

# Modified Bifurcating Neuron with Leaky-Integrate-and-Fire Model

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**Abstract.** The Modified Bifurcating Neuron (MBN) is a neuron model that is capable of amplitude-to-phase conversion and volume-holographic memory. Inputs are real valued and temporally spaced. This allows information to be coded in the temporal spacing of inputs and outputs as well as their values. At its core, the MBN incorporates a stateful leaky-integrate-and-fire neuron model. The MBN attempts to produce these properties by simulating mechanisms present in biological neural systems to a greater extent than is normally found in artificial neural networks. MBNs use an object model rather than the normal linear algebra approach. The MBN is conceptually based on the computational model presented in the “Bifurcating Neuron Network 2” by G. Lee and N. Farhat.

## 1 Introduction

The MNB is conceptually based on the Bifurcating Neuron (BN) [1] is a neuron model in which an integrate-and-fire neuron is augmented by coherent modulation from the neural environment. The BN is capable of amplitude to phase conversion and volume-holographic memory. Because of its integrate-and-fire activation model, it exhibits frequency response to incoming pulse timing. When used in a network, BNs have time delays between neuron connections that represent signal propagation latency [1]. A single BN is defined by the following three equations:

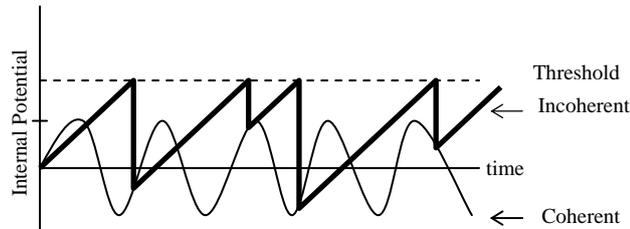
$$\theta_i(t) = 1 \quad (1)$$

$$\frac{dx(t)_i}{dt} = c_i \quad (2)$$

$$\rho_i(t) = \rho_0 \sin 2\pi ft \quad (3)$$

where  $\theta_i(t)$ , and  $\rho(t)_i$  are the threshold level, and the relaxation level of BN<sub>i</sub>, respectively. The potential  $x_i(t)$  rises at a constant rate,  $c_i$ , due to the incoherent signal, until it reaches the threshold level  $\theta_i$ . Then the internal potential immediately

drops to the relaxation level  $\rho(t)_i$ . The threshold level is constant. The relaxation level, Eq (3) is driven by the coherent signal and maintains a sinusoidal oscillation with maximum amplitude  $\rho_0$  and frequency  $f$ . See Figure 1. [1]



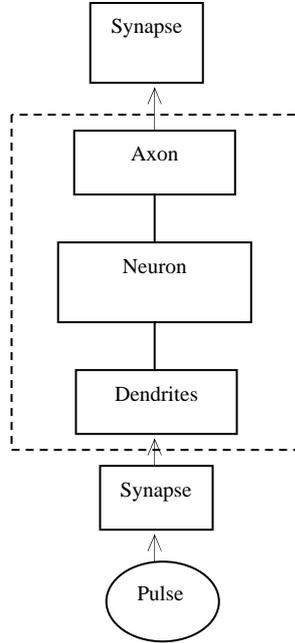
**Figure 1: Firing behavior of BN**

The MBN takes a simulation or mimetic approach and loosely attempts to use structures found in biological systems rather than pure mathematical solutions. This allows us to explore the role of some biological occurrences, as they pertain to neural computation. For instance, certain behaviors or rather behavioral modes have been correlated to the presence of Theta rhythms in the mammalian brain [1], [2]. MBNs are modeled using an object oriented paradigm and not the traditional linear algebra approach. The behavioral complexity of MBN processing elements is more complex than is normally found in neural networks.

## 2 Modified Bifurcating Neuron Object Model

Our Modified Bifurcating Neuron (MBN) attempts to approach the afore mentioned behavior of the BN using simulated biological mechanisms. Its structure is depicted in Figure 2.

