Electronic Journal of Differential Equations, Vol. 2017 (2017), No. 278, pp. 1–14.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu

EXISTENCE OF ATTRACTORS FOR THE NON-AUTONOMOUS BERGER EQUATION WITH NONLINEAR DAMPING

LU YANG, XUAN WANG

Communicated by Zhaosheng Feng

Abstract. In this article, we study the long-time behavior of the non-autonomous Berger equation with nonlinear damping. We prove the existence of a compact uniform attractor for the Berger equation with nonlinear damping in the space $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$.

1. Introduction

In this article, we consider the non-autonomous Berger equation with nonlinear damping,

$$\begin{gathered} u_{tt}+\gamma g(u_t)+\Delta^2u+(\Gamma-\int_\Omega|\nabla u|^2dx)\Delta u=p(x,t),\quad x\in\Omega,\\ u|_{\partial\Omega}=\Delta u|_{\partial\Omega}=0,\\ u(x,\tau)=u_\tau^0(x),\quad u_t(x,\tau)=u_\tau^1(x). \end{gathered}\tag{1.1}$$

Here $\Omega\subset\mathbb{R}^n$ is a bounded domain with a sufficiently smooth boundary; $\gamma>0$, and Γ are constants. The damping function $g\in C^1(\mathbb{R})$ satisfies

$$g(0)=0,\quad g \text{ is strictly increasing},\quad \liminf_{|s|\to\infty}g'(s)>0,\tag{1.2}$$

$$|g(s)|\le C(1+|s|^q),\tag{1.3}$$

with $1\le q<\infty$ if $n\le 4$, and $1\le q<\frac{n+4}{n-4}$ if $n>4$. The external force $p(x,t)$ satisfies

$$p(x,t)\in L^\infty(\mathbb{R};L^2(\Omega)),\tag{1.4}$$

$$\partial_tp\in L_b^r(\mathbb{R};L^r(\Omega)) \text{ with } r>\frac{2n}{n+4}.\tag{1.5}$$

Equation (1.1) describes the nonlinear oscillation of a plate. The function $u(x,t)$ measures the deflection of the plate at the point x and the moment of time t. The boundary condition implies that the edges of the plate are hinged. The function $p(x,t)$ describes the transverse load on the plate. The parameter Γ is proportional to the value of compressive force acting in the plane of the plate. The value γ describes the environment resistance.

2010 *Mathematics Subject Classification.* 35B40, 35B41, 35L70.
Key words and phrases. Uniform attractor; Berger equation; nonlinear damping.

Submitted September 23, 2016. Published November 8, 2017.

In this paper, we consider the non-autonomous system (1.1) via the uniform attractor of the corresponding family of processes $\{U_\sigma(t,\tau)\}$, $\sigma \in \Sigma$. For the Berger equation, the feature of the model (1.1) is that: (i) the equation does not account for rotational inertia, (i.e., Δu_{tt}), (ii) the damping is nonlinear, and (iii) the external forcing $p(x,t)$ is not translation compact in $L^2_{loc}(\mathbb{R}; L^2(\Omega))$.

For the autonomous case, if $n = 1$, equation (1.1) becomes a well-known beam equation which was treated by many authors, see, for example, [2, 3, 20] for the linear damping and [5] for the nonlinear damping. In [13], Marzocchi obtained the global attractor of beam equation with linear strong damping (i.e. u_{xxxxt}). Sell and You [18] showed the existence of the global attractor for (1.1) with linear damping in the one-dimensional case. In [14], Naboka considered the existence of the global attractor of two coupling berger plate equations with linear damping. Later, Lasiecka and Chueshov [6] gave a detailed discussion about the existence of the global attractor for the equation (1.1) in the space $(H^2(\Omega) \cap H^1_0(\Omega)) \times L^2(\Omega)$. Ma and Narciso [11] established the global attractor for the nonlinear beam equation with nonlinear damping and source terms. The existence of the exponential attractor for the plate equation was proved in [12].

In the case of non-autonomous system, for the wave equation, Sun et al [19] discussed the dynamical behavior of the non-autonomous wave equation. The random wave equation has been studied in [22]. The asymptotic behavior of the solution for the non-autonomous viscoelastic equation was considered in [15, 16].

The non-autonomous wave equation has attracted much attention in recent years. However, the non-autonomous plate equation with nonlinear damping is less discussed, especially for the Berger equation. This paper is devoted to the dynamical behavior of the solution of the equation (1.1).

In this article, inspired by the ideas in [6, 10, 19], we prove the existence of a compact uniform attractor for problem (1.1) in the apace $(H^2(\Omega) \cap H^1_0(\Omega)) \times L^2(\Omega)$. The main emphasis is placed on the external force and the nonlinear dissipation.

This article is organized as follows: In Section 2, we recall some results about function space and uniform attractor we will use in this paper; In Section 3, we give the existence of uniformly absorbing set in $(H^2(\Omega) \cap H^1_0(\Omega)) \times L^2(\Omega)$; In the last Section, we derive uniform asymptotic compactness of the corresponding family of processes $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ generated by the problem (1.1).

2. Preliminaries

In this section, we recall some fundamental concepts about the non-autonomous dynamical system, see more details in [4].

Let X be a Banach space, and Σ be a parameter set. The operator $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ is said to be a family of processes in X with symbol space Σ if for any $\sigma \in \Sigma$,

$$U_\sigma(t,s) \circ U_\sigma(s,\tau) = U_\sigma(t,\tau), \quad \forall t \geq s \geq \tau,\ \tau \in \mathbb{R}, \tag{2.1}$$

$$U_\sigma(\tau,\tau) = \mathrm{Id}, \quad \forall \tau \in \mathbb{R}, \tag{2.2}$$

where Id is the identity. Let $\{T(s)\}_{s\geq 0}$ be the translation semigroup on Σ, we say that a family of processes $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ satisfies the translation identity if

$$U_\sigma(t+s,\tau+s) = U_{T(s)\sigma}(t,\tau), \quad \forall \sigma \in \Sigma,\ t \geq \tau,\ \tau \in \mathbb{R},\ s \geq 0, \tag{2.3}$$

$$T(s)\Sigma = \Sigma, \quad \forall s \geq 0. \tag{2.4}$$

By $\mathcal{B}(X)$ we denote the collection of all bounded sets of X, and $\mathbb{R}_\tau = \{t \in \mathbb{R}, t \geq \tau\}$.

