Electronic Journal of Differential Equations, Vol. 2004(2004), No. 138, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

EXISTENCE RESULTS FOR ELLIPTIC SYSTEMS INVOLVING CRITICAL SOBOLEV EXPONENTS

MOHAMMED BOUCHEKIF, YASMINA NASRI

Abstract. In this paper, we study the existence and nonexistence of positive solutions of an elliptic system involving critical Sobolev exponent perturbed by a weakly coupled term.

1. Introduction

We establish conditions for existence and nonexistence of nontrivial solutions to the system

$$\begin{gathered} -\Delta u = (\alpha+1)u^{\alpha}v^{\beta+1} + \mu(\alpha'+1)u^{\alpha'}v^{\beta'+1} \quad \text{in } \Omega \\ -\Delta v = (\beta+1)u^{\alpha+1}v^{\beta} + \mu(\beta'+1)u^{\alpha'+1}v^{\beta'} \quad \text{in } \Omega \\ u>0, \quad v>0 \quad \text{in } \Omega \\ u=v=0 \quad \text{on } \partial\Omega, \end{gathered} \tag{1.1}$$

where Ω is a bounded regular domain of $\mathbb{R}^N$ $(N \geq 3)$ with smooth boundary $\partial\Omega$, $\mu \in \mathbb{R}$, α, β, α', β' are positive constants such that $\alpha+\beta = \frac{4}{N-2}$ and $0 \leq \alpha'+\beta' < \frac{4}{N-2}$.

In the scalar case, the problem

$$\begin{gathered} -\Delta u = u^p + \mu u^q \quad \text{in } \Omega \\ u>0 \quad \text{in } \Omega \\ u=0 \quad \text{on } \partial\Omega, \end{gathered} \tag{1.2}$$

has been considered by several authors. The paper of Brezis-Nirenberg [7] has drawn our attention.

In [7], they have obtained the following results: Suppose that Ω is a bounded domain in $\mathbb{R}^N$, $N \geq 3$, $p = \frac{N+2}{N-2}$, $q=1$ and let $\lambda_1 > 0$ denote the first eigenvalue of the operator $-\Delta$ with homogeneous Dirichlet boundary conditions.

(1) If $N \geq 4$, then for any $\mu \in (0, \lambda_1)$ there exists a solution of (1.2).
(2) If $N = 3$, there exists $\mu^* \in (0, \lambda_1)$ such that for any $\mu \in (\mu^*, \lambda_1)$ problem (1.2) admits a solution.

2000 *Mathematics Subject Classification.* 35J20, 35J50, 35J60.
Key words and phrases. Elliptic system; critical Sobolev exponent; variational method; moutain pass theorem.

Submitted July 6, 2004. Published November 25, 2004.

(3) If $N = 3$ and Ω is a ball, then $\mu^* = \frac{\lambda_1}{4}$ and for $\mu \le \frac{\lambda_1}{4}$ problem (1.2) has no solution.

They have also obtained the following results for $1 < q < \frac{N+2}{N-2}$:

(a) There is no solutions of (1.2) when $\mu \le 0$ and Ω is a starshaped domain.
(b) When $N \ge 4$, (1.2) has at least one solution for every $\mu > 0$.
(c) When $N = 3$, We distinguish two cases:
(i) If $3 < q < 5$, then for every $\mu > 0$ there is a solution of (1.2).
(ii) If $1 < q \le 3$, then for every μ large enough there is a solution of (1.2). Moreover, (1.2) has no solution for every small $\mu > 0$ when Ω is strictly starshaped.

In the vectorial case, Alves et al. [1] and Bouchekif and Nasri [4] have extended the results of [7] to elliptic system. A number of works contributed to study the elliptic system for example: Boccardo and de Figueiredo [3], de Thélin and Vélin [11] and Conti et al. [8].

