the greatest potential for a deeper understanding of noticing in these linguistically diverse mathematics classrooms. The five highest scoring teachers consisted of two eighth grade and three seventh grade teachers. The eighth grade teachers, Jaime and Patricia, were chosen as the eighth grade year is the farthest removed from elementary school which is where most current research into ELLs and teaching ELLs takes place. In this way the results of the study will contribute specifically to a better understanding of ELLs in *secondary* mathematics classrooms. The last teacher chosen was the highest scoring seventh grade teacher, María.

Instrumentation

Data for this study was gathered through the videotaped classroom observations of the three participant middle school mathematics teachers as well as an interview protocol, and information previously collected through the MIELL project. This section contains information about the tools used to collect data for this study, including pilot test results.

Interview protocol. Interviews are an integral part of doing qualitative research and considered the best data collection technique for a case study of several participants (Merriam, 2009). Interviews allow researchers to learn what cannot be observed, in the words of those experiencing what is under study (Patton, 2002; Rubin & Rubin, 1995). Classrooms observations are powerful, but any understanding of teacher's beliefs gained from that observation can only be a supposition. An in-depth, accurate knowledge of the beliefs held by the teacher participants of this research study can best be obtained through interviews. That information is what provides the context for answering the first and third research questions of this study.

The three teachers participating in this case study participated in face to face interviews with the researcher in order to gain understanding into their beliefs about mathematics and teaching mathematics, as research shows that these influence a teachers' classroom practice. Two of the teachers were interviewed in their classrooms, the third was interviewed in a colleague's classroom. The beliefs teachers hold about themselves as educators and about teaching ELLs were addressed during the interviews. The interview protocol utilized with the participants can be found in Appendix A. As part of the interview protocol, participants constructed their mathematics story, similar to the approach taken by Drake and Sherin's (2006) look into teacher beliefs. Their mathematics story interviews are based on McAdams (1993) life story interviews. As this research is interested in how the teachers themselves were positioned in their lives and how that translates to their current practices, it is believed that a mathematics story interview might help uncover this historical positioning.

Interview pilot study. The interview protocol was piloted with three central Texas middle school mathematics teachers in January and February of 2015. The purpose of the interview is to know more about teachers' beliefs about mathematics, mathematics teaching, positioning, and ELLs. Initially the interview protocol consisted of two main sections, first the mathematical life story and second a list of questions inspired by Pettit's (2011) review of teachers' beliefs towards ELLs. The mathematical life story portion stayed very true to McAdams (1993) protocol in which he asks interviewees to think of their life as a book or story with chapters. They are then asked to name the chapters of their lives and describe the major events of each chapter. This proved to be challenging for my pilot study teachers. They required a lot of scaffolding and

instructions in order to speak freely about the events of their life. One teacher in the pilot study even said, “I’m a math person. I don’t like stories.”

The interview protocol was revised and tested again based on this feedback. An outline of the chapters of a mathematical life story are now provided for interviewees as elementary school, middle school, high school, university, and becoming an educator. Interviewees are, of course, encouraged to speak to other facets of their lives that they feel are impactful and address their life stories in the chronology that makes the most sense to them. That is why the questions are not numbered. Although the protocol may be more structured than McAdams (1993), in order to create a narrative the researcher must be less controlling and more willing to follow where a participant’s story leads (Riessman, 2008). During the course of the pilot study, common themes emerged concerning parental involvement and episodes from childhood that have translated into or influenced their teaching. These themes are now included as specific questions in the protocol as well as a question that asks for specific recollections of working with ELLs. Because positioning is an important part of this study and a concept that many teachers are unaware of, it has also been highlighted in the revised interview protocol giving interviewees an opportunity to consider episodes they may have overlooked. Finally Yoon’s (2008) study of teacher positioning has inspired a question that investigates how teachers are positioning themselves in their classrooms as educators.

The most interesting and unexpected realization to come from the pilot study was the link between how these teachers were positioned as children learning mathematics and their chosen careers. Chet, one teacher, felt his teachers and classmates judged him as dumb at mathematics. He felt like he needed extra help that wasn’t available to him. Chet

now teaches mathematics to special education students. Simon, another teacher, felt ignored as a student, both at home and in the classroom. Simon now teaches at a school that delivers individualized mathematics instruction and attention to students. Finally, José is a teacher who grew up as a Spanish speaking minority and positioned as a stupid immigrant who could not learn. He now empowers other immigrants to be successful thinkers of mathematics, English, and their native languages.

The results of the pilot study showed that sufficient data would be collected to answer the research questions posed in this dissertation. The interview protocol is vital to answer the first research question which is, What do teachers of linguistically diverse mathematics classrooms believe about mathematics, mathematics teaching, and their linguistically diverse students of mathematics? How do teachers position themselves as teachers? The mathematics life story portion of the interview was used to understand teachers' beliefs about mathematics and mathematics teaching. The second portion of the protocol was used to discover how teachers position themselves and their beliefs about ELLs. Teachers use of beliefs, positioning, and research-based instructional strategies in the classroom is the focus of the third research question of this study and thus also relies on information uncovered by the interview protocol.

Classroom observations. All teachers participating in the MIELL project were video recorded teaching in their classroom three times during the 2013-2014 school year by an experienced, professional, educational videographer. He used a single camera and was instructed to maintain focus on the classroom teacher as much as possible. These video recordings represent three classroom observations for the three teachers that make-up this case study for a total of nine classroom observations. The video recordings took

place in October, January, and May of that school year. The video recordings were transcribed by the researcher to be used to answer the research questions. Segments of the transcripts that contained extensive conversation in Spanish were also reviewed and revised by a fluent Spanish speaker. In order to answer the first research question, the three participants were interviewed concerning their beliefs and positioning. After the interviews were completed the classroom observations were examined looking for how those beliefs and positioning are illustrated, or not, in the classroom.

The second research question focuses on professional mathematics teacher noticing in the middle school classroom of an ELL. An important component of this study is gaining insight into what professional mathematics teacher noticing looks like in linguistically diverse classrooms. These classroom observations were transcribed and coded to gain an understanding of what noticing means in a linguistically diverse mathematics classroom. The coding will be discussed in more detail in the forthcoming section titled, Analysis Plan.

The third research question brings together the results of the previous two and teachers' use of research-based instructional strategies to look at the positioning of students in the classroom. A sample of eleven videotaped lessons from eighth grade teacher participants in the MIELL project was viewed as part of this proposal to verify if the research-based instructional strategies considered in the literature review, and summarized in Table 1, could be witnessed in these classrooms. These initial viewings showed strong evidence of the following instructional practices: revoicing, language as resource, gestures, and scaffolding. Multiple representations and classroom mathematical discussion were seen less frequently. Using culture as a resource and multivoicedness, or

a give and take learning environment between teacher and student, were not noted in this sample. These recordings are, of course, just a few days captured over the course of an entire school year and it is unlikely that every instructional strategy would be captured. The absence of a strategy does not imply that it was never employed. Some strategies are likely to occur in greater frequency than others. Some strategies may only occur with a particular teacher. These instructional strategies, teacher beliefs, and positioning are the elements of the classroom observations that provided the context for and explain how students are positioned in the mathematics classroom. After viewing the eleven lesson sample, which is about 10% of the total videos recorded, there is reasonable assurance that the data exists to answer the research questions in this study.

The fourth research question looks at student engagement in these linguistically diverse classrooms, specifically what kinds of invitation moves the teachers are using in their classrooms. A research question that addresses student engagement was suggested by my dissertation committee members as a natural extension of a classroom investigation into teacher noticing. Student engagement is vital to creating the kind of classroom that NCTM (2000) lauds, which is also the kind of classroom where robust teacher noticing is able to take place.

Procedure

The video recorded classroom observations of teacher participants in MIELL were collected during the 2013-2014 school year and are currently available to researchers. The three participants were contacted during the summer of 2015 and agreed to be involved in this research study. A copy of the consent form can be found in Appendix B. The teacher interviews were conducted in August and September 2015 and

were then transcribed. The process of transcribing the nine videos took place after that and was also completed before the end of the summer.

Analysis Plan

Although the interview protocol used with the participants is more structured than McAdams' (1993) original the outcome was still a narrative of a participant's life and experiences as a student, teacher, and mathematician. For this reason Riessman's (2008) narrative analysis methods was employed to analyze the participant interviews, in particular thematic narrative analysis. After the first research question was answered the results were sent to the three teachers for member-check in order to validate the results.

The purpose of the second research question of this study is to gain insight into what professional mathematics teacher noticing looks like in linguistically diverse classrooms. The nine classroom observations were transcribed and instances of noticing coded to gain an understanding of what noticing means in an ELLs' mathematics classroom. A noticing episode is defined, for the purposes of this study, as a student-teacher interaction in which a student or students has produced some piece of mathematics that a teacher is motivated to interact with. This eliminates moments during a class when the teacher is lecturing or demonstrating a skill, students are talking amongst themselves, working independently, or similar. This purposeful definition is broad enough to capture noticing episodes with varying levels of attending and responding. This study uses Jacobs et al.'s (2010) three part definition of professional mathematics teacher noticing "as a set of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172). Their research also developed a coding

scheme to analyze elementary mathematics teachers' documented noticing in response to viewed classroom vignettes. This coding scheme can be found, presented in greater detail, in Appendix C.

Jacobs et al. (2010) developed their coding system to study carefully chosen classroom vignettes of student thinking. The participants in their study responded to these vignettes via writing prompts which were then coded and analyzed. In order to answer the second research question of this dissertation study, episodes of mathematical significance involving students in the classroom were noted. Each lesson and each teacher had a varying number of these episodes. Each noticing episode was coded by two researchers in accordance with Jacobs's et al. (2010) system for attending and deciding how to respond based on the actions a teacher took in these two components. Jacobs et al. (2010) score attending to a student's strategy as either a (0) Lack of evidence or (1) Evidence. Deciding how to respond based on student's understanding receives a score of (0) Lacking evidence, (1) Limited evidence, or (2) Robust evidence. Table 2 below is provided to demonstrate example scores via corresponding noticing episodes from this study. It is hoped this will clarify the process for readers and inform researchers interested in noticing.

Table 2. Coding of Noticing Examples.

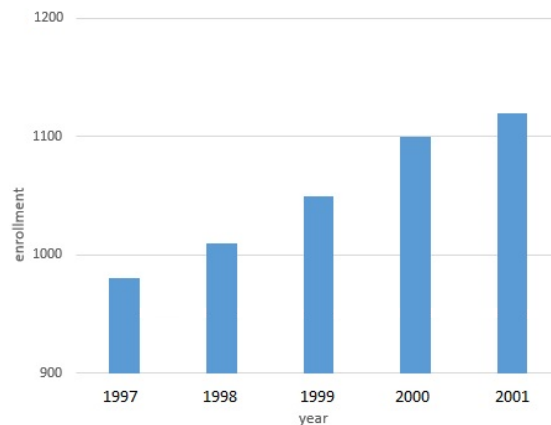
Attending Score: (0) Lack of evidence
<p>Example A: A pair of Patricia's students are calculating $(4.25)^2$ and have different answers. She does not investigate how her student arrived at the wrong answer. She recommends "making sense" of the product, but that advice does not help to differentiate whether 18.06 or 17.91 is correct.</p> <p>Patricia: How are you two doing? Student: Move the decimals 4 times 1, 2, 3, 4 put it there. I almost got the same answer, I got 18.06. Patricia: Which one makes sense? 4 times 4 is 16 right and it has a quarter so you should be, think about it, it's 4 and a fourth. So what is your mistake? You found your mistake? Student: Yup. Patricia: What was your answer? 17.91, very good. Now write the formula, write the formula here, with all the steps. Because that's the only way you can check if you are doing it correctly or not.</p>
<p>Example B: When teachers approach and engage a group of students that have not employed their own strategy for solving a problem this receives an attending score of (0). In these instances it is not possible to attend to a student's strategy.</p>
Attending Score: (1) Evidence
<p>Example A: Jaime checks with a student who has made a mistake in solving a proportion to find a percent increase. Through his questioning, Jaime discovers that his student's strategy was to incorrectly divide in his head by 25 and not 50.</p> <p>Jaime: What's number 1? Student: Increase of 12%. Jaime: Increase of 12%, let me see your work. Proportions, good. You multiplied then you're going to divide. Ok. How did you divide? Student: 50 into 300. Jaime: Ok, 50 into 300. Student: Uh 200 would be 8 and... Jaime: Whoa, how do you know that? Student: Because 50... Jaime: 50...</p>

Table 2. Continued.

Student: Oh, ok 50 and 100.
Jaime: Right, so every 2 is a 100.
Student: Oh ok. Then 50, 100, 150, 200, 250 oh 7.
Jaime: One more time.
Student: 50, 100, [counts on fingers]
Jaime: Uh huh.
Student: Then 150, 200,
Jaime: Uh-huh.
Student: Then 250, 300, oh.

Example B: María solicits a group of students to explain the method they used to answer the following question. Her comment confirms that she is processing their strategy.

The student enrollment at Johnson Junior High is shown in the graph below.



Based on the data in the graph, which statement about the student enrollment in 1996 would be true.

- F. No students were enrolled in 1996.
- G. More than 1,000 students were enrolled in 1996.
- H. Less than 800 students were enrolled in 1996.
- J. About 900 students were enrolled in 1996.

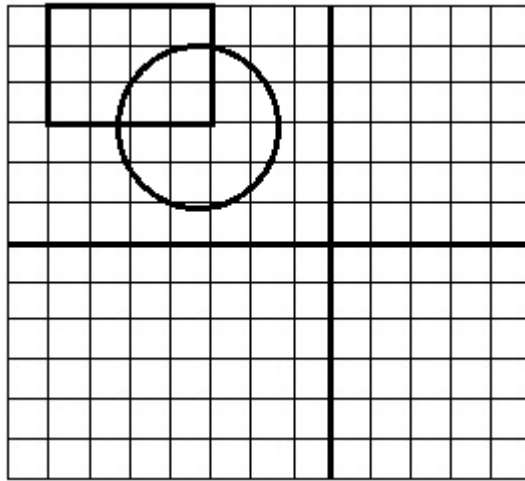
María: Really you finished? How did you prove that this one is correct? Can somebody explain to me? Sam will explain it to me.

Table 2. Continued.
<p>Sam: 1997 was 980 so it was 1996 on this one so it has to be lower than 980. And then so it's supposed to be less than 980 so it enrollment of how many people came in 1996 it had to be lower so about 900 were enrolled in 1996 cause it can't be less than 800 cause it would be like 1995, 1994.</p> <p>María: Oh, so this bar graph would be extended further to the right or maybe to the left? We could have done that. Ok, and I like what you did, you circled the word true. Because sometimes they switch it right? And what do they put instead of true?</p> <p>Sam: Not true.</p> <p>María: Not true, very good.</p>
Deciding how to respond Score: (0) Lack of evidence
<p>Example A: Patricia solicits a student, Ken, to explain the process for finding the surface area of a rectangular prism. His responses demonstrate his confusion, however Patricia's response ignores his misunderstandings and merely speculates as to the reason.</p> <p>Ken: You measure the two sides of the square or prism. You multiply then divide it by two. Then what ever you get is your surface area.</p> <p>Patricia: I'm not sure about this one. Ok, anybody wants to add something about what he said? He said to measure and then what else did he say?</p> <p>Ken: Multiply.</p> <p>Patricia: Multiply.</p> <p>Ken: And divide it by two.</p> <p>Patricia: And then divide it by two. Did we divide by two?</p> <p>Students: No.</p> <p>Ken: No, divide by, by...</p> <p>Patricia: Why didn't we divide by two in this case?</p> <p>Ken: Because it's to the second power?</p> <p>Patricia: It's the area, when do we divide by two...</p> <p>Students: When it's a triangle.</p> <p>Patricia: When it's a triangle. Were we working with triangles this time?</p> <p>Students: No.</p> <p>Patricia: No, when did we work on the triangles? When it's a...</p> <p>Students: Pyramid.</p> <p>Patricia: Pyramid, right? But today were rectangular prisms. Ok, you need to be very careful.</p>

Table 2. Continued.

Example B: When speaking with a group of students about translating ordered pairs, María dismisses a viable strategy offered by Francisca that is first translating the rectangle to determine the circle's eventual location. Seemly unaware that it will lead to a correct solution, María enforces her own strategy on the group.

Look at the image on the coordinate grid below.



If the image is translated 5 units to the right and 8 units down what ordered pair would only be inside the circle?

- A. (3, -5)
- B. (6, -2)
- C. (2, -2)
- D. (0,0)

Francisca: Yes, I translated this 5 times and I moved it down.

Maria: Which one? The one in the rectangle?

Student Francisca: Yes, this one this one this one this one. [She points to the corners of the rectangle.]

Maria: Why do you think I tell you to read, not only that I tell you to underline the question. And you haven't done it, you aren't following my strategy. What ordered pair will what?

Carrie: Be inside.

