A STUDY OF RADIAL RUNOUT FOR CIRCULAR GEOMETRIES

by

Devanda R. Lek, B.S.M.E.

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Committee Members:

Byoung Hee You, Chair

Yoo-Jae Kim

In-Hyouk Song

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ABSTRACT

A critical step in geometrical stacking for rotating machinery assembly requires mathematical representation of parts. Vector addition and Least-squares approximation were used to represent a part based on runout. Vector addition yielded a vector representing the net-effect of runout for a feature of a part. Least-squares yielded an approximate location of the physical center-point with respect to the ideal center-point. Representing a part mathematically would be a stepping stone to developing methods of geometrical stacking.

Two sets of 8-point runout data was generated to represent the forward and aft of a part. The data and collection was to mimic manual measurement techniques where dial indicators are used. After converting the points to polar form, vector addition and least-squares was applied. Both yielded vectors with angular location where inferences could be made with regards to the physical meaning.

For vector addition the resulting vector that represents the positions where runout had the greatest effect over the part. This could have been considered as a high point on the part. Least-squares was more easy to visualize as the vector represented the displaced of the physical center with respect to the ideal center of (0,0). It was noted that the angular locations of both methods were the same but this was due to the calculation methods used for least-square.

CHAPTER 1

INTRODUCTION

Motivation

The reduction of unbalance is a critical step in the assembly and operation of rotating assemblies and machinery. Unbalance is the uneven distribution of mass over the body of a part and is the result of variation. As unbalanced parts are rotated the uneven distribution of mass will create vibratory and centrifugal forces. These forces can deteriorate and increase stress on the parts (McMillan, 2003). Overtime excessive unbalance can lead to pre-mature or sudden failure of a part. In addition, unbalance will affect the operation efficiency of a part. Unbalance is reduced by attempting to redistribute mass over the part (McMillan, 2003; Sjoholm, 1998). This process is known as balancing and two main techniques are correction masses and geometrical stacking.

Balancing by mass addition or subtraction is a commonly used technique. The typical process requires a part or an assembly of parts to be rotated at or near operational speeds. As the part rotates its vibration characteristics are observed. These characteristics are be used to determine an amount of mass needed and where to place to properly redistribute the mass. Depending on the unbalance type it may require one or more correction masses. This process of measuring and adding mass is generally done until an acceptable amount of unbalance in achieved.

Correction mass addition is a commonly and commercially used technique. However its efficiency may be limited by the need for trial and error. The process is iterative requiring multiple instance of measuring, altering mass, assembly, and disassembly (Davidson and Wilcox, 1976; Forrestor and Wesling, 2002). Geometrical stacking was developed as a way to overcome or reduce the need for mass corrections. This is done by allowing the unbalance on one part to offset the unbalance on another part. Instead of measuring a part for unbalance a part is measured for its variation. The variations are used to represent the part mathematically and math is used to determine an assembly combination with an optimal or near-optimal amount of unbalance.

One limitation of geometrical stacking is as part count and position increase so the number of combinations. The sheer number of part combinations can reduce the ability to calculate an optimal assembly therefore practitioners may settle for a near-optimal assembly. In geometrical stacking runout is generally measured and used in the calculation process. However runout is not a dimension but a value that expresses the variation with respect to geometrical feature (ASME, 1994). For geometrical stacking to be applicable runout must be converted or translated so that it can represent a part.

Objective

The goal of this this study is to determine how a part could be represented based on its runout variation. Being able to simulate an assembly would help reduce the need for trial and error or the need for high speed balancing. The first objective is to determine if the net-effect or amount of runout can be approximated with vector addition. The second objective is to determine if the actual center can be approximated with least-squares. The third objective is to determine if offsetting the parts by 180° can reduce the net-effect of runout and displaced center-points. Representing a part mathematically on the basis of mechanical variation is a critical aspect for the development of stacking programs and techniques.

Scope of Study

The main goal was to determine if runout could be approximated using two calculations techniques. Runout data was generated to represent circular parts that were in a tolerance zone. The runout was considered to be only in the radial direction and planar variation was not considered. The generated parts were decomposed into polar coordinates to be applicable with the selected calculation methods. A part was represented by two sets of runout data; representing the forward and aft. The results of the calculations could be used to make inferences as to what physical meaning was being expressed by the calculation methods selected.

In this study only variation in the radial direction will be considered as circular geometries are the main focus. Therefore the study will exist in two-dimensions. Axial variation exists and is perpendicular to the radial variation. Axial variation requires the examination of flatness which would require moving into a third dimension (ANSI, 1972; ASME, 1994). Instead of fitting to a circle the measurements must be fitted to a plane.

As parts were mathematically represented, in vectors, assembly was simulated by adding the results to each other to see their net-effects. The effect of part variation over an entire assembly could be calculated. Combination could be generated altering the angles of the resulting vectors of each part. Combination by offsetting parts 180° was selected and other angular offsets were not considered. Offsetting correction mass 180° from the location of unbalance serves as the basis of offsetting parts during assembly (McMillan, 2003; Rao and Dukkipati, 1992).

CHAPTER 2

REVIEW OF LITERATURE

Unbalance

Rotating components will possess an: axis of rotation and a mass center. The axis of rotation is a line about which a part will rotate and the mass center is the point about which mass is evenly distributed, on any rotating object. If both the axis of rotation and mass center were along the exact geometric centerline of the rotating component, it is said to be perfectly balanced. For both characteristics to be perfectly centered, the mass of the part must be evenly distributed and geometrically perfect. However, it is unlikely that apart will possess even mass distribution.

The uneven distribution of mass will cause the axis of rotation and mass center to deviate from the geometric center point. Both the axis of rotation and mass center can independently deviate from the geometric center. The mass center can be translated from the geometric centerline while the axis of rotation can be translated or rotated from the geometric centerline. How the mass center and PIA are displaced and spatially related to each other will determine the type of unbalance.

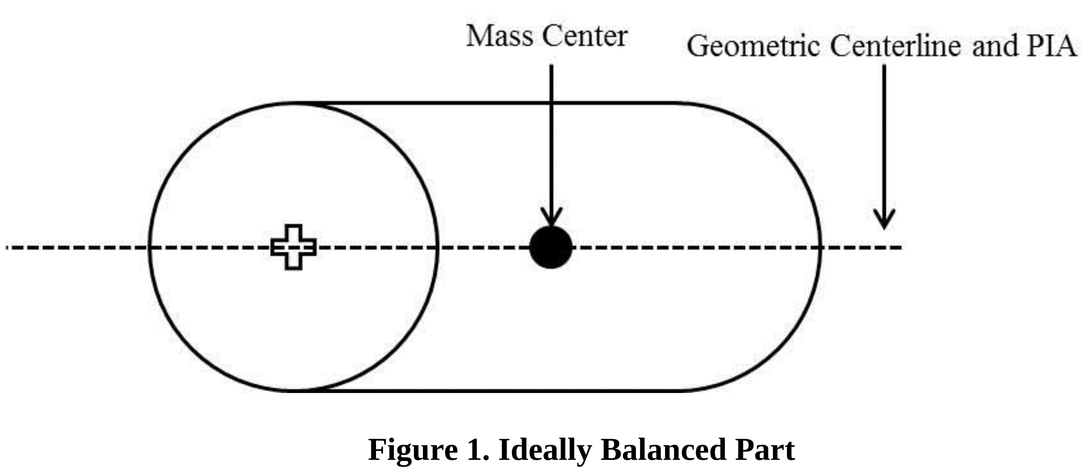

Figure 1. Ideally Balanced Part

Types of Unbalance

How the axis of rotation and mass center is displaced from the geometric centerline will create different types of unbalance. Therefore, unbalance is dependent of the spatial relationship among the axis of rotation, mass center, and geometric centerline. There are four basic types of unbalance a component can possess: static, couple, quasi-static, and dynamic (McMillan, 2003; Rao and Dukkipati, 1992). Static and couple unbalances are pure cases and less likely to occur while Quasi-static and dynamic unbalances are the result of a combination of displacements and are generally more likely to occur.