The neuron object is where most of the processing occurs. Information propagated by each neuron is modeled by pulse objects. Each pulse corresponds to a spike in a spike train i.e. neuron output. Each neuron contains an axon and a collection of dendrites. These objects serve as containers for incoming and outgoing pulses. The neuron object delegates direct control of pulse objects to these containers. Connections between neurons are managed by synapse objects. Synapses maintain a collection of pulses, which are waiting to arrive at the neuron in question, and contain information about the time delay between neurons and the connection strength for this particular connection. A synapse can be between at most two neuron objects. Time is defined in terms of a universal clock tick that all objects receive. When pulses that have been scheduled to arrive at a neuron held in a synapse actually arrive, they contribute to the internal potential of a neuron object. If this neuron reaches its threshold, a new pulse is generated and sent to every synapse connected to the axon of this neuron.



**Figure 2: MBN Object Model**

### 3 Modified Bifurcating Neuron Networks

MBNs are linked using synapse objects. These connections are one-way and may be redundant allowing the construction of high order networks. Pulse objects propagate along these connections. In real time they are held by synapse objects until their scheduled arrival time, at which time they contribute to the rise in potential of the neuron in question.

### 4 Modified Bifurcating Neuron Definition

We altered the basic BN definition to model our Modified Bifurcating Neuron (MBN). The form of the BN definition has been maintained to illustrate similarities and differences between the two models. Foremost, we see that the change in internal potential, Eq (5), is no longer constant and the resting potential, Eq (6), no longer oscillates. The set of equations to follow describe the behavior of an MBN, augmented with a coherent and incoherent signal input, without an actual data input. They describe the behavior of the  $i^{\text{th}}$  MBN, which we shall simply refer to as  $\text{MBN}_i$ :

$$\theta_i(t) = K_i \tag{4}$$

$$\frac{dx(t)_i}{dt} = \frac{d\psi(t)_i}{dt} + \frac{d\phi(t)_i}{dt} + \beta \frac{dx(t-1)_i}{dt} \quad (5)$$

$$\rho_i(t) = C_i \quad (6)$$

where  $x_i(t)$ ,  $\theta(t)_i$ , and  $\rho(t)_i$  are the internal potential, the threshold level, and the relaxation level of MBN<sub>*i*</sub>, respectively. Contributions to the internal potential of MBN<sub>*i*</sub> noted in Eq (5) come from the incoherent signal,  $\psi(t)_i$ , and coherent signal,  $\phi(t)_i$ . The potential  $x_i(t)$  changes in time due to contributions from the incoherent signal, the coherent signal, and the potential remaining from the previous time step. This remaining potential is modified by the 'leak' factor,  $\beta$  (a constant between 0 and 1) representing decay. This continues until the potential reaches the threshold level  $\theta_i(t)$ , which is some constant value  $K$ . The internal potential then immediately drops to the relaxation level  $\rho_i(t)$ , which is a constant value  $C_i$ . Eq (5) is a recursive function dependent on a discrete time step.