Definition 2.1 ([4])**.** A bounded set $B_0 \in \mathcal{B}(X)$ is said to be a bounded uniformly (w.r.t. $\sigma \in \Sigma$) absorbing set for $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ if for any $\tau \in \mathbb{R}$ and $B \in \mathcal{B}(X)$ there exists $T_0 = T_0(B,\tau)$ such that $\cup_{\sigma\in\Sigma} U_\sigma(t;\tau)B \subset B_0$ for all $t \geq T_0$.

Definition 2.2 ([4])**.** A set $\mathcal{A} \subset X$ is said to be uniformly (w.r.t $\sigma \in \Sigma$) attracting for the family of processes $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ if for any fixed $\tau \in \mathbb{R}$ and any $B \in \mathcal{B}(X)$

$$\lim_{t\to+\infty} \Big(\sup_{\sigma\in\Sigma} \operatorname{dist}(U_\sigma(t;\tau)B;\mathcal{A})\Big) = 0,$$

$\operatorname{dist}(\cdot,\cdot)$ is the usual Hausdorff semidistance in X between two sets.

Definition 2.3 ([4])**.** A closed set $\mathcal{A}_\Sigma \subset X$ is said to be the uniform (w.r.t $\sigma \in \Sigma$) attractor of the family of processes $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ if it is uniformly (w.r.t $\sigma \in \Sigma$) attracting (attracting property) and contained in any closed uniformly (w.r.t $\sigma \in \Sigma$) attracting set $\mathcal{A}'$ of the family of processes $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$: $\mathcal{A}_\Sigma \subseteq \mathcal{A}'$ (minimality property).

Definition 2.4 ([4])**.** A function φ is said to be translation bounded in $L^r_{loc}(\mathbb{R};X)$, if

$$\|\varphi\|_b^r = \sup_{t\in\mathbb{R}} \int_t^{t+1} \|\varphi\|_X^r ds < +\infty.$$

Denote by $L^r_b(\mathbb{R};X)$ the set of all translation bounded functions in $L^r_{loc}(\mathbb{R};X)$.

Next we recall some properties of the nonlinear damping function g.

Lemma 2.5 ([8, 10])**.** *Let $g(\cdot)$ satisfy condition* (1.3)*. Then for any $\delta > 0$ there exists $C_\delta > 0$, such that*

$$|u-v|^2 \leq \delta + C_\delta(g(u)-g(v))(u-v) \qquad \text{for } u,v \in R.$$

Hereafter, the norm in $L^2(\Omega)$ is denoted by $\|\cdot\|$. $H^s(\Omega)$ stands for the usual Sovolev space when $s \geq 0$ with the form $\|u\|_s$. C, C_i denote a general positive constant, $i = 1,\dots,$ which will be different in different estimates.

3. Existence of uniformly absorbing set

3.1. **Setting of the problem.** Similar to the autonomous case (e.g., see [6]), we can obtain the following existence and uniqueness results and the time-dependent terms make no essential complications.

Theorem 3.1. *Let Ω be a bounded domain of $\mathbb{R}^n$ with smooth boundary, g satisfies* (1.2)-(1.3)*, $p(x,t) \in L^\infty(\mathbb{R};L^2(\Omega))$. Then for any initial data $(u^0_\tau, u^1_\tau) \in (H^2(\Omega)\cap H^1_0(\Omega))\times L^2(\Omega)$, the problem* (1.1) *has an unique solution $u(t)$ which satisfies $(u(t),u_t(t)) \in C(\mathbb{R}_\tau;(H^2(\Omega)\cap H^1_0(\Omega))\times L^2(\Omega))$ and $\partial_{tt}u(t) \in L^2_{loc}(\mathbb{R}_\tau;H^{-2}(\Omega))$.*

Let $y(t) = (u(t),u_t(t))$, $y_\tau = (u^0_\tau,u^1_\tau)$, $E_0 = (H^2(\Omega)\cap H^1_0(\Omega))\times L^2(\Omega)$ with finite energy norm

$$\|y\|_{E_0} = \|\Delta u\|^2 + \|u_t\|^2.$$

Then system (1.1) is equivalent to:

$$\begin{gathered} \partial_t u_t = -\Delta^2 u - \gamma g(u_t) - (\Gamma - \int_\Omega |\nabla u|^2 dx)\Delta u + p(x,t), \quad \text{for } t \geq \tau, \\ u|_{\partial\Omega} = \frac{\partial}{\partial\nu} u|_{\partial\Omega} = 0, \quad u(x,\tau) = u^0_\tau(x), \quad u_t(x,\tau) = u^1_\tau(x). \end{gathered} \tag{3.1}$$

We can also rewrite (3.1) in the operator form:

$$\partial_t y = A_{\sigma(t)}(y), \quad y|_{t=\tau} = y_\tau, \tag{3.2}$$

where $\sigma(t) = p(t)$ is symbol of equation (3.2). We now define the symbol space for (3.2), take a fixed symbol $\sigma_0(s) = p_0(s)$, $p_0 \in L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega))$ for some $r > \frac{2n}{n+4}$, and set

$$\Sigma_0 = \{p_0(x, t+h) \quad h \in \mathbb{R}\}, \tag{3.3}$$

$$\Sigma \text{is the } *\text{-weakly closure of } \Sigma_0 \text{ in } L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega)). \tag{3.4}$$

Then we have the following properties.

Proposition 3.2. *Σ is bounded in $L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega))$, and for any $\sigma \in \Sigma$, the following estimate holds*

$$\|\sigma\|_{L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega))} \le \|p_0\|_{L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega))}.$$

Thus, from Theorem 3.1, we know that (1.1) is well posed for all $\sigma(s) \in \Sigma$ and generates a family of processes $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ given by the formula $U_\sigma(t,\tau)y_\tau = y(t)$. The $y(t)$ is the solution of (1.1)-(1.5) and $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ satisfies (2.1)-(2.2). At the same time, due to the unique solvability, we know $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ satisfies the translation identity (2.3)-(2.4).

In what follows, we denote by $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ the family of processes which is generated by (3.2)-(3.4). Next we recall some criterion developed in [19].

