HOW PLAY HELPS STUDENTS TO COMPREHEND MATHEMATICS

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ABSTRACT

In the realm of mathematics education, there is an emerging line of research which explores how to incorporate play-based practices into classrooms in order to better engage students in mathematics instruction. Play has traditionally been an important part of early childhood education, but only recently has the value of play in specific content areas been considered. This new research considering the integration of play-based methods in education has inspired the idea that play could be used in mathematics contexts in order to help students be more engaged with the subject and find it more applicable to their lives. This paper explores the incorporation of play-based methods into mathematics education. Play as a general term is defined as well as, more specifically, mathematics play. Mathematics play is analyzed in terms of effectiveness and benefits for students in both early childhood and elementary school education. As part of this project, research on play-based methods used in mathematics is synthesized to develop play-based mathematics activities designed for a 4th grade classroom. Each activity is described, linked to curricular standards, and explained in terms of how play is embedded into the activity. These activities were implemented for a short period of time in a 4th grade classroom and a reflection on the strengths, weaknesses, and overall effectiveness of these activities is shared. Implications for practice in upper elementary school contexts are discussed and contrasted with ways in which play is commonly used in early childhood classrooms. The complexities and tensions of defining and using play in classrooms are also explored throughout the paper.

I. WHAT IS PLAY?

When considering play, we often think of children acting out scenarios, building with blocks, or simply running around in circles during recess. However, defining and understanding play is often much more complex than one might expect. Parks (2015) explains "in working with classroom teachers and administrators…[that] a surprisingly wide range of activities are claimed as play" (p. 5). There are many ways children explore the world through play, and this fact is acknowledged and used often in early childhood classrooms around the world. The problem is that once students reach a certain age, play has been known to abruptly stop in classrooms. Even as soon as Kindergarten in the United States, "on a typical school day, kindergarteners spend four to six times as much time in literacy and numeracy instruction and taking tests or preparing to take them as in free play or 'choice time'" (Miller & Almon, 2009, p. 16). As students get older and they are "increasingly engaged with formal assessments of their academic performance" in various academic areas, students lose time they could be using to play and make their own choices (Parks, 2015, p. 3). When considering the benefits and uses of play, the fact that the amount of play in classrooms is decreasing in early childhood classrooms and beyond is important to consider. As the emphasis on standardized testing success intensifies in the upper grades, there is a resulting emphasis on formal assessments over play-based practices in many of the classrooms I have seen regardless of grade level. However, there is no age limit for play to be effectively used in classrooms. While "considerably less research has examined mathematical play in older children and adults… a playful orientation toward mathematics can be supported by designed situations" for any age group (Williams-Pierce & Thevenow-Harrison, 2021, p. 510). The discussion of play and math play in this paper in particular will focus on how math play can be utilized in upper elementary and middle school contexts, though it could potentially be extended to any level of education to produce beneficial results.

When thinking about play in any academic context, it is important to consider how play is defined. There are many different definitions in the world of play and education research as we aim to discover what makes an activity play. One definition of play is given by Elkind (2008), who specifies that play occurs when we "adapt our world to fit our imagination" rather than when we "adapt ourselves to our environment" (p.3). While these are not very technical terms for defining play, they do give us a general idea of how play can be understood. If a child is using tools in the world around them to suit their imaginative scenarios or ideas, then play is occurring. If a child is being convinced by an outside source to interact with the world in a particular way and children thus engage in this way in order to better fit in with the environment, it is no longer considered play. An example may be an adult asking a child to clean their room in order to make a good impression on house guests. This child is not voluntarily engaging in this activity in order to fit their imaginative vision but is doing so to fit in with the environment around them. While this definition can be difficult to apply considering the complex interaction of environmental and internal influences on decision-making, this gives some insight into the idea of choice being essential to play. If play is chosen and orchestrated by the child, their imaginative vision is being fully realized; in contrast, if an outside source imposes an activity upon the child, the child is required to adapt to the environment and is, therefore, not engaging in play.

Many definitions of play include choice as an important part of play, but also use a set of more specific criteria to classify an activity as play. For example, Burghardt (2012) describes play as an activity that is not completely functional in its context, occurs in an absence of stress, and is spontaneous, pleasurable, rewarding, or voluntary in nature. For something to be play it must not be completely functional in context meaning it may be similar to other more serious behaviors of adults or older children, but it differs from these serious behaviors in some way. An example would be a student putting on a show in which they sing a song. They are imitating the behavior of real musicians when they perform, but their behavior differs because they do not perform at an actual venue and rarely do they create their own songs to perform. Another part of this definition is that play occurs in an absence of stress and is spontaneous, pleasurable, rewarding, or voluntary. I would argue that play being stress-free, pleasurable, and rewarding are largely complimentary features. These aspects of play mean that the play results in something beneficial for the child—whether it is personal satisfaction or an external reward, neither of which can be accomplished if stress is present for the child. Burghardt's definition, as many others do, also includes voluntary engagement as playing a role in play; however, Burghardt does not include this as a necessary piece of the definition of play, but as one option for how the activity can be initiated. He argues that play could also be engaged in when it is pleasurable, rewarding, or reinforcing without necessarily being choice-based (Burghardt, 2011).

