Electronic Journal of Differential Equations, Vol. 2005(2005), No. 123, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

MIXED BOUNDARY-VALUE PROBLEMS FOR QUANTUM HYDRODYNAMIC MODELS WITH SEMICONDUCTORS IN THERMAL EQUILIBRIUM

JIANWEI DONG

Abstract. We show the existence of solutions for mixed boundary-value problems that model quantum hydrodynamics in thermal equilibrium. Also we find the semi-classical limit of the solutions.

1. Introduction

Models of ultra-small semiconductor devices with quantum effects include microscopic and macroscopic quantum models [1]. Microscopic quantum models include quantum kinetic equations and Schrödinger-Poisson systems [2]. Macroscopic quantum models include quantum hydrodynamic and quantum drift-diffusion equations [1]. Quantum hydrodynamic models give a fairly accurate account of the macroscopic behavior of ultra small semiconductor devices in terms of only macroscopic quantities such as particle densities, current densities and electric fields. The primary application of the quantum hydrodynamic equations has been in analyzing the flow of electrons in quantum semiconductor devices, such as resonant tunnelling diodes [3]. The quantum hydrodynamic equations have the form

$$
\begin{gathered}
\frac{\partial n}{\partial t} + \operatorname{div} J = 0,\\
\frac{\partial J}{\partial t} + \operatorname{div}\Big(\frac{J \bigotimes J}{n}\Big) + \nabla p(n) - n\nabla V - \delta^2 n \nabla\Big(\frac{\Delta\sqrt{n}}{\sqrt{n}}\Big) = -\frac{J}{\tau},\\
\lambda^2 \Delta V = n - C.
\end{gathered}
\tag{1.1}
$$

Here $n, J, p(n), V, C$ denote the electron density, electron current density, pressure function, electric potential and doping profile of the semiconductor, respectively; δ, τ, λ denote the scaled Planck constant, the scaled relaxation time and the scaled Debye length, respectively (see [4]). The case $p(n) = n$ is the isothermal case and the case $p(n) = n^\alpha$ with $\alpha > 1$ is the isentropic case. The equations of the thermal

2000 *Mathematics Subject Classification.* 35J55.
Key words and phrases. Quantum hydrodynamic; thermal equilibrium; mixed boundary value; existence; semi-classical limit.

Submitted July 18, 2005. Published November 3, 2005.

equilibrium states are obtained by setting $J = 0$ in the stationary QHD equations

$$\delta^2 \frac{\Delta\sqrt{n}}{\sqrt{n}} = h(n) - V,$$
$$\lambda^2 \Delta V = n - C,$$

where $h(n)$ is the enthalpy function satisfying $p'(n) = nh'(n)$ (see [4]). In the isothermal case, $h(n) = \ln(n)$ holds; for isentropic states, we have $h(n) = \frac{\alpha}{\alpha-1} n^{\alpha-1}$. Setting $w = \sqrt{n}$, we obtain

$$\delta^2 \Delta w = w(h(w^2) - V) \quad \text{in } \Omega,$$
$$\lambda^2 \Delta V = w^2 - C \quad \text{in } \Omega.$$

The existence and uniqueness of the solutions and the classical limit problem were investigated by Gasser and Jüngel [4], with the Dirichlet boundary conditions

$$w = w_0, \quad V = V_0 \quad \text{on } \partial\Omega,$$

where $w_0, V_0 \in H^1(\Omega) \bigcap L^\infty(\Omega)$, $w_0 \geq 0$ in $\Omega \subset \mathbb{R}^d$, $1 \leq d \leq 3$.

In thermal non-equilibrium states, Jüngel obtained the existence and uniqueness of solutions of stationary equations of the problem (1.1) by the truncation method (see [5]). Jüngel, Mariani and Rial [6] obtained the local existence of solutions to (1.1). For results on the stationary problem of (1.1) in the one-dimensional case, see [7, 8].