Definition 3.3 ([19])**.** Let X be a Banach space, B a bounded subset of X and Σ a symbol (or parameter) space. We call a function $\phi(\cdot,\cdot;\cdot,\cdot)$, defined on $(X \times X) \times (\Sigma \times \Sigma)$, to be a contractive function on $B \times B$ if for any sequence $\{x_n\}_{n=1}^\infty \subset B$ and any $\{\sigma_n\} \subset \Sigma$, there is a subsequence $\{x_{n_k}\}_{k=1}^\infty \subset \{x_n\}_{n=1}^\infty$ and $\{\sigma_{n_k}\}_{k=1}^\infty \subset \{\sigma_n\}_{n=1}^\infty$ such that

$$\lim_{k\to\infty} \lim_{l\to\infty} \phi(x_{n_k}, x_{n_l}; \sigma_{n_k}, \sigma_{n_l}) = 0.$$

We denote the set of all contractive functions on $B \times B$ by $\operatorname{contr}(B, \Sigma)$.

Theorem 3.4 ([19])**.** *Let $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ be a family of processes which satisfies the translation identity* (2.3)-(2.4) *on Banach space X and has a bounded uniformly (w.r.t. $\sigma \in \Sigma$) absorbing set $B_0 \subset X$. Moreover, assume that for any $\varepsilon > 0$ there exist $T = T(B_0, \varepsilon)$ and $\phi_T \in \operatorname{contr}(B_0, \Sigma)$ such that*

$$\|U_{\sigma_1}(T,0)x - U_{\sigma_2}(T,0)y\| \le \varepsilon + \phi_T(x, y; \sigma_1, \sigma_2), \quad \forall x, y \in B_0, \quad \forall \sigma_1, \sigma_2 \in \Sigma.$$

Then $\{U_\sigma(t,\tau)\}, \sigma \in \Sigma$ is uniformly (w.r.t. $\sigma \in \Sigma$) asymptotically compact in X.

Applying [17, Proposition 7.1], we obtain the following results.

Proposition 3.5. *Let $p \in L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega))$ $(r > \frac{2n}{n+4})$. Then there is an $M > 0$ such that*

$$\sup_{t\in\mathbb{R}} \|p(x, t+s)\|_{L^2(\Omega)} \le M \quad \textit{for all } s \in \mathbb{R}.$$

Proposition 3.6. *Let $s_i \in \mathbb{R}$ $(i = 1, 2, \dots)$, $p \in L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega))$ $(r > \frac{2n}{n+4})$, $\{u_n(t) : t \ge 0, n = 1, 2, \dots\}$ be bounded in $H^2(\Omega) \cap H_0^1(\Omega)$, and for any*

$T_1 > 0$, $\{u_{n_t}(t) \big| \ n = 1, 2, \dots\}$ bounded in $L^\infty(0, T_1; L^2(\Omega))$. Then for any $T > 0$, there exist subsequences $\{u_{n_k}\}_{k=1}^\infty$ of $\{u_n\}_{n=1}^\infty$ and $\{s_{n_k}\}_{k=1}^\infty$ of $\{s_n\}_{n=1}^\infty$ such that

$$\lim_{k\to\infty}\lim_{l\to\infty}\int_0^T\int_s^T\int_\Omega (p(x,\tau+s_{n_k})-p(x,\tau+s_{n_l}))(u_{n_k}-u_{n_l})_t(\tau)\,dx\,d\tau\,ds = 0.$$

3.2. **Uniformly absorbing set in** $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$**.** In this subsection, we start with the following result on the existence of uniformly (w.r.t. $\sigma\in\Sigma$) absorbing set in $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$. The proof is similar to the autonomous case [6], so we omit it here.

Theorem 3.7. *Assume that g satisfies (1.2)-(1.3). If*

$$p_0 \in L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega)) \quad \textit{for some } r > \frac{2n}{n+4}$$

and Σ is defined by (3.4), then the family of processes $\{U_\sigma(t,\tau)\}$, $\sigma\in\Sigma$ corresponding to problem (1.1) has a bounded uniformly (w.r.t. $\sigma\in\Sigma$) absorbing set in $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$.

4. Uniform (w.r.t. $\sigma\in\Sigma$) asymptotic compactness in $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$

In this section, we first prove some a priori estimates about the energy inequalities based on the idea presented in [6, 10, 19]. Then, we establish the uniform (w.r.t. $\sigma\in\Sigma$) asymptotic compactness in E_0.

For convenience, we denote by B_0 the bounded uniformly (w.r.t. $\sigma\in\Sigma$) absorbing set obtained in Theorem 3.7, and without loss of generality, we assume that $\gamma\equiv 1$ from now on. Hereafter, we use the notation

$$E_w(t) = \frac{1}{2}\int_\Omega |w_t(t)|^2 + \frac{1}{2}\int_\Omega |\triangle w(t)|^2.$$

4.1. **A priori estimates.** The main purpose of this part is to establish (4.14)-(4.16), which will be used to obtain the uniform (w.r.t. $\sigma\in\Sigma$) asymptotic compactness. Based on the technique in [6, 10, 19], we have the following subsequent procedure.

For any $(u_0^i, v_0^i)\in B_0$, let $(u_i(t), u_{i_t}(t))$ be the corresponding solution to σ_i with respect to initial data $(u_0^i, v_0^i), i = 1, 2$, that is, $(u_i(t), u_{i_t}(t))$ is the solution of the following equation

$$\begin{gathered} u_{tt} + g(u_t) + \Delta^2 u + (\Gamma - \int_\Omega |\nabla u|^2 dx)\Delta u = \sigma_i(x,t), \\ u|_{\partial\Omega} = \frac{\partial}{\partial\nu}u|_{\partial\Omega} = 0 \\ (u(0), u_t(0)) = (u_0^i, v_0^i). \end{gathered} \tag{4.1}$$

Lemma 4.1. *Assume that g satisfies (1.2)–(1.3). Then for any fixed $T > 0$, there exist a constant $C_{M,T}$ and a function $\phi_T = \phi_T((u_0^1, v_0^1), (u_0^2, v_0^2); \sigma_1, \sigma_2)$ such that*

$$\|u_1(T) - u_2(T)\|_{E_0} \le C_{M,T} + \phi_T((u_0^1, v_0^1), (u_0^2, v_0^2); \sigma_1, \sigma_2),$$

where $C_{M,T}$ and ϕ_T depend on T.