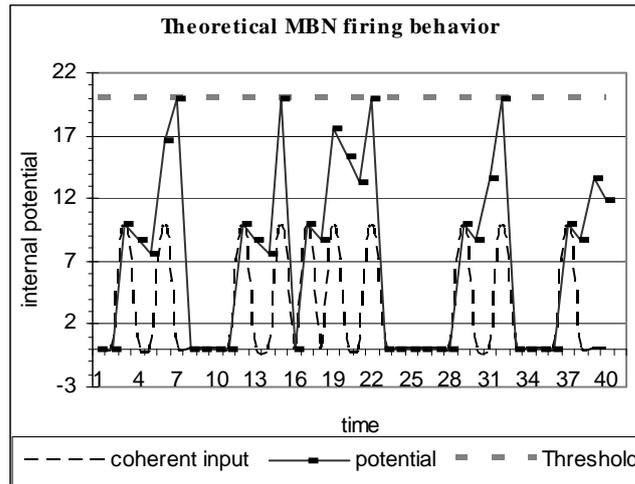
#### 4.1 Incoherent Signal

An incoherent signal is provided by a regular pulse of variable frequency. In an MBN network, this can be produced by an MBN that is connected to every other MBN and itself. This MBN produces a pulse on a constant time interval. This pulse arrives at every other MBN simultaneously and serves to increase the neuron potential as in Figure 1. The rhythmic pulse can be generated utilizing one recurrent connection, whose time delay serves to regulate the firing frequency. This is intended to be loosely synonymous with rhythmic pulses emanating from the hippocampal area in mammalian brains. Various studies have shown a correlation between frequency ranges of this rhythmic signal and general behavioral states in mammals such as heightened alertness, concentration and problem solving, hypnosis, sleep, etc. [3]. Different frequency ranges have been given different names such as Theta (7 – 10 Hz) and Gamma (60 – 100 Hz) [4]. Regardless of its specific role or roles in behavioral states, it is sufficient to say that this rhythmic pulse exists, is dynamic, and is propagated to various areas of the mammalian brain.

#### 4.2 Coherent Signal

The MBN coherent signal is not specifically sinusoidal as in the BN. It is a spike train, whose pattern repeats over a certain time interval. In other words, it is a series of pulses of various temporal spacing and possibly various amplitudes, which repeat over a time period. It can be spaced in time in such a way that it produces an approximate sinusoidal response in the internal potential of an MBN, but it is sufficient that this signal causes a change in the internal potential that is not constant overtime within the period, Eq (5). In Figure 3, we see a theoretical depiction of an

MBN stimulated by coherent signal alone. We can see that the presence of the coherent signal affects the system in a similar way as in the BN (Figure 1). It causes changes in the timing of threshold events i.e. neuron activation. This couples the temporal spacing of the output spike train to that of the coherent signal.



**Figure 3: Theoretical MBN firing behavior**

The internal potential is driven to the firing threshold by simulated Coherent input in this example. This figure illustrates coherent input of one period or less, therefore behavior arising from periodic behavior is not illustrated here. This figure is reminiscent of Figure 1 and illustrates how input leads to temporal spacing action potential generation. Additionally we see that internal potential falls to zero after a threshold event.

### 4.3 MBN Behaviors

The MBN is very similar to the BN in many respects. It exhibits similar behaviors, namely amplitude to phase conversion. However, the mechanisms by which these behaviors are achieved are dramatically different. The BN represents a neuron augmented by input from the neural environment. Therefore, its coherent input is represented as an internally generated sinusoidal wave that controls the neuron resting potential, whose frequency is supplied as a network input. The MBN does not seek to make this assumption but to an extent attempts to simulate the environment as well. The entire coherent signal is supplied externally as network input. The form of this input corresponds with neuron output so that output from one or more neurons could serve as coherent input for others. This adds flexibility to the use of this signal. Furthermore on a neuron level, the coherent input is not different from other inputs. More importantly, it contributes directly to the internal potential and has no direct effect on the resting potential. In this same vein, rise in potential

due to the incoherent signal is no longer constant but quasi-constant and supplied by a rhythmic pulse. Also, the internal potential of the MBN dissipates with time following the leaky-integrate-and-fire model, while the BN uses an integrate-and-fire model. The major differences between the BN and MBN are: 1) In the MBN, the coherent signal also contributes to the internal potential rise, where it does not in the BN. 2) The potential rise due to the incoherent signal is no longer constant, Eq (5), but merely quasi-constant. 3) The internal potential dissipates with time. This follows the leaky capacitor integrate-and-fire neuron model, where the internal potential of a given processing element decreases over time. Therefore, the contribution to the internal potential of a given processing element from two pulses will not be additive unless the arrival of these pulses are closely spaced in time.