Our aim is to generalize the results of [7] to an elliptic system when the lower order perturbation of $u^{\alpha+1}v^{\beta+1}$ for each equation is weakly coupled i. e.

$$-\vec{\Delta} U = \nabla H + \mu \nabla G,$$

where

$$\vec{\Delta} = \begin{pmatrix} \Delta \\ \Delta \end{pmatrix}, \quad H(u,v) = u^{\alpha+1}v^{\beta+1}, \quad U = \begin{pmatrix} u \\ v \end{pmatrix},$$

$G(u,v) = u^{\alpha'+1}v^{\beta'+1}$ and μ is a real parameter.

Our main results are stated as follows :

Theorem 1.1. *If $\alpha + \beta = \frac{4}{N-2}$; $0 \le \alpha' + \beta' < \frac{4}{N-2}$; $\mu \le 0$ and Ω is a starshaped domain, then* (1.1) *has no solution.*

Theorem 1.2. *We suppose that $N \ge 4$ and $\alpha + \beta = \frac{4}{N-2}$. We have:*

- *If $0 < \alpha' + \beta' < \frac{4}{N-2}$, then for every $\mu > 0$ problem* (1.1) *has at least one solution.*
- *If $\alpha' + \beta' = 0$, then for every $0 < \mu < \lambda_1$ problem* (1.1) *has a solution.*

Theorem 1.3. *Assume that $N = 3$ and $\alpha + \beta = 4$. We distinguish two cases:*

- *If $2 < \alpha' + \beta' < 4$, then for every $\mu > 0$ problem* (1.1) *has a solution.*
- *If $0 < \alpha' + \beta' \le 2$, then for every μ large enough there exists a solution to problem* (1.1).

The paper is organized as follows. Section 2 contains some preliminaries and notations. Section 3 contains the proof of nonexistence result. Section 4 deals with the existence theorems proofs.

2. Preliminaries

Lemma 2.1 (Pohozaev identity [10])**.** *Suppose that $(u,v) \in [C^2(\Omega)]^2$ is the solution to the problem*

$$\begin{gathered} -\Delta u = \frac{\partial F}{\partial u}(u,v) \quad \text{in } \Omega \\ -\Delta v = \frac{\partial F}{\partial v}(u,v) \quad \text{in } \Omega \\ u = v = 0 \quad \text{on } \partial\Omega, \end{gathered}$$

where Ω is a bounded domain in $\mathbb{R}^N$ $(N \geq 3)$ with smooth boundary $\partial\Omega$, $F \in C^1(\mathbb{R}^2)$, $F(0,0) = 0$, then we have

$$\int_{\partial\Omega} (|\frac{\partial u}{\partial \nu}|^2 + |\frac{\partial v}{\partial \nu}|^2) x\nu d\sigma + (N-2)\Big[\int_\Omega (u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v})dx\Big] = 2N\int_\Omega F(u,v)dx \quad (2.1)$$

where ν denotes the exterior unit normal.

We shall use the following version of the Brezis-Lieb lemma [6].

Lemma 2.2. *Assume that $F \in C^1(\mathbb{R}^N)$ with $F(0) = 0$ and $|\frac{\partial F}{\partial u_i}| \leq C|u|^{p-1}$. Let $(u_n) \subset L^p(\Omega)$ with $1 \leq p < \infty$. If (u_n) is bounded in $L^p(\Omega)$ and $u_n \to u$ a.e. on Ω, then*

$$\lim_{n\to\infty} \Big(\int_\Omega F(u_n) - F(u_n - u)\Big) = \int_\Omega F(u).$$

Let us define:

$$S_{\alpha+\beta+2} = S_{\alpha+\beta+2}(\Omega) := \inf_{u \in H_0^1(\Omega)\setminus\{0\}} \frac{\int_\Omega |\nabla u|^2 dx}{(\int_\Omega |u|^{\alpha+\beta+2}dx)^{\frac{2}{\alpha+\beta+2}}}$$

$$S_{\alpha,\beta} = S_{\alpha,\beta}(\Omega) := \inf_{(u,v)\in [H_0^1(\Omega)]^2\setminus\{(0,0)\}} \frac{\int_\Omega (|\nabla u|^2 + |\nabla v|^2)dx}{(\int_\Omega |u|^{\alpha+1}|v|^{\beta+1}dx)^{\frac{2}{\alpha+\beta+2}}}.$$