Proof. For convenience, we denote

$$\begin{gathered}
p_i(t) = \sigma_i(x,t), \quad t \geq 0,\ i = 1,2, \\
w(t) = u_1(t) - u_2(t).
\end{gathered}$$

Then $w(t)$ satisfies

$$\begin{gathered}
\begin{aligned}
&w_{tt} + g(u_{1_t}) - g(u_{2_t}) + \Delta^2 w - \Big(\int_\Omega |\nabla u_1|^2 dx \Delta u_1 - \int_\Omega |\nabla u_2|^2 dx \Delta u_2\Big) \\
&+ \Gamma \Delta w = p_1(t) - p_2(t),
\end{aligned} \\
w|_{\partial\Omega} = \frac{\partial}{\partial\nu} w|_{\partial\Omega} = 0, \\
(w(0), w_t(0)) = (u_0^1, v_0^1) - (u_0^2, v_0^2).
\end{gathered} \tag{4.2}$$

Multiplying (4.2) by w_t and integrating over $[s,T]\times\Omega$, we obtain

$$\begin{aligned}
&E_w(T) - E_w(s) + \int_s^T \int_\Omega (g(u_{1_t}(\tau)) - g(u_{2_t}(\tau))) w_t(\tau)\,dx\,d\tau \\
&= \int_s^T \int_\Omega (\|\nabla u_1(\tau)\|^2 \Delta u_1(\tau) - \|\nabla u_2(\tau)\|^2 \Delta u_2(\tau)) w_t(\tau)\,dx\,d\tau \\
&\quad + \frac{1}{2}\Gamma \int_\Omega |\nabla w(T)|^2 dx - \frac{1}{2}\Gamma \int_\Omega |\nabla w(s)|^2 dx + \int_s^T \int_\Omega (p_1 - p_2) w_t\,dx\,d\tau,
\end{aligned} \tag{4.3}$$

where $0 \leq s \leq T$. Then we have

$$\begin{aligned}
&\int_0^T \int_\Omega (g(u_{1_t}(\tau)) - g(u_{2_t}(\tau))) w_t(\tau)\,dx\,d\tau \\
&\leq E_w(0) + \frac{1}{2}\Gamma \int_\Omega |\nabla w(T)|^2 dx - \frac{1}{2}\Gamma \int_\Omega |\nabla w(0)|^2 dx \\
&\quad + \int_0^T \int_\Omega (\|\nabla u_1(\tau)\|^2 \Delta u_1(\tau) - \|\nabla u_2(\tau)\|^2 \Delta u_2(\tau)) w_t(\tau)\,dx\,d\tau \\
&\quad + \int_0^T \int_\Omega (p_1 - p_2) w_t\,dx\,d\tau.
\end{aligned} \tag{4.4}$$

Combining this with Lemma 2.5, we obtain that for any $\delta > 0$,

$$\begin{aligned}
&\int_0^T \int_\Omega |w_t(\tau)|^2\,dx\,d\tau \\
&\leq \delta T \operatorname{meas}(\Omega) + C_\delta E_w(0) + \frac{1}{2} C_\delta \Gamma \int_\Omega |\nabla w(T)|^2 dx \\
&\quad - \frac{1}{2} C_\delta \Gamma \int_\Omega |\nabla w(0)|^2 dx + C_\delta \int_0^T \int_\Omega \Big(\|\nabla u_1(\tau)\|^2 \Delta u_1(\tau) \\
&\quad - \|\nabla u_2(\tau)\|^2 \Delta u_2(\tau)\Big) w_t(\tau)\,dx\,d\tau + C_\delta \int_0^T \int_\Omega (p_1 - p_2) w_t\,dx\,d\tau.
\end{aligned} \tag{4.5}$$

Secondly, multiplying (4.2) by w and integrating over $[0,T]\times\Omega$, we obtain

$$\begin{aligned}
&\int_0^T\int_\Omega |\triangle w(s)|^2\,dx\,ds+\int_\Omega w_t(T)w(T)dx-\Gamma\int_0^T\int_\Omega |\nabla w(s)|^2\,dx\,ds\\
&=\int_0^T\int_\Omega |w_t(s)|^2\,dx\,ds-\int_0^T\int_\Omega (g(u_{1_t}(s))-g(u_{2_t}(s)))w(s)\,dx\,ds\\
&\quad+\int_\Omega w_t(0)w(0)dx+\int_0^T\int_\Omega \Big(\|\nabla u_1(s)\|^2\Delta u_1(s)\\
&\quad-\|\nabla u_2(s)\|^2\Delta u_2(s)\Big)w(s)\,dx\,ds+\int_0^T\int_\Omega (p_1-p_2)w\,dx\,ds.
\end{aligned}\tag{4.6}$$

So from (4.5)-(4.6), we have

$$\begin{aligned}
&\int_0^T E_w(s)ds\\
&\le \delta T\operatorname{meas}(\Omega)+C_\delta E_w(0)+\frac{1}{2}C_\delta\Gamma\int_\Omega|\nabla w(T)|^2dx-\frac{1}{2}C_\delta\Gamma\int_\Omega|\nabla w(0)|^2dx\\
&\quad+C_\delta\int_0^T\int_\Omega(\|\nabla u_1(s)\|^2\Delta u_1(s)-\|\nabla u_2(s)\|^2\Delta u_2(s))w_t(s)\,dx\,ds\\
&-\frac{1}{2}\int_\Omega w_t(T)w(T)dx+\frac{1}{2}\Gamma\int_0^T\int_\Omega|\nabla w(s)|^2\,dx\,ds\\
&\quad-\frac{1}{2}\int_0^T\int_\Omega(g(u_{1_t}(s))-g(u_{2_t}(s)))w(s)\,dx\,ds+\frac{1}{2}\int_\Omega w_t(0)w(0)dx\\
&\quad+\frac{1}{2}\int_0^T\int_\Omega(\|\nabla u_1(s)\|^2\Delta u_1(s)-\|\nabla u_2(s)\|^2\Delta u_2(s))w(s)\,dx\,ds\\
&\quad+C_\delta\int_0^T\int_\Omega(p_1-p_2)w_t\,dx\,ds+\frac{1}{2}\int_0^T\int_\Omega(p_1-p_2)w\,dx\,ds.
\end{aligned}\tag{4.7}$$

Integrating (4.3) over [0,T] with respect to s, we obtain

$$\begin{aligned}
&TE_w(T)\\
&\le\int_0^T\int_s^T\int_\Omega(\|\nabla u_1(\tau)\|^2\Delta u_1(\tau)-\|\nabla u_2(\tau)\|^2\Delta u_2(\tau))w_t(\tau)\,dx\,d\tau\,ds\\
&\quad+\int_0^T E_w(s)ds+\frac{1}{2}T\Gamma\int_\Omega|\nabla w(T)|^2dx\\
&\quad-\frac{1}{2}\Gamma\int_0^T\int_\Omega|\nabla w(s)|^2\,dx\,ds+\int_0^T\int_s^T\int_\Omega(p_1-p_2)w_t\,dx\,d\tau\,ds.
\end{aligned}\tag{4.8}$$

