Statistical Approach to Person Identification via Unique Properties of the Oculomotor Plant

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Abstract— This paper presents a statistical approach that **employs Hotelling's** T-square test for the purpose of novel biometric identification based on unique static and dynamic properties of the oculomotor plant (OP) represented by mechanics of the eyeball, surrounding tissues, ligaments, and extraocular muscles. Proposed statistical approach yielded False Acceptance Rate of 0% and False Rejection Rate of 9.1%, providing a significant improvement against previously published study where identification was performed with k-nearest neighbor classification (KNN) and decision tree (C4.5) approaches with dynamic and static properties of the OP extracted via a horizontal linear homeomorphic mathematical model of OP from recorded eye the movement trace. In the current research two dimensional linear homeomorphic mathematical model was employed, allowing to extract OP biometric information from two dimensional saccades therefore providing more accurate identification rates. Current study involved 46 subjects with eye movement recordings done at 1000Hz.

I. INTRODUCTION

The he need for identification of people arose since the start of the criminology. The methods of biometric identification evolved throughout history from basic measurements of head dimensions [1] to more advanced techniques involving fingerprints [2], iris [3], and face recognition [4]. Above mentioned techniques are not completely fraud-proof since they are based on a human's body characteristics that can be replicated with modern technological advances [2-5]. In addition, during a finger or iris scan the user has to be actively engaged in the biometric procedure to achieve high accuracy identification results therefore making those methods intrusive. As a result there is a significant need in biometrics community to identify methods that are both highly counterfeit resistant and non intrusive.

One of the approaches that has the potential to satisfy both needs is eye tracking (technology that allows to record spatial coordinates of the eye at a given time instance). Modern technological advancements provide the opportunity to perform the eye tracking at a comfortable distance with no hardware components attached to the human body [6].

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Human eye already provides significant amount of information useful for biometrics. The physical and behavioral properties of the eye are employed in biometrics based on the iris [7], face recognition [4], retina [8], periocular information [9], recordings of the raw eye positional, velocity signal and pupil dilation [10-11].

In terms of its anatomical structure, an eye provides a unique opportunity for the identification by containing a multitude of anatomical components that represent Oculomotor Plant (OP). These components are the eye globe and its surrounding tissues, ligaments, six extraocular muscles each containing thin and thick filaments, tendon-like components, various tissues and liquids [12]. As the results the dynamic and static properties of the OP are represented by the eye globe's inertia, dependency of an individual muscle's force on its length and velocity of contraction, resistive properties of the eye globe, muscles and ligaments, frequency characteristics of the neuronal control signal sent by the brain to the extraocular muscle and the speed of propagation of this signal. Individual properties of the extraocular muscles vary depending on role each muscle performs. There are two roles: agonist - muscle contracts and pulls the eye globe in the required direction and the antagonist - muscle expands and resists the pull [13]. Ability to numerically evaluate all OP properties would provide an opportunity to develop a highly counterfeit resistant biometric method due to the fact that these properties exist only in an alive individual. However, accurate estimation of the OP properties is challenging due to the secluded nature of corresponding anatomical components.

Komogortsev and colleagues [14] has proposed a biometric scheme that allows to estimate OP properties based on the eye movement recordings. Extraction of the OP properties was conducted via a horizontal linear homeomorphic mathematical model of the Oculomotor Plant [15]. Two popular classification methods were applied to the data in a form of k-nearest neighbor (KNN) [16] and decision tree (C4.5) [17]. The resulting system's accuracy was low with the best False Acceptance Rate (FAR) of 5.4% and False Rejection Rate (FRR) of 56.6% for KNN and FAR=80% and FRR=0% for C4.5. It was hypothesized that the KNN and C4.5 were not able to achieve accurate identification results due to the variability present in the estimated OP properties data and therefore a new method of person identification was required. Previous study was limited to the estimation of the OP properties that are responsible for the horizontal component of eye movement only, i.e., eye globe and just two extraocular muscles (superior and inferior recti). Hence, fewer OP properties were considered for the identification.

Therefore, the first objective of this paper was to alleviate the challenges associated with the variability of the estimated OP properties by applying statistical approach in a form of the Hotelling's T-square test. The second objective was to investigate the use of the two dimensional linear homeomorphic OP model providing the capability to employ all information from the 2D eye movement trace and increase the number of the estimated OP properties.