#### 4.4 General Form

It is important to remember that the MBN need not be augmented by incoherent and coherent inputs. Coherent and incoherent refer to a particular structure of the inputs. While this structure can result in rich and interesting or useful behavior, all inputs in general are handled in the same way. Thus we can construct a general form that makes no distinction between different types of network input. A single MBN is defined by the following three equations:

$$\theta_i(t) = K_i \quad (7)$$

$$\frac{dx(t)_i}{dt} = \frac{d\lambda(t)_i}{dt} + \beta \frac{dx(t-1)_i}{dt} \quad (8)$$

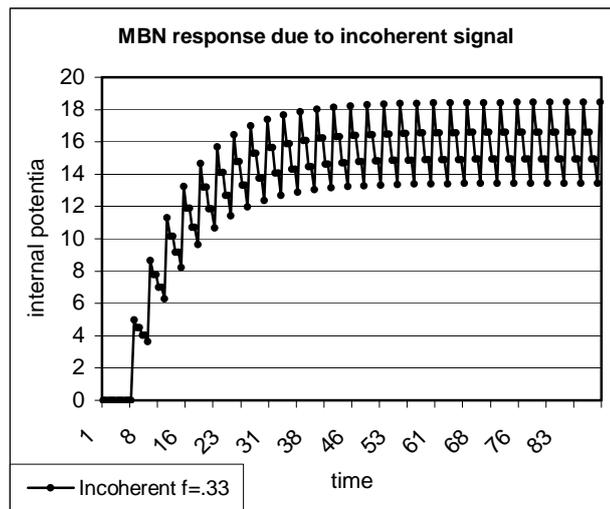
$$\rho_i(t) = C_i \quad (9)$$

where  $x_i(t)$ ,  $\theta(t)_i$ , and  $\rho(t)_i$  are the internal potential, the threshold level, and the relaxation level of MBN<sub>*i*</sub>, respectively. Here we make no distinctions as to the structure of the input,  $\lambda(t)_i$ . The potential  $x_i(t)$  changes in time due to contributions from an input,  $\lambda(t)_i$ , and the potential ‘leak’,  $\beta$ , from the previous time step. An interesting behavior emerges in Eq (8), if we consider the input as a growth rate and the ‘leak’ as a decay rate, where the decay rate is an exponential of the form  $\beta = \alpha e^{-\alpha}$ . We can see that when the growth rate equals the decay rate, the rate of change of the internal potential is zero. This implies that at some point an equilibrium condition will arise in which the internal potential will stabilize at or about some value. In general, it seems that there are two cases to consider: First, the case in which the growth rate is constant in time. In this case it can be shown, that the internal potential will asymptotically approach some static equilibrium point, if we cast, Eq (8) into a continuous form. Second, the case in which the growth is not constant in time. In this case, no static equilibrium exists. However, if we assume the growth rate periodically oscillates around some median value, it can be shown that the internal potential will reach an oscillatory equilibrium about some other

median value. We see that this oscillation of the growth rate can be said to drive the internal potential in a steady state. Thus we see a theoretical basis for making distinctions in the structure of the input. This approach to a stable state may also be construed as adaptive behavior, i.e. neural plasticity. It somewhat mimics the ability of biological neurons to adapt to sustained input.

## 5 Expected Outcome

We now examine the expected behavior of the MBN. First consider the MBN driven by the incoherent signal alone. The incoherent signal is synonymous with a signal that is constant in time. However, due to the discrete nature of the pulse data, this cannot be achieved. Therefore our incoherent signal is only quasi-constant in time. This is represented as a rhythmic pulse of constant amplitude and frequency. According to our discussion in section 4.4, the internal potential will approach a constant value in which the growth rate equals the decay rate given, a constant input. Therefore, we would expect our quasi-constant input to approach this behavior. However, according to section 4.4, a periodic input will result in a periodic oscillation of the potential.