Static unbalance is when the mass center will have translated from the geometric center line. When the mass center is radially displaced the PIA will also be displaced, but parallel, to the geometric center as shown in Figure 2 (McMillan, 2003). Static unbalance is be corrected by adding one mass to redistribute mass.

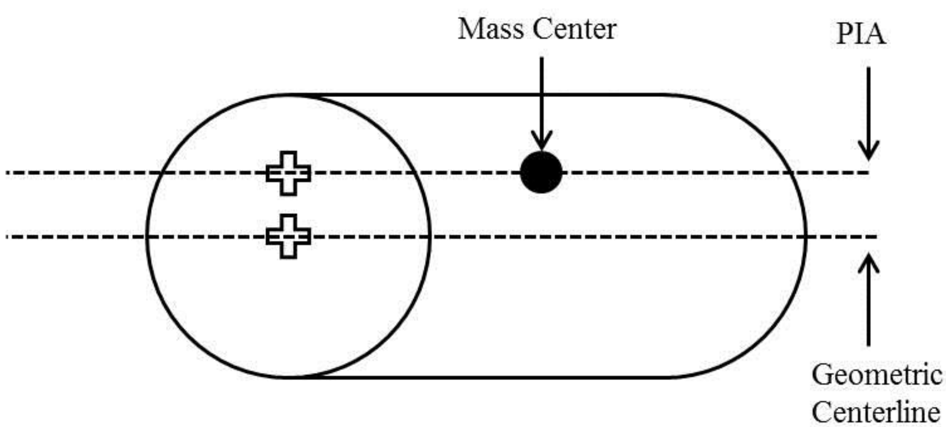

Figure 2. Static Unbalance

Couple unbalance is when the PIA intersects the mass center and is not parallel to the geometric center line as shown in Figure 3. The displacement is likely takes the form of a rotation about the mass center. Like static unbalance, pure couple unbalance is an ideal case and is unlikely. To correct couple unbalance an alteration of mass must be done at two points, 180 degrees apart, so that the PIA is essentially rotated so it is parallel to the geometric centerline.

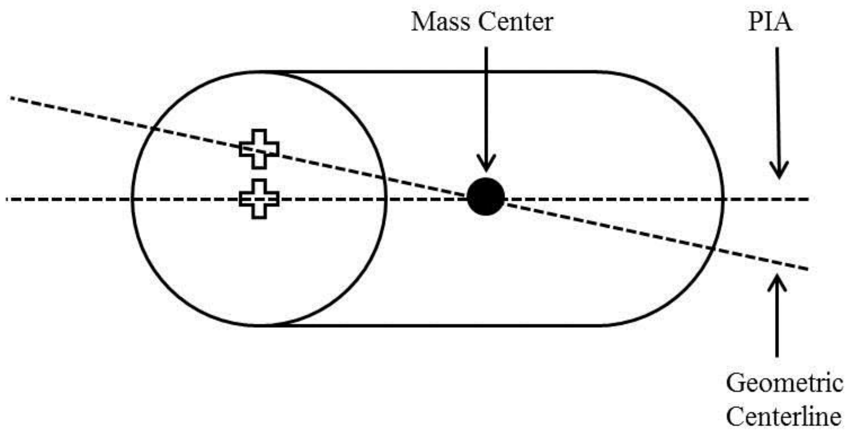

Figure 3. Couple Unbalance

Dynamic Unbalance the most common form of unbalance to exist in a rotating parts (Fox, 1980; McMillan, 2003). In dynamic unbalance the geometric centerline and PIA do not coincide with each other. The mass center is translated from the geometric centerline and the PIA is not parallel to the geometric centerline as shown in Figure 4.

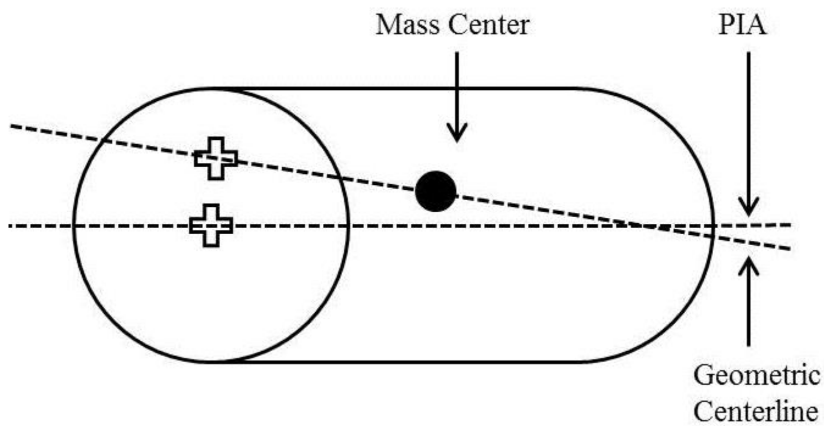

Figure 4. Dynamic Unbalance

Quasi-static is when the PIA intersect the geometric centerline but not at the mass center as shown in Figure 5 (McMillan, 2003). Unlike dynamic unbalance the PIA will intersect the shaft axis. Its rotation characteristics are similar to couple unbalance and require two correction masses to balance.

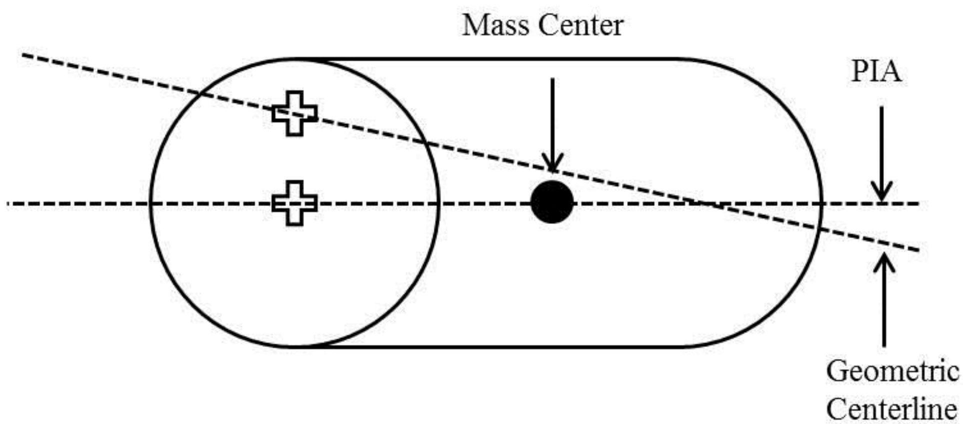

Figure 5. Quasi-Static Unbalance

<u>Issues with Unbalance</u>

Unbalance can significantly hinder the operation of rotating equipment and requires considerations during the assembly process. Any rotating component will generate a centripetal acceleration and the opposing centrifugal force (Rao and Dukkipati, 1989; Temple, 1983). The centrifugal force becomes an issue when mass is not evenly distributed over the part as it creates an unbalance forces. The resulting unbalance forces will then create vibratory noise and motion to be present during operation (Fox, 1980; Temple, 1983). In addition, the unbalance forces will increase the stress and loads on parts within the assembly.