Table 2. Continued.
<p>Maria: Inside what?</p> <p>Francesca: The circle.</p> <p>Maria: What are you going to move?</p> <p>Carrie: The circle.</p> <p>Maria: The circle. That is why I am asking Francisca, why are we moving the rectangle? If the question is only talking about the circle you see how important it is to read the question to underline the question to circle the key words, ok. What are the keywords?</p>
Deciding how to respond Score: (1) Limited evidence
<p>Example A: Patricia finds that a pair of students are repeatedly solving proportions correctly, but putting the decimal in the wrong place. She acknowledges their difficulties with decimals, but does not investigate the cause of their confusion. She encourages the students to show their work because next year it will please their math teacher.</p> <p>Patricia: Let me see it. Number 3, ok, erasers to cost, very good. 5 erasers 39 cents, that is correct and why did you put 60 over here?</p> <p>Pedro: For the 60 erasers.</p> <p>Patricia: Ok, and you are looking for the what?</p> <p>Students: The cost.</p> <p>Patricia: Ok, you did everything correct, you multiplied this part. Your problem continues to be with the decimal points. Where is the decimal going to be?</p> <p>Pedro: 2.340. [60 times .39 is worked out on the paper.]</p> <p>Patricia: How many times are you moving the decimal?</p> <p>Pedro: Two times.</p> <p>Patricia: Two times and where do you want to put?</p> <p>Pedro: So it's going to be 23.40.</p> <p>Patricia: 23.40, ok, 23 dollars and 40 cents. And that needs to be doing what? I have a problem guys you were supposed to show me. Where is the cross products in here? [Pointing to their paper.] You are supposed to show me here, point 39 times what?</p> <p>Pedro: 60.</p> <p>Patricia: And that equals to what?</p> <p>Pedro: 5x.</p> <p>Patricia: I need to see that one guys. Yes I know you are so good that you can do it mentally, but we're trying to connect this one with the test and Algebra 1 and in Algebra 1 you need to do this otherwise your teacher will be very upset.</p>

Table 2. Continued.
<p>Example B: During a discussion of the Real number system student Felipe defines terminating decimal with a unique connection to prior knowledge. Teacher Jaime acknowledges, but does not clarify or expand on this definition.</p> <p>Jaime: 3.10 is that a terminating decimal? Felipe: Um, I don't know. Jaime: You don't know. Felipe: No, cause I was going say something different. [another hand is up] Jaime: You think it's a yes, why? Vivian: Because I think a repeating decimal is the decimals that repeat the numbers. Jaime: Very good. Felipe: I was going say for dividing whenever you are dividing by decimals when you tell us to move it as many times to the right hand side until it becomes a whole number that's what I was thinking a terminating decimal was. Jaime: Oh, ok good so again a terminating decimal is one that we can do that. We divide and we move the decimal over to make it a whole number. Those that we can do it to are called terminating decimals 3.5, 6.25, numbers that terminate. Terminate, which means stop. Got it? Good job.</p>
Deciding how to respond Score: (2) Robust evidence
<p>Example A: Student Javier divides \$1.88 by 10 to determine the price of a single pencil. María bases her guidance and questioning on Javier's demonstrated knowledge of decimals and money.</p> <p>María: Let me check, ok what is the r for? Remainder? Did I show any remainder in my problem? Did I stop at my remainder? Javier: No. María: No, so we don't need this. Ok, why are you going from 8 to 10? Javier: Um, from, from the... María: Did I do something like this in my other problem? Javier: No. María: No, you were doing really good up to this. Continue, continue with your steps. What are you going to do? You get a remainder of 8, what do you do? [Javier adds a zero.] There you go, very good, there you go. Perfect. Now we got it right. Ok, now I want you to write for me, Javier. Write for me your complete answer the one that you want me to check. What are you going to write for me? Wherever you want. Zero</p>

Table 2. Continued.
<p>point, [Javier writes \$0.188.] Javier, we're talking about money. How much money is that that I see? [Javier amends his answer to \$0.19.] Ok good. And you know what I liked is you wrote you dollar sign. There we go. Ok, what does it mean? What does zero point nineteen mean?</p> <p>Javier: 19 cents per each pencil.</p> <p>María: Write it down. And say what you are writing.</p> <p>Javier: [writing] Zero point nineteen cents per each mechanical pencil.</p>
<p>Example B: Jaime and his class are classifying the number 5 as a natural, whole, integer, rational, and real number. Jaime knows his students neglected to classify 5 as a real number because the Real number system did not have a corresponding definition.</p> <p>This is communicated via a student.</p> <p>Felipe: I got natural number, whole number, rational number, real number, and integer.</p> <p>Jaime: Awesome job, awesome job. I think I know which one you left off, if you chose four. How many of you only had four? [hands go up] 1, 2, 3, ok. Vivian, which one did you leave off?</p> <p>Vivian: Real.</p> <p>Jaime: Real. John, which one did you leave off?</p> <p>John: Real.</p> <p>Jaime: Real. Sam, which one did you leave off?</p> <p>Sam: The same one.</p> <p>Jaime: The same one, real?</p> <p>John: There's no, like the definition out of all of those.</p> <p>Jaime: But if I ask you, John, is this a real number what would it be?</p> <p>John: Yes.</p>

How the participants of this study interpret student understanding, the third component of teacher noticing, was also a point of interest to this research. To that end, the participants of this study were asked, as part of their interview process, to watch the same video from the Annenberg Learner collection and then responded to three episodes of student thinking using writing prompts identical to those used in the Jacobs et al. (2010) study. These prompts are also included with the interview protocol in Appendix

A. In order to ensure reliability of the codes used for this research study, an independent rater was employed and discrepancies were discussed until consensus was reached.

The results of coding the nine teaching lessons of the three participants of this research will be disseminated and various exemplar episodes discussed in the subsequent chapters. In this way professional mathematics teacher noticing in linguistically diverse classrooms will be illuminated and made available for discussion in ways that are familiar to the mathematics education research community. This noticing analysis plan will form the basis of a framework for this study with the understanding that noticing and ELLs have not been studied extensively in the classroom. It is expected that techniques unique to linguistically diverse classrooms may emerge. For example, the sampling of videos viewed for instructional strategies illustrate literacy promoting teacher moves common to these linguistically diverse classrooms such as reading out loud, word walls, and the use of two languages.

The third research question takes what was gathered from questions one and two, combines it with the research-based instructional strategies and asks how these are used by the teacher to position students. Instructional strategies that participants utilized in the videotaped classroom lessons will be noted. Enyedy et al. (2008) studied revoicing and coined the phrase *revoicing to position* which they define as follows.

We use the term “positioning” to describe one potential of revoicing, by which a reutterance or reported speech has the strategic effect of explicitly placing the original speaker in relation to other people, the task, or the original speaker’s interpretation of his or her own utterance. (2008, p. 141)

Enyedy et al. chose and analyzed instances of revoicing to position by choosing episodes of revoicing that included “explicit verbal, gestural, and other non-verbal positioning moves by the teacher that could be located in the transcript of the video” (p. 141). Each of the revoicing to position episodes captured was then addressed using the eight questions that appear in Appendix C. In a similar fashion, each research-based instructional strategy employed by participants of this research study will be evaluated for its ability to position students. Instances of noticing identified in the transcript of the videos may also be discussed in terms of their ability to position as well as the beliefs a teacher holds and how they position themselves in the classroom

The fourth and final research question looks at student engagement in these linguistically diverse classrooms. Both instructional strategies and teacher noticing are highlighted in research questions two and three. Teacher noticing really requires a student centered classroom to thrive and student engagement is a useful way to investigate how “student centric” a classroom really is. Franke et al.’s (2015) six teacher invitation moves and three teacher support moves will be utilized to investigate whether teachers are promoting student engagement in their classrooms and how they do so. These six invitation moves were explain someone else’s solution, discuss differences between solutions, make a suggestion to another student about his or her idea, connect their ideas to other students’ ideas, create a solution together with other students, and use a solution that was shared by another students. Franke et al. found that teachers also used three follow up moves to support the students’ engagement with mathematical ideas. These support moves were probing, scaffolding, and positioning. The instructional strategies and teacher noticing performed by the participant teachers in these video recordings will

be examined for the density of these six invitation moves and three support moves as a measure of student engagement in the classroom.

IV. RESEARCH FINDINGS

The main goal of this study is to better understand what professional mathematics teacher noticing looks like in a linguistically diverse classroom. The study uses case studies and qualitative methods to analyze data collected from teacher interviews and previously recorded classroom lessons. This chapter describes the findings of this study through three case reports. Each report is divided into four sections: beliefs, professional mathematics teacher noticing, positioning, and student engagement. These sections correspond to the four research questions:

- 1) What do teachers of linguistically diverse mathematics classrooms believe about mathematics, mathematics teaching, and their linguistically diverse students of mathematics? How do teachers position themselves as teachers? What aspects of these beliefs and positioning can be witnessed in the classroom?
- 2) What does professional mathematics teacher noticing look like in a linguistically diverse middle school mathematics classroom?
- 3) How do teachers' use of research-based instructional strategies, their beliefs, and professional mathematics teacher noticing position students in the classroom?
- 4) How are teachers in linguistically diverse mathematics classrooms using invitation moves to promote student engagement?

The three participants for this study were chosen from a larger NSF funded research project, CAREER: Mathematics Instruction for English Language Learners (MIELL). The participants were chosen based on various assessments scores, including a Mathematics Knowledge for Teachers (MKT) assessment, a Mathematics Knowledge for Teaching English Language Learners (KT-MELL) assessment, and the Mathematics

Quality of Instruction (MQI) of their three recorded lessons. Portions of the MQI that would reflect a more student centered classroom were given greater emphasis in order to choose participant classrooms that were more likely to yield meaningful teacher noticing. The portions of the MQI used to do this were the overall score for working with students and mathematics and the average of the subcategory, common core accepted student practices. Figure 1 and Figure 2 below, show the standardized MKT scores of the three participants, Jaime, Patricia, and María, plotted on the horizontal axis, against both their MQI scores and a relevant sub-component of the MQI in relation to their peers. In both Figure 1 and Figure 2 below, the three participants of this study can be identified by a red square. The participants from left to right are María, Patricia, and Jaime in both graphs. Note that there are less data points than MIELL participants due to some repetition of scores.

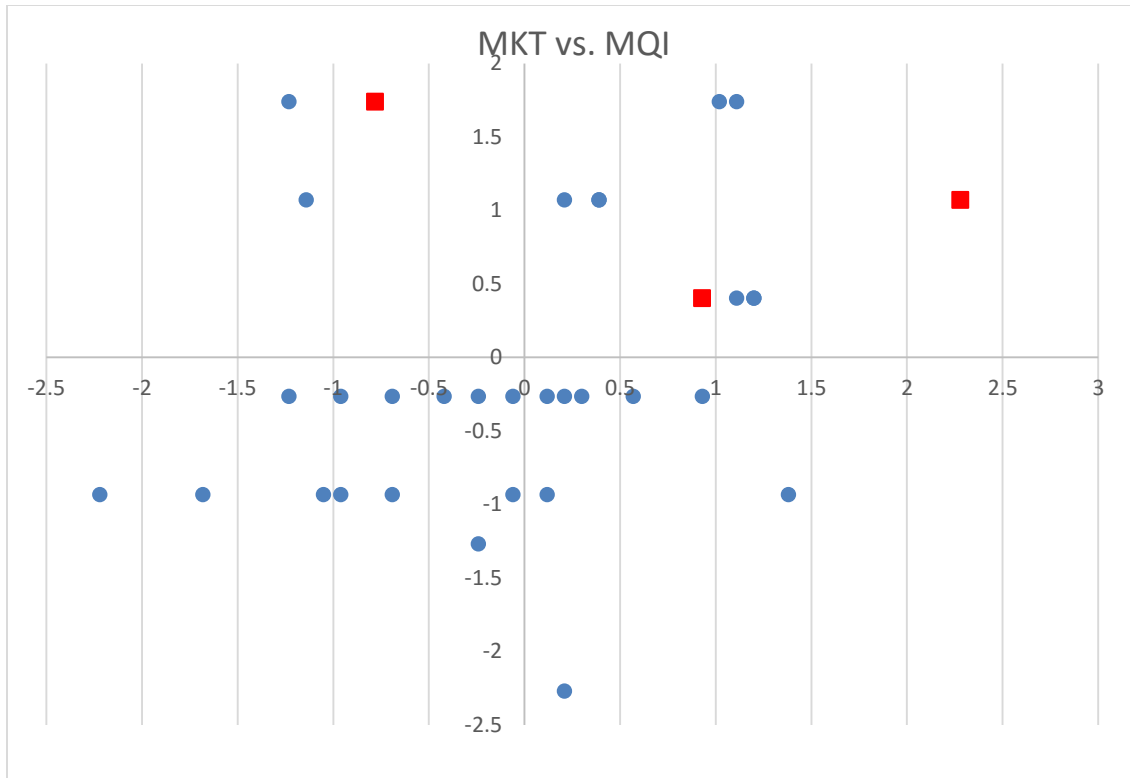


Figure 1. MKT vs. MQI Scores for MIELL Participants.

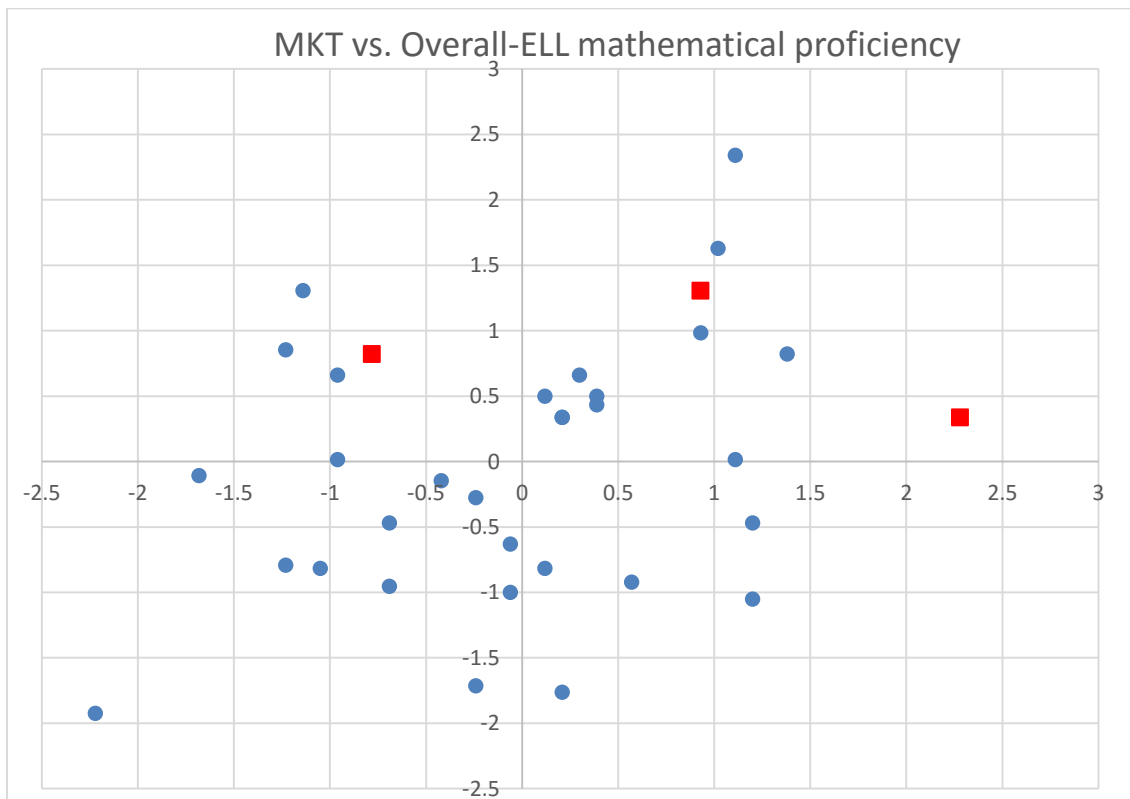


Figure 2. MKT vs. Overall-ELL mathematical proficiency for MIELL Participants.

Case 1: Jaime

Jaime is a 38 year old middle school mathematics teacher in a large south Texas public school district. He earned his degree in mathematics from the local public university and then received an emergency teaching certificate. At the time of this study he had taught middle school mathematics for 15 years, teaching all of those years at the same middle school where he himself was a student. He is currently the mathematics department chairperson at his middle school and teaches eighth grade. Jaime grew up in a Spanish speaking environment and considers himself a capable Spanish speaker, although he admits to feeling intimidated with some native speakers because there are words he does not know.

Beliefs.

Mathematics. Jaime believes that mathematics is a tool that allows us to make sense of the world around us by applying the appropriate rules. He first fell in love with the subject when he was a sixth grade student. “From there I liked math, I loved math, and found that I like conversing with math and I was good at it.” He looks at mathematics as a joyful activity replete with problem solving challenges. When asked to define mathematics however, Jaime was stumped. He had never considered the question before.

Mathematics teaching. As a student, Jaime most valued teachers that were patient, understanding, and that fostered discussion about the mathematics. He scorned teachers that he felt did not make an effort to get to know their students. Jaime believes it is most important to foster communication about mathematics in the classroom, therefore Jaime encourages and values student participation. “For me it’s everybody. They have to have a little time to shine even just when we ask questions.” He believes it is important to

get to know his students as people and individuals. “I think I am a patient person when it comes to my students and I do try to get to know my students.” Jaime admits the pressure teachers feel from high-stakes state testing and how it influences what he does in the classroom. “We have to find out what’s going to be on the test and gear our lectures, gear our strategies towards that.” Jaime admits to relying on direct teaching methods in his classroom, but also implements cooperative learning and project-based learning opportunities he has learned about during professional development sessions.

ELLs. Jaime views the bilingual program at the middle school level as a double edged sword. On the one hand it gives kids a sense of belonging, but it also labels a student. In his experience some students feel more comfortable with their ESL teacher and he believes that is a good relationship for the student. Jaime feels solely responsible for educating his students, particularly with regards to standardized test scores, but concedes that many people have a say in their education. He is unfamiliar with the term academic English and does not believe that it is important to encourage his students to speak their native language in the classroom, “in fact, I encourage them to speak more English even with each other, because I hear them in the hallways and they’re always talking in Spanish so I encourage them to speak in English.” He holds the same expectations for all of his students, regardless of their language status or abilities.

I hold them accountable, yes, and sometimes even if it’s in Spanish or Spanish and English put together, but they have to communicate... I believe that’s one of the things that they need the most is communication [about mathematics]. Even if it’s communication in Spanish and English, but that language will come eventually if they want it.

Position in the classroom. Jaime considered himself a teacher of students.

Because he values communication amongst his students he positions himself as the facilitator of these conversations, not the originator. Jaime does not want his students to view him as the sole receptacle of mathematical knowledge in the classroom.

I want them to hear that it's not just me that thinks like this. They also think like this, and I want them to hear each other. I want them to hear their thoughts... I want them to see there's always more than one way to solve a problem and I'd like it to come from them.

Observed beliefs. Jaime does use direct teaching methods in his classroom. The students sit in traditional rows and he uses PowerPoints and the whiteboard during class to share information with his students. His PowerPoints highlight the rules that the students need to follow in order to solve problems and the problems he chooses emulate standardized test questions. Although he relies on some methods associated with direct teaching, Jaime is careful to allow every student an opportunity to speak during class. He calls on each student multiple times over the course of a class period. Students can expect to answer questions, share their strategy, and help their neighbors in Jaime's class. Any classroom observer will quickly learn the name of every student in the room as he returns to them repeatedly. Jaime speaks English with his students, peppered with endearments such as "*mija*", but he almost never speaks about the mathematics in Spanish. Jaime's espoused beliefs are reflected in his classroom practices.

Professional mathematics teacher noticing. The topics that are covered during Jaime's three recorded classroom sessions are percent change, box-and-whisker plots, and the real number system. Each session is approximately 45 minutes in length. His

students sit in traditional rows, but will often be asked to check an answer with a neighbor or compare solution methods. Jaime includes the students during his introduction of new material by asking them to read definitions and problems, referencing their prior knowledge, and requiring them to provide solutions. He then assigns the students a handful of problems to try while he circulates and evaluates their progress. At the end of each lesson he assigns homework problems that reinforce the new concepts.

Jaime's three recorded classroom sessions yielded 12 instances of noticing. These instances of noticing were coded for two of the three components of teacher noticing, attending and deciding how to respond as described by Jacobs, Lamb, and Philipp (2010). Attending to student's strategies, was coded as either evidence of (1) or lack of evidence (0). Deciding how to respond, was coded as either lacking evidence (0), limited evidence (1), or robust evidence (2). Jaime's interpretation of student's understanding was evaluated through the use of a classroom video and is discussed in detail below. All of Jaime's noticing work was coded and discussed until agreement was reached by two raters, the author and an independent rater.