In this paper, we investigate the mixed boundary-value problem

$$\delta^2 \Delta w = w(h(w^2) - V) \quad \text{in } \Omega, \quad w = w_0 \quad \text{on } \Gamma_D, \quad \frac{\partial w}{\partial \nu} = 0 \quad \text{on } \Gamma_N, \tag{1.2}$$
$$\lambda^2 \Delta V = w^2 - C \quad \text{in } \Omega, \quad V = V_0 \quad \text{on } \Gamma_D, \quad \frac{\partial V}{\partial \nu} = 0 \quad \text{on } \Gamma_N, \tag{1.3}$$

where $w_0, V_0 \in H^1(\Omega) \cap L^\infty(\Omega)$, $w_0 \geq 0$in $\Omega \subset \mathbb{R}^d$, $1 \leq d < +\infty$, ν is the unit outward normal to Γ_N. Note that the mixed Dirichlet-Neumann boundary conditions are physically more realistic than the pure Dirichlet condition.

This paper is organized as follows: In section 2, we show the existence of solutions of the problem (1.2)-(1.3). In section 3, we perform the semi-classical limit.

2. Existence of solutions

For this section we have the following assumptions:

- (A1) $\Omega \subset \mathbb{R}^d$ is a bounded domain with $\partial\Omega \in C^{1,1}$, $\partial\Omega = \Gamma_D \cup \Gamma_N$, $\Gamma_D \cap \Gamma_N = \emptyset$, $\operatorname{meas}_{d-1} \Gamma_D > 0$, where $1 \leq d < +\infty$.
- (A2) $C \in L^\infty(\Omega)$, $\lambda, \delta > 0$, $w_0, V_0 \in H^1(\Omega) \cap L^\infty(\Omega)$, $w_0 \geq 0$ in Ω.
- (A3) $h \in C((0, \infty))$ is a non-decreasing function with $\lim_{x \to +\infty} h(x) = +\infty$, $-\infty < \lim_{x \to 0+} xh(x^2) \leq 0$.

The main result of this section is the following theorem.

Theorem 2.1. *Under assumptions (A1)-(A3), there exists a unique solution* (w, V) *in* $(H^1(\Omega) \cap L^\infty(\Omega))^2$ *of problem* (1.2)-(1.3)*. Furthermore,* $w \geq 0$ *a.e. in* Ω.

First we regularize the problem (1.2)-(1.3) using the truncation method, then we show the existence of solutions of the regularized problem by the Leray-Schauder fixed point theorem. Then we show that the solutions of the regularized problem are also solution to (1.2)-(1.3), by finding an $L^\infty(\Omega)$ estimate for u.

The proof of Theorem 2.1 is based on the following a priori estimate (see [4]).

Lemma 2.2. *Let (A1)-(A3) hold and let $(w,V)\in(H^1(\Omega))^2$ be a solution of*

$$\delta^2\Delta w = w_K(h(w_K^2)-V) \quad in\ \Omega, \quad w=w_0 \quad on\ \Gamma_D, \quad \frac{\partial w}{\partial\nu}=0 \quad on\ \Gamma_N, \tag{2.1}$$

$$\lambda^2\Delta V = w_K w - C \quad in\ \Omega, \quad V=V_0 \quad on\ \Gamma_D, \quad \frac{\partial V}{\partial\nu}=0 \quad on\ \Gamma_N, \tag{2.2}$$

where $w_K=\max(0,\min(w,K))$, $K>0$, w_0,V_0 are as in Theorem 2.1. Then there exists $K_0>0$ such that for all $K\ge K_0,\delta>0$ and $\lambda>0$ the following estimate holds

$$\delta^2\|w\|_{1,2,\Omega}^2+\lambda^2\|V\|_{1,2,\Omega}^2\le c\,,$$

where $c>0$ depends on Ω,h,w_0,V_0,C and λ such that $c\to\infty$ as $\lambda\to 0+$. Here, $\|\cdot\|_{m,p,\Omega}$ denotes the norm in the Sobolev space $W^{m,p}(\Omega)$.