Vibration is a critical issue for any rotating component and assembly. The vibration in itself is a disruptive force that can cause issues for performance and equipment life. As vibration is the result of centripetal forces it can exert excessive forces upon the components and the couplings (McMillan, 2003; Rao and Dukkipati, 1992). The excessive forces can increase the fatigue rate causing parts to degrade at a much faster rate than normal. Faster rate of degradation will increase the need for preventative maintenance and re-balancing procedures. Improper maintenance or balancing may lead to premature or even catastrophic failure.

Source of Unbalance: Variation

Variation is the deviation from a nominal specification as meeting them exactly is highly improbable (Fischer, 2011). Variation from design specification is will cause an uneven distribution of mass which will cause unbalance. All parts will possess some form of mechanical variation being slightly above or below the design specifications. In all assemblies variation requires consideration as it can affect the functionality. Since parts will not conform to specifications it is important to determine what amounts of variation is acceptable (ASME, 1994). The acceptable variation is called a tolerance and their analysis is critical to any assembly.

There are many causes of variation for any machined part. One significant source of variation is resolution of material processing techniques (Fischer, 2011; Ramaswami, 2011). Machine tools and their operators may be limited in their ability to precisely process the part to exact specifications. In addition to machining errors, the material properties and quality can cause defects as they may have previously existed or may be common to specific types of materials. Variation can also be a result of long periods of operation conditions (Fox, 1980). A rotating assembly that operates with some level of unbalance and high operation temperatures may experience part distortion and contaminations. As a part distorts the unbalance may increase even if the part had been previously balanced.

Circularity

For rotating components and assemblies there is an emphasis on circular geometry as the measurements tend to be related to the rotational axis. One measurement often used for rotating assemblies is circularity. An ideal component has circularity (roundness), meaning that all points along the circumferences are equidistant from the geometric center (ANSI, 1972; ASME, 1994). When variation between the radial measurements vary from the ideal radius, the part possess out-of-roundness (ANSI, 1972). Circularity, or lack thereof, will vary over the entire body of a part. Measuring the radius from multiple points may yield multiple measurements as shown in Figure 6.

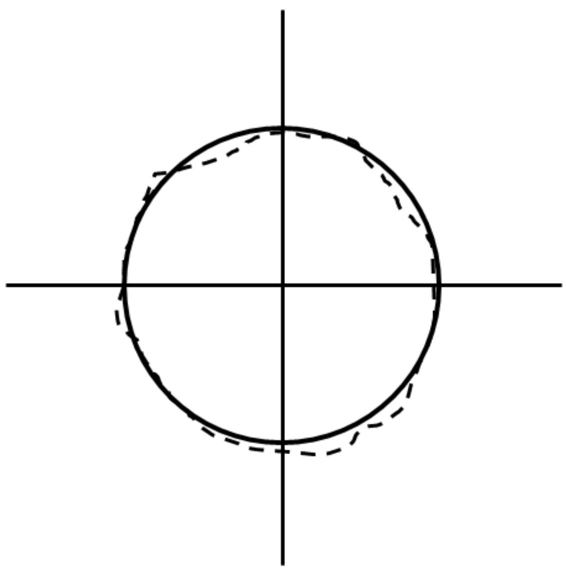

Figure 6. Circularity

Measurements for circularity are taken from a datum, generally the center point. The lack of circularity is referred to as "out-of-roundness" and is the difference between the smallest and largest radius measurement (ANSI, 1972). Measurements are often done with in-contact instruments where the part is rotated about its axis of rotation, the datum.

For rotating components it is necessary to conduct circularity measurements over the length of the part as opposed to one section. Typically measurements are taken in equal-sized cross-section lengths (ANSI, 1972). Measuring multiple cross sections can ensure that form tolerances are within limits (Ramaswami, 2011). With an out-of-roundness measurement a circular tolerance can be applied. A circular tolerance is a zone where the circular component must physically exist to conform to specifications, as shown in Figure 7.

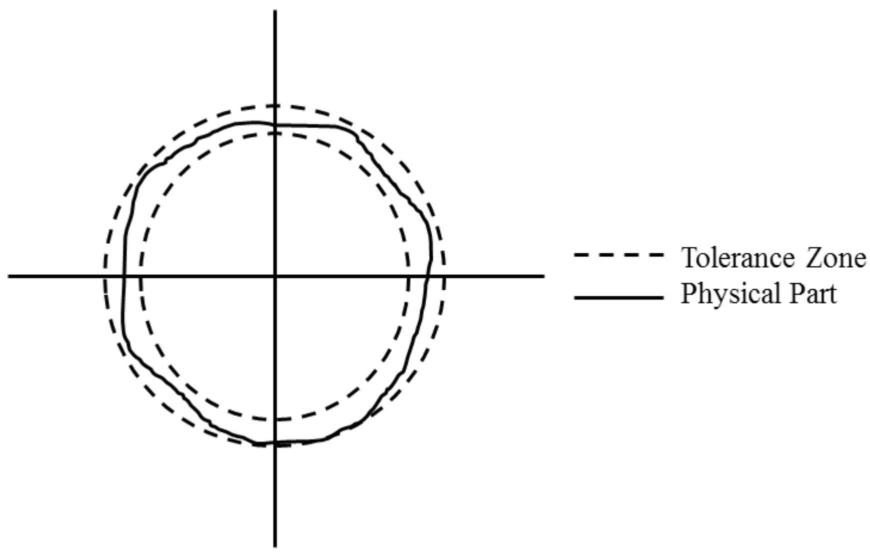

Figure 7. Tolerance Zone for Circular Geometry

<u>Runout</u>

Variation can exist at multiple points on a given part and a radius will vary along the entire circumference. In a rotating assembly variation over the entire part must be considered; not just specifically at one point. In addition a rotating component; its variation must be related and expressed with respect to its most basic geometrical shape. For rotating component, which typically embodies circular shapes, the variation must be related the datum, which is generally the center. To explain and express variation with respect to a datum, runout is a tolerance that is used to express the dimensional relations between a feature and a datum (ASME, 1994; Ramaswami, 2011). Runout is used to express the overall form of the part, and it can be applied to various geometries. The two emphasized runouts are circular and total (Ramaswami, 2011; Sjoholm, L., 1998). Circular runout will express the roundness of a specific cross section over the entire body. Total runout can be applied to express the variation over an entire body, with respect to certain geometry.

Circular runout is expressed by determining the difference between the largest and smallest measurement as the measurement gauge made one traverse over the entire circumference. Since it cannot be applied over the entire surface relying solely on circular runout may lead to under-specification (Ramaswami, 2011). To prevent under-specification it is better to take multiple circular runout measurements at different sections equally spaces in distance.

Total runout is can express radial variation of a circumference on a specific over the entire surface and is the highest level of runout control (Ramaswami, 2011). Measurement will occur over the entire surface of the part. It is calculated by determining the difference between the largest recorded measurement and smallest recorded measurement from any measurement set along the entire surface.

Measuring Runout

Measuring runout can be done either mechanically or electrically and is expressed in mil, which is a thousandth of an inch. Accurately measuring runout requires enough precision to measurements finer than 0.1 mils (Sjoholm, 1998). The most practical and commonly used methods are the graduated dial indicator and Linear Variable Differential Transformers (LVDT) (Littrel, 2005).

Dial indicators are the most basic and inexpensive instrument used to measure runout (Littrell, 2005). They consist of a spring-loaded tip to measure the part while a graduated dial is used to signal the corresponding reading. The general measurement process involves placing the dial indicator at initial position and zeroing out the dial. This step creates reference point where the remaining runout measurements will be related to. Depending on the technique the part or the dial indicator is rotated and measured in equal intervals. This is done until one complete rotation has occurred.