In another definition of play, Lifter & Bloom (1998) claim that play "consists of spontaneous, naturally occurring activities with objects that engage attention and interest" (p. 164). This echoes some of the same points as the Burghardt definition: primarily the idea that play must be engaging and pleasurable for students. This also includes activities being naturally occurring as an additional qualifier of play, something which is complementary, but not necessary in Burghardt's definition of play. Parks (2015), however, disagrees with parts of each of these definitions. Burghardt's definition would include the use of play materials in a structured academic setting as 'play' since, from his perspective, the criterion of play as voluntary is not necessary, but an optional way of engaging in play for which there are alternatives. For Parks, however, play must be freely chosen by the students and the element of choice is a defining feature of play. Lifter & Bloom's (1998) definition is similar to that of Parks (2015), though they use the terms "spontaneous" and "naturally occurring", which I interpret as their way of implying choice, rather than explicitly including choice itself as a requirement of play as Parks does. Lifter & Bloom also do not include any discussion in how play takes place over time as Parks does in addressing the way that play is varied.

The definition that this paper will use and focus on in its discussion of play is that of Amy Parks. As discussed previously, Parks (2015) emphasizes that the "opportunity to make choices is critical" in most researchers' definitions of play (p. 6). As seen in Burghardt's definition, making choices is included as an important part of play in some researchers' definitions, but not necessarily a defining factor. Some researchers, however, include that the engagement in the activity itself is choice-based, or voluntary, as a defining part of play beyond simply some choices being offered within the activity. Parks' definition considers other key features common across many researchers' definitions of play including that play is "voluntary, pleasurable, and varied" (Parks, 2015, p.6). These researchers include Lifter & Bloom, Burghardt, and Elkind, whose work together emphasizes the following key features of play: (a) chosen freely by the child (though Parks specifies that the activity should be voluntary rather than just including some choice elements), (b) pleasure-producing for the child, and (c) varied in nature when considering how the specific play activity can be used across a period of time.

I would thus define *play*, considering these different perspectives on how students engage in play, as a voluntary activity that children engage in that is overall pleasurable for them and varied in nature. Since the amount of play differs across activities based on the amount of choice, variation, and pleasure it includes, I refer to any activity that completely meets these three criteria as *pure play* and other activities which include a subset of these elements as *play-based*. For an activity to be considered play in any capacity, an essential part of the definition is that the activity is in some way chosen by the person engaging in play. The voluntary nature of play is the most foundational piece of play, both in that which is pure play (i.e., chosen completely by the participant), and in that which is play-based, where some pieces may be chosen by someone else, but the engagement in the activity is still voluntary in nature. The fact that play should occur in an absence of stress is related to the idea that play should be voluntary. For example, if a student feels any sort of external pressure to engage in play it is no longer entirely voluntary and thus no longer pure play. It is important to consider the role of evaluation in this criterion for play because evaluation can cause pressure to engage in an activity a child would not choose on their own, and thus can make it more challenging to implement an evaluated activity which would also be considered pure play. I also include that play must be pleasurable as a requirement to be considered play. Play should be overall pleasurable, but not necessarily entirely pleasurable. For example, a student can engage in a play activity in which there are pieces of the process that do not produce joy. Learning to interact with others or create things can be frustrating for a child, but if the activity overall produces some sort of satisfaction or pleasure, the activity would still be considered play. It is important that play is both voluntary and pleasurable—the two work together to create an environment that the child wants to be in and will continually engage with. Lastly, the third criterion in my definition of play is that it is varied in some way. When an activity is repeated over time, as it often is in play contexts, there should be variation in how a student engages with the activity as well as variation in how different students will engage with the activity. As a part of the varied nature of play, I would also argue that pure play does not necessarily need to be spontaneous, as is suggested in some definitions of play. A way that play can vary in different scenarios or at different times is, in fact, that children often plan their play before engaging in it by assigning roles, developing a set of rules, or gathering materials beforehand. However, it is important in discussing the planning of play to remember that any authority figure planning play for students would limit the playfulness of the activity by determining the nature of play for students and thus diminishing both the free choice of play and the varied nature of the play, making an activity more play-based than pure play.