Therefore, from (4.7) and (4.8), we have

$$
\begin{aligned}
&TE_w(T)\\
&\le \delta T\,\mathrm{meas}(\Omega) + C_\delta E_w(0) + \frac{1}{2}C_\delta\Gamma\int_\Omega |\nabla w(T)|^2dx - \frac{1}{2}C_\delta\Gamma\int_\Omega |\nabla w(0)|^2dx\\
&\quad + C_\delta\int_0^T\int_\Omega (\|\nabla u_1(s)\|^2\Delta u_1(s) - \|\nabla u_2(s)\|^2\Delta u_2(s))w_t(s)\,dx\,ds\\
&\quad + \frac{1}{2}T\Gamma\int_\Omega |\nabla w(T)|^2dx - \frac{1}{2}\int_\Omega w_t(T)w(T)dx + \frac{1}{2}\int_0^T\int_\Omega (p_1-p_2)w\,dx\,ds\\
&\quad - \frac{1}{2}\int_0^T\int_\Omega (g(u_{1_t}(s)) - g(u_{2_t}(s)))w(s)\,dx\,ds + \frac{1}{2}\int_\Omega w_t(0)w(0)dx\\
&\quad + \frac{1}{2}\int_0^T\int_\Omega (\|\nabla u_1(s)\|^2\Delta u_1(s) - \|\nabla u_2(s)\|^2\Delta u_2(s))w(s)\,dx\,ds\\
&\quad + \int_0^T\int_s^T\int_\Omega (\|\nabla u_1(\tau)\|^2\Delta u_1(\tau) - \|\nabla u_2(\tau)\|^2\Delta u_2(\tau))w_t(\tau)\,dx\,d\tau\,ds\\
&\quad + \int_0^T\int_s^T\int_\Omega (p_1-p_2)w_t\,dx\,d\tau\,ds + C_\delta\int_0^T\int_\Omega (p_1-p_2)w_t\,dx\,ds.
\end{aligned}
\tag{4.9}
$$

Next, we need to study $\int_0^T\int_\Omega (g(u_{1_t}) - g(u_{2_t}))w\,dx\,ds$. The following estimate can be derived by using similar arguments as in [6, Chap. 5]. However, for the sake of completeness we give the proof. From condition (1.3), we have

$$
|g(s)|^{\frac{q+1}{q}} = |g(s)|^{1/q}\cdot|g(s)| \le C(1+|s|)|g(s)|,
$$

combining this with (1.2), we obtain

$$
|g(s)|^{\frac{q+1}{q}} \le \begin{cases} C, & |s|\le 1,\\ 2Cg(s)s, & |s|\ge 1,\end{cases} \tag{4.10}
$$

where C is a constant which is independent of s. Multiplying (4.1) by $u_{i_t}(t)$, we obtain

$$
\frac{1}{2}\frac{d}{dt}\int_\Omega \big(|u_{i_t}|^2 + |\Delta u_i|^2\big) + \int_\Omega g(u_{i_t})u_{i_t} + \int_\Omega (\Gamma - \|\nabla u_i\|^2)\Delta u_i u_{i_t} = \int_\Omega p_i u_{i_t},
$$

which, combined with the existence of bounded uniformly absorbing set, implies

$$
\int_0^T\int_\Omega g(u_{i_t})u_{i_t} \le C_{\rho,T}, \tag{4.11}
$$

where $C_{\rho,T}$ is a constant which depends on the size of B_0 in $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$ and T. Therefore, from (4.10) and (4.11), we have

$$
\begin{aligned}
&|\int_0^T\int_\Omega g(u_{i_t})w| \\
&\le \int_0^T\int_{\Omega(|u_{i_t}|\le 1)}|g(u_{i_t})w| + \int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}|g(u_{i_t})w| \\
&\le C\int_0^T\int_{\Omega(|u_{i_t}|\le 1)}|w| + \int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}|g(u_{i_t})||w| \\
&\le C\int_0^T\int_{\Omega(|u_{i_t}|\le 1)}|w| + \Big(\int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}|g(u_{i_t})|^{\frac{q+1}{q}}\Big)^{\frac{q}{q+1}} \\
&\quad\times\Big(\int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}|w|^{q+1}\Big)^{\frac{1}{q+1}} \\
&\le C\int_0^T\int_{\Omega(|u_{i_t}|\le 1)}|w| + 2C\Big(\int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}g(u_{i_t})u_{i_t}\Big)^{\frac{q}{q+1}} \\
&\quad\times\Big(\int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}|w|^{q+1}\Big)^{\frac{1}{q+1}} \\
&\le C\int_0^T\int_{\Omega(|u_{i_t}|\le 1)}|w| + C_{\rho,T}\Big(\int_0^T\int_{\Omega(|u_{i_t}|\ge 1)}|w|^{q+1}\Big)^{\frac{1}{q+1}}.
\end{aligned}
\tag{4.12}
$$

Combining (4.9) and (4.12), we obtain

$$
\begin{aligned}
&TE_w(T) \\
&\le \delta T\,\mathrm{meas}(\Omega) + C_\delta E_w(0) + \frac12 C_\delta\Gamma\int_\Omega|\nabla w(T)|^2dx - \frac12 C_\delta\Gamma\int_\Omega|\nabla w(0)|^2dx \\
&\quad + C_\delta\int_0^T\int_\Omega(\|\nabla u_1(s)\|^2\Delta u_1(s) - \|\nabla u_2(s)\|^2\Delta u_2(s))w_t(s)\,dx\,ds \\
&\quad + \frac12 T\Gamma\int_\Omega|\nabla w(T)|^2dx - \frac12\int_\Omega w_t(T)w(T)dx + \frac12\int_0^T\int_\Omega(p_1-p_2)w\,dx\,ds \\
&\quad + C\int_0^T\int_\Omega|w|\,dx\,ds + C_{\rho,T}\Big(\int_0^T\int_\Omega|w|^{q+1}\,dx\,ds\Big)^{\frac{1}{q+1}} + \frac12\int_\Omega w_t(0)w(0)dx \\
&\quad + \frac12\int_0^T\int_\Omega(\|\nabla u_1(s)\|^2\Delta u_1(s) - \|\nabla u_2(s)\|^2\Delta u_2(s))w(s)\,dx\,ds \\
&\quad + \int_0^T\int_s^T\int_\Omega(\|\nabla u_1(\tau)\|^2\Delta u_1(\tau) - \|\nabla u_2(\tau)\|^2\Delta u_2(\tau))w_t(\tau)\,dx\,d\tau\,ds \\
&\quad + \int_0^T\int_s^T\int_\Omega(p_1-p_2)w_t\,dx\,d\tau\,ds + C_\delta\int_0^T\int_\Omega(p_1-p_2)w_t\,dx\,ds.
\end{aligned}
\tag{4.13}
$$