The paper is organized in the following way: section II presents brief overview of the biometric identification via a mathematical model of the oculomotor plant and provides the description of the Hotelling's T-square test, section III presents methodology of the experiment performed, section IV presents the conclusion and future work.

II. BIOMETRIC IDENTIFICATION BY OCULOMOTOR PLANT MATHEMATICAL MODEL

A. Overview

This section provides a brief overview with the general scheme illustrated by Figure 1. The recorded eye movement signal u(t) from a single individual is supplied to the "Eye Movement Classification" module that classifies eye position signal into fixations (movements that keep an eye focused on the stationary object of interest) and saccades (extremely rapid eye rotations between the points of fixation). We focus on the detected saccade trajectories, represented by Θ(t) in the diagram. The detected saccade properties such as the onset, the offset, and the amplitude, depicted by h(t), are sent to the Oculomotor Plant Mathematical Model (OPMM) module for the purpose of generating simulated saccade trajectories represented by the signal x(t), based on the default values for the OP properties. The difference between the detected saccade trajectories Θ(t) and the simulated saccade trajectories x(t) is computed by the "Error Function" module and the resulting error e(t) is produced. The magnitude of the e(t) signal serves as a command to the "Minimization Algorithm" module to optimize the OP properties and generate new saccade trajectories minimizing the error e(t). After several iterations, the values for the OP properties that produce minimum e(t) are supplied to the "Person Identification" module which performs the actual identification. Brief descriptions of each module is provided below with a detailed description provided of the "Person Identification Module" that contains KNN, C4.5 and Hotelling's T-square test identification algorithms.

B. Eye Movement Classification

Velocity-Threshold (I-VT) algorithm [18] was employed to process eye movement trace into fixations and saccades.

C. Oculomotor Plant Mathematical Model

Two types of the OPMM models were investigated in our research. First model was horizontal linear homeomorphic OP (1D-OP) [15] and the second model was two dimensional linear homeomorphic OP (2D-OP) [19].

The 1D-OP is capable of simulating horizontal eye movements including saccades by considering physical

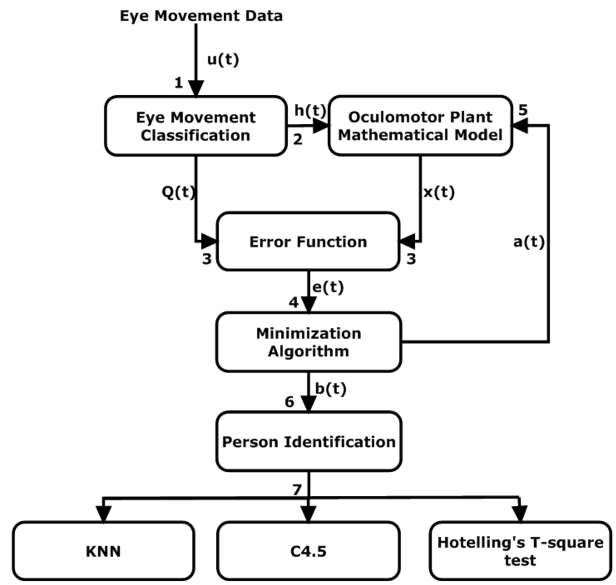

Fig. 1. Biometric identification via an oculomotor plan mathematical model

properties of the eye globe and two extraocular muscles: medial and lateral recti. The 1D-OP mathematically represents dynamic properties of the OP via a set of linear mechanical components such as springs and damping elements. Specifically following properties are considered: *active state tension* – tension developed as a result of the innervations of a muscle by neuronal control signal, *length tension relationship* – the relationship between the length of a muscle and the force it is capable of exerting, *force velocity relationship* - the relationship between the velocity of a muscle extension/contraction and the force it is capable of exerting, passive elasticity – the resisting properties of a muscle not innervated by the neuronal control signal, *series elasticity* – resistive properties of a muscle while the muscle is innervated by the neuronal control signal, passive elastic and viscous properties of the eye globe due to the characteristics of the surrounding tissues. Neuronal control signal command that is sent by the brain to the extraocular muscles in a form of the neuronal discharge is approximated as a pulse-step signal where step part of the signal determines the eye position prior and after the saccade and pulse part of the signal determines saccadic amplitude. More detailed description of these properties can be found in [15]. The 1D-OP employs only OP properties that are contributing to the horizontal component of eye movement, describing dynamics of the eye globe's rotation via six differential equations [15, 20].