My definition of play takes into account both the definitions of experts discussed above, and my personal observations of children engaged in play. I have seen that children are often more enthusiastic, passionate, and engaged with their activity if they have chosen for themselves to complete an activity of their own free will. "Self-initiated play nourishes the child's curiosity, imagination, and creativity" in a unique way that must be fostered in children (Elkind, 2008, p.2). If students are prescribed a task or behavior, they are not engaging in play when completing it and that can be evident in the amount of enthusiasm they have for the activity. My definition of play also addresses that stress or a lack of pleasure from an activity diminishes its playfulness because it means that children are no longer able to focus on using their imagination to explore and create in the real world but are instead worried about meeting standards.

Play-Based Activities vs Pure Play

An important point to remember in considering play being used in classrooms is that pure play is not something that can be initiated by a teacher or other outside authoritative source. The purest form of play is that which comes from a child's mind and their interactions with the environment surrounding them. When children are at school, some choice is already removed because they are not offered the choice to be in school, and they are not offered a choice in participating in classroom activities. Pure play cannot be created by teachers because it has to be from the child's own free will and interests. Our goal as educators, therefore, is not to create an environment of pure play. Not only would this produce chaos, it would also not achieve any desired academic goals. Our goal, instead, is to implement as much play-based content as possible in children's learning in order to better engage them. Play, I would argue, is not either present or absent in a certain activity, it can be understood as a continuum. And while teachers have no control over whether students engage in play when working with the content they are taught and encouraged to explore, they do have control over play-based elements or playful materials that they can include in their classrooms and give students the space to learn with and through.

Play-based is a term I will use in this paper to describe something that resembles common childhood play behaviors without meeting all three of the necessary defining characteristics of the purest form of play as discussed above. Anything involving role playing, game playing, project creation, or collaboration in a team would involve certain play elements, and consequently an activity using these tools would be considered play-based. Similarly, something being playful may mean that it incorporates elements that are similar to play or that encourage free exploration, but that is not pure play.

II. WHAT IS MATHEMATICAL PLAY

Mathematical play is play in which students engage in some kind of mathematical thinking, not necessarily requiring math vocabulary or any pre-defined math processes. This means students are engaging in some kind of voluntary, pleasurable, and varied activity that involves some kind of mathematics content or skill relating to math. Abstract thinking or problem solving are skills that can be developed through play that will help with and which are part of math learning (Parks, 2015).

While play could be considered as a tool for teaching any content area, particularly in early childhood contexts, the focus of this paper is on mathematical play specifically. Using play in mathematics is important in how we can help students to better enjoy and appreciate the subject. As said by Ginsberg, "if we drill preschoolers in number facts, we may increase their current and subsequent scores on tests that emphasize this topic… but we may at the same time fail to foster their current more genuine mathematical interest," (Ginsberg, 2006, p. 158). It is motivating for students to understand and contribute to the context in which they are learning. Van Oers discusses that "in order to be meaningful and to be stimulating for development, learning necessarily should be meaningful in… both a cultural and personal dimension at the same time," which is a concise explanation for how unique play situations in which the child is in charge of the decisions can be more stimulating because "personal meaning… relates to the involvement of pupils in practices that make sense to them," (2010, p. 26). Through non-play-based instruction, students may have more trouble seeing how skills like algebra or geometry will be applicable to their lives or their futures. This is why mathematics play has such potential for mathematics education. The purpose of this specific kind of play is to allow students to explore within their current world view and set of understandings from home in order to attempt to expand their knowledge of the world through a variety of joy-producing and easily understandable activities. Applying play to mathematics contexts may be the key for some students to understand why math is an important part of their lives and to truly engage with the subject in their learning of it. Play, since it is "so intimately connected to both children's home lives and to their early mathematical learning… provide[s] an invaluable opportunity to strengthen bonds between homes and schools and to support a pleasurable entry into early mathematics" (Parks, 2015, p. 22).