Lemma 2.3 ([1])**.** *Let Ω be a domain in $\mathbb{R}^N$ (not necessarily bounded) and $\alpha+\beta \leq \frac{4}{N-2}$, then we have*

$$S_{\alpha,\beta} = \Big[\big(\frac{\alpha+1}{\beta+1}\big)^{\frac{\beta+1}{\alpha+\beta+2}} + \big(\frac{\alpha+1}{\beta+1}\big)^{\frac{-\alpha-1}{\alpha+\beta+2}}\Big] S_{\alpha+\beta+2}.$$

Moreover, if $S_{\alpha+\beta+2}$ is attained at ω_0, then $S_{\alpha,\beta}$ is attained at $(A\omega_0, B\omega_0)$ for any real constants A and B such that $\frac{A}{B} = (\frac{\alpha+1}{\beta+1})^{1/2}$.

We adopt the following notation:

- For $p > 1$, $\|u\|_p = [\int_\Omega |u|^p dx]^{\frac{1}{p}}$;
- $H_0^1(\Omega)$ is the Sobolev space endowed with the norm $\|u\|_{1,2} = [\int_\Omega |\nabla u|^2 dx]^{1/2}$;
- $\|(u,v)\|_E^2 := \|u\|_{1,2}^2 + \|v\|_{1,2}^2$;
- $E := [H_0^1(\Omega)]^2$;
- E' denotes the dual of E;
- $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent;
- $u^+ := \max(u, 0)$ and $u^- = u^+ - u$.

The functional associated to problem (1.1) is written as

$$J(u,v) := \frac{1}{2}\|(u,v)\|_E^2 - \int_\Omega (u^+)^{\alpha+1}(v^+)^{\beta+1}dx - \mu\int_\Omega (u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx. \quad (2.2)$$

3. Nonexistence result

Theorem 1.1 is a direct consequence of the Pohozaev identity.

Proof of Theorem 1.1. Arguing by contradiction. Suppose that problem (1.1) has a solution $(u,v) \neq (0,0)$, applying Lemma 2.1 and putting

$$F(u,v) = H(u,v) + \mu G(u,v),$$

the expression (2.1) becomes

$$\int_{\partial\Omega}(|\frac{\partial u}{\partial\nu}|^2+|\frac{\partial v}{\partial\nu}|^2)x\nu\,d\sigma=\mu\,[2N-(N-2)(\alpha'+\beta'+2)]\int_\Omega|u|^{\alpha'+1}|v|^{\beta'+1}dx.$$

Since $2N-(N-2)(\alpha'+\beta'+2)>0$ and the fact that Ω is starshaped with respect to the origin, we get

$$0\leq\int_{\partial\Omega}(|\frac{\partial u}{\partial\nu}|^2+|\frac{\partial v}{\partial\nu}|^2)x\nu\,d\sigma<0.$$

A contradiction. Hence (1.1) has no a solution for $\mu\leq 0$. □

4. Existence results

The proof of Theorems 1.2 and 1.3 are based on the following Ambrosetti-Rabinowitz result [2].

Lemma 4.1 (Mountain Pass Theorem)**.** *Let J be a C^1 functional on a Banach space E. Suppose there exits a neighborhood V of 0 in E and a positive constant ρ such that*

(i) *$J(u,v)\geq\rho$ for every U in the boundary of V.*
(ii) *$J(0,0)<\rho$ and $J(\varphi,\psi)<0$ for some $\Psi:=(\varphi,\psi)\notin V$.*

We set

$$c=\inf_{\phi\in\Gamma}\max_{t\in[0,1]}J(\phi(t))$$

with $\Gamma=\{\phi\in C([0,1],E):\phi(0)=0,\,\phi(1)=\Psi\}$. Then there exists a sequence (u_n,v_n) in E such that $J(u_n,v_n)\to c$ and $J'(u_n,v_n)\to 0$ in E'.