Set

$$
\begin{aligned}
C_{M,T} &= \delta T\,\mathrm{meas}(\Omega) + C_\delta E_w(0) + \frac12 C_\delta\Gamma\int_\Omega|\nabla w(T)|^2dx \\
&\quad - \frac12 C_\delta\Gamma\int_\Omega|\nabla w(0)|^2dx - \frac12\int_\Omega w_t(T)w(T)dx + \frac12\int_\Omega w_t(0)w(0)dx,
\end{aligned}
\tag{4.14}
$$

$$
\begin{aligned}
&\phi_{\delta,T}((u_0^1,v_0^1),(u_0^2,v_0^2);\sigma_1,\sigma_2)\\
&= C_\delta\int_0^T\int_\Omega(\|\nabla u_1(s)\|^2\Delta u_1(s)-\|\nabla u_2(s)\|^2\Delta u_2(s))w_t(s)\,dx\,ds\\
&\quad+\frac{1}{2}\int_0^T\int_\Omega(\|\nabla u_1(s)\|^2\Delta u_1(s)-\|\nabla u_2(s)\|^2\Delta u_2(s))w(s)\,dx\,ds\\
&\quad+\int_0^T\int_s^T\int_\Omega(\|\nabla u_1(\tau)\|^2\Delta u_1(\tau)-\|\nabla u_2(\tau)\|^2\Delta u_2(\tau))w_t(\tau)\,dx\,d\tau\,ds\\
&\quad+C\int_0^T\int_\Omega|w|\,dx\,ds+C_{\rho,T}\Big(\int_0^T\int_\Omega|w|^{q+1}\,dx\,ds\Big)^{\frac{1}{q+1}}+\frac{1}{2}T\Gamma\int_\Omega|\nabla w(T)|^2dx\\
&\quad+\int_0^T\int_s^T\int_\Omega(p_1-p_2)w_t\,dx\,d\tau\,ds+C_\delta\int_0^T\int_\Omega(p_1-p_2)w_t\,dx\,ds\\
&\quad+\frac{1}{2}\int_0^T\int_\Omega(p_1-p_2)w\,dx\,ds.
\end{aligned}\tag{4.15}
$$

Then we have

$$
E_w(T)\le\frac{C_{M,T}}{T}+\frac{1}{T}\phi_{\delta,T}((u_0^1,v_0^1),(u_0^2,v_0^2);\sigma_1,\sigma_2).\tag{4.16}
$$

□

4.2. **Uniform asymptotic compactness.** In this subsection, we prove the uniform (w.r.t. $\sigma\in\Sigma$) asymptotic compactness in $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$, which is given in the following theorem.

Theorem 4.2. *Assume that g satisfies* (1.2)-(1.3). *If*

$$
p_0\in L^\infty(\mathbb{R};L^2(\Omega))\cap W_b^{1,r}(\mathbb{R};L^r(\Omega))\quad\text{for some } r>\frac{2n}{n+4}
$$

and Σ is defined by (3.4)*, then the family of processes $\{U_\sigma(t,\tau)\}$, $\sigma\in\Sigma$ corresponding to problem* (1.1)*, is uniformly (w.r.t. $\sigma\in\Sigma$) asymptotically compact in $(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega)$.*

Proof. Since the family of processes $\{U_\sigma(t,\tau)\}$ $\sigma\in\Sigma$ has a bounded uniformly absorbing set and from the Lemma 4.1, for any fixed $\varepsilon>0$, we can choose first $\delta\le\frac{\varepsilon}{2\,\mathrm{meas}(\Omega)}$, and let T so large that

$$
\frac{C_{M,T}}{T}\le\varepsilon.
$$

Hence, thanks to Theorem 3.4, it is sufficient to prove that $\phi_{\delta,T}(\cdot,\cdot;\cdot,\cdot)$ defined in (4.15) belongs to $\mathrm{contr}(B_0,\Sigma)$ for each fixed T.

From Theorem 3.7, we can deduce that for any fixed T,

$$
\cup_{\sigma\in\Sigma}\cup_{t\in[0,T]}U_\sigma(t,0)B_0 \text{ is bounded in } E_0,\tag{4.17}
$$

and the bound depends on T.

Let (u_n,u_{n_t}) be the solutions corresponding to initial data $(u_0^n,v_0^n)\in B_0$ with respect to symbol $\sigma_n\in\Sigma, n=1,2,\dots$. From (4.17), without loss of generality (at most by passing subsequence), we assume that

$$
u_n\to u\quad\text{weakly star in } L^\infty(0,T;H^2(\Omega)\cap H_0^1(\Omega)),\tag{4.18}
$$

$$u_{n_t} \to u_t \quad \text{weakly star in } L^\infty(0,T;L^2(\Omega)), \tag{4.19}$$
$$u_n \to u \quad \text{in } L^2(0,T;L^2(\Omega)), \tag{4.20}$$
$$u_n \to u \quad \text{in} \quad L^{q+1}(0,T;L^{q+1}(\Omega)), \tag{4.21}$$
$$u_n(T) \to u(T) \quad \text{strongly in } H_0^1(\Omega), \tag{4.22}$$

for $q < \frac{n+4}{n-4}$, where we use the compact embeddings $H^2 \hookrightarrow H_0^1$ and $H^2 \hookrightarrow L^{q+1}$.