Some define math play with secondary students as a "way of engaging with mathematics that has emerged naturally for mathematicians and some students," (Williams-Pierce & Thevenow-Harrison, 2021, p. 510). This definition is limiting because it only considers formal mathematics learning that students then engage with in a playful way. I would argue that younger students can also engage in math play through play-based activities with mathematics embedded within it. In a more comprehensive sense, math play is the act of playing in a way that produces or is related to mathematical thinking. There is great variety in this definition because the mathematical aspect of the definition is open-ended. According to Ginsberg, we can consider three types of mathematical play, "[mathematics may be] deeply embedded within play, play may center on mathematical ideas and objects, and play may center on the mathematics that the teacher has taught" (2006, p. 148). The first way of mathematical play occurring is through a situation in which mathematics is embedded within play. Play is foregrounded in this type of activity. In this kind of math play, students engage in some play scenario or exploration that through some circumstance requires mathematics knowledge to make the play more believable or a scenario in which students encounter mathematics unexpectedly. This may include pretending to exchange money when playing restaurant, pretending that time has passed after fake-sleeping in an acted-out scene, or having to think about distances or measurements in playing a sport of some kind. I would consider mathematics embedded in play as a kind of 'play-centered mathematics' since, as the name suggests, the play is the primary activity being engaged with. Play-centered mathematics is a common engagement in mathematics play for children, and the type of mathematics play that I focused on in preparing activities for this project (See appendix).

According to Ginsberg (2006), another related kind of play, which I would also consider a type of play-centered mathematics, though one distinct from mathematics embedded in play, is play centering on mathematical objects or ideas. Play centering on mathematics ideas is when students are playing with mathematics directly, that is, students understand some mathematical concept and play with it through voluntary, pleasurable, and varied action. This is similar to mathematics embedded in play in that the center of the activity is the play rather than the mathematics. It is different from mathematics embedded in play because when students engage in mathematics embedded in play they find themselves engaging in some math learning without necessarily recognizing it whereas in play centered on mathematical objects or ideas the mathematical applications are more explicit (though students may still not be fully aware of that mathematics). An example of this kind of math play would be students deciding to count objects rhythmically or see who can run a farther distance. The mathematics being engaged with is not formalized for students in this kind of play, but there is significant mathematics content being used or considered.

The third type of mathematical play according to Ginsberg is what I would consider the reverse of these two kinds of 'play-centered math': 'math-centered play'. As the name suggests, math is foregrounded in this type of activity. Math-centered play occurs when a student is engaged in mathematics learning, and they voluntarily play with the content they are learning by choosing to manipulate numbers, graphs, or other math pieces in a way that was not taught to them in a way that is voluntary and pleasurable (Ginsberg, 2006). I would argue that this is what mathematicians engage in when exploring new ideas in the field. They choose to play with the knowledge that they already have of mathematics in order to make new discoveries about the world or simply enjoy that which they already understand about the realm of mathematics. The difference between this and other kinds of math play is that the core of this type of play is the mathematics. Students are engaging with mathematics concepts and then, through exploring within the realm of mathematics, find themselves playing with the ideas they have been taught explicitly.

I would suggest that these three kinds of play proposed by Ginsberg can be considered in two primary ways that students engage in mathematical play. These two ways are separated by the focus of the play, whether it is "play-centered math" or "math-centered play". It is important to remember that, regardless of whether an activity focuses primarily on the math or the play, that for something to be considered play, it must be engaged in voluntarily. My definition of *mathematical play* is thus any activity falling into one or both of these categories. Any activity which meets the criteria for play in some respect, ranging from pure play to play-based, and includes some exploration of mathematics content, regardless of whether it is knowingly engaged in or not, is math play. I define math play in this way to both make it inclusive of many voluntary, joyful experiences that people have with math, but also to ensure that things which are not play (that is, not at all voluntary, varied, or joy-producing) are not included in my definition. Math play, just like the broader term play, is a spectrum of experiences, the purest of which is completely voluntary and initiated by the individual engaging in the activity. It is also important to consider that math can also be engaged in with a spectrum of awareness of the mathematical knowledge being learned through the experience. A student may compare two distances without realizing they are engaging in mathematical thinking, or a student may be completely engaged in exploring deeper ideas within mathematics itself, and both would be considered math play by my definition. It is also important to remember that, even if an activity is not purely initiated or chosen by a student, it can still have valuable applications in a classroom, even if not in a way that would be considered pure play. A strategy of "combining playful child-directed activity with teacher or adult supported or guided learning objectives" can be helpful in mathematics education (Parker & Thompsen, 2019, p. 17). The important thing to consider is not the specifics of where within the definition of math play an activity fits in any given scenario, but rather how to best implement math play in a classroom in a way that sustains the voluntary part of play without sacrificing the mathematics.

In early childhood contexts, math is often unconsciously embedded in the classroom simply by providing students with blocks, numbers, puzzles, and other math materials to engage with in their play. These things are important and helpful for students in beginning to explore where mathematics occurs in the world, but the helpfulness of these activities is limited when considering formalized math which must be taught. PreK classrooms provide significant time to explore and introduce concepts like spatial awareness, comparisons of whole numbers, and other simple math concepts as they arise in play. These concepts are addressed in elementary classrooms as well, but the play-based contexts in which they more naturally arise are often missing. It is incredibly powerful for young students to practically experience these mathematical ideas before learning formal mathematics, but the question of how to extend this kind of exploratory and voluntary learning is difficult to translate to upper grades.