The proof of the above lemma can be found in [4].

Proof of Theorem 2.1. Let $K\ge K_0$ where K_0 is the constant of Lemma 2.2. Let $u\in L^2(\Omega)$, and let $V\in H^1(\Omega)$ be the unique solution of

$$\lambda^2\Delta V = u_K u - C \quad \text{in } \Omega, \quad V=V_0 \quad \text{on } \Gamma_D, \quad \frac{\partial V}{\partial\nu}=0 \quad \text{on } \Gamma_N.$$

Let $w\in H^1(\Omega)$ be the unique solution of

$$\delta^2\Delta w = \sigma u_K(h(u_K^2)-V) \quad \text{in } \Omega, \quad w=\sigma w_0 \quad \text{on } \Gamma_D, \quad \frac{\partial w}{\partial\nu}=0 \quad \text{on } \Gamma_N,$$

with $\sigma\in[0,1]$. Then the operator $T: L^2(\Omega)\times[0,1]\to L^2(\Omega)$, $(u,\sigma)\mapsto w$ is well defined. It holds $T(u,0)=0$ for $u\in L^2(\Omega)$. An estimate similarly as in the proof of Lemma 2.2 gives the bound

$$\|w\|_{1,2,\Omega}\le c$$

for all $w\in L^2(\Omega)$ with $T(w,\sigma)=w$ where $c>0$ is independent of w and σ. Some standard arguments show that T is continuous and compact. Therefore, we can apply the Leray-Schauder fixed point theorem to get a solution $(w,V)\in(H^1(\Omega))^2$ of (2.1)-(2.2). It remains to find an $L^\infty(\Omega)$ bound on w.

First use $w^-=\min(0,w)\in H^1(\Omega)$ as a test function in (2.1) to obtain

$$\delta^2\int_\Omega|\nabla w^-|^2 = -\int_\Omega w^- w_K h(w_K^2)+\int_\Omega V w_K w^- = 0,$$

thus $w\ge 0$ a.e. in Ω. Let $U_0=\sup_{\Gamma_D}V_0$, $U\ge U_0$ and take $(V-U)^+=\max(0,V-U)$ as a test function in (2.2) to obtain

$$\begin{aligned}\lambda^2\int_\Omega|\nabla(V-U)^+|^2 &= -\int_\Omega w_K w(V-U)^+ + \int_\Omega C(V-U)^+\\ &\le \int_\Omega C(V-U)^+\\ &\le c\|(V-U)^+\|_{1,2,\Omega}(\mathrm{meas}(V>U))^{1/2}.\end{aligned}\tag{2.3}$$

Let $r>2$ be such that the embedding $H^1(\Omega)\hookrightarrow L^r(\Omega)$ is continuous. It is well known that for $W>U$,

$$(\mathrm{meas}(V>W))^{1/r}(W-U)\le c(\Omega)\|(V-U)^+\|_{1,2,\Omega},$$

see [9, Ch. 4]. Therefore, we get from (2.3), for $W>U\ge U_0$,

$$\mathrm{meas}(V>W)\le\frac{c}{(W-U)^r}(\mathrm{meas}(V>U))^{r/2}.$$

Since $\frac{r}{2} > 1$, we can apply Stampacchia's Lemma (see [9, Ch. 4] or [10, Ch. 2.3]) to get

$$V(x) \le \overline{V} := U_0 + c(\Omega, d, \lambda)\|C\|_{0,\infty,\Omega},$$

where $c(\Omega, d, \lambda) > 0$. Let $\overline{w} \ge \|w_0\|_{0,\infty,\Gamma_D}$ and $K > \overline{w}$, use $(w - \overline{w})^+$ as a test function in (2.1) to obtain

$$\begin{aligned}
\delta^2 \int_\Omega |\nabla(w - \overline{w})^+|^2 &= -\int_\Omega w_K(h(w_K^2) - V)(w - \overline{w})^+ \\
&= -\int_\Omega w_K(h(w_K^2) - h(\overline{w}^2))(w - \overline{w})^+ \\
&\quad + \int_\Omega (V - h(\overline{w}^2))w_K(w - \overline{w})^+ \le 0.
\end{aligned}$$

where $\overline{w}$ satisfies

$$h(\overline{w}^2) \ge \overline{V} := U_0 + c(\Omega, d, \lambda)\|C\|_{0,\infty,\Omega}.$$