Although dial indicators are useful the graduated dials do not possess high resolution. Most dial indicators can only provide precision to a tenth of a mil, providing limited resolutions to the human eye (ANSI, 1972; Littrel, 2003). The accuracy of measurements is therefore reliant on the skill and experience of the operator. Modern balancing techniques, based on runout, are automating the measurement process. The manual nature of dial indicators does not lend itself to automated data collection as measurements must be recorded manually. LVDT's are applicable to instances where automation as the have the ability to take continuous measurements.

Approximating Runout

As a circular component varies from circularity its geometric centerline will change. Depending on the variation over the circumference, a geometric center can be approximated from runout measurements. To approximate the center point, least-squares method for circular geometries and general vector addition were used. To be compatible with both methods it was necessary to convert the part into a polar form. This can be done by setting a datum for the radial and angular measurements. The datum should be the same for all parts

Least-Squares Approximations

To approximate the center of circle based on two-dimensional coordinate points, least-squares fitting can be applied. Least-squares is a mathematical process to reduce the mean square distances for the fitting circle to the data points (Anton and Rorres, 2003; Lay, 2003). A theoretical circle can be fitted to the coordinate points with a calculated center-point and radius. Least-squares fitting can be applied to measurements that are taken by an analog device or digital as long as the measurements are decomposed into horizontal and vertical components (ANSI, 1972).

A rotating components center and radius can be calculated based on the variations measurements given the the data is converted and expressed in polar form. Given a point on a circle, with respect to the ideal origin, and its angular location from the origin its x- and y-coordinate can be calculated using the trigonometric identities: cosine and sine. The x- and y-coordinates are then placed into Equations 1 and 2. Variables “a” and “b’ will represent the x and y displacement from the ideal center point (0,0), respectively (ANSI, 1972). Equations 1 and 2 show the American National Standard Institute (ANSI) least-square method where x and y are the horizontal and vertical components, respectively, and n is the amount of data points.

$$a = \frac{2\sum x}{n} \tag{1}$$

$$b = \frac{2\sum y}{n} \tag{2}$$

Vector Addition

A plurality of vectors can be added together to calculate the overall magnitude or net-effect. A matrix containing only one column with two real numbers is referred as a vector (Anton and Rorres, 2003; Lay, 2003). In a polar graph the vector can express the x- and y-values, where x_n is the horizontal component and y_n is the vertical component as seen in equation 3.

$$w_n = \begin{bmatrix} x_n \\ y_n \end{bmatrix} \tag{3}$$

Given a plurality of vectors their sum can be calculated by adding their corresponding values the x- and y-coordinates. Equation 4 shows vector addition where $x_1 \cdots x_n$ and $y_1 \cdots y_n$ are all the horizontal and vertical components, respectively.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \dots \begin{bmatrix} x_n \\ y_n \end{bmatrix} \tag{4}$$

The resulting x- and y-coordinates can be used to infer certain characteristic of the vector. The coordinates can be either negative or positive allowing the direction to be inferred based on a four-quadrant graph. The magnitude of the vector can be determined by placing the final x- and y-coordinates in the Pythagorean Theorem and finding the square root of the result.

Based on a four-quadrant graph and the x- and y-coordinates the angular location of the vector can be calculated using trigonometric identities. A general method is to use the inverse tangent function. The angle calculated will be with respect to one of the four axes that border a quadrant. The actual angular location should be calculated with respect the origin, 0°.

Correcting Unbalance

Correcting unbalance is a critical step in the assembly or operation of rotating machinery. Balancing is the attempt to reduce or offset the unbalance by evenly redistributing mass over the part (Fox, 1980; McMillan, 2003; Rao and Dukkipati, 1992). The most prevalent balancing technique is altering the overall mass of the component, typically done by adding additional mass. Altering the mass can be limited by its efficiency so methods of “geometrical stacking” have been developed. Geometrical Stacking is the process of assembling the parts so that variation one part will offset the variation on the preceding and subsequent part. Completely eliminating unbalance is unlikely so practitioners will seek to achieve a near optimal amount; essentially reducing unbalance to its lowest achievable or acceptable amount.

Depending on the method balancing can take place individually or in-situ (Pitsoulis et al., 2001; Temple, 1983). Individual balancing occurs before assembly or after disassembly as it involves measuring and balancing each part independently of each other. In-situ allows allow for an individual component to be balances while still in assembly. The entire assembly may be operated, measured, and corrected without any disassembly.

Traditional Techniques

The most basic way to correct unbalance is to redistribute the mass over the part by adding mass to the component. Field balancing is a commonly used process for balancing rotating components or assembled (Temple, 1983). The process of field balancing involves rotating the component, measuring the vibration, and calculating the correction mass amount and location. After placing the correction mass, the component is allowed to rotate and the process is repeated until an optimal amount of vibration is achieved (Fox, 1980; McMillan, 2003). Typically balancing can occur in either single or double planes. The basic process of rotating, measuring, and altering the rates is the general process of both.

Single plane balancing can be applied to parts that possess pure static unbalance (McMillan, 2003; Rao and Dukkipati, 1989). The part is first allowed to rotate near operational speed so that the vibration can be observed. Typically a strobe light is used to track the amplitude of vibration and its phase location; which are to be recorded (McMillan, 2003; Pitsoulis et al., 2001). A trial is placed on the part while taking into account the amount, distance from the center, and phase location. With the trial weight the part is allow to rotate again. The trial mass will alter the amplitude and phase location of the generated vibration. The amplitude and phase angle can be represented as a vector on a polar graph. Using vector mathematics the amount of correct mass and its location can be calculated. The trial mass is removed and the correction mass is placed, and the part is rotated and measured for vibration. This process can be done until an acceptable amount of vibration is met. In real application, unbalance in exist in more than one plane. The process of multi-plane balancing is similar to single except; multi-plane requires more than one correction mass at different location (McMillan, 2003).

Geometrical Stacking

Unlike tradition methods, geometrical stacking does not alter the mass of each individual part. Instead it will assemble the parts so that the variations can offset each other (Davidson and Wilcox, 1976; Forrestor and Wesling, 2002). Unlike traditional methods it does not necessarily require the act of manually balancing a part. This process is inherently more complex as now variation on one part must be related to variation on the next and subsequent part. The process of geometrical requires the use of simulations as multiple combinations and orientations need to be analyzed. Since an optimal assembly is based on the radial and axial runout; it is necessary to convert the runout measurements into a form that can be used to determine a stack combination (Davidson and Wilcox, 1976).

As technology increases rotating assemblies now possess more parts and geometrical stacking offers a less time consuming method of balancing as the need for disassembly, reworking, and reassembly is reduced or eliminated (Davidson and Wilcox, 1976; Forrestor and Wesling, 2002). In addition time is saved as multiple assembly combinations can be simulated and analyzed. Programs can be developed to carry large amounts of iterations and can run until an optimal assembly combination is met. Its ability and need to simulate so many iterations is also a limitation. As parts and part orientation increase so do the amount of possible combinations. Large computational loads may be required to determine an assembly.

Gleason Optistack

Davidson and Wilcox (1976) developed a method to use runout measurements of gas turbine as the basis for assembly process. The method relied on total indicated runout (TIR) in the radial and axial directions for all the components and couplings. The high point of each part was considered the TIR and its location was expressed as an angle. The TIR and their angular location were then mathematically transformed into part dimensions to determine stacking combination. The position of each part was dictated by the amount of bolt holes present. Based upon the newly calculated part dimensions a program was developed to determine the combination for minimized runout. All parts possess equally space bolt holes which denote position.