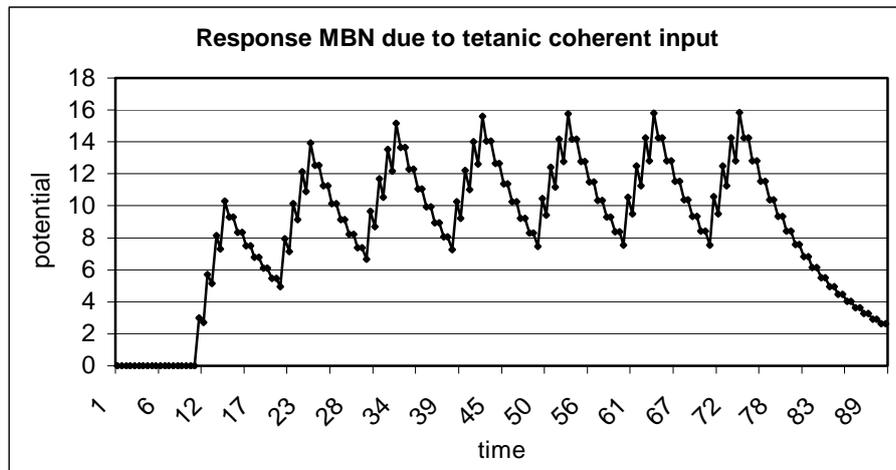


**Figure 4: MBN response due to incoherent signal**

An MBN is driven by a quasi-constant input of frequency 0.33 Hz. The internal potential is seen to oscillate periodically around some median value, which asymptotically approaches an equilibrium constant.

Figure 4 depicts a sample output of an MBN supplied with this incoherent input. The result is as expected. The internal potential approaches an oscillatory equilibrium in which it oscillates predictably about some median value that

asymptotically approaches a constant value. Likewise for the coherent input, we would expect a similar response. Figure 5 depicts the response of an MBN to coherent input. In this instance, coherent input is titanic, which means that the signal is composed of a series of pulse inputs closely spaced in time (one time step) followed by a period of no input. The whole cycle is then repeated. While this is not sinusoidal, it fits our requirement that the signal is not constant in time and is periodic. The fact that a titanic signal is acceptable here is interesting, considering the prevalence of titanic signals in biological systems.



**Figure 5: Response of MBN due to titanic coherent input**

Titanic coherent input consists of multiple closely spaced pulses (titanic), which occur periodically. The internal potential is seen to oscillate periodically around some median value. The trailing tail marks the conclusion of the input signal. The potential then falls off exponentially.