This implies $w \le \overline{w}$ a.e. in Ω. Now use $(-V - U)^+$ with $U \ge U_0 = -\inf_{\Gamma_D} V_0$ as a test function in (2.2) to get

$$\begin{aligned}
\lambda^2 \int_\Omega |\nabla(-V - U)^+|^2 &\le \int_\Omega (w_K w - C)(-V - U)^+ \\
&\le c\int_\Omega (-V - U)^+ \\
&\le c\|(-V - U)^+\|_{1,2,\Omega}(\mathrm{meas}(-V > U))^{1/2},
\end{aligned}$$

where $c > 0$ depends on C and $\overline{w}$. Using Stampacchia's method as above allows to conclude that

$$V(x) \ge -\underline{V} := -U_0 - c(\Omega, d, \lambda)(\|C\|_{0,\infty,\Omega} + \overline{w}^2).$$

We get $w_K = w$ by taking $K > \max(K_0, \overline{w})$. Therefore the solution of (2.1)-(2.2) is the solution of (1.2)-(1.3). The proof of uniqueness of the solution we can see [4]. $\square$

3. The semi-classical limit

We need the following assumptions in this section.

(A4) $w_0 = \sqrt{C}$, $V_0 = h(C)$ on $\Gamma_D, C \in H^1(\Omega)$, $C(x) \ge \underline{C} > 0$ on Γ_D.

Theorem 3.1. *Let (A1)-(A4) hold and let (w_δ, V_δ) be a solution of* (1.2)-(1.3). *$h(x) = h_1(x) = \ln x$ (isothermal). Then there exists a subsequence, also denoted by (w_δ, V_δ), such that as $\delta \to 0$,*

$$w_\delta \rightharpoonup w, \quad V_\delta \rightharpoonup V \quad \text{in } H^1(\Omega),$$

where (w, V) is a solution of

$$\lambda^2 \Delta V = w^2 - C \quad \text{in } \Omega, \quad V = V_0 \quad \text{on } \Gamma_D, \quad \frac{\partial V}{\partial \nu} = 0 \quad \text{on } \Gamma_N, \tag{3.1}$$

$$0 = w(h_1(w^2) - V) \quad \text{in } \Omega. \tag{3.2}$$

Proof. Because of the $L^\infty(\Omega)$-estimate of V_δ in Theorem 2.1 there exists $a > 0$ such that $h_1(a^2) \le -\|V_\delta\|_{0,\infty,\Omega}$. Take $\underline{w} = \min(\sqrt{\underline{C}}, a)$ and use $(w_\delta - \underline{w})^-$ as a test function in (1.2) to get

$$\begin{aligned}
&\delta^2 \int_\Omega |\nabla(w_\delta - \underline{w})^-|^2 \\
&= -\int_\Omega (h_1(w_\delta^2) - h_1(\underline{w}^2))w_\delta(w_\delta - \underline{w})^- + \int_\Omega (V_\delta - h_1(\underline{w}^2))w_\delta(w_\delta - \underline{w})^- \le 0.
\end{aligned}$$

This implies $w_\delta \ge \underline{w}$ a.e. in Ω. Using $h_1(w_\delta^2) - V_\delta$ as a test function in (1.2), we obtain

$$\begin{aligned}
&-\int_\Omega w_\delta(h_1(w_\delta^2) - V_\delta)^2 \\
&= \delta^2 \int_\Omega \nabla w_\delta \cdot \nabla h_1(w_\delta^2) - \delta^2 \int_\Omega \nabla V_\delta \cdot \nabla(w_\delta - w_0) - \delta^2 \int_\Omega \nabla V_\delta \cdot \nabla w_0.
\end{aligned}$$