The method offered the increased efficiency of assembling a gas turbine. A near optimal assembly combination could be generated by the program, without the need for individual balancing (Davidson and Wilcox, 1976). One limitation of the method was the amount of combinations possible. Large assemblies with multiple parts which had multiple positions necessitated the need for large calculation loads. To reduce calculation loads, the program broke up the optimization process into smaller segments. For example, the program would calculate the assembly combination for three parts at a time. The segmented approach may only yield a near-optimal assembly combination.

Loop Stacked Rotor Assembly

Forrestor and Wesling (2002) of General Electric developed a method represent eccentricity of a component as a single vector. These representational vectors can be used to mathematically determine an assembly combination that minimizes the unbalance stack. A part in placed on a turntable and rotated about its geometric center. Four digital probes are placed into contact with the component to measure variation in radial and planar directions. A computer program will automatically determine a single vector and its angular location to represent all the eccentricity of every component in the assembly.

The vectors added to each other by placing them in a loop changing their angular location. A closed loop is preferred as eccentricity of the last part can be reduced to zero. After the loop is formed, the net eccentricity if calculated (Forrestor and Wesling, 2002). This process can be completed until an acceptable net eccentricity is achieved.

Many combination can be mathematically represented iterated therefore many possible combinations can be analyzed. Such an approach would reduce the need for assembly, measurements, dis-assembly, and reassembly based on the measurements. Instead the loop combinations can be automated so that a minimal net eccentricity is found (Forrestor and Wesling, 2002). Like all geometrical stacking operations this can be limited by the need for determining combinations.

CHAPTER 3

METHOD

Generating Runout Data

In typical measurements, the dial indicator is placed at the initial position and is set to 0; essentially creating a reference point (Davidson and Wilcox, 1976; Littrell, 2003). The subsequent measurements after that point will be made with respect to the initial position. Therefore, the first measurement will indicate no present runout. To mimic this procedure a set of eight measurement points will be generated along a circumference of a circle (ANSI, 1972). The first point will indicate no present variations and the remaining seven will be randomly generated from a specified tolerance zone. The goal is to mimic analog measurements which occur in intervals not a single continuous measurement.

The tolerance zone was set to be a minimum of -5 mils and a maximum of 5 mils. Based on this range seven data points could be generated in the tolerance zone using Excel. For one component two sets of runout measurements were generated to represent the forward and aft of a single part. The randomly generated points were then biased by 10,000. After the bias the runout values were normalized to 20. This process essentially made 20 the zero point. Data was generated such that it mimics measuring a component manually at 45° intervals. This technique is commonly used for gas turbine assemblies. Table 1 shows the data for Assembly 1 after being biased and normalized to a value of 20.

Table 1. Biased Generated Radial Runout Data

	Part 1 Runout (mils)		Part 2 Runout (mils)		Part 3 Runout (mils)		Part 4 Runout (mils)	
	Front	Aft	Front	Aft	Front	Aft	Front	Aft
Point 1	20	20	20	20	20	20	20	20
Point 2	21	21	17	18	18	21	24	17
Point 3	16	18	23	17	17	22	18	17
Point 4	18	23	20	25	22	24	24	21
Point 5	21	19	20	24	21	20	20	19
Point 6	23	23	21	22	20	19	19	24
Point 7	19	20	16	16	23	23	22	20
Point 8	22	18	18	19	18	17	21	18

The next step was to convert all the measurement to a polar form using the trigonometric identities cosine and sine. For specific point, the runout was multiplied by the cosine and sine with respect to its angular location as shown in figure. That allowed the creation of vertical and horizontal components. Figure 8 shows a representation of parts based on normalized runout data.

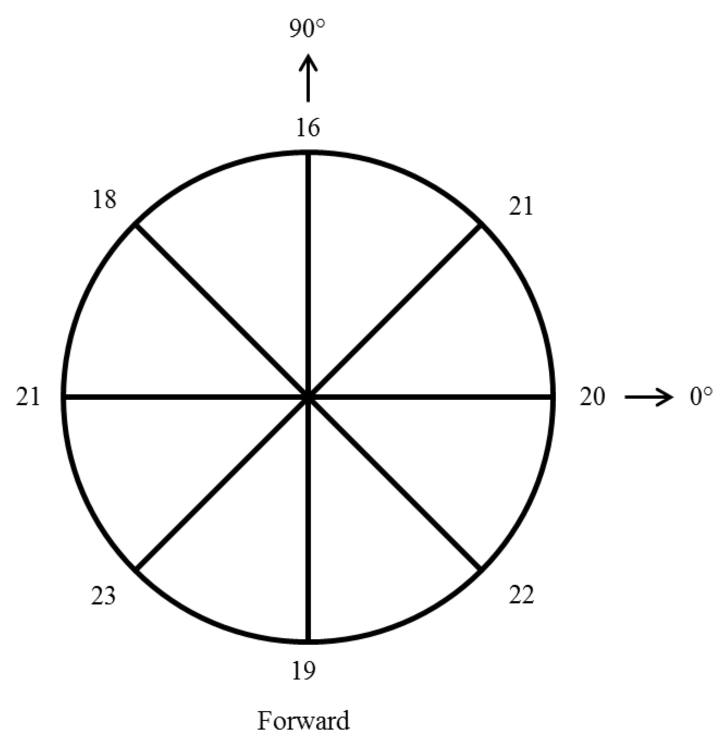

Figure 8. 8-Point Measurement Format

Approximating with Vector Addition

Given the x- and y-components of the runout measurements vector addition could be applied. All the x-components were summed and all the y-components were summed. Next the magnitude of runout and angular location were approximated. Using Pythagorean Theorem, the result was assumed to represent the net-effect of the runout (Anton and Rorres, 2000; Lay, 2003). Using the arctangent function, the location of the net-runout could be approximated. Since the arctangent gives an angle relative to its quadrant has to be related to the origin. The net approximate and angular location could be used to represent the forward and aft of a rotating component. The vector and its angular location would represent the point of most eccentricity as shown in Figure 9.

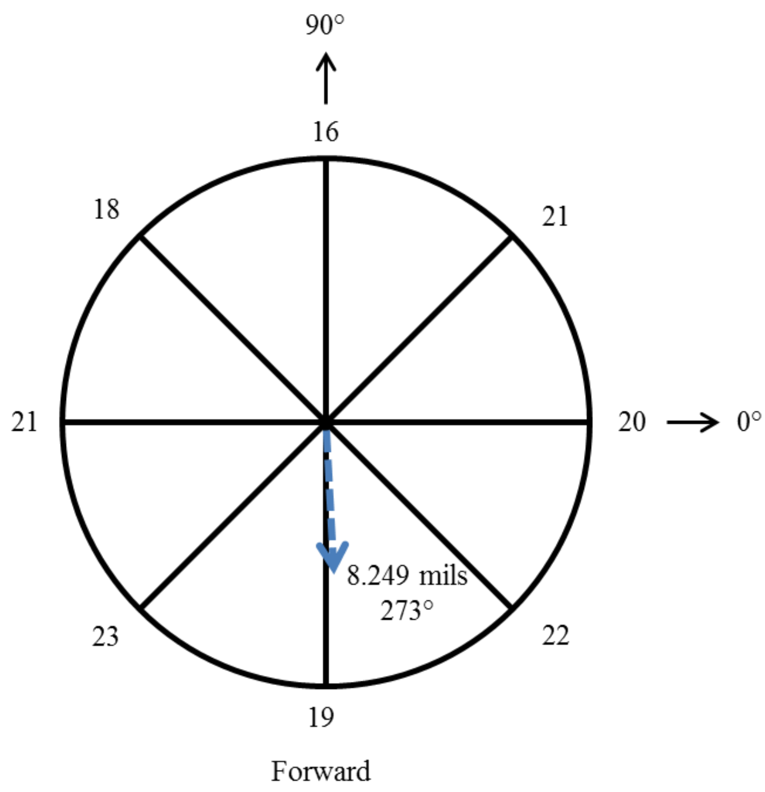

Figure 9. Part Representation by Vector Addition

Approximating with Least-Squares Fitting

Given the x- and y-components of the runout measurements least-squared could be applied. All the x-components were summed and all the y-components were summed and multiplied by two over the amount of measurement points. The result was of approximation of how much the physical center is displaced from the ideal center-point as shown in Figure 10. Using Pythagorean Theorem, the distance from the center to the displaced center could be calculated (ANSI, 1972; Anton and Rorres, 2000). Using the arctangent function, the angular location of the center point could be calculated with respect to the initial measurement point. Since a part should rotate at its geometric center, the approximate center-point will be considered the point at which rotation should occur based on runout.