The 2D-OP additionally considers two extraocular muscles (superior and inferior recti) that are primarily responsible for the vertical component of the eye movements, their static and dynamic properties and the neuronal control commands sent to each muscle. Also, the model contains the eye globes properties contributing to the vertical dynamics of movement as well. Twelve differential equations describe the eye's rotation in 2D space [19]. As a result 2D-OP is capable of simulating accurate saccadic signal on the two dimensional plane, therefore allowing to estimate all OP properties contributing to the eye rotation. Consequently, the 2D-OP has higher potential in producing more accurate identification results due to the more accurate representation of the OP with larger number of anatomical components included in the model.

Two dimensional linear homeomorphic representation of the OP is beneficial because a) it is able to produce 2D eye movement signal (projection of the line of sight on a computer screen) with characteristics of normal humans, therefore allowing for a close match between the simulated and the recorded signal, b) it contains the representation for the major anatomical components of the OP, allowing to estimate those components from the eye movement trace, c) it has linear design speeding up the estimation procedure for OP properties.

D. OP Properties

The OP properties for the 1D-OP consist of the following values: the width of the pulse of the neuronal control signal for the agonist muscle (LR_p), pulse height of the neuronal control signal for the agonist muscle (LR_s), length tension (K_{LT}), series elasticity (K_{SE}), passive viscosity of the eye globe (B_p), and force velocity relationship in the agonist muscle represented by the damping component (B_{AG}), combined passive elasticity of the eye globe and all extraocular muscles (K_p), eye globe inertia (J). These properties were selected as the most influential in terms of the resulting saccadic trajectory [21].

The use of the 2D-OP doubles the number of the OP properties due to the addition of properties responsible for the vertical component of the eye movement.

E. Error Function

Error function was implemented as the Root Mean Square Error (RMSE) computed between the simulated by the OPMM and the recorded by the eye tracker saccadic trajectories.

F. Minimization Algorithm

The goal of the Minimization Algorithm module is to select the values for the OP properties that would produce a minimum error e(t) between the simulated and recorded saccadic trajectory. To achieve this goal minimax optimization approach involving sequential quadratic programming (SQP) [22] was employed to sequentially optimize the values in the OP properties to produce the minimum error e(t).

G. Person identification

The input to the Person Identification module consists of a set of optimal values for the OP properties estimated for each qualifying saccade. The output is an authorization score classifying each saccade as belonging to an authorized user or an imposter. Three methods were employed in our paper: 1) K-nearest neighbor (KNN), 2) decision tree (C4.5), and 3) Hotelling's T-square test. KNN and C4.5 were previously described elsewhere [14, 23], therefore we provide detailed description of the person identification based on the Hotelling's T-square test. It was important for us to consider the KNN and C.45 classification even in case of their poor performance of those methods in the previous study [14], due to the fact that the current study employed a more accurate eye tracker equipment with a higher sampling frequency, therefore allowing for a possibility of the better classification results.

The oculomotor plant literature has extremely limited experimentation that allows to infer the actual values for the OP properties. Majority of the "default" values for the 1D-OP and 2D-OP models were deduced with the help of the data from a strabismus surgery performed on several patients [24], and for the lack of data even cat studies [25]. As a result it is hard to estimate a priory the amount of the variability of the values for the OP properties in a large pool of normal humans. Nevertheless, we have assumed that there is enough variability between persons to provide reliable identification. Based on this assumption we required a statistical test that is able to consider all OP properties with all their recorded values at the same time. The Hotelling's T-square test [26] perfectly fitted that purpose, by assessing the variability in the data for a single individual as well as across a set of individuals.

Specifically, 2 kinds of tests were performed to determine whether two given datasets belong to the same person or two different people. In the acceptance test, a dataset of a specific individual was split into 2 different sets on which the test was performed (Figure 2). The rejection test, however, involved datasets from 2 different people (Figure 3). Both the rejection and acceptance tests resulted in certain level of statistical significance, therefore determining the outcome of the test.