Now we deal with each term corresponding to that in (4.15). First, from Proposition 3.5 and (4.21), we can obtain

$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T\int_\Omega (p_n(x,s)-p_m(x,s))(u_n(s)-u_m(s))\,dx\,ds = 0, \tag{4.23}$$

and from Proposition 3.6 we can get

$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T\int_\Omega (p_n(x,s)-p_m(x,s))(u_{n_t}(s)-u_{m_t}(s))\,dx\,ds = 0, \tag{4.24}$$
$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T\int_s^T\int_\Omega (p_n(x,\tau)-p_m(x,\tau))(u_{n_t}(\tau)-u_{m_t}(\tau))\,dx\,d\tau\,ds = 0. \tag{4.25}$$

Secondly, from (4.18) and (4.21), we can get that

$$\lim_{n\to\infty}\lim_{m\to\infty}\|\nabla u_n(T)-\nabla u_m(T)\|^2 = 0, \tag{4.26}$$
$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T\int_\Omega |u_n(s)-u_m(s)|\,dx\,ds = 0, \tag{4.27}$$
$$\lim_{n\to\infty}\lim_{m\to\infty}\Big(\int_0^T\int_\Omega |u_n(s)-u_m(s)|^{q+1}\,dx\,ds\Big)^{\frac{1}{q+1}} = 0. \tag{4.28}$$

Since $\{(u_n,u_{n_t})\}_{n=1}^\infty$ is bounded in $C(0,T;(H^2(\Omega)\cap H_0^1(\Omega))\times L^2(\Omega))$ and the embedding $H^2 \hookrightarrow C(\bar\Omega)$ is compact, by the Arzela theorem $\{u_n\}_{n=1}^\infty$ is compact in $C(0,T;C(\bar\Omega))$.

On the other hand, $\{u_n\}_{n=1}^\infty$ converges weakly star in $L^\infty(0,T;(H^2(\Omega)\cap H_0^1(\Omega)))$. Thus $\{u_n\}_{n=1}^\infty$ strongly converges in $C(0,T;C(\bar\Omega))$ and then we find that

$$\begin{aligned} &\Big|\int_0^T\int_\Omega (\|\nabla u_n\|^2\Delta u_n - \|\nabla u_m\|^2\Delta u_m)(u_n-u_m)\,dx\,ds\Big| \\ &\le C_{R,T}\|u_n-u_m\|_{C(0,T;C(\bar\Omega))}. \end{aligned} \tag{4.29}$$

From (4.29), we obtain

$$\lim_{n\to\infty}\lim_{m\to\infty}\int_0^T\int_\Omega (\|\nabla u_n\|^2\Delta u_n - \|\nabla u_m\|^2\Delta u_m)(u_n-u_m)\,dx\,ds = 0. \tag{4.30}$$

Finally, Since (for smooth solutions) we have

$$\int_\Omega \|\nabla u\|^2\Delta u u_t\,dx = -\frac{1}{4}\frac{\partial}{\partial t}\|\nabla u\|^4,$$

from the above equality, we obtain

$$\begin{aligned} &\int_0^T\int_\Omega (\|\nabla u_n(s)\|^2\Delta u_n(s) - \|\nabla u_m(s)\|^2\Delta u_m(s))(u_{n_t}(s)-u_{m_t}(s))\,dx\,ds \\ &= \int_0^T\int_\Omega \|\nabla u_n(s)\|^2\Delta u_n(s)u_{n_t}(s)\,dx\,ds + \int_0^T\int_\Omega \|\nabla u_m(s)\|^2\Delta u_m(s)u_{m_t}(s)\,dx\,ds \end{aligned}$$

$$
\begin{aligned}
&- \int_0^T \int_\Omega \|\nabla u_n(s)\|^2 \Delta u_n(s) u_{m_t}(s)\, dx\, ds \\
&- \int_0^T \int_\Omega \|\nabla u_m(s)\|^2 \Delta u_m(s) u_{n_t}(s)\, dx\, ds \\
&= \frac{1}{4}\ \big[\ \|\nabla u_n(0)\|^4 - \|\nabla u_n(T)\|^4 + \|\nabla u_m(0)\|^4 - \|\nabla u_m(T)\|^4\ \big] \\
&- \int_0^T \int_\Omega \|\nabla u_n(s)\|^2 \Delta u_n(s) u_{m_t}(s)\, dx\, ds \\
&- \int_0^T \int_\Omega \|\nabla u_m(s)\|^2 \Delta u_m(s) u_{n_t}(s)\, dx\, ds,
\end{aligned}
$$

using (4.18), (4.19) and (4.22), taking first $m \to \infty$, then $n \to \infty$, we obtain

$$
\begin{aligned}
&\lim_{n\to\infty} \lim_{m\to\infty} \int_0^T \int_\Omega \Big(\|\nabla u_n(s)\|^2 \Delta u_n(s) \\
&\quad - \|\nabla u_m(s)\|^2 \Delta u_m(s)\Big)(u_{n_t}(s) - u_{m_t}(s))\, dx\, ds \\
&= \frac{1}{2}[\|\nabla u(0)\|^4 - \|\nabla u(T)\|^4] - 2\int_0^T \int_\Omega \|\nabla u(s)\|^2 \Delta u(s) u_t(s)\, dx\, ds = 0.
\end{aligned} \tag{4.31}
$$

Similarly, we have

$$
\begin{aligned}
&\int_s^T \int_\Omega (\|\nabla u_n(\tau)\|^2 \Delta u_n(\tau) - \|\nabla u_m(\tau)\|^2 \Delta u_m(\tau))(u_{n_t}(\tau) - u_{m_t}(\tau))\, dx\, d\tau \\
&= \frac{1}{4}\ \big[\ \|\nabla u_n(s)\|^4 - \|\nabla u_n(T)\|^4 + \|\nabla u_m(s)\|^4 - \|\nabla u_m(T)\|^4\ \big] \\
&\quad - \int_s^T \int_\Omega \|\nabla u_n(\tau)\|^2 \Delta u_n(\tau) u_{m_t}(\tau)\, dx\, d\tau \\
&\quad - \int_s^T \int_\Omega \|\nabla u_m(\tau)\|^2 \Delta u_m(\tau) u_{n_t}(\tau)\, dx\, d\tau.
\end{aligned}
$$

At the same time, $|\int_s^T \int_\Omega (\|\nabla u_n(\tau)\|^2 \Delta u_n(\tau) - \|\nabla u_m(\tau)\|^2 \Delta u_m(\tau))(u_{n_t}(\tau) - u_{m_t}(\tau))\, dx\, d\tau|$ is bounded for each fixed T, by the Lebesgue dominated convergence theorem we have

$$
\begin{aligned}
&\lim_{n\to\infty} \lim_{m\to\infty} \int_0^T \int_s^T \int_\Omega \Big(\|\nabla u_n(\tau)\|^2 \Delta u_n(\tau) \\
&\quad - \|\nabla u_m(\tau)\|^2 \Delta u_m(\tau)\Big)(u_{n_t} - u_{m_t})\, dx\, d\tau\, ds \\
&= \int_0^T \Big(\lim_{n\to\infty} \lim_{m\to\infty} \int_s^T \int_\Omega \Big(\|\nabla u_n(\tau)\|^2 \Delta u_n(\tau) \\
&\quad - \|\nabla u_m(\tau)\|^2 \Delta u_m(\tau)\Big)(u_{n_t} - u_{m_t})\, dx\, d\tau\Big) ds \\
&= \int_0^T 0 ds = 0.
\end{aligned} \tag{4.32}
$$