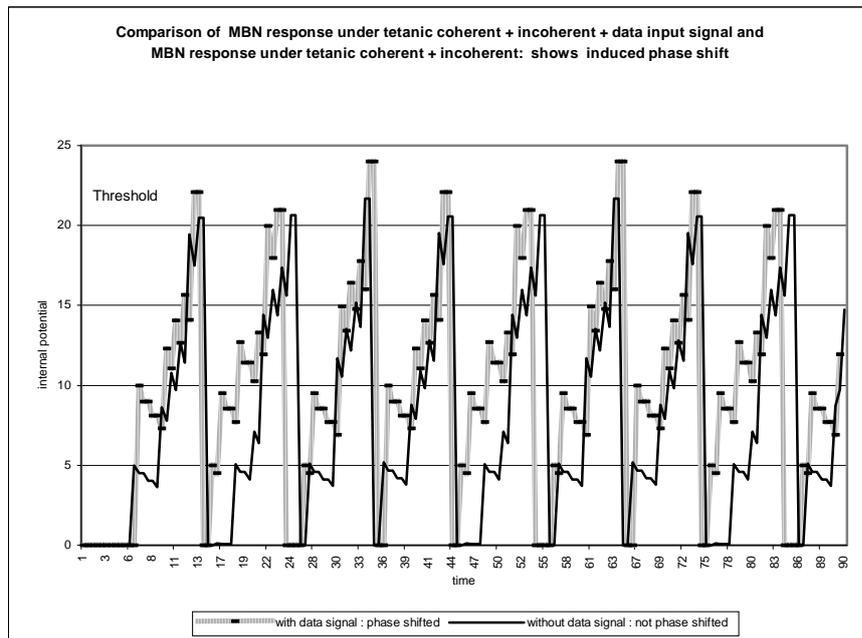
## 5.1 Amplitude to Phase Conversion

In Amplitude to phase conversion, we start with input patterns that contain both incoherent and coherent signals. These are arranged in such a way that the MBN produces output spikes on regular intervals that correspond to the period of the coherent input. If an additional input or inputs are introduced during a coherent input period, this causes a phase lead in the output spike for that period. In other words, the MBN fires before its normal firing time by some amount proportional to the amplitude of the additional input signal. If this signal is repeated in the next period this phase lead will appear again. This effect can be summarized with the addition of a new term to Eq (5), representing the contribution to the internal

potential from the additional input denoted by  $v(t)$ , referred to as the data input.

$$\frac{dx(t)_i}{dt} = \frac{dv(t)}{dt} + \frac{d\psi(t)_i}{dt} + \frac{d\phi(t)_i}{dt} - \beta \frac{dx(t-1)_i}{dt} \quad (10)$$

Figure 6 depicts an example of the MBN response due to incoherent and coherent input compared to the response of incoherent, coherent, and data input signals. The output pulse timing of response including the data signal is phase shifted.



**Figure 6: Response of MBN due to tetanic coherent input**

Input containing a data signal as well as coherent and incoherent input is phase shifted with respect to the same coherent and incoherent input without the data signal. The firing threshold is twenty. The firing time phase shift is not constant, but this pattern of irregularity repeats over several firing intervals. Notice that the phase shift of the firing time at  $t = 23$  is the same as the spike at  $t = 54$  and  $t = 85$ . This is due to the discrete nature of the time sampling

## 5.2 Holographic Paging

Now consider a more general case where the contributions to the internal potential from the data section at time  $t$  are given by  $v(t)$ . If  $v(t)$  is maintained, but the coherent signal,  $\phi(t)_i$ , is altered, it is apparent that the rate of increase of the internal potential and thus the time spacing of output spikes will be altered. In fact, changing the coherent signal should change intervals on which the MBN fires even without a

data input. Thus, the output spike pattern will only be the same, if both the coherent signal and the data input pattern are unaltered. Significant changes in either will result in an output pattern that does not match the original. Thus a particular output pattern can only be recreated given the appropriate coherent signal. The second major requirement is that storing a new data pattern with a different coherent input does not disturb the original pattern. The original BN accomplished this in a nearest neighbor pulse coupled neural network PCNN, using higher order synaptic connections. This requires that one or more new connections be added for each stored pattern. The time delays of new synaptic connection are offset by an amount proportional to the induced phase shift.

## 6 Conclusions

The MBN more closely mimics biological neural systems than conventional neural models. MBNs exhibit diverse temporal behavior. The MBN responds differently to different structured inputs. Using these strategies, we have demonstrated amplitude to phase conversion and a theoretical basis for holographic memory. Furthermore the MBN is capable of higher order recurrent connections. This allows construction of complex temporal patterns. As stated in BNN 2 by Lee and Farhat [1], these complex temporal patterns are possible examples of how and why the brain uses multiple recurring connections. There are approximately  $10^4$  recurrent connections in the brain for each neuron. The reduction of these connections to one seems to be an oversimplification.

Neural Networks, in general, have proven to be powerful computational tools. Perhaps borrowing more behaviors from biological systems can lead to even more powerful tools and a better understanding of the systems that inspired them.

## 7 References

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