Proof. Using Holder's inequality and Sobolev injection, we obtain that

$$\begin{aligned}J(u,v)&=\frac{1}{2}\|(u,v)\|_E^2-\int_\Omega(u^+)^{\alpha+1}(v^+)^{\beta+1}dx-\mu\int_\Omega(u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx\\&\geq\frac{1}{2}\|(u,v)\|_E^2-A\|(u,v)\|_E^{2^*}-B\|(u,v)\|_E^{\alpha'+\beta'+2}\end{aligned}$$

where A and B are positive constants.

If $\alpha'+\beta'>0$ then (i) is satisfying for small norm $\|(u,v)\|_E=R$. If $\alpha'+\beta'=0$, we have

$$J(u,v)\geq\frac{1}{2}\big(1-\frac{\mu}{\lambda_1}\big)\|(u,v)\|_E^2-A\|(u,v)\|_E^{2^*}$$

and condition (i) is still satisfied for $\mu<\lambda_1$ and $R<(\frac{1-\frac{\mu}{\lambda_1}}{2A})^{\frac{1}{2^*-2}}$. For any $(\varphi,\psi)\in E$ with $\varphi\neq 0$ and $\psi\neq 0$, we have that $\lim_{t\to+\infty}J(t\varphi,t\psi)=-\infty$. Thus, there are many (φ,ψ) satisfying (ii). It will be important to use with a special $(\varphi,\psi):=(t_0\varphi_0,t_0\psi_0)$ for some $t_0>0$ chosen large enough so that $(\varphi,\psi)\notin V$, $J(\varphi,\psi)<0$ and $\sup_{t\geq 0}J(t\varphi,t\psi)<\frac{2^*}{N}(\frac{S_{\alpha,\beta}}{2^*})^{N/2}$. Then there exists a sequence $(u_n,v_n)\in E$ such that $J(u_n,v_n)\to c$ and $J'(u_n,v_n)\to 0$ in E'. □

Lemma 4.2. *Suppose $\mu>0$ and let (u_n,v_n) be a sequence in E such that $J(u_n,v_n)\to c$ and $J'(u_n,v_n)\to 0$ in E' with*

$$c<\frac{2^*}{N}(\frac{S_{\alpha,\beta}}{2^*})^{N/2}=\frac{2}{N-2}(\frac{S_{\alpha,\beta}}{2^*})^{N/2}$$

Then (u_n,v_n) is relatively compact in E.

Proof. We show that the sequence (u_n, v_n) is bounded in E. Since (u_n, v_n) satisfies:

$$\frac{1}{2}\|(u_n,v_n)\|_E^2 - \int_\Omega (u_n^+)^{\alpha+1}(v_n^+)^{\beta+1}dx - \mu\int_\Omega (u_n^+)^{\alpha'+1}(v_n^+)^{\beta'+1}dx = c + o(1) \quad (4.1)$$

and

$$\begin{aligned}&\|(u_n,v_n)\|_E^2 - 2^*\int_\Omega (u_n^+)^{\alpha+1}(v_n^+)^{\beta+1}dx - \mu(\alpha'+\beta'+2)\int_\Omega (u_n^+)^{\alpha'+1}(v_n^+)^{\beta'+1}dx\\&= \langle \varepsilon_n, (u_n,v_n)\rangle\end{aligned} \quad (4.2)$$

with $\varepsilon_n \to 0$ in E'. Combining (4.1) and (4.2), we obtain

$$\begin{aligned}&(\frac{2^*}{2}-1)\int_\Omega (u_n^+)^{\alpha+1}(v_n^+)^{\beta+1}dx + \mu(\frac{\alpha'+\beta'}{2})\int_\Omega (u_n^+)^{\alpha'+1}(v_n^+)^{\beta'+1}dx\\&\le c + o(1) + \|\varepsilon_n\|_{E'}\|(u_n,v_n)\|_E.\end{aligned} \quad (4.3)$$