A Null Hypothesis (H_0) was formulated based on the fact that 2 datasets from subject i and j were being compared: "H_0:There is no difference between the set of vectors from subject i and j". In order to make a conclusion about the difference between 2 subjects, the statistical significance (P_{level}) resulting from the test was compared to the significance level. A significance level of 0.05 was chosen in this work. If the resulting P_{level} was smaller than the significance level, the H_0 was rejected indicating that the

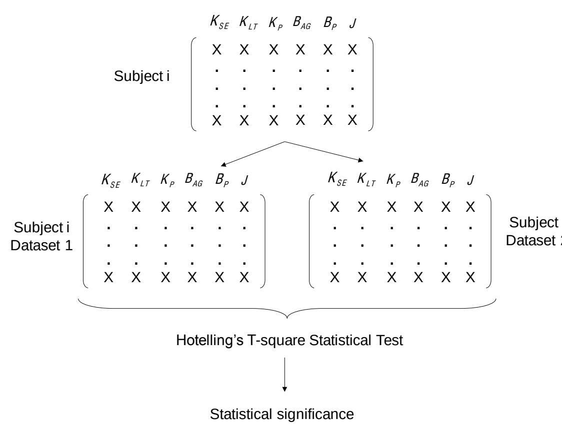

Fig. 2. The acceptance test is performed on 2 datasets from the same subject.

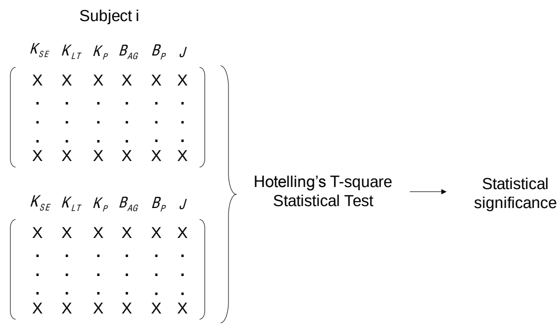

Figure 3. The rejection test is performed on 2 datasets from 2 different subjects.

datasets compared belonged to different people. Otherwise, the H_0 was accepted indicating that the datasets belonged to the same person.

The acceptance test failed when the H_0 was rejected when both datasets belonged to the same person. This test was run on the data extracted from every subject who conducted the experiment. The rejection test failed, however, when the H_0 was accepted while comparing datasets belonging to different people. It was run on every possible pair of different subjects. The failure rates resulting from the tests reflect the performance of the identification methodology presented in this work.

III. Methodology

A. Apparatus

The data was recorded using the EyeLink 1000 eye tracker with a sampling frequency of 1000Hz [27]. EyeLink 1000 provides drift free eye tracking with a spatial resolution of 0.01º, and 0.25-0.5º of positional accuracy. EyeLink 1000 enables an eye to camera distances between 60 and 150cm and horizontal and vertical operating range of 55° and 45° respectively. The eye tracker was connected to a 20' CRT monitor capable of presenting visual stimulus with the refresh rate of 160Hz. During the recording screen resolution was set to 1024x768pix. To ensure high accuracy of the eye movement recording a chin rest was employed. The chin rest was positioned to assure 70cm distance between the display surface and the eyes of the subject.

B. Eye Movement Invocation Task

The goal of the stimulus was to invoke a large number of vertical and horizontal saccades. During the experiment, each subject was presented with a step stimulus displayed as a jumping dot, consisting of a grey disc sized approximately 1º with a small black point in the center. The dot performed 100 jumps horizontally and 100 jumps vertically, with a spatial amplitude of 20º in each direction. Before each sub-sequent jump the dot was displayed for the period of 1s.

C. Participants:

A total of 46 participants (24 males/ 22 females), ages 18 – 25 years with an average age of 24.9 (SD=6.06), volunteered for the project from the Texas State University campus. Mean positional accuracy of the recordings was 0.76º (SD=0.61º) along the x-axis and 1.74º ((SD=2.73 º) and a mean invalid data percentage of 12.43% (SD=17.22%). Only saccades with amplitudes of 17-22º were employed for biometric identification. Classified saccade trajectories were downsampled to 120Hz for faster execution of the minimization algorithm.

D. Performance evaluation metrics

Two metrics were employed for the assessment of the accuracy of each biometrics scheme:

False Acceptance Rate (FAR) – expresses, in general, the probability that a given individual is falsely accepted into the system while he shouldn't. In the following work, this rate was computed as the number of rejection tests that failed while identifying 2 different subjects divided by the total number of rejection tests performed. A rejection test fails when it identifies 2 different subjects as being the same person.

False Rejection Rate (FRR) – expresses, in general, the probability that a given individual is falsely rejected from the system while it should be accepted. In this work, the FRR was computed as the number of acceptance tests that failed while identifying the same person divided by the total number of acceptance tests performed. An acceptance test fails when it identifies 2 datasets from the same person as being from different people.