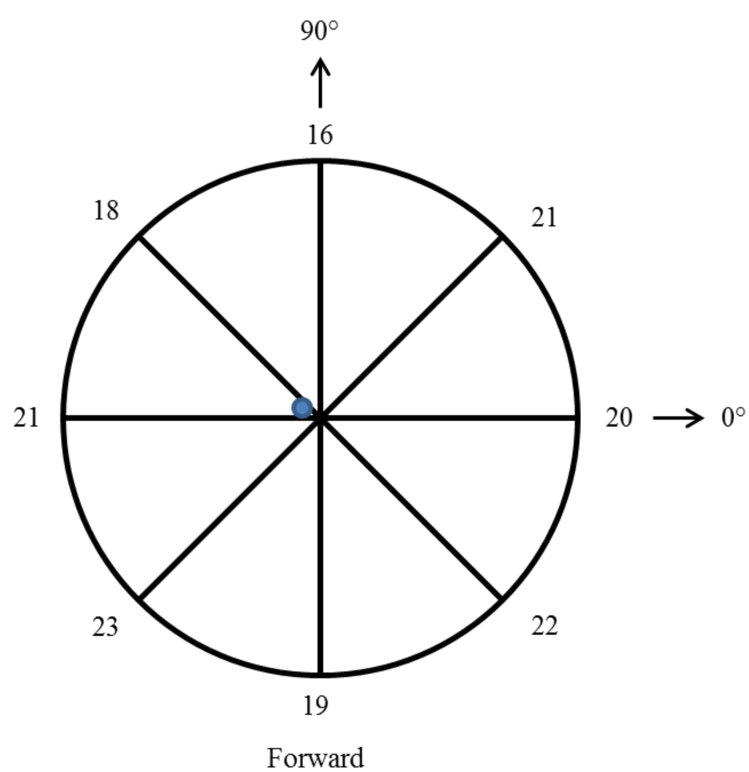

Figure 10. Representation Based on Least-Squares

Approximating Assembly Initial Net-Eccentricity

Using the vectors calculated by the vector addition the net-eccentricity of an assembly could be calculated by using the results. The resulting vectors were summed using vector addition with respect to the angular location. Summation requires decomposing the vector to vertical and horizontal components. The initial net-runout values were summed to represent an assembly where the initial measurement points were aligned. This would represent assembly without considering part positions. The result was an approximation of the runout had the highest effect over the entire assembly (Forrestor and Wesling, 2002).

Approximating Assembly Initial Assembly Center-Point

By treating the center-points as vector, with angular location, they could be summed using vector addition. After the points were decomposed into x- and y-coordinates summation occurred (ANSI, 1972; Anton and Rorres, 2000). The summation occurred as if the parts were assembled without rotating them from their initial measurement position. The result is single vector with angular location representing the approximate centerline of the entire assembly.

Offsetting Parts

To offset eccentricity the couplings between mating parts were set 180° apart based on the calculations of the parts. Offsetting parts angularly will assist in reducing assembly variation (Forrestor and Wesling, 2002). The procedure for offsetting was the same for parts calculated with vector addition and least-squares. Part 1 was left in its initial position and served as the reference for the assembly. The critical point of Part 1 aft was at 178° from the origins. The critical point of Part 2 forwards was at 127° and was rotated to 358° to be offset from the aft of Part 1. When rotating Part 2 the aft critical point changed from 96° to 327°. Since the aft of Part 2 changed, the forward of Part 3 needed to be set to 147° to be 180° from 327°. The new location of the Part 3 aft critical point was now 49° and the Part 4 forward was rotated to 229° to be offset from it. The location of Part 4 aft was noted. Offsetting between forwards and aft occurred; in assembly variation will stack especially near assembly points.

Approximating Assembly Characteristics after Offsetting

Based on their new angular locations, the high-points were decomposed into their x- and y-coordinates using their new angular location and the trigonometric identities. The x-components were summed as were the y-components for both the results from vector addition and least-squares. For the vector addition results, Pythagorean Theorem could approximate how the point at which runout has the greatest effect (ANSI, 1972). In addition, Pythagorean Theorem could approximate how far the center-point deviates from the ideal center-point (ANSI, 1972). In both instance the arctangent function can be used to locate the angular location of the net-effect or center-point with respect to the initial measurement location. The initial results were compared to the results from the offsetting of parts. Percent difference was calculated to quantify how much the effect changed with respect to each other.

CHAPTER 4

RESULTS

Vector Addition for Approximation of Runout

After decomposing all the components into x- and y-coordinates vector addition occurred. One part would be represented by two separate vectors; one vector represents the forward while the other represents the aft. The vector represented the point at which runout had the greatest effect and presence (Davidson and Wilcox, 1976).

The Forward of Part 1 could be represented as a vector with a magnitude of 7.254 mils at an angle of 273°. The Aft of Part 1 could be represented as a vector with a magnitude of 3.952 at 178°. The effect of variation was most significant on the forward when compared to the aft. Figure 11 shows the vector representation of the forward and aft of Part 1. The entire part would have introduced a 7.952 mils to the assembly.

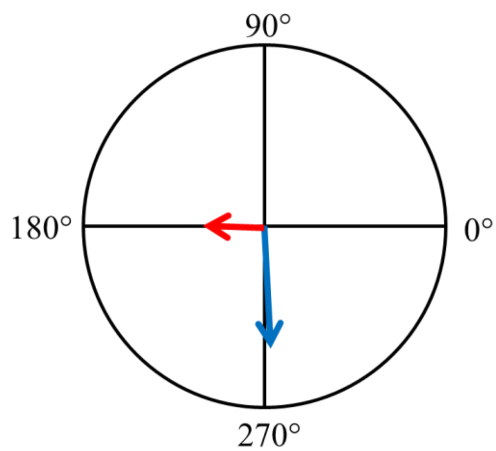

Figure 11. Vectors of Part 1

The Forward of Part 2 could be represented as a vector with a magnitude of 7.014 mils at an angle of 127°. The Aft of Part 2 could be represented as a vector with a magnitude of 2.429 mils at 96°. Similar to part 1, the forward possessed the highest net-effect of the variation. Figure 12 shows the vector representation represent the forwards and aft of Part 2.

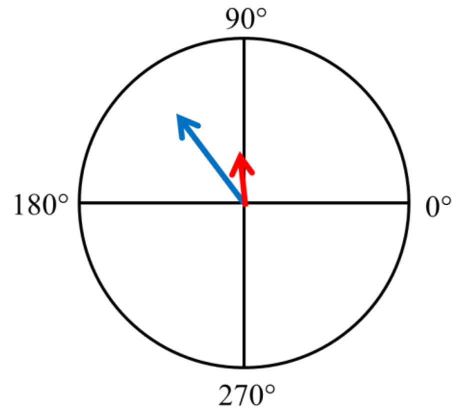

Figure 12. Vectors of Part 2

The Forward of Part 3 could be represented as a vector with a magnitude of 6.966 mils at an angle of 221°. The Aft could be represented as a vector with a magnitude of 6.424 mils at 123°. Figure 13 shows the vector representation Part 3 with both vectors.