Using $w_\delta - w_0$ as a test function in (1.3), we obtain

$$-\delta^2 \int_\Omega \nabla V_\delta \cdot \nabla(w_\delta - w_0) = \delta^2 \lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0).$$

Therefore,

$$\begin{aligned}
&-\int_\Omega w_\delta(h_1(w_\delta^2) - V_\delta)^2 \\
&= \delta^2 \int_\Omega \nabla w_\delta \cdot \nabla h_1(w_\delta^2) + \delta^2\lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0) - \delta^2 \int_\Omega \nabla V_\delta \cdot \nabla w_0,
\end{aligned}$$

i.e.

$$\begin{aligned}
&\int_\Omega \nabla w_\delta \cdot \nabla h_1(w_\delta^2) \\
&= -\delta^{-2} \int_\Omega w_\delta(h_1(w_\delta^2) - V_\delta)^2 - \lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0) + \int_\Omega \nabla V_\delta \cdot \nabla w_0 \\
&\le -\lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0) + \int_\Omega \nabla V_\delta \cdot \nabla w_0 \le c_1,
\end{aligned}$$

where c_1 is independent of δ. Then we have

$$2 \int_\Omega \frac{|\nabla w_\delta|^2}{w_\delta} \le c_1.$$

Hence

$$\int_\Omega |\nabla w_\delta|^2 \le \frac{c_1 \overline{w}}{2} \le c,$$

where c is independent of δ. By Lemma 2.2, we know $\int_\Omega |\nabla V_\delta|^2 \le c$, where c is independent of $\delta \to 0$. Thus there exists a subsequence, also denoted by (w_δ, V_δ), such that as $\delta \to 0$,

$$\begin{gathered}
w_\delta \rightharpoonup w, \quad V_\delta \rightharpoonup V \quad \text{in } H^1(\Omega), \quad w_\delta \to w, \quad V_\delta \to V \quad \text{in } L^2(\Omega), \\
w_\delta \to w \quad \text{a.e. in } \Omega.
\end{gathered} \tag{3.3}$$

By $0 < \underline{w} \le w_\delta \le \overline{w}$ and (3.3), for a subsequence,

$$w_\delta h_1(w_\delta^2) \rightharpoonup w h_1(w^2) \quad \text{in } L^2(\Omega).$$

Therefore,

$$\begin{gathered} -\lambda^2 \int_\Omega \nabla V \cdot \nabla \varphi = \int_\Omega (w^2 - C)\varphi, \\ 0 = \int_\Omega w(h_1(w^2) - V)\varphi \end{gathered}$$

for any $\varphi \in H_0^1(\Omega \cup \Gamma_N)$, i.e. (w, V) is a solution of (3.1)-(3.2). □

Theorem 3.2. *Let (A1)-(A4) hold, $d \le 2$ and (w_δ, V_δ) be a solution of* (1.2)-(1.3)*. Set $h(x) = h_\alpha(x) = \frac{\alpha}{\alpha-1} x^{\alpha-1}$ with $\alpha > 1$ (isentropic). Then there exists a subsequence, also denoted by (w_δ, V_δ), such that, as $\delta \to 0$, we have*

$$w_\delta \rightharpoonup w, \quad V_\delta \rightharpoonup V \quad \text{in } H^1(\Omega),$$

where (w, V) is a solution of

$$\lambda^2 \Delta V = w^2 - C \quad \text{in } \Omega, \quad V = V_0 \quad \text{on } \Gamma_D, \quad \frac{\partial V}{\partial \nu} = 0 \quad \text{on } \Gamma_N, \tag{3.4}$$