Attending to students' strategies. Out of 12 total instances of noticing, Jaime's attending score was a 0.92 or in 11/12 instances he showed evidence of attending to students' strategies. Jaime does not allow students to provide answers without supporting justification. He will commonly say things like, "Explain your process.", "What would be next?", "From there where did you go?", "You know how to tell? Tell me.", "You think it's a yes, why?" In this way he attends to the students' thinking behind their mathematics. When a student answers a question incorrectly he is able to guide them through their

mistake solely through revoicing their own statements. In this way the students are realizing their mistake and reconstructing solutions themselves. In the follow example, Jaime is soliciting the students to see who has determined the cost of one ticket at the rodeo.

The prices for tickets to the rodeo are listed on the table. Which equation could be used to represent the table?

Rodeo	
Tickets (t)	Cost per Person (c)
2	\$24.50
3	\$36.75
4	\$49.00
5	\$61.25

F. $c = t + 12.25$

H. $c = 20t + 4.50$

G. $c = 12.25t$

J. $c = 12t$

Jaime: Can you figure it out, Alexa?

Alexa: Isn't it like 12, like 12 dollars?

Jaime: Is it 12 dollars?

Alexa: The half of the first price? [Student points at problem on white board from her seat.]

Jaime: Ok, half of the first price?

Alexa: Because it's saying that there's two tickets and it's the cost per person is \$24.50 and isn't it like, um what's it called? Each ticket costs 12 dollars?

Jaime: Each ticket costs 12 dollars. Now what's 12 and 12?

Students: 24.

Alexa: Then it's \$12.25.

Jaime: Ah, ok. You all agree?

Students: Yes.

Interpreting students' understanding. Jaime watched a video of fifth grade students from Brookline, Massachusetts working in groups with equivalent fractions. This educational video is available, among many others, from Annenberg Learner at www.learner.org/resources. Annenberg Learner seeks to improve education and assist teachers using video clips and other forms of media. In the clip used for this study, the students were presented with a game board that had seven number lines divided into halves, thirds, fourths, fifths, sixths, eighths, and tenths respectively. A picture of the game board can be found in Appendix A. The object of the game is to move seven games pieces from the left side of each number line to the right side, or one whole. In order to move the pieces the students draw a fraction card and decide how to proceed. For example, if a student drew the card $\frac{4}{8}$ they could choose to move their marker forward four eighths on the eighths number line or they could move the fourths marker to one fourth and the eighths marker to two eighths. Students continue to draw cards until all of their game pieces have reached the right side of their number lines.

In the video clip shown, the students' teacher checks on three groups' progress giving the students an opportunity to discuss their strategies. The first pair of students discuss animatedly and correctly the numerous ways they found to represent $\frac{4}{10}$ on their game board. They proceed to write down the different ways. The second group contains three students. They draw the card $\frac{6}{10}$. One group member suggests moving the game

piece to $\frac{2}{5}$ plus $\frac{6}{10}$. His teammate informs him that they cannot move forward more than $\frac{6}{10}$. The student amends his answer and decides to move forward $\frac{1}{10}$ plus $\frac{5}{10}$. The third group of students is a pair that demonstrate for their teacher how they correctly used classroom manipulatives, in this case colored plastic rods, to verify that $\frac{3}{8}$ plus $\frac{2}{4}$ is equivalent to $\frac{7}{8}$.

After considering what three student groups did in the video, Jaime was asked to explain in writing what he learned about these children's understandings. Interpreting student understanding is coded identically to deciding how to respond, that is as either lacking evidence (0), limited evidence (1), or robust evidence (2). Jaime's interpreting score was only a 0.66, meaning he showed some evidence in two out of three instances. This was based on the three groups shown working in the video clip. Jaime's answers read as a summary of events rather than an interpretation of the student thinking. "Joe had to communicate his findings to his classmate. They had to realize their findings made sense." He did correctly identify the second groups' misconception about $\frac{6}{10}$, but an in-depth analysis of the thinking behind the student utterances was lacking.

Deciding-how-to-respond. Jaime received a score of 1.25 over the 12 instances of noticing from his recorded classroom lessons. He received seven scores of (1) which indicates limited (not robust) evidence of responding based on students' strategies. Scenarios when he received a score of 1 include rushing with a student as the dismissal bell rang or not fully explaining his reasoning to his students. That being said, Jaime did demonstrate several different approaches to responding to his students. When a student provides and justifies a correct solution he may just praise them. In one discussion of the median of a set of data a student correctly describes a process for finding the median. At

the time he is praised, but the strategy is referenced later in the lesson when Jaime solicits the class for a new strategy to find the median.

Jaime: Do we have someone right in the middle? What do you think? Who does not know if there is someone right in the middle? [Raises hand to indicate what he would like the students to do.] Who does not know? Raise your hand, it is ok if you do not know. Who does not know if there is one in the middle or how to tell when there is one in the middle? [Fred has his hand up.] You don't know?

Fred: No, I know how to tell.

Jaime: You know how to tell, tell me.

Fred: Um, you take off the first two and then the second two and then you go down towards the middle and then you see which one stays in the middle

[Indicates with fingers his technique.]

Jaime: Awesome job I like that. I like that. Does that make sense to everybody?

Ok, so right now is there one in the middle?

Students: Yes, 28.

Later Jaime and the class revisit median and after some discussion a new strategy emerges, contributed by a student named Daniel.

Jaime: Right off the bat without crossing out the first one and the last one and the second and the second to last, without doing that who does not know if we have one number in the middle? Honestly, who does not know? [Puts his hand in the air.] Anybody, Francisco?

Francisco: 83.

Jaime: Oh, so there's one in the middle? Everybody knows how to do that?

[Ismael mumbles.] Tell me how, Ismael.

Ismael: You start like counting up.

Jaime: Ok, I'm gonna stop you there Ismael because I asked you without crossing out the first one and the last one and second one and second to last one. [Students raise hands.] Ahh, let me have Daniel this time.

Daniel: 13 is an odd number.

Jaime: 13 is an odd number. Ok, how does that help us?

Daniel: Because for even numbers there's always gonna have a pair, but with 13 there's only gonna be one that's left.

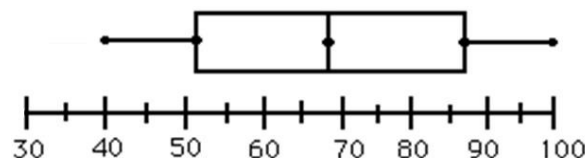
Jaime: Ah, *andale*. Very good.

Ismael: So 6 and 6 and 1.

Jaime: Exactly now. There you go. So it's easy to know if there is one number in the middle or there's two numbers in the middle. Are we good with that?

Anybody have a question?

When a student provides a wrong solution he may point out the error and also use it as a teaching tool to warn the rest of the class. In the following excerpt the class is learning to read values from a box-and-whisker plot. Jaime's asked Vivian for the least value.



Jaime: Now we're gonna read it. Now they gave us one and we're gonna read that information. [Points to the five number summary written on the whiteboard]

earlier.] Let's go ahead and do this and maybe that's enough to answer those questions. All right, ah Vivian?

Vivian: 30.

Jaime: I'm sorry?

Vivian: 30, least? Oh, 40.

Jaime: Good job, it's all right. Did everyone hear her mistake? Did you hear her mistake? [asks Ismael.]

Ismael: No.

Jaime: No? She said 30.

Ismael: Oh.

Jaime: Would that be an easy mistake to do?

Ismael: Yes.

Jaime: Yes, you gotta be careful. Don't worry about it. 30.

Positioning. Jaime positions himself in the classroom as a facilitator who elicits mathematical ideas from his students. When students provide ideas, strategies, and solutions in front of their peers, these are opportunities for the students to be positioned as knowledgeable, capable mathematicians. Depending on how the teacher responds, these same opportunities could position contributors as poor students, not worth listening to.

Instructional strategies. The research-based instructional strategies recommended for use with ELLs that are observable in Jaime's class are revoicing, language as resource, multivoicedness, and classroom mathematical discussions. The extent to which these strategies are positioning the students varies.

Revoicing. Jaime revoices virtually every utterance a student makes. Because Jaime uses revoicing so automatically it feels like a regular part of the classroom experience rather than a special opportunity to position students in some way. Jaime uses revoicing as a means to simply confirm what a student has said or a way to broadcast the statement for the back row rather than positioning. In the following example, Jaime is beginning a class discussion of the real number system and heavily revoices.

Jaime: All right, good morning, today we're going to be talking about the real numbers. What does that mean to anyone? [Raises his hand.] Raise your hand, raise your hand.

Juan: Numbers that are real.

Jaime: [revoices] Numbers that are real, ok.

Vivian: Wanna make sure they're the real ones not the fake ones.

Jaime: [revoices] The real ones and not the fake ones. Got it got it got it. Nichole, real numbers? What do you think? What do you think?

Student Nichole: [shakes her head and shrugs.] Like the even and the odds?

Jaime: [revoices] The evens and the odds, ok. Um, Frank? What do you think, real numbers? Can't think of any right now?

Frank: No.

Jaime: Ok. Is this a real number? [Writes a 5 on the whiteboard.]

Students: Of course. Yes.

Jaime: [revoices] Yes.

Language as resource. Although Jaime's considers himself a competent Spanish speaker he chooses not to speak academic Spanish in the classroom. He only speaks two

mathematical terms in Spanish during these sessions. When discussing rounding a decimal to the nearest whole numbers with a student, Jaime says “*enteros*” [wholes] to emphasize that they want the answer to be a whole number. When introducing the various parts of the five-number summary he calls the median, “*medio* in Spanish,” however this is not the correct translation for median. The proper term for median in Spanish is *mediana*. Although observed speaking academic Spanish in class, it is limited.

Multivoicedness. Multivoicedness is a recommended instructional strategy for use with ELLs in which there are opportunities for teacher and student to be repositioned as learner and educator. This allows students have the opportunity to be seen by their teacher and their peers as contributors of knowledge. In his own words Jaime stated that he prefers for mathematical ideas to come from his students and he solicits them throughout class. At one point his students are finding the percent decrease and Jaime calls on a student, Rafael, to discuss the steps involved. At the end of his explanation a second student questions his work and Jaime puts Rafael in charge, giving Rafael ownership of his mathematical ideas. This happens for a second time a few minutes later.

Fred: Where did we get 25 from?

Jaime: Good question. Rafael? Why?

Rafael: Because, do I tell him?

Jaime: Yes.

Rafael: Because you have to subtract your two numbers, your original and the one you are left with.

Chrissy: To see if it's an increase or a decrease.

Jaime: Right, right here. [Points at whiteboard.] We're looking for the difference of 45 and 20. We're good? Ok? You're done? [Goes to student's desk.] Awesome job.

[a few moments pass before Ismael questions the value used.]

Ismael: Yeah. But why, why 25?

Jaime: Oh you're not paying attention. Rafael they need your help. Rafael?

Another person wasn't paying attention and doesn't know where you got 25 from.

Rafael: Oh, cause you need to subtract the original number from the one you're left off with.

Ismael: Oh the increase.

Jaime: Yes well, actually the what?

Ismael and others: The decrease.

During a different lesson a student's paper is broadcast to the class via the document camera and overhead projector. This student's work is used as the exemplar of the proper way to solve a proportion and elevates the student into the position of teacher or authority. During the class discussion of real numbers one student, Vivian, is called to the office. When she returns Jaime catches her up with the class using her fellow students for help.

Jaime: You missed a little bit Vivian. We went over, were you here with rationals?

Vivian: No.

Jaime: No? Ok, uh Daniel what are rational number real quick?

Daniel: Numbers that can be turned into fractions.

Jaime: We're good? Can you repeat that?

Daniel: Numbers that can be turned into fractions.

Jaime: Can Vivian repeat that?

Vivian: Do I have to?

Jaime: Please.

Vivian: Numbers that can what? [Turns to neighbor for help who mumbles the words to her.] Numbers that can be turned into a fraction.

Jaime: I like that, thank you. Ok, so with that definition it helped us to find irrational. What was irrational?

Students: Numbers that can't be turned into fractions.

Jaime: Can't be turned into fractions and Vivian an example is π , ok. Can anyone think of another example?

Multivoicedness occurs in Jaime's classes as a part of his regular mathematical discussions. It allows the students to act as teacher and positions them accordingly.

Classroom mathematical discussions. Whole class discussions are a hallmark of Jaime's classes and reflect his desire to hear his students communicating mathematically. During these discussions student ideas are contrasted and evaluated. In the follow brief exchange student solutions are contrasted and evaluated. Jaime positions Chrissy as a good student for paying attention and creating the correct proportion. Simultaneously Fred, who initially offers the wrong answer, is addressed later in the exchange as Jaime checks for understanding. In this case Fred may feel that Jaime positioned him poorly as his turn was usurped by another student who did a better job answering the question.

Jaime: Fred, how do I set up my proportion?

Fred: Um, put 50 over 80 and x over 100.

Jaime: Uh, [Sees hand in air.] Chrissy what do you think?

Chrissy: You have to subtract 80 and 50...

Jaime: [interrupts] Awesome job.

Chrissy: And you get 30 and put 30 over 50.

Jaime: 30 over 50. Why 50? [Writes proportion on whiteboard.]

Chrissy: Cause it's the original price.

Jaime: I like that you are paying attention.

Chrissy: And then um, x over 100.

Jaime: Beautiful job, excellent job. Good, excellent job. Does anyone have any questions, Fred? Do you understand why it should be 30 over 50? [Points to proportion on whiteboard.]

Fred: [Mumbles inaudible.]

Jaime: Right, ok.

Jaime does not hesitate to exploit his students' mistakes. He uses them as learning opportunities as was demonstrated in the deciding-how-to-respond component of noticing. This aspect of his classroom mathematical discussion might make some student uncomfortable and feel positioned as bad students. Another example of this takes place when discussing the attributes of the number five in class. The students have already communicated their answers to Jaime by using personal sized whiteboards and holding them up towards the front of the room for Jaime to read, but he still upsets the anonymity.

Jaime: [Raises hand in air.] How many of you knew it was an integer? Raise your hand. Raise your hand if you knew it was an integer. [No one raises their hand.]

No, ouch. Ok, guys it's all right. Ok, how many know what an integer is? [A few hands go up. He points to one girl.] Talk to me.

Mathematical ideas can be expanded as we saw earlier with the different strategies for finding the median of a set of data. Jaime's questioning can also allow for the steps of a task to be revealed one at a time and created by multiple voices. Classroom mathematical discussion is an instructional strategy that allows for the most varied functions of positioning to occur. If Jaime was not as invested in hearing his students voices then these opportunities would be absent from the learning environment.

Teacher beliefs about mathematics teaching. Jaime promotes communication in his mathematics classes as the most important aspect of student learning. Students with opportunities to communicate are also presented with opportunities to be positioned. Jaime's beliefs increase the likelihood that positioning moves will occur.

Noticing episodes. Jaime is not afraid to attend to students' mathematical thinking and make his deciding-to-respond a public matter. Students will be praised for right answers, but may also be recognized for their wrong answers. Jaime reassures his students that wrong answers can happen to anyone. For example, after the students independently arranged data from least to greatest he tells the students, "All right, go ahead and check if you agree with me. Check because I make mistakes. Did I make a mistake?" Jaime tries to downplay the negative stigma of making mistakes, but these public noticing episodes are still opportunities for students to feel as though they are being positioned as good students or poor students.

Student Engagement. In order to investigate student engagement in Jaime's classroom, his three recorded classroom lessons were searched for evidence of Franke et

al.'s (2015) six invitation moves and three support moves. The six invitation moves are as follows: explain someone else's solution, discuss differences between solutions, make a suggestion to another student about his or her idea, connect their ideas to other students' ideas, create a solution together with other students, and use a solution that was shared by another student. The three support moves are probing, scaffolding, and positioning.

Throughout his recorded classroom lessons Jaime made fifteen invitation moves with his students. In descending order of use they were discuss differences between solutions (6), make a suggestion to another student about his or her idea (3), and use a solution that was shared by another student (3). The remaining invitation moves were each used one time. He shows a preference for discussing differences between solutions which is in keeping with the whole class discussions that Jaime favors. During one class the students use personal sized whiteboards to record their answers and then share them with the rest of the class by raising them over their heads. This occurs when the students are determining which subsets of the real numbers $\sqrt{25}$ belongs in. Jaime remarks, "Ok, as I look around I see two people that wrote down irrational. Now why do you think someone would chose irrational for this number?"

Jaime also missed numerous opportunities to support student engagement. When a student shares a strategy in class Jaime gives ownership of that strategy to the student. The student becomes the expert in that strategy and others are rarely asked to comment on that strategy or take ownership of it for themselves. At one point the class is determining what equation would best represent a table of values. (This question can be found on page 88.) Daniel explains why one equation is the correct answer by applying the equation to a set of values from the table. A classmate wonders aloud if the equation

will hold for the rest of the data. Instead of asking a new student or probing the student who asked the question Jaime returns to Daniel to explain.

Recall that Jaime relies on whole class discussions and individual seat work. During individual seat work the students have no opportunities to engage with others' ideas. During whole class discussions Jaime will often call on students by name, even though others may be trying to contribute. He will literally stifle the proffered engagement in order to give the named student a chance to participate. For example, when Jaime asks Vivian to explain what a question is asking another student interrupts with an answer. Jaime shushes the student and points towards Vivian expectantly. On the one hand Jaime must maintain order in the classroom and teach students to listen respectfully, but on the other hand numerous student voices engaged in mathematical thinking are being silenced.

The three teacher support moves are probing, scaffolding, and positioning. Jaime does position his students by acknowledging their ownership of a solution method, but as previously mentioned he may be doing this too well. When he refers back to the "expert" student for explanation or confirmation the rest of the class is missing an opportunity to interact with the idea and make it their own. Because he relies heavily on revoicing to forward student thinking, Jaime does not use scaffolding often. Jaime does support student engagement by probing. For example, on two occasions when Fred has trouble explaining a strategy Jaime gets another student to explain theirs and then does probe Fred again to make sure he understands what was said by his classmate.

Summary. Jaime is a teacher who believes in the importance of communication in the classroom, but also believes that he overly relies on more traditional direct teaching

methods. This is reflected in his classroom where he fosters classroom discussions and student mathematical thinking, but only on a whole class basis. Although all students are included through his questioning process there are some student voices that are louder than others and are willing to volunteer information. Others only contribute when prompted. Jaime notices his students' thinking and uses numerous instructional strategies like revoicing and classroom mathematical discussions to reach his ELLs.