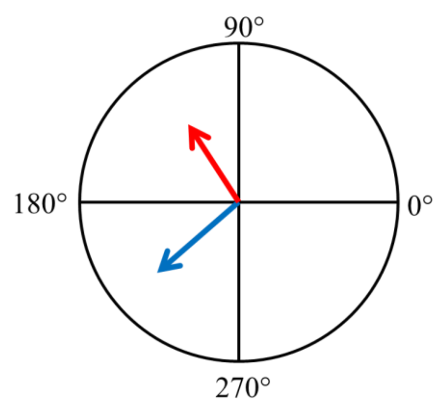

Figure 13. Vectors of Part 3

The Forward of Part 4 could be represented as a vector with a magnitude of 2.178 mils at 49°. The Aft of Part 4 could be represented as a vector with a magnitude of 8.416 mils at 223°. The difference in angular location was nearly 180° offset which may allow the parts own variation to offset itself. Figure 14 shows the vector to represent the forwards and aft of Part 4.

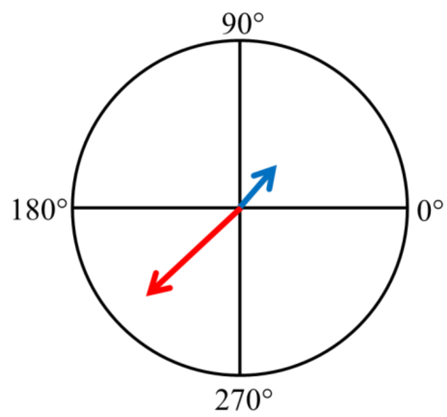

Figure 14. Vectors of Part 4

<u>Assembly Net-Eccentricity Based on Vector Addition</u>

To determine the net-eccentricity the vectors were summed together using vector addition. The results could predict how well variation between parts could offset each other. The majority of the vectors exist in the second and third quadrant. Figure 15 shows the vectors of all the parts in their initial positions.

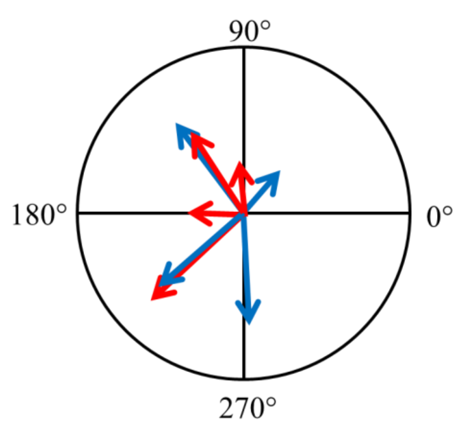

Figure 15. Vectors in Initial Positions

After the forwards of subsequent parts were offset 180° from the aft of the proceeding part; net-eccentricity. Figure 16 shows which vectors were 180° offset and show a more balance distribution. The net-eccentricity reduced to 9.519 mils at an 25.475°. The net-eccentricity reduced by 77.74% when the parts were offset from each other.

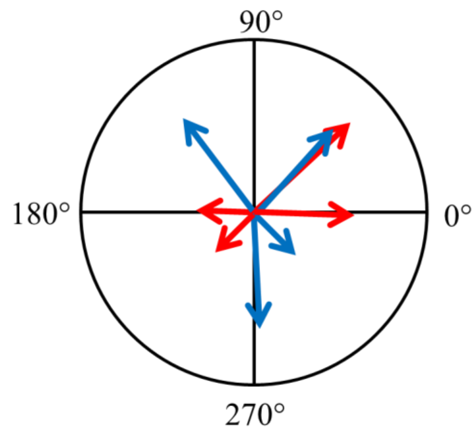

Figure 16. Vectors in Offset Positions

Least-Squares for Approximation of Center-Point Displacement

Least-square methods were used to provide a general location of a parts geometrical center. For Part1 the center-point was displaced .104 mils and -1.811 mils in the x- and y-direction, respectively. The aft was displaced 0.987 mils and 0.030 mils in the x- and y-direction, respectively. The center of the Forward was displaced by 1.814 mils at 273° while the Aft was displaced by 0.988 mils at 178° as shown in Figure 17.

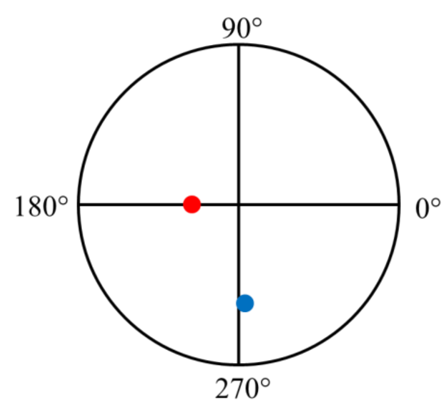

Figure 17. Centers of Part 1

On Part 2 forward it was shown to be displaced -0.987 mils and -0.030 mils in the x- and y-direction, respectively. The center-point of the aft was location displaced -0.066 mils and 0.604 mils in the x- and y-direction, respectively. The Forward center existed at 1.754 mils from the center at 127° and the Aft's center 0.607 mils away and at 96°. Figure 18 shows the displacements of both the forward and aft of Part 2.

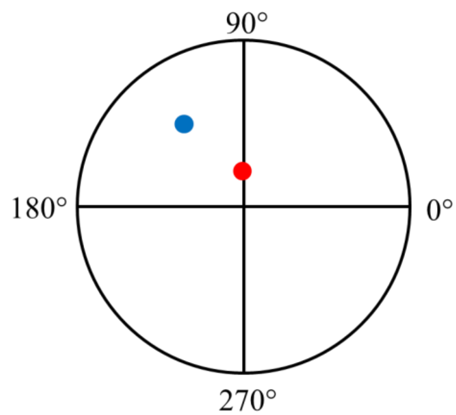

Figure 18. Centers of Part 2

On Part 3 center-point was displaced -1.311 mils and -1.146 mils in the x- and y-direction, respectively 1.741 mils at 221°. The aft of Part 3, center-point was displaced -0.884 mils and -1.341 mils in the x- and y-direction, respectively. Figure 19 shows the displacements Part 3 for both the forward and aft.

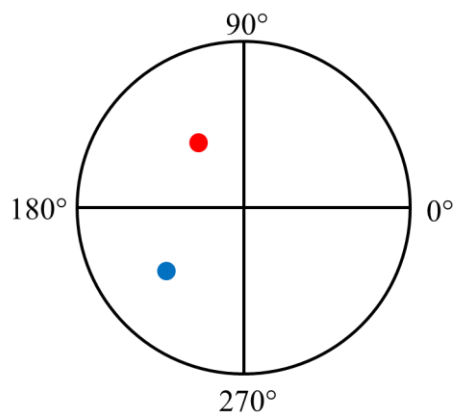

Figure 19. Centers of Part 3

On Part 4 the center-point of the forward was shown to be displaced 0.354 mils and -0.414 mils in the x- and y-direction, respectively, 0.545 mils at 49°. The aft center-point was displaced 1.518 mils and -1.457 mils in the x- and y-direction, respectively. A vector would show 2.104 mils displacement at 223° as shown in Figure 20.