$$0 = w(h_\alpha(w^2) - V) \quad \text{in } \Omega. \tag{3.5}$$

Proof. By (1.2) and [11] we know there exists $p > 2$ such that $w \in W^{1,p}(\Omega)$. We suppose that there exists a set ω such that $\omega \subset\subset \Omega$, $\operatorname{meas}(\omega) > 0$ and $w = 0$ in ω. Let $\omega_n \subset\subset \Omega$ be a sequence of sets such that $\omega \subset \omega_n$ and $\omega_n \to \Omega$ as $n \to \infty$. By (1.2) and Harnack's inequality (see [12, p.199]),

$$0 \le \sup_{\omega_n} w \le c \inf_{\omega_n} w = 0.$$

Therefore, $w = 0$ in ω_n, $n \in \mathbf{N}$. Because $W^{1,p}(\Omega) \hookrightarrow C^0(\overline{\Omega})$ for $d \le 2$ and $p > 2$, we have $w = 0$ in $\overline{\Omega}$, this contradicts (A4). Then there exists a $\underline{w} > 0$ such that $w \ge \underline{w}$ in Ω. Using $h_\alpha(w_\delta^2) - V_\delta$ as a test function in (1.2) we obtain

$$\begin{aligned} &- \int_\Omega w_\delta (h_\alpha(w_\delta^2) - V_\delta)^2 \\ &= \delta^2 \int_\Omega \nabla w_\delta \cdot \nabla h_\alpha(w_\delta^2) - \delta^2 \int_\Omega \nabla V_\delta \cdot \nabla (w_\delta - w_0) - \delta^2 \int_\Omega \nabla V_\delta \cdot \nabla w_0. \end{aligned}$$

Using $w_\delta - w_0$ as a test function in (1.3),

$$-\delta^2 \int_\Omega \nabla V_\delta \cdot \nabla (w_\delta - w_0) = \delta^2 \lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0).$$

Hence

$$\begin{aligned} &- \int_\Omega w_\delta (h_\alpha(w_\delta^2) - V_\delta)^2 \\ &= \delta^2 \int_\Omega \nabla w_\delta \cdot \nabla h_\alpha(w_\delta^2) + \delta^2 \lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0) - \delta^2 \int_\Omega \nabla V_\delta \cdot \nabla w_0, \end{aligned}$$

i.e.

$$\begin{aligned} &\int_\Omega \nabla w_\delta \cdot \nabla h_\alpha(w_\delta^2) \\ &= -\delta^{-2} \int_\Omega w_\delta (h_\alpha(w_\delta^2) - V_\delta)^2 - \lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0) + \int_\Omega \nabla V_\delta \cdot \nabla w_0 \\ &\le -\lambda^{-2} \int_\Omega (w_\delta^2 - C)(w_\delta - w_0) + \int_\Omega \nabla V_\delta \cdot \nabla w_0 \le c_1, \end{aligned}$$

where c_1 is independent of δ. Therefore,

$$2\alpha \int_\Omega w_\delta^{2\alpha-3} |\nabla w_\delta|^2 \le c_1.$$

If $\alpha > 3/2$, then

$$2\alpha \int_\Omega \underline{w}^{2\alpha-3} |\nabla w_\delta|^2 \le c_1.$$

Hence

$$\int_\Omega |\nabla w_\delta|^2 \le \frac{c_1}{2\alpha \underline{w}^{2\alpha-3}}.$$

If $\alpha = 3/2$, then

$$2\alpha \int_\Omega |\nabla w_\delta|^2 \le c_1.$$

Thus

$$\int_\Omega |\nabla w_\delta|^2 \le \frac{c_1}{2\alpha}.$$

If $1 < \alpha < 3/2$, then

$$2\alpha \int_\Omega \overline{w}^{2\alpha-3} |\nabla w_\delta|^2 \le c_1.$$

Therefore,

$$\int_\Omega |\nabla w_\delta|^2 \le \frac{c_1}{2\alpha \overline{w}^{2\alpha-3}}.$$