Case 2: Patricia

Patricia is a 57 year old middle school mathematics teacher in a large south Texas public school district. Patricia was born in the United States, but moved to Mexico with her family when she was 8 years old. After graduating high school she earned her undergraduate degree in civil engineering in Mexico. At that time her family returned to the United States and she went with them. Her university credits did not transfer so she earned another degree in mathematics and her teaching certificate at the local public university. At the time of this study Patricia has taught middle school mathematics for 23 years, all at the same school. She is currently the mathematics department chairperson at her middle school and teaches eighth grade.

Beliefs.

Mathematics. Patricia defines mathematics as patterns, making comparisons, and evaluating commonalities and differences. She learned first-hand the value of mathematics in Mexico through problem solving opportunities that incorporated real life tasks such as surveying plots on her university campus and a high school teacher who concurrently worked at Petroleos Mexicanos as an engineer.

Mathematics teaching. As a student Patricia preferred teachers that were caring and patient, “those teachers that taught because they like to teach.” It is important to Patricia that teachers value their students and treat them respectfully. She lists accountability as the most important thing in her classroom and with pride retells the story of a student who slept every day of his seventh grade mathematics class. When he passed the eighth grade standardized test on his second try he told her, “Ma’am, you’re the only one that always bugged me. The rest let me sleep.”

Patricia feels the pressure of the high stakes standardized testing that is required by the state and admits that sometimes her students are unable to study a topic as fully as she would like. “We are training them to just answer, not really going in-depth.”

ELLs. Patricia is very familiar with the kinds of difficulties that her ELL students face. She can talk all day about students who struggle in class because their families have been fragmented by the Mexico-United States border or parents who have died trying to cross it. Patricia also has experienced tension in her classroom between students who speak Spanish and students who speak English.

Patricia feels that bilingual education programs can be good because students are able to keep a bit of their home language, but unless language acquisition is reinforced at home the students will never be fully bilingual.

You have to give the same value to both languages, so the kids can say, “Okay, both languages are important. I need to learn English so I can defend myself and I need to know Spanish because of my culture.” A lot of parents don’t do that [give the same value to both languages].

Patricia will speak Spanish to ELLs that are new as a way to help them bridge the language barrier and learn the mathematics concepts. She does not encourage her students to speak in Spanish however, believing that they need to practice their English when in class. “If they step outside of my classroom, the first thing you’re going to hear is them speaking in Spanish.” Although unfamiliar with the term academic English she was aware of the concept and knew how long it takes for ELLs to acquire. Patricia holds the same expectations for all of her students, regardless of their classification saying, “The test is the same. The expectation is the same.” When it comes to her students’ success on the state standardized tests she feels solely responsible, but otherwise believes educating her students is a team effort.

Position in the classroom. Patricia considered herself a teacher of students. Her emphasis on accountability means that in her classroom everybody, regardless of language, has a voice. “It’s opportunities. I guess you have to give them the opportunity to learn. You have to give them the opportunities to shine.” Patricia positions herself as the task-master who ensures there is participation and learning, but she is not unfeeling. Patricia cares deeply about her students. She seeks to understand their personal struggles and help them however she can. She is not just their teacher, but has also positioned herself as a supportive figure in her students’ lives. She tells of one former student who repeated the eighth grade because she failed the reading portion of the state standardized testing. Although she wasn’t a strong mathematics student either, Patricia chose to challenge the girl by giving her the algebra curriculum. “I said, ‘You’re going to pass reading and you’re going to pass math. You’re going to pass the EOC [end of course exam] of algebra.’... She did pass.” Speaking of another former student Patricia says, “She

needed to have a lot of attention and love because her mother passes away, very sadly. She drowned when they were crossing the river. To that student I always say, ‘Andale, andale, come on. You can do it. You can do it. Come on.’”

Observed beliefs. When interviewed, Patricia frequently cited the pressures of the standardized testing that her students must pass in order to advance to high school. It affects her teaching and her students. During class, Patricia tailors her instruction to the standardized test. The problems her students work on are versions of possible test questions. She gives them experience measuring with the standardized test ruler and using the formula chart. Patricia will discuss with students different “tricks” they might expect test makers to use when writing test questions.

Patricia’s students sit and work in pairs which allows her to quickly circle the room, visiting each group and evaluating their progress. Patricia holds each student responsible for the learning, if one student writes up a problem she will ask the other to explain it. When discussing unit rates she discusses with the students that, logically, if one value is increasing then it’s counterpart in the proportion must do the same. Patricia does speak Spanish in her class, but it is rarely mathematical in nature even though her students will often utilize Spanish to explain their reasoning, pose a question, and work cooperatively. Patricia’s espoused beliefs are observable in her classroom.

Professional mathematics teacher noticing. Patricia’s students work on setting up proportions, finding the surface area of rectangular prisms, and finding the volume of rectangular prisms and cylinders during her three recorded classroom sessions. Each session is approximately 45 minutes in length. At the beginning and end of each class Patricia will lead a whole class discussion about the work and she will chose students to

answer questions about the topic or read questions out loud. Her students sit in pairs and work together during each of the sessions. Patricia continuously circulates the room and checks for understanding with each pair.

Patricia's three recorded lessons yielded 15 noticing episodes. These episodes were coded for the attending and deciding how to respond components of noticing as described by Jacobs, Lamb, and Philipp (2010). Attending to student's strategies, was coded as either evidence of (1) or lack of evidence (0). Deciding how to respond, was coded as either lacking evidence (0), limited evidence (1), or robust evidence (2). Patricia's interpretation of student's understanding was evaluated through the use of a classroom video and is discussed in detail below. All of Patricia's noticing work was coded and discussed until agreement was reached by two raters, the author and an independent rater.

Attending to students' strategies. Patricia received an attending score of 0.80 over 15 episodes, or there was evidence of attending to student' strategies in 12/15 instances. She received a score of (0) for three instances because she engaged with student groups that had not yet begun to solve a mathematics problem. It is possible they would not have begun without her intervention, but for purposes of attending this does not equate to evidence of attending to a students' strategy. Patricia's students work in pairs and she circulates the room checking on their understanding. She typically opens with an evaluative statement by saying "Let me see; what is your answer here?", "How are we doing? This is correct.", and "Did you fix it?" Then she will encourage the students to either explain the steps they took or address their error, "Explain to me what you did step by step... Why do you think it was wrong?" As an alternative to listening to

the students explain their steps Patricia might attend to their strategy by asking questions like, “Could you have done it a different way?”

Regardless of the correctness of a students’ answer, if their written work does not meet Patricia’s standards she will address it. Patricia will attend to a student’s strategy through improving their written communication and reading skills. When visiting a pair of students working on setting up and solving a proportion Patricia sees that one student, Gabby, has the correct answer, but did not show the steps properly. The other student, Linda, has not answered the problem yet. Patricia praises Gabby for answering the question and then begins working with Linda on the solution. After setting up the proportion with Linda, Patricia returns to Gabby.

Patricia: And Gabby, you did it there excellent. I like it you were able to do it without setting it up because this is already the unit rate. [Points back to Linda’s paper.] We want to do this one for a little practice, equals what? *Por acá* [over here], x over 3.5. Why do you think she put 3.5 over here? Three and a half. Cause what is in the denominator? [Points to word ratio.] What is going to be in the denominator? Read it over here from her. What is it she has? What does she have here, in this word ratio?

Gabby: Miles and hours.

Patricia: Ok, she put 3.5 over here in the denominator. Why do you think she put 3.5 in the denominator? What does 3.5 represent?

Gabby: Hours.

Patricia: Hours, *verdad* [right]? Ok, set up the third one.

Even though Patricia saw that Gabby had found the correct answer she was able to reinforce the concepts with Gabby by including her in the construction of the solution that Patricia made with her partner, Linda. Patricia attends to Gabby's strategy of using the unit rate and contrasts it with the proportion that Patricia sets up with Linda.

Interpreting students' understanding. Patricia watched a video of fifth grade students from Brookline, Massachusetts working in groups with equivalent fractions. This educational video is available, among many others, from Annenberg Learner at www.learner.org/resources. Annenberg Learner seeks to improve education and assist teachers using video clips and other forms of media. In the clip used for this study, the students were presented with a game board that had seven number lines divided into halves, thirds, fourths, fifths, sixths, eighths, and tenths respectively. A picture of the game board can be found in Appendix A. The object of the game is to move seven games pieces from the left side of each number line to the right side, or one whole. In order to move the pieces the students draw a fraction card and decide how to proceed. For example, if a student drew the card $\frac{4}{8}$ they could choose to move their marker forward four eighths on the eighths number line or they could move the fourths marker to one fourth and the eighths marker to two eighths. Students continue to draw cards until all of their game pieces have reached the right side of their number lines.

In the video clip shown, the students' teacher checks on three groups' progress giving the students an opportunity to discuss their strategies. The first pair of students discuss animatedly and correctly the numerous ways they found to represent $\frac{4}{10}$ on their game board. They proceed to write down the different ways. The second group contains three students. They draw the card $\frac{6}{10}$. One group member suggests moving the game

piece to $\frac{2}{5}$ plus $\frac{6}{10}$. His teammate informs him that they cannot move forward more than $\frac{6}{10}$. The student amends his answer and decides to move forward $\frac{1}{10}$ plus $\frac{5}{10}$. The third group of students is a pair that demonstrate for their teacher how they correctly used classroom manipulatives, in this case colored plastic rods, to verify that $\frac{3}{8}$ plus $\frac{2}{4}$ is equivalent to $\frac{7}{8}$.

After considering what three student groups did in the video, Patricia was asked to explain in writing what he learned about these children's understandings. Interpreting student understanding is coded identically to deciding how to respond, that is as either lacking evidence (0), limited evidence (1), or robust evidence (2). Patricia's interpreting score was only a 0.66, meaning she showed some evidence in two out of three instances. This was based on the three groups shown working in the video clip. Patricia wrote her explanations for the understanding of three student groups working with equivalent fractions. For two of the groups she partially addressed the underlying understanding, but also reported a narrative of what took place within the groups. Patricia did not note the third student group whose understanding was lacking.

Deciding-how-to-respond. Patricia received a deciding score of 1.13 over 15 instances of noticing. Her scores were evenly distributed with four (0)'s, five (1)'s, and 6 (2)'s. Patricia responds to her students' understandings a variety of ways. Sometimes her responses, although connected to the material, are not in keeping with the student's strategy or understanding. Patricia talked with one student, Ben, who was finding the total surface area of a rectangular prism. She noticed that Ben added the surface area of two adjacent faces and then multiplied the sum by two, but didn't take the opportunity to

discuss what made this possible. Instead she turns the conversation into a vacuous discussion of units.

Patricia: Explain to me why you only did 209 plus 94 and then you multiplied by two.

Ben: Um, because it would be the same if I add these two and these two and add them all I just had the same answer.

Patricia: Cause the answer will give you the same. So it's a shortcut. And those are the lateral?

Ben: Lateral faces.

Patricia: Yes. And why are we using centimeters squared in the units?

Ben: Um, like what?

Patricia: Why did you put centimeters squared and not inches squared? And not feet squared?

Ben: We measured by centimeters.

Patricia: We measured by centimeters. Good *muy bien* [very good]. Correct.

Often Patricia will support students' work by helping them look at the problem logically.

In the following excerpt, students Juan and Pablo set up and solved a proportion, but ultimately put the decimal point in the wrong place. Patricia begins by discussing the mechanics of multiplying decimals, but turns it into a logical argument for why their answer must be larger. Figure 3 immediately follows their transcript and shows the work Juan and Pablo have done. The problem stated: At the store this week the russet potatoes cost 5 pounds for 15 dollars. How much will 2.4 pounds of potatoes cost?

Patricia: Let me see what is your answer here? Ok, decimal point. Come on guys.

You multiply here you move the decimal from the right to the left or the left to the right? Right to the left. So where is it going to be between the 3 and the 6 or the 6 and the zero?

Juan: 6 and the zero.

Patricia: 6 and the zero. So the decimal place is in the wrong spot. You have everything correct, it's just that one. And guess what they do when they do standardized tests, they find a way to confuse you. So it's going to be the decimal here or what is going to be?

Pablo: It's going to be 7.2.

Patricia: It's going to be 7.20. Now let's go look at the problem for a moment, let's look at the problem. Because something had told you from 5 pounds I went to 2 pounds, it's almost what?

Students: Two and a half.

Patricia: Ok, so does it make sense that my cost is gonna be decreasing? But it went from 15 dollars to 72 cents? So that's one hint that it tells you that you are doing some mistake somewhere else. This is almost half of that one right so my cost has to be almost half of 15 dollars.

Juan: Yes.

Patricia: If I give you 15 and I ask you for the half, I don't want 72 cents.

Pablo: It's 7.2.

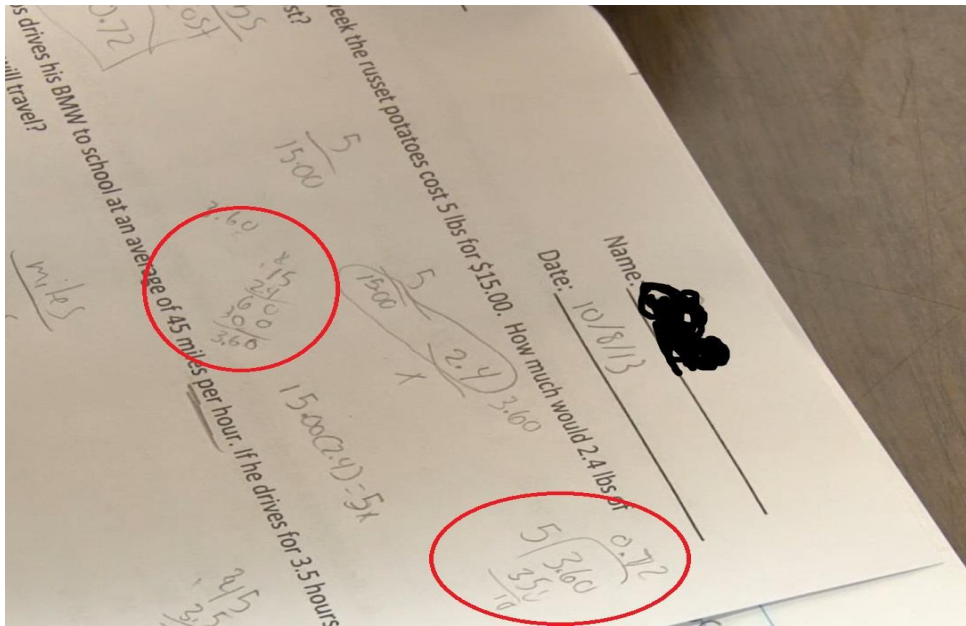


Figure 3. Work of Students Juan and Pablo.

When responding to students' strategies, Patricia will also take the opportunity to reinforce proper vocabulary and terminology so that students who can solve a problem on paper can also communicate that answer to others. At one point Patricia stops at a group of three students, Neo, Sue, and Tania, who have finishing finding the total surface area of a rectangular prism. Patricia asks the boy, Neo, to explain it to her.

Patricia: Explain to me what you did. You explain to me what you did step by step.

Neo: You multiply the faces.

Patricia: To find what?

Neo: To find the total.

Patricia: To find the total? [Sue leans over and underlines the word area on his paper.]

Neo: The area.

Patricia: The area *muy bien* [very good] and once I find the area of what [flips the box over.]

Neo: The base.

Patricia: The faces. What I did after that one?

Neo: Multiply.

Patricia: Once I multiply I found the what?

Neo: Total, the how much, how dimensions...

Patricia: Ok, you measure. Done, done, done, right? [Points to edges with her finger.] Then you did what? [She turns his paper over to see the net.] You found 15 by 15 here and you wrote 225. What is the 225 represent? My 15 by 15, what is the 225 represents?

Neo: Oh, the total.

Patricia: Read on your table. Your table is telling you what it is. Can you help me out here? [Speaking to Sue and Tania.] What is the 225 represents?

Sue and Tania: The area of face A.

Patricia: *Otra vez* [Again], what does 225 represent?

Neo: The area of the face.

Patricia: The area of the face A, right? Then you did what?

Sue: Did the same to all the sides.

Patricia: You did the same to all the ...

Sue: Faces.

Patricia: Faces, right? And then these are called the what? [Pats the ends of the box.]

Sue and Tania: Bases.

Patricia: Bases, right? But if I flip it this way which one is the base?

Neo, Sue, and Tania: The bottom and the top.

Patricia: Uh huh, so it doesn't matter actually we can call them bases we can call them...

Neo, Sue, and Tania: Faces.

Patricia: Faces, ok.

Positioning. Patricia positions herself in the classroom as a facilitator and an observer who will introduce topics to her students and then watch as they grapple with the mathematics in pairs. Because accountability is important to Patricia she makes it a point to touch base with every group and every group member, especially if she thinks the workload is not being shared equitably. Working in groups allows Patricia's students to converse about mathematics privately which decreases opportunities for class embarrassment that could create negative feelings and negative positioning. It also allows many students opportunities to teach their partner, be taught by their partner, and feel successful. Moreover, Patricia never compares the progress of different groups or shares information about one group to another.

Instructional strategies. Patricia can be seen using the following research recommended instructional strategies for use with ELLs in her classroom: revoicing, language as resource, gestures, multiple representations, and classroom mathematical discussions.

Revoicing. Patricia revoices with great regularity, particularly when a student gives her a mathematical term she would like to accentuate. She will also revoice as a

means to translate a student's Spanish into English. For example when constructing proportions she asked a student where a particular number would go and the student answered, "*arriba*." Patricia revoiced, "On top, numerator. Say that again." Patricia's revoicing turns into an opportunity for the student to practice the proper term in English.

Patricia also gets other students to revoice from Spanish to English for her.

When looking for the value that belongs in the denominator of that same proportion one student offered the correct answer, "*ciento veinte*." Patricia responded, "Ok, *muy bien*.

What number am I going to put in there?" In this way she gets another student to provide the answer one hundred twenty in English, a revoicing of the first answer. Revoicing is a tool for Patricia to emphasis mathematical terminology and to translate Spanish into English, but it happens with such regularity it seems to blend with the rhythm of the class without positioning the students.

Language as resource. Patricia is a fluent Spanish speaker, having been educating in Mexico through college. However the Spanish she speaks in her classroom is largely endearments such as "*muy bien*" and "*mija*." Patricia does speak to one student about how she may have already learned proportions in Mexico and references the term "*la regla de tres simple*" as a way to make the connection. The interaction is brief and neither Patricia nor her student capitalize on it. (During her interview Patricia volunteered that she isn't sure how mathematics is taught in Mexico anymore and admitted that when she refers to *la regla de tres simple* [the rule of three] her students from Mexico do not know what she is talking about.) *La regla de tres simple* refers to the procedural steps of setting up three known values in a proportion and solving to determine the remaining unknown quantity.