IV. Results

The identification results were broken into the three categories: KNN, C4.5, and Hotelling's T-square test. Within each category the results are separated to the OP properties obtained by the 1D-OP and the 2D-OP models.

A. KNN

1) 1D-OP

Table 1 presents the identification results. Prefix "h_" indicates that corresponding parameter is estimated from the horizontal component of movement. Parameter 1D indicates that the vector distance computed by the KNN method was obtained employing all horizontal OP properties.

OP param.	h_LR_p	h_LR_s	h_K_{LT}	h_K_{se}	h_B_P	h_B_{AG}	h_K_p	h_J	1D
FAR	34%	38.5%	26.4%	29.7%	12.5%	24.5%	33%	36.3%	39.6%
FRR	84.2%	94.7%	65.8%	73.7%	61.5%	65.8%	81.6%	89.5.5%	97.4%

Table 1. Identification results for KNN with 1D-OP model

The results confirm poor performance of the KNN classification method even in case of a more accurate eye tracking hardware which was employed for this study. Poor performance for the 1D metric can be explained due to contribution of the larger distances between samples introduced by some metrics, e.g, h_LR_s, h_J, which inflate overall vector distance reducing identification performance.

2) 2D-OP

Table 2 presents identification results. Prefix "v_" indicates that corresponding parameter is estimated from the vertical component of movement. Parameter 2D indicates that the vector distance computed by the KNN method was obtained employing all horizontal and vertical OP properties.

OP param.	v_LR_p	v_LR_s	v_K_{LT}	v_K_{se}	v_B_P	v_B_{AG}	v_K_p	v_J	2D
FAR	0%	0%	29.2%	14.6%	37.5%	45.8%	22.9%	41.7%	35.4%
FRR	34.2%	34.2%	73.7%	52.6%	84.2%	94.7%	63.2%	86.8%	78.9%

Table 2. Identification results for KNN with 2D-OP model

The results indicate that the vertical OP properties can provide higher accuracy of the identification than the vertical properties. The use of all OP properties yielded better identification results than the case when only horizontal properties were considered. However, individual OP properties provided better accuracy than the combined distance, i.e, 1D and 2D cases, due to the reasons explained in the previous subsection.

B. C4.5

C4.5 classification algorithm did not provide acceptable identification results achieving the FAR of 79% and the FRR of 8% in the 1D-OP case. Similar to the KNN classification, better eye tracking equipment did not improve the identification results. The use of the 2D-OP model decreased the accuracy of the identification providing the FAR of 0% and the FRR of 100%. We hypothesize that this reduction in the accuracy was due to the multi level nature of the decision tree, where at each level of the tree the testing sample should meet the criteria (minimum distance to the computed average) to be considered in the next level or otherwise be rejected. High amount of variability present in the OP properties does not allow to meet the criteria therefore resulting in poor identification results. The difference between the 1D-OP and the 2D-OP results can be attributed to the shorter height of the decision tree in the 1D-OP case and the smaller number of the OP properties considered. Shorter height translates to the smaller number of tests during tree traversal. Therefore, an individual is less likely to be rejected. At the same time, the distinction between two individuals is weaker because less properties are taken into consideration. As a result, an non-rejected individual is more likely to be mistakenly identified as someone else, explaining higher FAR.

C. *Hotelling's T*-square test

Figure 1 illustrates the ranges (vertical bars), standard deviations (rectangle dimensions) and the means (crosses) for the selected values of the OP properties within a single subject and between subjects. It is possible to see that the range and standard deviations are smaller within subjects and larger between subjects, therefore empirically confirming the applicability of the statistical approach to the person identification problem using the OP properties. Our analysis indicates that such properties as passive elasticity (K_p), series elasticity (K_{se}), length tension (K_{lt}), and force velocity properties in the agonist muscle (B_{AG}) have higher variability between individuals, therefore providing higher potential for