From this inequality, we obtain

$$\begin{gathered}\int_\Omega (u_n^+)^{\alpha+1}(v_n^+)^{\beta+1}dx \le C\,,\\ \int_\Omega (u_n^+)^{\alpha'+1}(v_n^+)^{\beta'+1}dx \le C\,.\end{gathered}$$

Where C is any generic positive constant. Therefore, the sequence (u_n, v_n) is bounded in E. By the Sobolev embedding Theorem, there exists a subsequence again denoted by (u_n, v_n) such that

- $(u_n, v_n) \to (u, v)$ weakly in E
- $(u_n, v_n) \to (u, v)$ strongly in $L^r \times L^q$ for $2 \le r, q < 2^*$
- $(u_n, v_n) \to (u, v)$ a. e. on Ω.

Since $w_n := u_n^\alpha v_n^{\beta+1}$ and $t_n := u_n^{\alpha+1}v_n^\beta$ are bounded sequences in $[L^{\frac{2^*}{2^*-1}}(\Omega)]^2$, these sequences converge to $w := u^\alpha v^{\beta+1}$ and to $t := u^{\alpha+1}v^\beta$ respectively. Passing to the limit, we obtain

$$\begin{aligned}-\Delta u &= (\alpha+1)(u^+)^\alpha(v^+)^{\beta+1} + \mu(\alpha'+1)(u^+)^{\alpha'}(v^+)^{\beta'+1}\\ -\Delta v &= (\beta+1)(u^+)^{\alpha+1}(v^+)^\beta + \mu(\beta'+1)(u^+)^{\alpha'+1}(v^+)^{\beta'}\end{aligned}$$

i.e

$$\|(u,v)\|_E^2 = 2^*\int_\Omega (u^+)^{\alpha+1}(v^+)^{\beta+1}dx + \mu(\alpha'+\beta'+2)\int_\Omega (u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx$$

Moreover,

$$J(u,v) = (\frac{2^*}{2}-1)\int_\Omega (u^+)^{\alpha+1}(v^+)^{\beta+1}dx + \mu(\frac{\alpha'+\beta'}{2})\int_\Omega (u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx \ge 0.$$

We put

$$u = u_n + \varphi_n,\ v = v_n + \psi_n \quad \text{and} \quad H(u_n,v_n) = u_n^{\alpha+1}v_n^{\beta+1}$$

Applying Lemma 2.2 for $H(u_n, v_n)$ and the following two relations (Brezis-Lieb [6])

$$\begin{gathered}\|u_n\|^2 = \|u-\varphi_n\|^2 = \|u\|^2 + \|\varphi_n\|^2 + o(1)\,,\\ \|v_n\|^2 = \|v-\varphi_n\|^2 = \|v\|^2 + \|\psi_n\|^2 + o(1),\end{gathered}$$

we obtain

$$J(u,v)+\frac{1}{2}\|(\varphi_n,\psi_n)\|_E^2-\int_\Omega H(\varphi_n^+,\psi_n^+)dx=c+o(1) \tag{4.4}$$

and

$$\begin{aligned}\|(\varphi_n,\psi_n)\|_E^2+\|(u,v)\|_E^2&=2^*\big[\int_\Omega (H(u^+,v^+)+H(\varphi_n^+,\psi_n^+))dx\big]\\&\quad+\mu(\alpha'+\beta'+2)\int_\Omega (u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx+o(1).\end{aligned} \tag{4.5}$$