Although the mathematics Patricia speaks in Spanish is very limited, some of the pair groups in her class speak exclusively in Spanish when working on their assignment. These students will address Patricia using a mixture of Spanish and English, typically using English for the mathematical terms and Spanish for the rest of the sentence. For example after reading the directions that call for a sketch on his worksheet one student asked Patricia, “*Pero vamos a tener aquí questions que tenemos asi a sketch?* [But we are going to have questions where we have a sketch?] ” Patricia answers all students in English. There are two instances during the three recorded lessons when Patricia verifies what she said during her interview and demonstrates that she places a higher value on answers spoken in English. At the end of the January lesson she asks the students to complete an exit ticket by explaining how to find the surface area of a rectangular prism. She tells her students, “Explain how to determine the surface area of a rectangular prism. As much as you can in English. *No me lo pongas en espanol.* [Don’t give it to me in Spanish.]” During the April lesson she prompts a student to explain their work by asking, “To find the radius, what you need? Tell me what you did. In Spanish if you have to.”

Patricia encourages her students to speak English and thus models the behavior she would like them to emulate. However she does not stop them from speaking Spanish with each other or occasionally making a reference that she believes might help a student. In these ways Patricia is allowing language to be a resource in her classroom. If she chastised her students for speaking Spanish and never used it herself it may have the effect of positioning the students negatively, but in this context it does not have that power.

Gestures. The lessons captured in these videos offer Patricia many opportunities to use gestures as a tool for student understanding. She repeatedly indicates the dimensions of the shapes her students are studying with her hands. When discussing proportions and the idea that a quantity can grow or shrink she uses her hands as a visual aid to demonstrate the growing and shrinking. These gestures are helpful learning tools, but are not positioning individual students.

Multiple representations. The lessons that Patricia and her students encounter in these videos offer opportunities for multiple representations. When the students are studying the dimensions and measures of rectangular prisms and cylinders they employ nets and sketches of the objects. They also use actual 3D objects that can be found at home like cereal boxes and soup cans during their investigations. These allow the students to realize the equality that is inherent to the measurements of prisms and gives them strategies to use during standardized testing. While helpful, and perhaps confidence building, the presence of multiple representations is not positioning students.

Classroom mathematical discussions. At the beginning and end of each class Patricia does take a few moments to discuss the activity of the day or to work on a few multiple choice problems as a whole class. Therefore, there are times during class when students' strategies are compared or contrasted and evaluated for validity. Patricia will call on students, but more often students will call out answers individually or as a group. In this way, and as a class, the students and Patricia have brief opportunities to discuss and construct step by step solutions to problems. Patricia praises correct answers and strategies, but rarely singles out a particular student. During the start of one class Patricia asks a student, Tomás, to read the question out loud, but then immediately chooses

another to read because she knows Tomás has trouble reading in English. Although this may position Tomás negatively she immediately follows up with Tomás to answer a question about what was just read and gives him a chance to practice his English.

Patricia: What is the first ratio that you see? Can you read that first sentence?

[Points to a boy in the front row, Tomás.] No, Tomás because I know that you are going to have a hard time. [Points to a different boy, Brandon.] Can you read the first sentence?

Brandon: [reading.] The school cook uses 10 pounds of potatoes to make mashed potatoes for 40 students.

Patricia: Ok, that's the first sentence. Within the first sentence can you see the ratio? [Returns to Tomás now.] What is the first ratio? What I know?

Tomás: 10 pounds of potatoes.

Patricia: [revoices] 10 and what?

Tomás: 40.

Patricia: 40 ,very good. Now that number is hard for you but it is ok, 10 over 40.

[She gets Tomás to repeat after her.] We read it 10 to 40. *Muy bien*. [very good] 10 to 40.

Patricia's students spend most of the class period working together in pairs. We can hope and assume they are having mathematical discussions together, but there is limited evidence available to support this. The lack of evidence is due to the fact that the videos were recorded so that the teacher was the focus of the videographer's attention. There were a few instances where the students do not appear to be working together and

Patricia has to encourage them to do so or, as we saw with Linda and Gabby, uses it as an instructional opportunity.

Teacher beliefs about mathematics teaching. Patricia holds strongly to the belief that accountability is the most important element in her classroom. She expects all students to be working and learning. Patricia's students sit and work in pairs which gives the students ample opportunities for interacting with their peers about the material. In order to evaluate her students' learning and monitor that accountability, Patricia circulates the room and discusses the work with each group. Keeping her students accountable for their learning also gives her the ability to praise their work which can position students positively. Because the students are working in pairs and she is only addressing the pair, mistakes are not broadcast to the entire class which minimizes embarrassment and possible negative positioning. Working in pairs also increases the number of students who are given the opportunity to explain a correct answer and instruct a peer which also increases positive positioning instances.

Noticing episodes. Because Patricia's students work in groups nearly all of the teacher noticing that takes place in the classroom is not made public. Patricia never takes an idea from one group and shares it with another or encourages two group to discuss differing mathematical ideas. Patricia does not elevate one student at the expense of their partner. If a pair of students do not have the same work she responds by helping the weaker student. The stronger student might participate by reading the problem out loud, answering a question their partner cannot, or interjecting a realization.

Even though she does not compare her students to each other, Patricia is not afraid to compare her students' intelligence to her own and always positions them as the

winner. During one class Patricia is quickly going from group to group and mistakenly tells a student that he made an error labeling the dimensions of a rectangular prism. The following exchange occurs between Patricia and the student.

Student: Ma'am, look look.

Patricia: No. We'll try again. Ok, this is what? The length, right? [Indicates each part with her finger.] The width, the height. [Students repeat the parts with her.]

Oh, yes you're right.

Student: *Ya, ver* [You see] ma'am, I'm not that dumb.

Patricia: No, You're smarter than me.

Student: Yes, I know.

Patricia: Yes, you are.

Student Engagement. In order to investigate student engagement in Patricia's classroom, her three recorded classroom lessons were searched for evidence of Franke et al.'s (2015) six invitation moves and three support moves. The six invitation moves are as follows: explain someone else's solution, discuss differences between solutions, make a suggestion to another student about his or her idea, connect their ideas to other students' ideas, create a solution together with other students, and use a solution that was shared by another student. The three support moves are probing, scaffolding, and positioning.

Throughout her recorded classroom lessons Patricia made 18 invitation moves with her students. In descending order of use they were create a solution together with other students (8), make a suggestion to another student about his or her idea (6), explain someone else's solution (3), and connect their idea to others students' ideas (1). She shows a preference for using create a solution together with other students which is in

keeping with the partner work that she favors. While walking around the room she says things like, “You can ask your partner for help.” and “Set up the next one, together. Is that clear?” Patricia was more likely to use, encourage students to make a suggestion to another student about his or her ideas, during the briefer whole class discussions that occur at the start or end of class. At the start of one class Patricia said, “Bradley, do you agree with Travis?” and “Albert, can you help him out?” These are the kinds of things she says that encourage students to augment the mathematical ideas offered by classmates.

As the school year progressed Patricia used markedly less invitation moves with her students. In October she made twelve, in January five, and in May just one. Because she expects her students to work together in pairs it is likely that by May she already believes them to be properly “trained” in classroom expectations and the students require less reminders to work together. The student groups do appear to work together easily and more readily in May as compared to October. While Patricia monitors the student groups she does use support moves, mainly probing and to a lesser degree scaffolding. However, these moves are not really used in the context of supporting students’ engagement with the ideas of peers. Patricia probes and scaffolds the student groups as they discover mathematical ideas together. Patricia considers her student pairs to be akin to a team and she does nothing to position one student differently from their partner. She also does not carry information from one pair to the next so students are not positioned in that way either.

Summary. Patricia is an experienced mathematics teacher who values student accountability and fosters it in her classroom. Her students work together to solve

mathematics problems which provides Patricia numerous opportunities to interact with her students and attend to student strategies. She ensures that all students are participating in the learning and that they are showing the work according to her standards, and evaluates solutions for correctness. Patricia responds to her students' understanding include reinforcement of mathematical terms and discussion about the mathematical underpinnings of their work. Although she speaks Spanish fluently and will use that ability to interpret her students' words, Patricia does not herself speak about the mathematics in Spanish. However, she does not discourage Spanish speaking peers from offering assistance. Patricia makes it clear that speaking English is valued in her classroom.

Case 3: María

María is a 54 year old middle school mathematics teacher in a large south Texas public school district. María was born in Mexico and moved to the United States when she was 8 years old. She earned degrees in mathematics and Spanish at the local public university. At the time of this study María has taught middle school mathematics for 31 years in several different schools in the district. She is teaching seventh grade during her recorded lessons and at the time of her interview.

Beliefs.

Mathematics. María has enjoyed doing mathematics since she moved to the United States and amazed her third grade teacher by demonstrating mastery of all four operations. She talks about mathematics as a collection of interesting ideas waiting to be discovered and considers it a tool that can be used for the betterment of people's lives.

Mathematics teaching. María values a classroom atmosphere that fosters student engagement. She disparages some classrooms she knew as a child in which students were afraid to ask questions. “I didn’t like that class because it was super, super strict. Very strict. You could not even turn around like that. [Turns her head slightly.] Don’t dare ask a question. If you asked a question, forget it. You would feel this big.” She describes the type of teaching she prefers as, “moving around, very energetic. ‘Come on honey, if you have any questions honey, ask.’ I loved it.” María values learning by discovery, letting her students do projects such as investigating the value of π by encouraging them to bring in circular objects from home.

ELLs. As a new immigrant María was called to the principal’s office and chastised for speaking Spanish in school. She was warned that if it happened again she would be hit with a ruler. Needless to say this left a lasting impression,

I always cry when I remember that, but then I start thinking about the positives...

It was for a reason and the reason was that I needed to pick up the language faster.

I think I did. Nowadays, I see my ESL kids, it takes them years to learn the language. Why? The laws have changed. You can’t force the kid.

This echoes her opinion of bilingual education programs, that students are not learning English as quickly as they used to. When asked if she encourages her students to speak their native language in class she assumed I was asking if she encourages her students to speak English in the classroom. “I tell the kids, ‘You need to practice. You need to practice the English.’” Later she elaborated on her role in the classroom,

Sometimes I feel like I shouldn’t speak Spanish to them, but sometimes I say to myself, ‘If I don’t speak Spanish to them, they’re not going to understand what

I'm saying,' I have to. I have to speak Spanish to them at the beginning and then hopefully later on they will understand what I'm saying. I do tell them that they need to. 'Yes it's okay to speak Spanish. We're from the same culture,' I tell them, 'but we're in the United States. We need to learn the language.'

María holds the same expectations for all of her students, regardless of language or background. She feels solely responsible for educating the students in her classroom, although she acknowledges receiving help from others such as the special education inclusion teacher. She was not familiar with the term academic English.

Position in the classroom. María considered herself a teacher of students. "I hear the kids say, 'Oh man, some classes are just a waste of time because some teachers, they don't teach.' It's sad. I tell them, 'Hey guys, we're not going to think about that. We're going to do this.'” She wants her students to feel safe and know that she supports their learning. María does not tolerate rudeness or disrespect from the students in her classroom. Thus, she positions herself as the arbiter of student behavior, but because she utilizes learning by discovery she is also the facilitator of student learning. She wants the mathematics to come from the students, not herself.

What I love to do is to tell them, 'Look, there's not one way of solving the problems. There's several ways. If you have another strategy, I would love for you to share it with us. If I like it, I'll use your strategy.'

María is also a motivator through the organization of mathematics competitions in the classroom that foster student involvement as well as a raffle system that maintains students' interest. María will distribute raffle tickets during class to students who ask insightful questions or she feels deserve recognition for something. At the end of the

week students turn in their tickets and if their name is drawn in Friday's lottery they can choose an item from María's "store." The store has school supplies, candies, and small toys. María also uses a lottery system to fairly select a small group of students to be invited to her home for her annual Christmas party.

Observed beliefs. María speaks about mathematics as a tool that successful people need in life. She uses real life examples in class to show students how mathematics can help them. María's students work in group of three or four in which she designates one student as the "teacher". The "teacher" is responsible for making sure everyone in their group reads the question and understands the solutions they find. If the "teacher" experiences difficulty they may ask María for help. The "teachers" do not appear nervous to approach María with their questions and listen to her advice. María does speak Spanish in class when she is talking with ELLs about mathematics. She also relies on Spanish speaking peers to help unpack the mathematical concepts. As the school year progresses María encourages the ELL students and their English speaking peers to communicate their ideas in both languages. Although María did not speak about the impact of standardized testing on her classroom practices, it is very evident. Many of the problems that the student groups are witnessed solving are basic, procedural questions or multiple choice standardized test type questions. María's espoused beliefs are in keeping with her classroom practices albeit with the looming presence of standardized testing.

Professional mathematics teacher noticing. María's three recorded classroom sessions cover the following topics: unit rate, general problem solving/standardized test preparation, and scientific notation. Each session lasts approximately 45 minutes. During the first session her students sit in traditional rows and will answer questions as she poses

them about unit rate. María uses real grocery products such as sandwich bags and drink mixes to explore unit rate with her class. The students are also called upon to read problems out loud. María assigns a few problems for the students to try and she walks the room checking on their progress. She encourages neighbors to check solutions with each other. During the second two classroom sessions the students sit in groups of three or four students and work together to solve the assigned work. Each group has a leader or teacher responsible for their peers' progress. María continuously circulates from group to group checking on understanding, progress, and teamwork.

There were 13 episodes of teacher noticing observed during María's recorded classroom lessons. These instances of noticing were coded for two of the three components of teacher noticing, attending and deciding how to respond as described by Jacobs, Lamb, and Philipp (2010). Attending to student's strategies, was coded as either evidence of (1) or lack of evidence (0). Deciding how to respond, was coded as either lacking evidence (0), limited evidence (1), or robust evidence (2). María's interpretation of student's understanding was evaluated through the use of a classroom video and is discussed in detail below. All of María's noticing work was coded and discussed until agreement was reached by two raters, the author and an independent rater.

Attending to students' strategies. María received an average attending score of 0.77 that is, she showed evidence of attending to students' strategies in 10 out of 13 instances. Because María uses cooperative learning extensively during class most of the teacher noticing that takes place in her classes is in a small group setting. However there were two instances of whole class noticing. The first occurs when María asks a student to

explain his work at the whiteboard. The second happens when she asks a student to explain his reasoning when comparing two unit rates.

As previously discussed, María's students work in small groups. She assigns a student "teacher" and charges them with explaining solutions to their groupmates. This is an excellent opportunity, which María takes advantage of, to attend to student's strategies. She may solicit the "teacher" to explain a solution or happen upon a group where such a conversation is already taking place. María will engage the students with questions and statements like, "How did you prove this one is correct?" , "How did you teach it to her?", "If you know you can give me an explanation.", "But you need to explain it."

In three cases María did not show evidence of attending to students' strategies. This occurred when María invalidated an appropriate student strategy and imposed her own. It happens with two different groups when the students are solving the following problem: Denise ran $\frac{5}{8}$ of a race that was 24 miles long. What percent of the race did she not finish?

F. 9%

G. 12.5%

J. 62.5%

H. 37.5%

María: Where are we? Ok, see this one was a tough problem I think because what happened when you guys read the problem and you were looking for a percent, we thought about the box right away, right? Ok. On this one we don't necessarily need the box because what does the question say?

Students: She not finish.

María: Not finish ok, um so what do we do? We want the answer as percents so we have several options, what is one of them?

Luke: 62.5%.

María: Where did you get the 62.5%?

Luke: Um, cause you divide it out 24 times.

María: No, we don't need 24. We're only working on the fraction. $\frac{5}{8}$. What I was telling to other group guys is that when I see fractions yes my first instinct is to turn them into decimals right but I can also use the pie graph. Ok, into how many equal pieces am I going to divide this pie graph?

Students: 8.

María: 8, that's the total. Divide it into 8, draw it and divide it into 8. [Students do it.] Ok, how much of this am I going to shade in?

Students: 5.

María: 5, shade em in. What are you shading, the what?

Brandon: The part that she did.

María: That she ran right, that she has finished. Ok, but what is the question say?

Students: Not finished.

María: Not finished, so what are you going to do?

Brandon: $\frac{3}{8}$.

María: $\frac{3}{8}$, she has not finished $\frac{3}{8}$.

Brandon: We turn that into a percent.

María: How do you do that?

Miguel: Divide it.

María: And how do you turn it into a decimal?

Students: Divide it.

María: Ok, well show me your work, divide. Remember the cowboy and the horse, the cowboy is sitting on top of the horse cowboy goes inside the house and the horse is waiting outside. Your division do you need help with the division?

Nope, we're doing ok. Ok, ok so you have zero point what?

Miguel: 375.

María: But we're looking for a percent.

Miguel: Move the decimal two times to the right.

María: Ok, how many times do we move it?

Student: Two.

María: Two times why, Miguel? Why is it two times? Why not three times why not one time?

Student Miguel: Because the hundred, the percent is actually 100 so it belongs in the hundreds.

María: Right and one hundred is how many places?

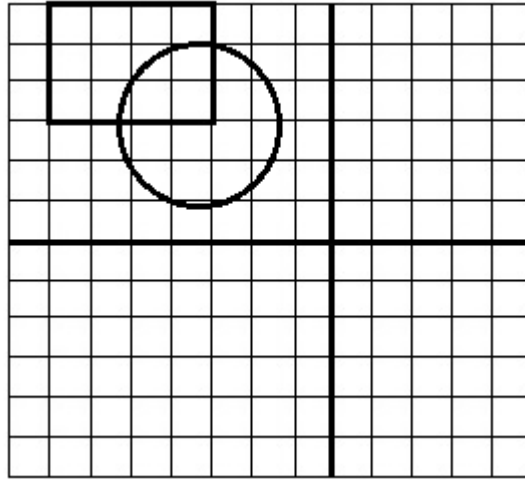
Miguel: Two.

María: Two places very good. Ok so did we just find the answer?

Students: Yes.

Another example of María's inattention to student's strategy occurred with a group of students that were led by Francesca in solving the following problem.

Look at the image on the coordinate grid below.



If the image is translated 5 units to the right and 8 units down what ordered pair would only be inside the circle?

- A. (3, -5)
- B. (6, -2)
- C. (2, -2)
- D. (0,0)

Francesca's strategy was to first move the rectangle which could legitimately be used as a tool for approximating the translated image of the circle. María criticized this approach and accused Francesca of not following directions. María then instructs the group to solve the problem by translating ordered pairs from inside the circle, a suggestion that seems to surprise the entire group. Francesca does not speak for the rest of the interaction.

Francesca: Yes, I translated this 5 times and I moved it down.

María: Which one? The one in the rectangle?

Francesca: Yes, this one, this one, this one, this one. [She points to the corners of the rectangle.]

María: Why do you think I tell you to read, not only that I tell you to underline the question. And you haven't done it, you aren't following my strategy. What ordered pair will what?