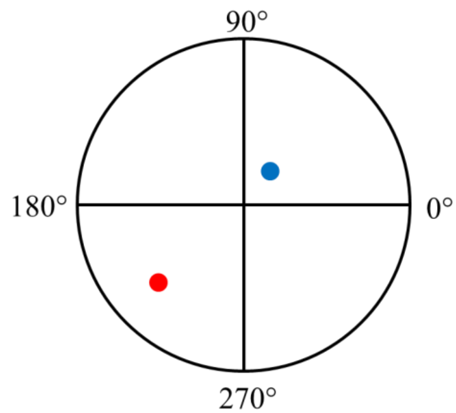

Figure 20. Centers of Part 4

Assembly Net-Eccentricity Based on Least-Squares

Least-square could be interpreted as the displacement from the ideal center-point of (0,0). Summing them can show the center line of the assembly. The assembly center-point was displaced by 5.406 mils. Offsetting decreased total displacement to 2.380 mils, a 77.74% reduction. Figure 21 shows the centers in initial positions to visualize assembly.

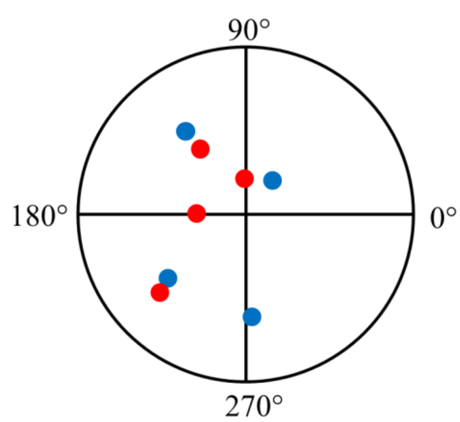

Figure 21. Centers in Initial Positions

The majority of center point existed in the second and third quadrants. By using least-square and deduction could be made as to where the center of masses was located with respect to an ideal assembly line. Similar to vectors addition, offsetting showed a more spatially balance distribution of centers. Figure 22 shows the mathematical assembly with offset centers.

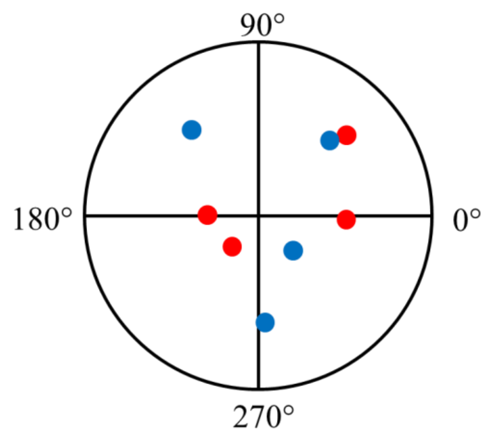

Figure 22. Centers in Offset Positions

CHAPTER 5

CONCLUSION

Discussion

Two methods were applied to mathematically representing a circular component based on radial runout. Vector-Addition was applied to determine the net-effect of the runout on a part. Least-squares approximated the physical center of the part based on the runout. Both could be used to represent a part as a vector. The angular locations of the calculation were constant throughout the calculations whether in initial or offset positions. This was because least-squares only multiplied the result of vector addition. So when calculating the angles the ratio of y- to x-component was equal and yielded the same angle.

Vector addition was a general way to represent the part based on runout. However it was difficult to interpret the meaning of the vector. One interpretation is it could represent the location at which the runout has the greatest effect.

The least-square method was a general technique and was prescribed by ANSI. Using least-squares provided a more explainable effect of the radial runout of a part. It provided the ability to visualize where the physical or true center was located with respect to the ideal center point of (0,0). The physical center-point can also be viewed as the center-of-mass, one of the points at which a part should rotate.

It should be noted that the angular location of the vector was constant for each part during the initial and offset calculations. The same angular results indicate that vector addition was the key component even in the least-square calculations. Other least-squares methods should be selected as to provide more distinction between vector addition and least squares method.

Geometric stacking is dependent on finding the assembly combination that provides an acceptable or the least amount possible of unbalance and eccentricity. Offsetting by 180° can create instance where unbalance and eccentricity on one part can offset the unbalance and eccentricity on another. However, it can only reduce eccentricity so much due to limited combinations. Instead of doing just 180° offset between parts, it may be better to determine angular combinations that produce the smallest amount of net-eccentricity of net-center-point. However, such method would require multiple iterations especially as the number of parts in an assembly increase.

Future Considerations

For this study on the radial runout on mating ends were considered. To achieve a more accurate representation of a part it may be necessary to consider runout over the entire body and in other directions. In rotating assembly variation in the planar direction is also very critical to the overall balance and assembly quality. Planar variation or runout is perpendicular to radial direction. It is important because the mating surfaces are not flat, planar runout may cause the parts to tilt with respect to the variation. To include planar variation vector addition and least-squares need to be applied to planar geometries instead of circular. Only the variation at the ends of a part was examined. To fully represent a part more instances of measurements should be taken. This would be particularly useful for least-square as multiple center calculations could lend itself to approximation of a relative of the entire part.

Offsetting by 180° is a reasonable but limited way to offset the variation between parts as it creates only a few possible combinations. If the parts variation was equal in magnitude then a zero net effect would be possible. Since that is highly unlikely the varying runout can only be offset and reduced by so much. If parts are to be represented by vectors it may be more beneficial to create vector combinations that offset at varying angles. Doing so would allow for many more distinct combinations. The ability to generate multiple combinations might require dedicated program.

Conclusion

For geometrical stacking part representation is a critical step in the process. Variation on multiple parts in multiple directions must be measured and related to each other. Least-squares was the most applicable method as its results could be visualized. Applicability of least-square is significant as characteristics of various geometries can be selected. Further expansion would require studying variation in three-dimensions and increasing combinations.

REFERENCES

American National Standard Institute (ANSI). (1972). *Measurement of out-of-roundness.* (ANSI B89.3.1-1972). New York, NY: The American Society of Mechanical Engineers.

American Society of Mechanical Engineers (ASME). (1994). *Dimensioning and Tolerancing.*(ASME Y14.5-1994). New York, NY: American Society of Mechanical Engineers.

Anton, H., and Rorres, D. (2000). Elementary linear algebra applications version. *Least Squares Fitting to Data* (pp.437-442). Hoboken NJ: John Wiley and Sons, Inc
.

Davidson, J. and Wilcox, L., (1976). Minimizing assembly runout in turbo-machines made with curvic couplings. *Journal of Power Engineering.* 98(1), 37-46.

Fischer, B. (2011). Mechanical tolerance stackup and analysis. Boca Raton, FL: CRC Press.

Forrester, J. and Wesling, R. (2002). *U.S. Patent No. 6,341,419 B1.* Washington, DC: U.S.Patent and Trademark Office.

Fox, R. (1980). Dynamic balancing. *The Proceedings of the ninth tubormachinery Symposium.* 151-183

Lay, D. (2003). Linear algebra and its applications. New York, NY: Addison Wesley

McMillan, R., (2003). Rotating machinery: practical solutions to unbalance and misalignment. Lilburn, GA: Fairmont Press, Inc.

Pitsoulis, L., Pardalos, P., Hearn., (2001). Approximate Solutions to the turbine balancing Problem. *European Journal of Operational Research.* 130, 147-155.

Ramaswami, H., Turek, S., Rajmohan, S., Ananda, S. (2011). A Comprehensive methodology for runout tolerances evaluation using discrete data. *International Journal Advanced Manufacturing Technology*. 56, 663-676.

Rao, J., Dukkipati, R. (1992). Mechanism and machine theory. New Dehli, India: Wiley Eastern Limited.

Sjoholm, L. (1998). Rotor evaluations regarding runout. *International Compressor Engineering Conference,* 589-594. Retrieved from http://docs.lib.purdue.edu/cgi/viewcontent.cgi?article=2303andcontext=icec

Temple. D., (1983). Field balancing large rotating machinery. *Facilities Instruction Standards and Techniques.*