Hence, combining (4.23)-(4.32), we obtain that $\phi_{\delta,T}(\cdot,\cdot;\cdot,\cdot) \in \mathrm{contr}(B_0,\Sigma)$ immediately. $\square$

4.3. **Existence of a compact uniform attractor.**

Theorem 4.3. *Assume that g satisfies* (1.2)-(1.3). *If*

$$p_0 \in L^\infty(\mathbb{R}; L^2(\Omega)) \cap W_b^{1,r}(\mathbb{R}; L^r(\Omega)) \quad \text{for some } r > \frac{2n}{n+4}$$

and Σ is defined by (3.4), *then the family of processes $\{U_\sigma(t,\tau)\}$, $\sigma \in \Sigma$ corresponding to problem* (1.1) *has a compact uniform (w.r.t. $\sigma \in \Sigma$) attractor $\mathcal{A}_\Sigma$ in $(H^2(\Omega) \cap H_0^1(\Omega)) \times L^2(\Omega)$.*

Proof. Theorem 3.7 and Theorem 4.2 imply the existence of a compact uniform attractor immediately. □

Remark 4.4. For the autonomous case of (1.1), that is $p(x,t) = p(x)$, the growth order of nonlinear damping g is equal to $\frac{n+4}{n-4}$ if $n > 4$. As for the non-autonomous system, the constant $C_{\rho,T}$ in (4.11) depends on T, which is different from the autonomous case, and to some extent, (4.11) requires that the growth order of g is strictly less than $\frac{n+4}{n-4}$ with $n > 4$.

Remark 4.5. The technique (scheme) used in this paper is also applicable to another non-autonomous plate models, e.g., the model of non-autonomous extensible beam with nonlinear damping and source terms.

Acknowledgments. This work was supported by the NSFC Grants (11361053, 11471148), by the Fundamental Research Funds for the Central Universities Grant (lzujbky-2016-98), and by the Young Teachers Scientific Research Ability Promotion Plan of Northwest Normal University (NWNU-LKQN-11-5).

References

[1] J. M. Ball; *Initial and boundary value problems for an extensible beam,* J. Math. Anal. Appl., 42(1973), 61-90.
[2] J. M. Ball; *Stability theory for an extensible beam,* J. Differential Equations, 14(1973), 399-418.
[3] G. Bianchi, A. Marzocchi; *Asymptotic properties of solutions to semilinear damped equations for elastic beams,* Riv. Mat. Univ. Parma, 4(1995), 241-258.
[4] V. V. Chepyzhov, M. I. Vishik; *Attractors for Equations of Mathematical Physics*, Amer. Math. Soc., Providence, RI, 2002.
[5] P. L. Chow; *Asymptotic solutions of a nonlinear stochastic beam equation,* Discrete Contin. Dyn. Syst. Ser. B, 6 (2006), 735-749.
[6] I. Chueshov, I. Lasiecka; *Long-time behavior of second order evolution equations with nonlinear damping,* Mem. Amer. Math. Soc., 2007.
[7] I. Chueshov; *Introduction to the Theory of Infinite-Dimensional Dissipative Systems,* Acta, Kharkov, 2002.
[8] E. Feireisl; *Global attractors for damped wave equations with supercritical exponent,* J. Differential Equations, 116(1995), 431-447.
[9] M. Feckan; *Free vibrations of beams on bearings with nonlinear elastic responses,* J. Differential Equations, 154(1999), 55-72.
[10] A.Kh. Khanmamedov; *Global attractors for von Karman equations with nonlinear interior dissipation,* J. Math. Anal. Appl., 318(2006), 92-101.
[11] T. F. Ma, V. Narciso; *Global attractor for a model of extensible beam with nonlinear damping and source terms,* Nonlinear Anal., 73 (2010), 3402-3412.
[12] Q. Z. Ma, Y. Yang, X. L. Zhang; *Existence of exponential attractors for the plate equations with strong damping,* Elec. J. Differential Equations, 114(2013), 1-10.
[13] A. Marzocchi, E. Vuk; *Golbal attractor for damped semilinear elastic beam equations with memory,* Z. Angew. Math. Phys., 54(2003), 224-234.

[14] O. Naboka; *Synchronization of nonlinear oscillations of two coupling Berger plates,* Nonlinear Anal., 67 (2007), 1015-1026.
[15] Y. M. Qin, B. W. Feng, M. Zhang; *Uniform attractors for a non-autonomous viscoelastic equation with a past history,* Nonlinear Anal., 101(2014), 1-15.
[16] Y. M. Qin, J. P. Zhang, L. L. Sun; *Upper semicontinuity of pullback attractors for a non-autonomous viscoelastic equation,* Appl. Math. Comput., 223(2013), 362-376.
[17] J. C. Robinson; *Infinite-dimensional dynamical systems, An introduction to dissipative parabolic PDEs and the theory of global attractors,* Cambridge University Press 2001.
[18] G. R. Sell, Y. You; *Dynamics of Evolutionary Equations,* NewYork, Springer, 2002.
[19] C. Y. Sun, D. M. Cao, J. Q. Duan; *Uniform attractors for non-autonomous wave equations with nonlinear damping,* SIAM J. Appl. Dyn. Syst., 6(2007), 293-318.
[20] C. Y. Sun, L. Yang, J. Q. Duan; *Asymptotic behavior for a semilinear second order evolution equation,* Tran. Amer. Math. Soc., 363 (2011), 6085-6109.
[21] S. Woinowsky-Krieger; *The effect of an axial force on the vibration of hinged bars,* J. Appl. Mech, 17 (1950), 35-36.
[22] M. H. Yang, J. Q. Duan, P. E. Kloeden; *Asymptotic behavior of solutions for random wave equation with nonlinear damping and white noise,* Nonlinear Anal., RWA, 12(2011), 464-478.

Lu Yang (corresponding author)
School of Mathematics and Statistics, Lanzhou University, Lanzhou, 730000, China.
Key Laboratory of Applied Mathematics and Complex Systems, Gansu Province, China
E-mail address: yanglu@lzu.edu.cn

Xuan Wang
College of Mathematics and Statistics, Northwest Normal University, Lanzhou, 730070, China
E-mail address: wangxuan@nwnu.edu.cn