Carla: Be inside.

María: Inside what?

Francesca: The circle.

María: What are you going to move?

Carla: The circle.

María: The circle. That is why I am asking Francesca, why are we moving the rectangle? If the question is only talking about the circle you see how important it is to read the question to underline the question to circle the key words, ok. What are the keywords?

María: Inside, very good.

Carla: Circle.

María: Circle, very good. What else?

Carla: Translate.

María: Translate what?

Carla: 5 units.

María: To the what?

Carla: Right.

María: Yes, we need that information. So where is your translation here?

Carla: Right here.

María: Where are you going to start? You can choose any point that you want where? From where? [María is tracing the circle's circumference with her pencil.]

Carla: Outside.

María: You just told me that you were circling,

Carla: Oh, inside.

María: There you go. So what point do you like inside the circle? You can choose what point you want. Chose any point that you want. This one? You like this one? Yeah you can choose this one, you can choose this one, you have a lot of choices. You like this one? Ok, so from here where did you go?

Carla: It's cause I started on the outside.

María: Because you did not read. Correct? You didn't circle keywords. Now you know what to do, right?

There are also numerous examples from María's classroom where she is attending to student's strategies about mathematics topics that she has not previously discussed with the students. Instead of dissuading them she encourages them to explain their reasoning and praises their adaptation of prior mathematical knowledge. At the end of one such conversation María says to the group,

So you mean to tell me that when I start covering this I'm not going to work that hard because you guys already know some of this information. That's cool because when I introduce these lessons you're going to be able to help me with the rest of the students. Yes?

Interpreting students' understandings. María watched a video of fifth grade students from Brookline, Massachusetts working in groups with equivalent fractions. This educational video is available, among many others, from Annenberg Learner at www.learner.org/resources. Annenberg Learner seeks to improve education and assist teachers using video clips and other forms of media. In the clip used for this study, the students were presented with a game board that had seven number lines divided into halves, thirds, fourths, fifths, sixths, eighths, and tenths respectively. A picture of the game board can be found in Appendix A. The object of the game is to move seven game pieces from the left side of each number line to the right side, or one whole. In order to move the pieces the students draw a fraction card and decide how to proceed. For example, if a student drew the card $\frac{4}{8}$ they could choose to move their marker forward four eighths on the eighths number line or they could move the fourths marker to one fourth and the eighths marker to two eighths. Students continue to draw cards until all of their game pieces have reached the right side of their number lines.

In the video clip shown, the students' teacher checks on three groups' progress giving the students an opportunity to discuss their strategies. The first pair of students discuss animatedly and correctly the numerous ways they found to represent $\frac{4}{10}$ on their game board. They proceed to write down the different ways. The second group contains three students. They draw the card $\frac{6}{10}$. One group member suggests moving the game piece to $\frac{2}{5}$ plus $\frac{6}{10}$. His teammate informs him that they cannot move forward more than $\frac{6}{10}$. The student amends his answer and decides to move forward $\frac{1}{10}$ plus $\frac{5}{10}$. The third group of students is a pair that demonstrate for their teacher how they correctly

used classroom manipulatives, in this case colored plastic rods, to verify that $\frac{3}{8}$ plus $\frac{2}{4}$ is equivalent to $\frac{7}{8}$.

After considering what three student groups did in the video, María was asked to explain in writing what he learned about these children's understandings. Interpreting student understanding is coded identically to deciding how to respond, that is as either lacking evidence (0), limited evidence (1), or robust evidence (2). María interpreting score was a 0.66. María responded in writing to a video that showed three student groups and their work with equivalent fractions. María provided a simple narrative of the events that took place for the first clip. "They are trying to add fractions, a combination of two fractions so their chips move forward." In the second clip she did not reference the error a student made. In the third clip María provided more evidence that she had interpreted the children's understanding. "The boys understood that using the rods to add two fractions was easier to understand. They saw the relationship with the $\frac{2}{4}$ and $\frac{1}{2}$ or that $\frac{2}{4} + \frac{3}{8} = \frac{7}{8}$ without even getting any common denominators."

Deciding-how-to-respond. María received a deciding score of 1.33, meaning that over 13 instances of noticing she consistently showed some or robust evidence of deciding to respond based on students' strategies. The only instance in which she failed to show evidence occurred during the translation of the circle problem previously discussed. One reason María scores well is that she emphasizes the importance of students reporting their solution and she will frequently request this of the student "teacher" or another member in each group. This allows her to hear her students thinking and ensure that the entire group has heard the solution. If she is not pleased with her students' reasoning she will impose her own. After Jesús correctly determines the unit

rate for one pencil in a pack of ten he divides \$1.88 by 10 at the whiteboard, María does not let him sit back down before the following exchange.

María: Ok, now I want you to write for me, Jesús. Write for me your complete answer the one that you want me to check. What are you going to write for me? Wherever you want. Zero point, [Jesús writes \$0.188.] Jesús, we're talking about money. How much money is that that I see? [Jesús amends his answer to \$0.19.] Ok good. And you know what I liked is you wrote you dollar sign. There we go. Ok, what does it mean? What does zero point nineteen mean?

Jesús: 19 cents per each pencil.

María: Write it down and say what you are writing.

Jesús: [writing.] Zero point nineteen cents per each mechanical pencil.

María: Ok, just write pencil. Now my question to you is this, do I have to write per and each?

Students and Jesús: No.

María: Ok, which one would you like better, each? Ok? Whatever you want.

Maybe I like per better than each. Do they still mean the same thing?

In the previous exchange María stresses the proper way to express, in English and mathematics, the answer. When responding to her students, María takes the opportunity to probe their familiarity with the proper mathematical terminology and English language. In the following excerpt, a group of students have been working on writing 31.3×10^6 in standard notation. María listens to Miguel's method, then makes sure the rest of the group has found the answer and that they can report and verbalize it properly.

Miguel: First put the number, this one. The decimal you have to move it to the right the number of the exponent. So 1, 2, 3...

María: How many times is that?

Miguel: Six.

María: Ok.

Miguel: 4 5, 6, and the empty spaces you have to add zeros.

María: How many empty spaces do you have?

Miguel: Five.

María: Five, ok. What happened to that decimal that you moved?

Miguel: It's gone.

María: It's gone, it's at the end, ok.

Miguel: You're supposed to put commas, 1, 2, 3, comma, 1, 2, 3 comma.

María: Did Matthew get there?

Miguel: Yes. Wait, yes you have 5.

María: You got it? Count 1, 2, 3 comma 1, 2, 3 comma. So how do you read that number?

Matthew and María: 31 million 300 thousand.

María: 31 million 300 thousand. That's what you got, very good.

When a group in María's class contains an ELL she does alter her approach. Classmates who are proficient in both languages are expected to step up and teach their peers. María will even rely on students to teach new topics to their peers in Spanish. The following exchange takes place after Annie asks María if they were supposed to skip a question involving the circumference of a circle. María replies that yes they are supposed

to skip it because the topic has not been covered yet in class. Annie tells her teacher that she can find the circumference of a circle and proceeds to prove this by demonstrating her mathematical knowledge for the benefit of her Spanish speaking classmate, Bella.

María: For right now yes skip it unless you know how to work the circumference. Cause remember I haven't covered it, but if you know you can give me an explanation I can tell you.

Annie: Well since the formula for circumference um, $2 \pi r$, radius. So it was multiplied, so the radius was 4 cm I multiplied...

María: Where is the radius in the circle?

Annie: It's this one. [Traces the circle with pencil.]

María: Ok, can you explain it to Bella, too?

Annie, Mm, *como el* circumference. [like the]

Bella: ¿*Circunferencia*?

María: ¿*Qué es circunferencia*? [What is circumference?]

Bella: *Circunferencia, es...* [Circumference is...]

María: Uh huh, *qué significa*? [Yes, what does it mean?] Circumference, *circunferencia*.

Bella: *No sé la verdad*. [I don't know.]

María: ¿*No sabes circunferencia*? [You don't know circumference?] Ok, can you explain to her what is circumference?

Annie: Um sure, well *circunferencia es como la area de un círculo, no?*

[Circumference is like the area of a circle, right?]

María: *La distancia*. [The distance.]

Annie: *La distancia*. [The distance.]

María: Good, oh good. *¿Entendistes?* [Do you understand, Bella?]

Bella: *Sí*.

María: *Enseñame circunferencia de círculo*. [Show me the circumference of the circle.] [María points to circle on her paper. Bella traces it.] *Que lo que hizo aquí*. [Yes, what you did here.] Uh huh, *muy bien, es alrededor eso es circunferencia*, ok so you continue on explaining to her. [very good, the circumference is around this]

Positioning. María positions herself in the classroom as the facilitator of student collaboration and as the ultimate authority in matters of mathematics and behavior. She will not tolerate students that are not working together and helping their peers. She does not tolerate any actions that could create a hostile classroom environment like teasing. María relies on student groups and elects a student as “teacher” within their small group. This and her students working together and teaching each other serves to naturally, positively position students as mathematical leaders in their groups. Different groups may feel competitive or positioned in relation to each other as María does broadcast information from one group to the next. There are also times when María will criticize the work done by a group’s “teacher” which may result in a negative positioning.

Instructional strategies. María will use the following research recommended instructional strategies for use with ELLs in her classroom: revoicing, language as resource, multivoicedness, and classroom mathematical discussions.

Revoicing. María revoices her students’ utterances as a matter of course during whole class activities. She will also revoice to translate for the class when a student

answers in Spanish. Finally, María will revoice to reinforce the mathematical vocabulary she wants to emphasize. During whole class discussion María will revoice the incorrect assumptions of students from other classes in contrast to the correct answers of the students in front of her. “We’re gonna use PEMDAS. That’s awesome, because you know what happened earlier in the other classes? They said, ‘ use KCC.’” “Somebody in the other class said there was five and I was like, ‘No... There is ten.’” Whether these incorrect assumptions are genuine or fabricated, they may serve to elevate the classroom as superior to others.

During cooperative learning María will revoice as students explain their reasoning. This revoicing serves several purposes. It reiterates the strategy for all group members and supports the speaker as they present their solution. Students working in group tend to work together to contribute pieces of the solutions they have found so when María revoices she does not assign ownership of what she hears to one particular student. It becomes the group’s work and can serve to position the entire group. Several times during class María will carry the news of one group’s success, or lack thereof, to the rest of the classroom, as she compares their progress. She tells one group, “Ok, you guys are getting ahead of the others and that is good.” When addressing a question that another group has already solved María says, “I did go over it with the girls because they had already solved it and they had a very good explanation about it.”

Language as resource. María does not speak Spanish when she is addressing her students in a whole class setting, however she will speak Spanish when speaking with individual students who may need language support. This includes speaking about mathematics in Spanish. María encourages students that are comfortable in both

languages to speak in Spanish about the mathematics to their ELL peers. It is during these times that María's multilingual students can be positioned positively. In one instance, in the springtime, a group of three girls are working together to solve problems. One of the students, Bella, moved from Mexico and joined the class the previous October. The other two girls in the group, Annie and Francesca both speak Spanish although Francesca's academic language abilities are at the beginning level. Annie and Francesca discuss which problem to do next.

Annie: Number seven so *escribe allí*. [write it there]

Francesca: Let's do, no not number seven. She [Bella] won't get.

Annie: No? Well, I'll explain it to her if she doesn't get it.

Annie has strong Spanish speaking skills, and although other episodes demonstrate that her mathematics vocabulary in Spanish is limited she feels herself capable of using her language abilities and her mathematical knowledge to help her classmate. Through her discussions with María and Bella, Annie learns more mathematical vocabulary in Spanish. Using language as a resource enables Annie to position herself as an authority in her learning community. Later during the same class María tells Annie and Francesca, "I think she [Bella] already got the concept in Spanish right because she understood what I just asked her. So now talk to her in English. More English less Spanish, ok?"

During a whole class discussion of unit rate, María asks the class the meaning of the word "servings". She asks Hector who is an ELL from Honduras. He is unsure what servings means and María uses the opportunity to educate her students about differences that can be found in the Spanish language. "*Que la palabra?* [What does the word

mean?] Es que [It's that] Hector is from Honduras, right? Hector is from Honduras and sometimes the words that he says in Spanish, even me sometimes I don't recognize them because some words are different right. *Ok, son diferentes son las palabras.* [Ok, the words are different.]” This brief aside helps Hector's peers better understand him and assist him during collaborative work.

During a later class, María checks in with a group of four which includes Hector. María checks in with the group regularly and witnesses how the boys are supporting Hector's learning and using language as a resource to do so. At one point the group disagrees about the answer to a problem and María comes over to talk them through the solution. When María confirms the answer a student in the group, Miguel, says triumphantly, “Ah, *que les dije a todos!* [Ah, I told them all!]” Miguel expresses, in Spanish, his pride at getting the correct answer on his own and uses Spanish to position himself as a leader in the group.

Multivoicedness. At the start of two out of three of María's recorded lessons the students are already sitting in groups and she instructs them as follows.

Today we are going to be working in groups. There will be one student in each group that is going to be assigned as the teacher. Your responsibility will be to solve the problems in your group and you are going to explain it. Now if you are not the teacher it doesn't mean that you don't have an input. The rest of you can bring up your ideas.

María's student teachers are expected to choose a student to read each question. They then look for and identify key words and numbers. Next, the students employ problem solving strategies that María has modeled for them, and some she has not, to solve

various problems like proportions, geometric transformations, statistics, and scientific notation. María use of cooperative learning and the rotating assignment of a student as teacher allows the students the opportunity to shed their traditional student role. María's student teachers perform their roles with aplomb. They assign tasks to their peers, judge their methods, share their own strategies, and help each other. The teacher in each group gets the opportunity to act as a knowledgeable authority figure which can position students positively.

Classroom mathematical discussions. María typically begins her lessons with a short whole class discussion of the mathematical ideas that will be featured that day. Students will call out answers, but for the most part María will call on students. She calls on students to read the question to be answered or to contribute a problem solving strategy which the class will then discuss the details of or the strategy's appropriateness. María uses these opportunities to reinforce the proper mathematical terms she would like her students to use.

María encourages her students to work together. In the first recorded lesson the students sit individually but she as they solve problems she tells them, "When you have your answer I want you to share with the person that is sitting close to you. I want you to share when you have an answer, share your information." In later classes, María's students work in groups of three or four students. These small group mathematical discussions allow more student voices to be heard and more ideas to be shared. As addressed above, María's use of small groups and student leaders allows for students to be positive positively in the mathematics classroom.

Teacher beliefs about mathematics teaching. It is important to María that she create a classroom space that allows for student questioning and investigation. She uses cooperative learning which allows more students' questions to be voiced and considered. It also allows for more student engagement with mathematical investigations. If necessary, María will settle disputes or facilitate explanations among group members. María's belief in the importance of students' connections with mathematics has created a student-centered learning environment where students are positioned regularly as important voices in the learning community.

Noticing episodes. María's use of cooperative learning allows for the existence of a student authority figure in each group. When María speaks with each group she often is attending to their strategies and responds by eliciting more information from them about their solutions. This allows her to learn more about their thinking and supports the learning of quieter students who may not volunteer their ideas so eagerly. If María is not in agreement with the mathematics she hears from a student she is capable of reproaching them in ways that may position a student negatively. While responding to her students, María may compare groups as being faster or slower workers or praising the solution of group A to group B. In this way she is positioning her students groups in relation to each other.

Student Engagement. In order to investigate student engagement in María's classroom, her three recorded classroom lessons were searched for evidence of Franke et al.'s (2015) six invitation moves and three support moves. The six invitation moves are as follows: explain someone else's solution, discuss differences between solutions, make a suggestion to another student about his or her idea, connect their ideas to other students'

ideas, create a solution together with other students, and use a solution that was shared by another student. The three support moves are probing, scaffolding, and positioning.

Throughout her recorded classroom lessons María made 13 invitation moves with her students. Three of the invitation moves were each used three times by María. They are explain someone else's solution, discuss differences between solutions, and connect their ideas to other students' ideas. She used create a solution together with the other students twice. The remaining moves were used one time each, that is make a suggestion to another student about his or her ideas and use a solution that was shared by another student. María's students often work in groups with a designated student "teacher" and the potpourri of moves that María utilizes reflects their needs as she visits each group.

When María's students are working in groups she will circulate the room and sometimes engage a group's "teacher" about the mathematics or choose another group member to speak. While the student explains the mathematics it is uncertain if they are explaining their own work, someone else's work, or a co-creation of the group. Therefore it is difficult to say to what degree the student speaker is engaging with the mathematics. María's own instructions are, "There will be one student in each group that is going to be assigned as the teacher. Ok, I will assign you as a teacher in your group. Your responsibility will be to solve the problems in your group and you are going to explain it." Based on these instructions it appears that the student "teacher" is engaged with their own mathematics, but the rest of the group may have a more passive relationship with the material. María's instructions to the class do continue on to encourage a discussion of differing solution methods, "Now if you are not the teacher it doesn't mean that you don't

have an input. The rest of you can bring up your ideas, ‘Well, this is the way I solved the problem’ or ‘This is the method that I used.’”

Despite María’s verbal instructions, there are numerous instances where it is clear that the groups are not co-creating the mathematics, particularly in the spring lesson. During this lesson, the students are working on simple conversions from standard to scientific notation and María comments multiple times that group members are not at the same place on the worksheet. The students admit that they are working independently and then comparing answers after a time. If her students do not arrive at the same solutions that a discussion of differences between solutions may result. It is also apparent that Bella, an ELL, is being taught the material by her peers, at María’s insistence. María will use the support move probing to check for understanding with Bella, but Bella’s interaction with the mathematics never elevates to the level of debating different solutions, adding to a classmate’s solution, or a connection of Bella’s ideas with others’.

After María asks a student to explain a solution (which may or may not be an explanation of someone else’s result) she will frequently use the support move, probing, to check for understanding with a different group member. María probes her students with questions like, “Did you understand what she said?” “What’s another way of saying it?” “Did you see that?” She usually engages in some scaffolding to advance the discussion and, less frequently, positioning to connect mathematical ideas with a student speaker. An instance where María positioned a student by connecting them with their mathematical idea was when she stated, “Germán already explained why that was the answer.” A brief example of scaffolding occurs when María is constructing a circle graph with student that represents a word problem in which $\frac{5}{8}$ of a race is completed.

María: How much do you have left over[after shading five]?

Student: Three.

María: Ok, three out of what? Three out of...

María and Students: Eight.

María: Ok, so write it, three out of eight. What does that three out of eight mean?

Student: How much she didn't run.

María: How much she didn't run, is that what they want?