By setting

$$c = \max\Big(\frac{c_1}{2\alpha \underline{w} 2\alpha - 3}, \frac{c_1}{2\alpha}, \frac{c_1}{2\alpha \overline{w}^{2\alpha-3}}\Big),$$

we obtain

$$\int_\Omega |\nabla w_\delta|^2 \le c,$$

where c is independent of δ. By Lemma 2.2 we know $\int_\Omega |\nabla V_\delta|^2 \le c$, where c is independent of $\delta \to 0$. Therefore, there exists a subsequence, also denoted by (w_δ, V_δ), such that as $\delta \to 0$, we have

$$\begin{gathered} w_\delta \rightharpoonup w, \quad V_\delta \rightharpoonup V \quad \text{in } H^1(\Omega), \quad w_\delta \to w, \quad V_\delta \to V \quad \text{in } L^2(\Omega), \\ w_\delta \to w \quad \text{a.e. in } \Omega. \end{gathered} \tag{3.6}$$

By $0 < \underline{w} \le w_\delta \le \overline{w}$ and (3.6) we know, that for a subsequence,

$$w_\delta h_\alpha(w_\delta^2) \rightharpoonup w h_\alpha(w^2) \quad \text{in } L^2(\Omega).$$

Hence

$$\begin{gathered} -\lambda^2 \int_\Omega \nabla V \cdot \nabla \varphi = \int_\Omega (w^2 - C)\varphi, \\ 0 = \int_\Omega w(h_\alpha(w^2) - V)\varphi \end{gathered}$$

for any $\varphi \in H_0^1(\Omega \cup \Gamma_N)$, i.e. (w, V) is a solution of (3.4)-(3.5). $\square$

References

[1] A. Jüngel, *Nonlinear problems in quantum semiconductor modeling,* Nonlinear Analysis **47** (2001), no. 9, 5873–5884.

[2] P. A. Markowich, C. A. Ringhofer and C. Schmeiser, *Semiconductor equations,* Springer-Verlag, Vienna, 1990.

[3] C. L. Gardner, *The quantum hydrodynamic model for semiconductor devices,* SIAM J. Appl. Math. **54** (1994), no. 2, 409–427.

[4] I. Gasser and A. Jüngel, *The quantum hydrodynamic model for semiconductors in thermal equilibrium,* Z. Angew. Math. Phys. **48** (1997), no. 1, 45–59.

[5] A. Jüngel, *A steady-state quantum Euler-Poisson system for potential flows,* Commun. Math. Phys. **194** (1998), no. 2, 463–479.

[6] A. Jüngel, M. C. Mariani and D. Rial, *Local existence of solutions to the transient quantum hydrodynamic equations,* Math. Models Methods Appl. Sci. **12** (2002), no. 4, 485–495.

[7] M. T. Gyi and A. Jüngel, *A quantum regularization of one-dimensional hydrodynamic model for semiconductors,* Adv. Differential Equations **5** (2000), no. 4-6, 773–800.

[8] B. Zhang and J. Jerome, *On a steady-state quantum hydrodynamic model for semiconductors,* Nonlinear Anal. **26** (1996), no. 4, 845–856.

[9] G. Stampacchia, *'Equations elliptiques du second ordre à coefficients discontinus,* Les Presses de l'Université de Montréal, Que., 1966.

[10] G. M. Troianiello, *Elliptic differential equations and obstacle problems,* Plenum Press, New York, 1987.

[11] K. Gröger, *A $W^{1,p}$-estimate for solutions to mixed boundary value problems for second order elliptic differential equations,* Math. Ann. **283** (1989), no. 4, 679–687.

[12] D. Gilbarg and N. S. Trudinger, *Elliptic partial differential equations of second order,* 2nd ed., Springer, Berlin, 1983

Jianwei Dong
Department of Basic Courses, Zhengzhou Institute of Aeronautical Industry Management, Zhengzhou 450015, China
E-mail address: `dongjianweiccm@163.com`