From this equality, we deduce

$$\|(\varphi_n,\psi_n)\|_E^2=2^*\int_\Omega H(\varphi_n^+,\psi_n^+)dx+o(1).$$

We may therefore assume that

$$\|(\varphi_n,\psi_n)\|_E^2\to k \quad\text{and}\quad 2^*\int_\Omega H(\varphi_n^+,\psi_n^+)dx\to k.$$

By the Sobolev inequality,

$$\|(\varphi_n,\psi_n)\|_E^2\geq S_{\alpha,\beta}\Big(\int_\Omega \left(\varphi_n^+\right)^{\alpha+1}(\psi_n^+)^{\beta+1}dx\Big)^{\frac{2}{2^*}}.$$

In the limit, $k\geq S_{\alpha,\beta}(\frac{k}{2^*})^{2/2^*}$. It follows that either $k=0$ or $k\geq 2^*(\frac{S_{\alpha,\beta}}{2^*})^{N/2}$.

We show that $(u_n,v_n)\to(u,v)$ strongly in E i. e. $(\varphi_n,\psi_n)\to(0,0)$ strongly in E. Suppose that $k\geq 2^*(\frac{S_{\alpha,\beta}}{2^*})^{N/2}$. Since

$$J(u,v)+\frac{k}{N}=c$$

and $J(u,v)\geq 0$, then $\frac{k}{N}\leq c$ i.e. $c\geq\frac{2^*}{N}(\frac{S_{\alpha,\beta}(\Omega)}{2^*})^{N/2}$ in contradiction with the hypothesis. Thus $k=0$ and $(u_n,v_n)\to(u,v)$ strongly in E. □

Proof of Theorem 1.2. It suffices to apply the mountain pass theorem with the value $c<\frac{2^*}{N}(\frac{S_{\alpha,\beta}(\Omega)}{2^*})^{N/2}$. We have to show that this geometric condition on c is satisfied. Following the method in [7]. Without loss of generality we assume that $0\in\Omega$, we use the test function

$$\omega_\varepsilon(x)=\frac{\varphi(x)}{(\varepsilon+|x|^2)^{\frac{N-2}{2}}}\,,\quad \varepsilon>0$$

where φ is a cut-off positive function such that $\varphi\equiv 1$ in a neighborhood of 0. Let A and B be positive constants such that

$$\frac{A}{B}=(\frac{\alpha+1}{\beta+1})^{1/2}$$

then $(A\omega_\varepsilon,B\omega_\varepsilon)$ is a solution of

$$\begin{gathered}-\Delta u=(\alpha+1)u^\alpha v^{\beta+1}\quad\text{in }\mathbb{R}^N\\-\Delta v=(\beta+1)u^{\alpha+1}v^\beta\quad\text{in }\mathbb{R}^N\\u(x)=0,\quad v(x)=0\quad\text{as }|x|\to+\infty\end{gathered}$$

By [7, lemma 1], we obtain

$$\sup_{t\geq 0}J(tA\omega_\varepsilon,tB\omega_\varepsilon)\leq\frac{2^*}{N}\big(\frac{S_{\alpha,\beta}}{2^*}\big)^{N/2}+O\big(\varepsilon^{\frac{N-2}{2}}\big)-\mu K\varepsilon^\theta$$

where K is a positive constant independent of ε, and $\theta := (4-(\alpha'+\beta')(N-2))/4$.

For $\theta < \frac{N-2}{2}$ if $N > 4$ the inequality is satisfying for all $0 \le \alpha' + \beta' < \frac{4}{N-2}$. Thus we obtain

$$\sup_{t\ge 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) < \frac{2^*}{N}\Big(\frac{S_{\alpha,\beta}}{2^*}\Big)^{N/2} \quad \text{for } \varepsilon > 0 \text{ small enough.}$$

Then problem (1.1) has a solution for every $\mu > 0$.