Summary. María values a classroom where students are able to discuss mathematics, ask questions, and makes discoveries. Her students typically work in small groups with one student designated as the “teacher.” Each young teacher is responsible for their group’s understanding which María monitors by circulating the room and questioning. The students that María chooses to act as “teacher” are positioned as authority figures in their groups. In addition, they can be positioned positively or negatively depending on María’s reaction to their mathematics during episodes of noticing. She will speak to her students about the mathematics in Spanish, but depends heavily on Spanish speaking students to assist in the task of educating ELLs.

V. DISCUSSION

This was a qualitative case study looking at professional mathematics teacher noticing in linguistically diverse classrooms. Participants of this study were three middle school mathematics teachers who also participated in a NSF funded research project, Mathematics Instructional for English Language Learners (MIELL). Four research questions guided this research study:

- 1) What do teachers of linguistically diverse classrooms believe concerning mathematics, mathematics teaching, and their linguistically diverse students of mathematics? How do teachers position themselves as teachers? What about these beliefs and positioning can be witnessed in the classroom?
- 2) What does professional mathematics teacher noticing look like in a linguistically diverse middle school mathematics classroom?
- 3) How do teachers use of research-based instructional strategies, their beliefs, and professional mathematics teacher noticing position students in the classroom?
- 4) How are teachers in linguistically diverse mathematics classrooms using invitation moves to promote student engagement?

To answer these questions I used data that was collected during the MIELL project (2011-2015), chiefly nine video recorded mathematics classes, three per teacher. I also used various knowledge assessment scores that the teachers were administered as MIELL participants to aid in the selection process for this study (see Chapter III). Additionally, I interviewed each teacher in person during which I also administered a short activity to better discern each teacher's interpretation of children's understanding. The findings from these data sources were presented in Chapter IV. In this chapter, I present a

discussion of these findings and recommendations for future research into linguistically diverse classrooms and teacher noticing.

Discussion of Findings

Teacher beliefs. The traditional vision of a mathematics classrooms that many of us have is a place where teachers lecture, students frantically copy into notebooks, and little in the way of conversations take place. In a classroom where some students cannot speak English, one might think that mathematics conversations would have an even more diminished presence. The teacher participants in this research study all teach in classrooms where 30-40% of students are classified as ELLs, but each teacher expressed that the belief most important to them required student's speaking about mathematics. Jaime stated that communication was the most important thing in his classroom and María said, students asking questions. Patricia felt most strongly about student accountability, which in her classroom means all students verbalizing with her about the mathematics. This fits with the research that asks teachers to hold ELLs to high standards in the mathematics classroom, instead of letting them slide with a deficit mentality. The teacher in this study were not going to let students remain silent. The beliefs that these teachers professed were enacted in their classrooms which we know from research is not always the case (Philipp, 2007).

Research into educating ELLs emphasizes the importance of using students' home language as a resource in the classroom and encouraging the use of that language in the classroom (Campbell et al., 2007; Pettit, 2011). During the pilot study for this dissertation I was not surprised when both Chet and Simon, two monolingual

mathematics teachers with the least experiencing teaching ELLs, said that native language use should not be encouraged in the classroom. I expected a different answer when I interviewed the three participants for this study, all of them Hispanic, Spanish speaking teachers with years of experience teaching ELLs. I was wrong.

Although the teachers in this research study use language as a resource to varying degrees, none of them believed that speaking Spanish should be encouraged in the classroom. Patricia, a native Spanish speaker with a college degree in engineering from Mexico, did not speak about mathematics in Spanish in her classroom. Although she allowed her students to speak about mathematics in Spanish with each other and to her, she elevated English by encouraging them to practice their English both in writing and when speaking. María, the only seventh grade teacher in the study, may feel less pressure from the high stakes standardized testing than the two eighth grade teachers that participated in this study. That being said, María, who did engage in some mathematical discussions in Spanish with her students, was still heard, in April, encouraging her ELLs to practice their English with peers.

All of the teachers in this research study believed that students had enough opportunities to speak Spanish outside of the classroom and thus it should not be encouraged inside the classroom. In fact, when María was asked whether students should be encouraged to speak Spanish in the classroom she automatically assumed the question was, “Should students be encouraged to speak English in the classroom” and began to answer accordingly. This results in students who speak Spanish, but cannot speak academic Spanish fluently. Unfortunately, this is the reality of their school district. These teachers view their classrooms as one of the few places where their students will practice

their English. The role of an educator is ultimately to equip their students with skills they will need in the world. Without the ability to speak and write English fluently, these teachers believe that their students will not be as successful in the United States. It will be harder for them to work, go to school, and move away from their predominately Spanish speaking county. This is one characteristic that may be unique to a linguistically diverse classroom where 99% of the student body and teachers are Hispanic.

Professional mathematics teacher noticing. The three teacher participants of this research demonstrate teacher noticing is possible in a variety of classroom arrangements. Jaime engages his students in whole class discussions and then individually as they work on problems. Patricia's students are always found in pairs and most of her interactions with students also happen in these pairs. María's student work primarily in small groups and this is when most of her teacher noticing takes place.

Attending to students' strategies. Jacobs et al. (2010) found that experienced teachers were more skilled at attending to children's strategies. Although the noticing occurs in different student configurations, these teachers do attend to students' strategies and their methods are similar. Of course, they listen to what students say and look at their written work, but they also ask the probing questions necessary to begin an interpretation of student understanding.

The noticing framework used by Jacobs et al. (2010) was administered to teachers as part of a professional development cohort and included pre-service teachers, teachers with limited exposure to the professional development, and experienced teachers already invested in the professional development program. Teachers in Jacobs et al.'s

study who had both teaching experience and two years of professional development were classified as advanced participants. All of the teachers that participated in this dissertation study are experienced educators and participate in regular professional development opportunities, however it is not to be expected that the teacher participants in this dissertation study would score as high as the experienced teachers in Jacobs et al.'s study as their teachers were participating in a professional development that specifically focused on children's mathematical thinking.

Only Jaime's average score of 0.92 aligned him with the experienced educators (0.90) invested in professional development from Jacobs et al.'s participants. Jaime consistently attended to student's strategies through his questioning and revoicing techniques and broadcasts the strategies for the whole class to digest. Patricia's score of 0.80 was just shy of Jacobs et al.'s experienced classification. On several occasions Patricia would engage with students who had yet to make any attempt to solve the given problem. Therefore she did not receive credit for attending to student's strategy because she did not give them the time to employ a strategy. It is entirely possible that these students would never have made the effort without her guidance, but for the purposes of this research those are considered missed opportunities. María received a score of 0.65 which equals the average that would categorize her in Jacobs et al.'s study as a practicing teacher without the professional development background. María's score reflects the fact that she regularly dismissed the work her students had done and the strategies they had chosen and imposed her own methods upon the students. Her students had good ideas that would result in valid solutions, but she did not take the time to consider their validity and this served to shut down her students.

Interpreting students' understandings. Interpreting is the component of professional mathematics teacher noticing that can be difficult to quantify and study. Interpreting takes place in the minds of teachers and cannot be viewed in a video clip or witnessed from the back of a classroom. Some researchers have gone so far as to equip teachers with body cameras that they activate when interpreting is taking place. In this way at least the researchers can quiz the teachers about what happened during a future debriefing (Sherin et al., 2011b).

In order to speak to their interpreting Jaime, Patricia, and María agreed to watch a video clip of students working in groups on an equivalent fractions activity. They were then asked to explain in writing what they learned about the students' understandings. This exercise was meant to mimic the interpreting exercise created by Jacobs et al. and administered to their participants. Jaime (0.66), Patricia (0.66), and María (0.77) all scored much lower than Jacobs et al.'s experienced teachers (1.19), although higher than the mean for pre-service teachers (0.47). I believe this exercise had limitations capturing the intended component of noticing. Like Jacobs et al. they were only allowed to view the video once, but it came at the end of a long interview. Unlike Jacobs et al., these teachers had no previous experience with the topic of children's mathematical thinking, had not completed a similar attending exercise, and had little support outside of the directions. Most of the teachers' explanations were essentially a description of what was happening in the videos instead of a true interpretation of the students' understanding. I do believe these teachers are capable of successfully interpreting their students' understandings based on the attending and responding they exhibited in their own classrooms.

Mathematics education researchers are still debating various ways to better capture the interpreting component of noticing. In the future, if a similar study is conducted I believe more valuable information concerning interpreting students' understanding may be gathered from participants by asking them to watch clips of their own teaching. Although current research claims interpreting is a noticing component that cannot be witnessed I believe there is still more discussion to be had over the accuracy of that statement. I believe my participants did interpret their students understandings and I think it is evidenced by the responses they choose to make.

Deciding-how-to-respond. When applying Jacobs et al.'s framework to prerecorded classroom lessons one question I asked myself was whether the teacher's response left the student capable of solving a similar problem on their own or not. Was the teacher's response confusing, off target, or unrelated to their student's demonstrated understanding? Did the teacher's words empower the student and build confidence? Ultimately, all of the teachers in this study scored higher than Jacobs et al.'s experienced teachers at deciding-how-to respond. They demonstrate the wide variety of responses a teachers may have to choose from when working in a linguistically diverse classroom.

Jaime received a score of 1.25 which was higher than the 0.84 mean of Jacobs et al.'s experienced teachers. Jaime's classes are run as a whole class discussion. When Jaime attends to a student's strategy he will ensure that it is shared with the class and made a point of discussion. Students will accept ownership of their strategy and explain it to peers. Their strategy might be compared to another or used as an example of what not to do. Jaime encourages right and wrong answers as learning opportunities. Although he may be responding to a single student's strategy, Jaime will use that as a gauge for the

entire classroom's understanding and respond in ways meant to help all of his students. Jaime rarely concludes a topic without some assurance that the entire class is comfortable and ready to begin something new.

Patricia scored a 1.13 which was also higher than the 0.84 mean of Jacobs et al.'s experienced teachers. Patricia also advances her goals of students working together and accountability when responding to her students. Patricia frequently will address one student's work, but include their partner in the discussion. The partner may be used to reinforce what Patricia is saying or assist in advancing the conversation. When responding, Patricia emphasizes the importance of using a strategy, showing the work, and adhering to guidelines demanded by the standardized testing. Patricia was seen to respond in ways that incorporated relevant mathematics vocabulary and extended student understanding. However, and not surprisingly, when Patricia's attending to a particular student's strategy lacked evidence, the associated response did not score well. When this happened the discussion Patricia led became convoluted, confusing, and the student did not appear capable of solving a similar problem successfully because Patricia did not get to the root of their confusion. This also occurred in instances where Patricia did attend to a student's strategy, but did not invest in it. This was seen to happen twice when Patricia was running out of class time.

María scored a 1.3 which was higher than the 0.84 mean of Jacobs et al.'s experienced teachers. María is a very experienced teacher, with over thirty years in the classroom. Having said that, she can be quite particular about how her students solve problems. On several occasions she dismisses her students' methods and imposes her own. This does not translate into a robust response to students' understanding. These

episodes notwithstanding, María does attend to the strategies being used by her student groups throughout the class period. She believes in the importance of students asking questions and feeling comfortable in her classroom and this is supported by her responding. When responding, María will include group members and encourage everyone to contribute. She is very concerned that all group members understand what is happening with the mathematics. Once she has attended to a student's strategy she will nearly always ask them to explain it to a group member, especially if that group member does not speak English fluently. This cements for María the student's understanding, provides the student another opportunity to communicate effectively, and provides another opportunity for group members to think about the solution. María's responses serve to make students responsible for their peers' learning.

The current available research into teacher noticing in linguistically diverse classrooms is minimal and the research that is relevant is dominated by K-5 classrooms such as Turner et al. (2013a) and Turner et al. (2013b). However, it is likely that the comparative maturity of their students allows middle school teachers more options when deciding how to respond to their students as well as more opportunities to positively position their students as a result. Mathematics teachers in linguistically diverse classrooms have additional, relevant ways they may respond to a student's strategy. It is possible these teachers scored higher than those in the Jacobs et al. study for this reason. All of these teachers respond to their students in ways that promote the classroom environment they are fostering whether it is whole class discussion, working in pairs, or working in small groups.

ELL Teacher Noticing. All the teachers in this study engage in literacy centric strategies throughout their classes. They will ask students to read problems or definitions out loud. They will even ask multiple students to read the same thing. This repetition allows all students to hear the words and gives more students an opportunity to practice reading in English. Students that are not strong English speakers are still able to participate and might be asked to read or answer questions that are in keeping with their current abilities. This happens in Patricia's class when she chooses not to let a student, Tomás, read a problem stem out loud, but immediately returns to him to help identify parts of the proportion. This illustrates how important it is for teachers to understand their students' language abilities in order to support their inclusion in classroom activities and discussion.

These literacy strategies are not confined to whole class discussions and frequently happen when the teacher is noticing a student's thinking. When students are working on problems alone or with peers the teachers will often focus their thinking or remediate mistakes by asking the student to focus on the words in the problem. They will ask the students to read the questions out loud, to read a particular sentence, and to employ reading comprehension strategies such as underlining, circling, and highlighting key words. In a mathematics classroom where most or all students are monolingual English speakers these literacy strategies are not prevalent. Mathematics teachers asked to incorporate literacy strategies in the classroom might not know what that looks like. Reading out loud is typically an isolated occurrence emphasized in English literature classes.

The teachers in this study employ a specific kind of noticing we have coined, ELL teacher noticing. They know the language abilities of all their students. María, who was born in Mexico, knows that her student, Hector, is originally from Honduras and that the way she speaks Spanish may not be the same as his. Moreover, María does not have this conversation in private with Hector, but in front of the entire class. There is no stigma placed on Hector for being “different” and when his peers speak with him in Spanish they will also be aware that some words may be unexpected. Teachers like María do not just consider the language ability of their ELLs, but also the students who speak English fluently and also speak some Spanish. In this way, María can group students to maximize learning and support for her ELLs.

In fact, it is a very common strategy for these teachers to group their students and encourage them to work together. Even Jaime, whose students mostly work individually, will sometimes prod his students to compare answers with a neighbor. María relies heavily on students helping students, which has many benefits, but may result in student fatigue. After watching Annie helping Bella for 90 minutes it is clear that Annie has a good understanding of the mathematics and is a competent Spanish speaker. It is also clear that Bella, who moved from Mexico to Texas in October, is eager to learn and picks up the language as the year progresses. That being said, during the third lesson María asks Annie to explain a concept to Bella and Annie replies, “I’m not good at explaining things to Bella.” Although she does an admirable job of it, for whatever reason Annie requests a reprieve from her job as teacher/translator. A few minutes later Annie cannot help but jump back into it, but it seems important to consider how students are being used in the classroom.

Positioning. The theoretical framework for this study was positioning theory – the idea that classrooms are social spaces where teachers and students engage in constant interactions. The theory purports that these constant interactions have the power to position participants in different ways – as teacher/learner, as a contributor/audience member, as smart/dumb, as capable/incapable, as a member/non-member, etc. I believed, before this research began, that noticing could be used by teachers to position students, hopefully in a positive manner. In addition, I hypothesized that the research-based instructional strategies outlined in Chapter II were effective with ELLs because they are capable of being used by teachers to position students.

Instructional strategies. My research showed that most of the research-based instructional strategies that I witnessed being utilized by the three participants were not positioning the students. Future research may demonstrate that additional research-based instructional strategies are being used by teachers to position students, but it was not the case for these teachers. The data gathered in this study show that the strategies that can position students involve communication like multivoicedness and classroom mathematical discussion. This may not be surprising given the theoretical framework for this study, positioning theory, is described by Harré and van Langenhove (1999a) using words like speaking, acting, conversation, and discourse. Students in the middle grades are capable of more sophisticated mathematical discussions than elementary students. For this reason teachers should have higher expectations for their students' abilities to debate logical arguments and reason together. In the middle grades students can work alone and in groups with less supervision and for longer periods of time. Thus middle school mathematics teachers of ELLs should pay even closer attention to using language as a

resource and fostering classroom mathematics discussions than the current literature such as Khisty and Chval (2002) and Mustani et al. (2009), derived largely from elementary grades, recommends.

The teachers in this study do use many of the instructional strategies recommended for use with ELLs that were discussed in Chapter II. Revoicing is used, particularly by Jaime and Patricia, to such an extent that nearly every student utterance is repeated by the teacher. Using revoicing in this manner is akin to a strategy called “Organize Mathematical Contributions” by Hand et al. (2015, p. 263). The teachers in this study use revoicing to “bring students’ sense making into the flow of classroom conversation” (p. 264). Using revoicing in this manner may be a good strategy for fostering and enabling classroom mathematical discussions teachers to use with students, but does not serve to position the students.

As expected, some instructional strategies were not present in these classrooms. As this study began to take shape I assumed that multidiscursivity, or the inclusion of students’ cultural experiences, would be a strategy that these teachers would utilize and this also was not the case. Although these classrooms fit the definition of linguistically diverse they were also 99% Hispanic and all of the teachers were Hispanic and Spanish speaking. The teachers and students, for the most part, are part of the same culture, in contrast to the kind of novel cultural exchanges written about in the research.

Teachers using multiple representations was seen in many videos collected during the MIELL study, but multiple representations is heavily dependent on the topic under study. Patricia presented surface area and volume of prisms and incorporated 3D models and 2D nets. She also guided her students through the process of setting up proportions,

which yielded less opportunity for multiple representations. This also seemed to be the case for gestures as some mathematical processes lend themselves more to physical representations than others. For example, Patricia models shrinking and growing with her hands, but how would Jaime incorporate gestures into a discussion of the Real number system?

Teacher noticing. The noticing exhibited by these participants in their middle school classrooms leads to students being positioning in many instances. How? The teachers accomplish this by attaching good ideas and strategies to the students who offer them. They also have high expectations for their students and treat them as peers capable of assisting their classmates. These students not only can help each other learn mathematics, but they can also learn to speak better English and Spanish together. The teachers include ELLs as part of classroom discussions and give them the time and assistance they require to participate. Because these teachers have high expectations for all students, an important belief according to Pettit (2011), these students are not allowed to melt into the background. Teacher noticing affords teachers an opportunity to including ELLs as part of the discussion which in turn elevates and positions them in class as part of the mathematical conversation (Turner et al., 2013b). This connection between positioning and teacher noticing that occurs in this middle school mathematics classrooms is an important aspect of this research.

These teachers couple noticing with their chosen learning environment- whole class discussion, pairs, and small groups. Teacher noticing creates opportunities for teachers to position their students. Of course, students can be positioned both positively and negatively. Jaime solicits his students by attending to their strategies. The students

takes ownership of their strategy as the class discusses it. They may be asked to explain it multiple times to classmates. When Jaime sees that a student needs help he employs a peer by saying something like, “This student wasn’t paying attention and needs your help.” Jaime does not say, “This student can’t do the work and needs your help.” This subtle choice of words on Jaime’s part is the difference between negatively positioning a student as incapable versus a student temporarily engaged in another task. Jaime also communicates to his students that wrong answers are no reason to be upset. In fact, he treats wrong answers as learning opportunities which may help students’ from feeling negatively positioned.