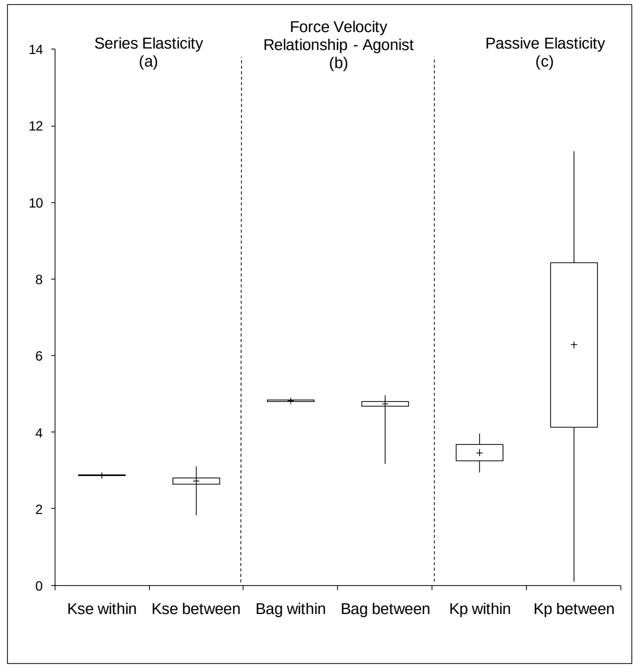

Fig. 4. Properties of the OP properties for between and within subjects. Part (a) illustrates series elasticity - K_{se}, (b) damping component responsible for the force velocity relationship in the agonist muscle - B_{AG}, (c) passive elasticity - K_p

the identification.

In the following subsections results of the identification performed by the Hotelling's T-square test are presented for the combinations of the OP properties that provide five best performance for each type of the OPMM model employed.

1) 1D-OP

OP param	h_K_{se}, h_K_p	h_K_{lt}, h_K_{se}, h_B_{AG}	h_K_{se}, h_B_p
FAR	0.5%	0.8%	1.2%
FRR	13.6%	13.6	13.6%

Table 3. Identification results for the Hotelling's T-square test with 1D-OP model

Hotelling's T-square test allowed to achieve an order of magnitude improvement in identification accuracy when compared to the KNN and the C4.5 classification methods.

2) 2D-OP

OP param	v_K_{lt}, v_K_{se}, v_K_p	h_B_{AG}, v_K_{lt}, v_B_{AG}	h_K_{se}, h_K_p, v_B_{AG}, v_K_p
FAR	3.2%	1.9%	0%
FRR	4.5%	6.8%	9.1%

Table 4. Identification results for Hotelling's T-square test with 2D-OP model

The use of the 2D-OP model allowed to improve the accuracy of the identification by considering the OP properties that are responsible for the vertical component of movement. In the best case, such approach reduced false acceptance rate to 0% and false rejection rate to 9.1%. These results support the original hypothesis that a multivariate statistical test is the right tool to address the variability issue and to provide better identification results.

The results allow us to identify OP properties with the higher potential for biometrics. These properties are series elasticity (K_{se}), passive elasticity (K_p), length tension (K_{lt}), and force velocity properties in the agonist muscle (B_{AG}). The results confirm original variability results illustrated by Fig. 5.

V. Conclusion and Further Work

We have introduced a statistical approach to the biometric identification via static and dynamic properties of the Oculomotor Plant (OP). Specifically such properties as the eye globe's inertia, dependency of an individual muscle's force on its length and velocity of contraction, resistive properties of the eye globe, muscles and ligaments, frequency characteristics of the neuronal control signal sent by the brain to the extraocular muscles were employed for the biometric purposes. This novel method of biometric identification is highly counterfeit resistant due the fact that the OP properties only exist in an alive individual.

As a result of our experiments we have confirmed that previously employed methods such as the k-nearest neighbor (KNN) and the decision tree (C4.5) do not show improvement in identification accuracy if a more precise eye tracking equipment is used. The final identification results are not sufficient to rate the performance of these methods as acceptable. The use of the two dimensional model of the OP slightly improved the identification results compared to the one dimensional model for the KNN. However, in case of the decision tree (C4.5) approach identification resulted in a failure rejecting all authentic users and accepting all impostors.

We have proposed and employed Hotelling's T-square test for the identification purposes addressing the variability of the estimated OP properties. As a result there was the order of magnitude improvement in the identification results, when compared to the best performing cases of KNN and C4.5. The use of the two dimensional OP model further increased the accuracy of the identification preventing all imposters from accessing the system and rejecting just 9.1% of the authentic users. These results indicate that novel biometrics method based on the dynamic and static properties of the OP is a promising direction of research, requiring larger user studies, different eye tracking setups, various types of stimulus presentation and the studies that would investigate the usability of the proposed method.

As a closing note we would like to state that that in addition to the eye gaze position coordinates, the eye tracker provides the picture of the iris, therefore, allowing to combine biometric identification based on the iris and the OP properties together potentially improving the reliability and the security of the eye based biometrics.

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