For $N = 4$, we distinguish two cases. Case 1: We have $\theta < 1$ for all $\alpha' + \beta' > 0$. Case 2: If $\alpha' + \beta' = 0$, we obtain

$$\sup_{t\ge 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) \le \Big(\frac{S_{\alpha,\beta}}{4}\Big)^2 + O(\varepsilon) - \mu K\varepsilon|\log\varepsilon|,$$

so for $\varepsilon > 0$ small enough, $\sup_{t\ge 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) < (\frac{S_{\alpha,\beta}}{4})^2$.

Note that the maximum principle ensures the positivity of solution. $\square$

Proof of Theorem 1.3. In three dimension the situation is different. We have

$$\sup_{t\ge 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) \le 2\Big(\frac{S_{\alpha,\beta}}{6}\Big)^{3/2} + O(\varepsilon^{1/2}) - \mu K\varepsilon^\theta.$$

In this case we distinguish two cases.

(i) $0 < \theta < \frac{1}{2}$ if $2 < \alpha' + \beta' < 4$,

(ii) $\theta \ge \frac{1}{2}$ if $0 < \alpha' + \beta' \le 2$.

In case (i) we have the same conclusion as in the previous proof for $(N \ge 4)$. So for the case $0 < \alpha' + \beta' \le 2$, the existence of positive solution is assured for μ large enough. It follows that $\sup_{t\ge 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) < 2(\frac{S_{\alpha,\beta}}{6})^{3/2}$. Thus (1.1) has a solution. $\square$

References

[1] C. O. Alves, D. C. de Morais Filho and M. A. S. Souto; *On Systems of Elliptic equations involving subcritical or critical Sobolev exponents*, Nonlinear Anal. N. 5, Ser. Theory Methods, 42 (2000), 771–787.

[2] A. Ambrosetti and P.H. Rabinowitz; *Dual variational methods in critical point theory and applications*, J. Funct. Anal., 14 (1973), 349-381.

[3] L. Boccardo, D.G. de Figueiredo: *Some remarks on system of quasilinear elliptic equations*, Nonlinear diff. eq. appl. 9 (2002), pp. 309-323.

[4] M. Bouchekif and Y. Nasri; *On a class of elliptic system involving critical Sobolev exponent*, Preprint Tlemcen 2004.

[5] H. Brezis; *Some Variational Problems with Lack of Compactness*, Proceedings of Symposia in Pure Mathematics, Vol. 45 (1986), 165-201.

[6] H. Brezis and E. Lieb; *A relation between point wise convergence of functions and convergence of functionals*, Proc. A.M.S. Vol. 48, No. 3 (1993), 486-499.

[7] H. Brezis and L. Nirenberg; *Positive Solutions of Nonlinear Elliptic Equations Involving Critical Exponents*, Comm. Pure Appl. Math, Vol. 36 (1983), 437-477.

[8] M. Conti, L. Merizzi and S. Terracini; *On the existence of many solutions for a class of superlinear elliptic system*, J. Diff. Eq. 167 (2000), 357–387.

[9] S. I. Pohozaev; *Eigenfunctions of the equation* $\Delta u + \lambda f(u) = 0$, Nonlinearity Doklady Akad. Nauk SSRR 165, (1965), 9-36.

[10] P. Pucci and J. Serrin; *A general variational identity*, Indiana University Mathematics Journal, (1986), 681-703.

[11] F. de Thélin and J. Vélin; *Existence and nonexistence of nontrivial solutions for some nonlinear elliptic systems*, Revista Mathematica Universidad complutense de Madrid, vol. 6 (1993), 153-193.

Mohammed Bouchekif
Departement of Mathematics, University of Tlemcen B. P. 119 Tlemcen 13000, Algeria
E-mail address: m_bouchekif@mail.univ-tlemcen.dz

Yasmina Nasri
Departement of Mathematics, University of Tlemcen B. P. 119 Tlemcen 13000, Algeria
E-mail address: y_nasri@mail.univ-tlemcen.dz