Patricia’s students work in pairs and noticing occurs when she fulfills her goal of student accountability and discusses the mathematics with her student pairs. She does not compare groups or foster a competitive spirit which may position some groups as “smarter”, while simultaneously positioning others negatively. Patricia consistently praises her students and does not discourage the use of two languages when necessary, meaning that students who prefer to speak in Spanish are not positioned negatively. Patricia couches much of her criticism as preparation for standardized testing which may serve to ease the blow for students. “So what are you gonna have to do when you take the test?... You did improve a lot lately so that work *así*, [just like that] done step by step is what I want.” With her students in pairs, Patricia will often respond to student A’s strategy with their partner, student B. This can help student B’s understanding and satisfy Patricia’s desire for accountability, but it also elevates student A as an exemplar mathematician.

The positioning opportunities that are created by teacher noticing are most prominent in María's classroom. This likely results from the fact that she utilizes small groups and appoints a student "teacher" for each. This student is positioned from the start as the knowledgeable leader and they are tasked with communicating the mathematics to their peers. Noticing occurs when María circulates the room and interacts with each group's leader. When evidence of attending to students' strategies is lacking in these interactions, María may be positioning students negatively. She is communicating that their solution method or strategy is not worthwhile, although it is perfectly reasonable mathematics. The interaction between Francesca and María, which begins on page 130 of Chapter IV, illustrates this. Francesca, an animated and involved student, withdraws and does not contribute again after María rebuffs her method for translating a circle.

When María's response to a student includes the task of communicating to peers, which it usually does, many of María's students assume the burden enthusiastically. Some students preemptively announce that they have already helped a group member with the problem. When a peer expresses the opinion that her ELL classmate will not understand a question, group leader Annie replies, "Well, I'll explain it to her if she doesn't get it." Annie is positioned, by María's class structure and María's responses, as a capable mathematician.

Student Engagement. Teacher noticing requires a classroom environment where student are actively engaging with the mathematics, which is why student engagement was a consideration in this dissertation. All of the case studies that make up this dissertation research demonstrate teachers using invitation and support moves to encourage student engagement with mathematics and, more specifically, the

mathematical ideas of their peers. The participants of this study demonstrate the use of invitation moves that encourage student engagement and participation which Edwards (2015) considers the most important thing that can be fostered in the middle grades. Although the three teachers use a similar amount of these moves, they seem to have varying levels of effectiveness.

Jaime, who relies on whole class discussions and independent seat work in his mathematics classroom, also relies on Franke et al.'s (2015) invitation move, make a suggestion to another student about his or her idea. He uses this move twice as often as any other. This is his go-to move for facilitating class discussion, but his decreased reliance on students connecting their own ideas or creating solutions together leaves some students disengaged. In fact, student who speak up are often silenced by Jaime as he waits for a particular student to give an answer. Thus, students in Jaime's class who are not currently being called on are learning that they do not need to be engaged with the mathematics.

Jaime believes with some chagrin that he reverts back to the lecture style classrooms he experienced as a student. His classroom may not epitomize the lecture style classroom of his youth, but it is true that his students do not have the opportunities to engage in mathematical ideas with their peers that might be possible given a change in classroom expectations and organization. As he grows in the profession Jaime may move more and more in this direction as he already expresses distain for his reliance on more traditional teaching methods. If Jaime is able to eventually combine invitation and support moves in a more student centered classroom they will likely be more effective.

María has already made the move to a classroom that is based on students working with mathematics collaboratively. She also uses invitation and support moves as she speaks with her student groups and, like many teachers, she assigns roles to certain students, specifically she elects one student in each group as the “teacher”. This “teacher” is responsible for explaining any solutions to their group members. Unfortunately, the groups in María’s class are not engaging in many of the behaviors that the invitation moves would be used to foster, that is connecting their ideas and creating the mathematics jointly. In fact, the group members are frequently shown to be working independently and then comparing answers after a time. This is akin to student working independently at home and simply checking their answers in the back of the book. Furthermore, María’s elected “teacher” being tasked with explaining the solutions to other members lets the rest of the group members off the hook from engaging with the mathematics and each other. A solution to this may be to rotate the position of “teacher”, eliminate it altogether, or devise a new system for responsible group membership.

Patricia also maintains the expectation in her classroom that students will work collaboratively on the mathematics. With a total of eighteen, Patricia also used more invitation moves than Jaime or María while circling the room to check her students’ progress. She favors the moves, inviting students to create solutions together and making suggestions to another student about their mathematical ideas. When Patricia speaks with a group she will address one student, but then draw their partner into the conversation. Oftentimes the partner cannot help but contribute voluntarily. Patricia treats each group as a unit and does not position one student as the leader or differently abled than their partner. Having student grouped in pairs makes it harder for a student to disengage and in

Patricia's class they are witnessed working together, amid near constant encouragement to do so from Patricia.

When a student speaks out of turn Patricia does not dismiss their contribution, but she does stress polite classrooms practices. This occurs during a whole class discussion when Patricia asks a student, Sarah, to answer a question. When a male student shouts out the correct answer Patricia does not ignore him or chastise him, rather she says with a smirk, "Thank you, Sarah." This causes the students to giggle, but it also communicates to the class that their ideas and engagement are important, but raising their hand is also important.

Summary

This research project demonstrates how professional mathematics teacher noticing is happening and can happen in linguistically diverse mathematics classrooms. At first glance these classrooms and their teachers seems very homogeneous – these classrooms are in the same school district, the teachers share the same ethnicity, and similar educational experiences. Interestingly though the classroom experiences are quite diverse from the ways that the teachers group their students, to the level of student engagement, to the instructional strategies used, and how teachers notice and respond to their students' mathematics.

I think this study shows that teacher noticing is a tool that can be seamlessly utilized in classrooms by teachers, especially by teachers that may be wary of another educational initiative in their classroom. In fact, teacher noticing can be molded to fit different teachers and different classroom designs. Witnessing three different, diverse

teachers, with not training or awareness of professional mathematics teacher noticing, attending to strategies and responding to their students shows that all teachers can purpose to notice their students' strategies. Moreover, I believe teachers actively engaged in a noticing mindset can foster this classroom behavior via communication with peers and through a continuous improvement program.

Suggestions for Future Research

Linguistically Diverse Classrooms. The definition of linguistically diverse classroom that appears in Chapter I of this research study is as follows, “classrooms containing a mixture of monolingual English speaking students and students classified as English Language Learners that may speak a language other than English outside of school.” The term “mixture” is broad. Let class A include one monolingual English speaker and twenty students that speak a language other than English. Then let class B include one student that speaks a language other than English and the majority of the class are monolingual English speaking students. The needs of ELLs may be the same, but the resources, opportunities, and classroom climates in class A and class B are surely different. It begs the question, should these classrooms be treated the same? Spanish speaking students in the classrooms that were studied as part of this research had teachers that spoke Spanish and peers that spoke Spanish. They had other classmates who were also working to learn mathematics and English simultaneously. When they left their mathematics classroom these ELLs could speak Spanish or English with their friends after school or with the checkout girl at the supermarket. Contrast this with an ELL in small town New England who recently emigrated with his family from China. He can speak Chinese at home with his parents, but his teacher does not know Chinese, his peers

cannot communicate with him nor his neighbors. His might be the only Asian family in town.

These starkly different scenarios both result in a linguistically diverse classroom by definition, but the classrooms look very different. There are different strata of linguistically diverse classrooms and future research should compare and contrast these classrooms. It may be that some strategies are more effective with one type of classroom. There may be currently unknown strategies that could be introduced to some stratum of linguistically diverse classroom.

Professional Development. When Chet and Simon, two of the participants in the pilot study for this dissertation, were asked how they felt about professional development geared towards ELLs they both balked. Neither had ELLs regularly in their classrooms and didn't see the need for additional training. As the number of non-English speaking students in our classrooms increase, classrooms and teachers that have never experienced an ELL will suddenly have one or two. Classrooms with the occasional ELL will regularly have a handful. Classrooms that are familiar with ELLs may now have a majority of students that speak a different language. Classrooms and teachers tasked with teaching mathematics may now have ELLs that speak many different languages, instead of just one or two. A natural result of any study into English Language Learners should be to promote and encourage professional development opportunities for teachers about working with these students. The results of the MIELL project will, in part, be used to seed new research into improving professional development opportunities for teachers. Finding ways to encourage awareness of the needs of ELLs and interest in their education should be part of the not too distant future of mathematics education research.

Noticing. Teacher noticing has received a significant amount of attention in the education research community. Research declares that noticing is an important part of a teacher's classroom practices. Research has shown that, yes, noticing can be taught, in particular to pre-service mathematics teachers. Companies that create teacher assessments have even started incorporating noticing into their instruments (C. Mejía Colindres, personal communication, October 25, 2015). It seems likely that future researchers will use teacher noticing as a gauge for quantifying student centered classrooms.

I hope that future research into professional mathematics teacher noticing will consider noticing that takes place in the classroom, particularly linguistically diverse classrooms. This is an undeveloped source of information about noticing and effective teaching for ELLs. This dissertation, like most, was conducted within various limitations of time and money. I hope that future research into noticing in linguistically diverse mathematics classrooms will look to Yoon (2011) and imbed a teacher/researcher for a long period of study that can interact fully with the ELL and non-ELL students.

APPENDIX SECTION

APPENDIX A

Interview Guide

Hello, thank you for being here. I would like to gather some basic background information about you.

Can you tell me your name and age?

Where did you grow up?

How many years have you been teaching? Teaching middle school mathematics?

How would you introduce your career to a stranger?

Can you tell me how you define mathematics?

Tell me about your journey towards becoming a middle school mathematics teacher.

When did you know you wanted to become a teacher?

What events or people, if any, from elementary school stand out to you in terms of your mathematics education and teachers? Please discuss what you consider to be high points, low points and turning points of this time. Also consider any particular challenges you faced.

What events or people, if any, from middle school stand out to you in terms of your mathematics education and teachers?

Were your parents involved in your education? How, attending parent conferences, helping with homework, critiquing report cards, etc.? Did they have high expectations for you?

What events or people, if any, from high school stand out to you in terms of your mathematics education and teachers?

What events or people, if any, from university stand out to you in terms of your mathematics education and teachers?

How do you feel your own experiences as a student have translated into what you do in your classroom? What things from your childhood do you still embrace? What have you cast aside?

Positioning is the idea that in a social interaction people can be positioned in different ways. In a classroom for example a teacher might say to a student: "Adam that is a great idea; I would like you to share it with the class. Class, listen to the thoughtful idea that Adam has." In this scenario the teacher is positioning Adam as a contributor and a

student with ideas worth considering. When I was in fourth grade my teacher told me I was too smart and to stop answering questions in class. I felt this positioned me as a non-member of the learning community.

Do you remember a time you were positioned positively or negatively by classmates or your teacher?

Are you careful in how you position your students? Please provide specific examples of the considerations you make, if possible.

Think of an ELLs(s) that you have had. In what ways do you see them change/grow through a school year?

What is your opinion of/experience with bilingual education programs?

Thank you. I would like to finish this part of the interview with a few questions more specific about your teaching and classroom.

- 1) What expectations do you hold for your students? How do you feel this is reflected by what you do in your classroom? Do you hold the same or different expectations for ELLs?
- 2) What kind of outside help do you get in your classroom perhaps from parents, team coordinators, subject specialists, school psychologists, counselor or ESL teachers?

How much do you work together with these people?

How much do they contribute to your classrooms?

Do you feel solely responsible for educating the students in your classroom or do these people share some of that responsibility?

- 3) Is it important to you to encourage ELLs to speak their native language? Why or why not?

In what ways do you encourage ELLs in your classroom to speak their native language in class?

In what ways do you encourage ELLs in your classroom to speak their native language at home?

- 4) Are you familiar with the term *academic English*?

Do you have an idea how long it takes a new ELL in your classroom to learn academic English?

How do you arrive at this number?

- 5) What are your opportunities for professional development?

Are you able to request professional development in particular areas of interest to you?

How much professional development do you receive that is specific to the education of ELLs?

Would you prefer more, less, or the same amount?

Last question!

Is there anything else you think I should know that you would like to tell me?

Thank you so much for being a part of this research.

The last part of this interview includes a video clip of a classroom lesson using fractions.

Please view the following clips of a classroom lesson on fractions using the game board shown below.

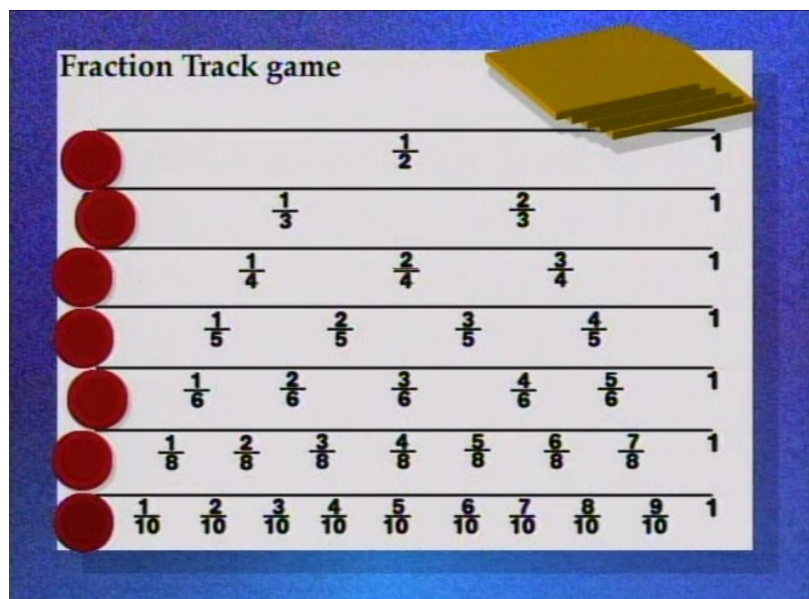
1:12-3:00 Intro to Activity

4:12-5:55 Working in Groups

Sean's $\frac{4}{10}$ Group

Rebecca's $\frac{6}{10}$ Group

9:15-10:03 Joe and the rods



http://www.learner.org/vod/vod_window.html?pid=916

Consider what you think each child did in response to this activity. Please explain in writing below what you learned about these children's understandings.

4:12-5:55 Working in Groups

Sean's $\frac{4}{10}$ Group

Rebecca's $\frac{6}{10}$ Group

9:15-10:03 Joe and the rods

APPENDIX B

CONSENT FORM

Consent Form

You are being asked to be part of a research project. I am trying to learn more about teachers' experiences learning and teaching mathematics. If you agree to be part of this research, I will conduct an interview that will last approximately 2 hours. The research is being conducted by Rachel Bower, a doctoral student at Texas State University, XXXXX@txstate.edu (000-000-0000).

I don't think that there are any serious risks to you, but some of the questions may be personal (for example, questions about your childhood and difficult times in your life.) You may choose not to answer any question(s) for any reason.

The results of your interview are anonymous; I am not publishing your name or other identifying information. Only the researcher, Rachel Bower, and my advisor, Dr. M. Alejandra Sorto will be aware of the identity of project participants.

This project [EXP2015E988009F] was exempted by the Texas State IRB on [06-04-15]. Pertinent questions or concerns about the research, research participants' rights, and/or research-related injuries to participants should be directed to:

Institutional Review Board
Office of Research Compliance
Texas State University-San Marcos
JCK 489
601 University Drive, San Marcos, TX 78666
(ph) 000/000-0000 / (fax) 000/000-0000 / xxxxx@txstate.edu /

Your participation is voluntary, and refusal to participate will involve no penalty or loss of benefits to which you are otherwise entitled. There are no direct benefits to you for participating in this research. However, society may benefit from the results. You will not receive anything for participating.

A summary of the findings will be provided to participants upon completion of the study, if requested. To accessing results of the study, contact Rachel Bower.

Date: _____

Name of Participant: _____

Signature of Participant: _____

Name of Researcher: _____

Signature of Researcher: _____

APPENDIX C

The following documents the coding scheme developed by Jacobs et al.'s (2010) to analyze the three components of elementary mathematics teachers' professional noticing of children's mathematical thinking. In this study, 131 pre- and in-service elementary teachers were asked to view a 9-minute video clip, examine three classroom artifacts, and then answer three questions aimed at assessing their noticing. The questions they were posed appear below as well as the coding scheme used by the researchers.

Attending prompt. To assess participants' expertise in attending to children's strategies, we requested, "Please describe in detail what you think each child did in response to this problem." The specific names of the children whose strategies were shown in the video clip or in the written work were listed after the prompt to ensure that participants commented on each strategy. Coding responses was a three-step processes. First for each of the six strategies we identified the mathematically significant details. ... Second for each of the six strategies we determined whether the response demonstrated attention to most of these mathematical details or only a few. Third, for each artifact, we aggregated the three strategy codes to identify whether we had evidence for each participant's attention to children's strategies: evidence (1) or lack of evidence (0). Participants who provided most details for at least two of the three strategies were considered to have provided evidence of attention to children's strategies for that artifact.

Interpreting prompt. To assess participants' expertise in interpreting children's understandings, we requested, "Please explain what you learned about these

children's understandings." We coded responses on a 3-point scale that reflected the extent of the evidence we had of participants' interpretation of children's understandings: robust evidence (2), limited evidence (1), or lack of evidence (0)...

Deciding-how-to-respond prompt. To assess participants' expertise in deciding how to respond on the basis of children's understandings, we asked, "Pretend that you are the teacher of these children. What problem or problems would you pose next?" ... We coded responses on a 3-point scale that reflected the extent of the evidence we had of participants' deciding how to respond on the basis of children's understandings: robust evidence (2), limited evidence (1), or lack of evidence (0). ... We recognize that selecting a next problem is only one of the many ways that a teacher can respond to a child. Other types of responses include probing existing strategies, facilitating comparison of strategies, purposefully pairing children to share ideas, and so on. (p. 178-179)

APPENDIX D

The eight questions used to consider *revoicing to position* episodes as developed by Enyedy et al. (2008, p. 142) are as follows:

1. Is the reported speech being juxtaposed against another student or group?
2. Is the reported speech being evaluated for its mathematical validity – that is, is it positioned against a mathematical norm?
3. Is the reported speech being challenged or expanded – that is, positioned against a social norm?
4. Is the reported speech being placed in the context of previous or next steps – that is positioned in relation to the task structure?
5. Is the reported speech being placed in the context of the teacher's (or the class') goals – that is, positioned in relation to the goal structure?
6. What identifiable roles for participation are being created – that is, is there also a positioning of the student as a role model for others?
7. Does the episode occur in (a) a public, whole class discussion; (b) in local, small group interaction; or (c) a combination of local and public settings?
8. Does the episode take place in English, Spanish, or a combination of both languages?

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