In upper elementary school, there are often so many standards, tests, and regulations placed on students and teachers that open-ended play becomes less practical and, unfortunately, unrealistic. To encourage mathematical play in these grades, activities may need to have more structure than would be ideal to fit the criteria for 'play' in order to fit into the school day as it is currently structured in most public schools. These types of activities will no longer be pure play, but in designing play-based activities and explorations, teachers can still make an effort to keep the activities as open-ended, joyful, and new as possible while still focusing on required curriculum and standards.

III. BENEFITS OF PLAY

In considering the benefits of using play, and math play in particular, it is important to remember that play is a voluntary experience for children where many activities throughout most school days are not chosen by the children themselves. This creates a very different dynamic in play-based activities than in other instructional methods because the nature of the activity is determined by the child's experiences at home and in society. Children's expectations for how the world works will thus take on a large role in how play takes place in the classroom, and there are some ways in which this trend is beneficial for student learning and success. Choice-based and pleasurable activities being implemented can help students to better appreciate mathematics in their own cultural and home contexts, understand math content and develop problem solving skills, learn how to work well with others, and can engage students in unique ways.

Mathematics play, in its many different forms, can help students to better comprehend and appreciate mathematics ideas in how they relate to their home life and how these ideas may be useful to them. Van Oers (2010) claimed that "the emergence of mathematical actions and concepts in young children's activities is at first essentially dependent on the adults' cultural reaction to the child's actions and utterances," meaning that at first, students do not have the language to formally label the math they are engaging with (p. 29). This leads to the conclusion that in initial interactions with mathematical ideas "mathematical thinking is 'projected' into the child's mind in the process of cultural interactions with others, wherein the mathematical meanings of children's actions are negotiated with the help of symbolic means" (van Oers, 2010, p. 29). Playing with mathematics concepts cannot give students the formal language they need to engage with mathematical ideas, but it can give students fundamental experiences with math ideas that they are not yet aware of. This is an opportunity for teachers to make connections between schoolwork and students' cultural understandings that they bring from their lives at home in order to promote mathematical understandings. As can be seen in any play scenario, "through interacting with everyday materials, such as containers, collections of small objects, lines, and sticks, children build their conceptions of what mathematics is," by engaging with different math ideas using these materials (Parks, 2015, p. 12). There are many things which can be considered play materials for mathematics learning. Even commonplace items can be playful in nature when they are freely chosen by students and used in a variety of ways in pleasurable activities. These materials can encourage students to think about spatial awareness, counting numbers, and geometric ideas while still promoting play-based explorations.

In thinking about these mathematical ideas and concepts in their own contexts through play, students are able to also develop better understandings of the math content itself. There have been several links found in play research between problem solving and creativity in children (Holton et al., 2001). Since creativity is a foundational piece of the pleasurable and variety requirements in defining play, we can thus see that play can help students to develop problem solving skills that they will need for both mathematics and other subjects. Even in less recent conceptualizations of play, "the notion that play is related casually to mastery over one's environment is reaffirmed," indicating that for many years and through different play theories, a link between learning about one's environment and play has been noted (Rubin, 1982, p. 12). Even in casual contexts, we can see how students who play with objects or ideas have more refined ideas about such subjects because "mathematical play provides a non-threatening environment where incorrect solutions are not read as mistakes" (Holton et al., 2001, p. 404). There have been many explorations into the effects of play on understanding and on student success, as previously discussed. These explorations indicate that a variety of factors such as a comfortable environment or an open-ended prompt that make an activity play "may lead to a better understanding of the problem and/or the confrontation of misconceptions" (Holton et al., 2001, p. 404).

Another benefit of math play is that any kind of play, including math play, can help students to develop social and emotional skills in the classroom because there is so much choice and variation involved. It is fairly common for play to be a collaborative effort between multiple students, and this is an excellent way for children to develop skills in working together and interacting with others. One of the primary ways that students engaged in learning in a play-based curriculum which was used by van Oers was through learning to communicate with one another in culturally appropriate ways for different scenarios (2010, p. 30). Because culture plays such a large role in the choice aspect of play, it is thus also an important part of how communication skills are developed. When students are faced with differences in expectations due to different home lives, they must work together in order to truly enjoy the play and be engaged with one another.

Engagement is another benefit of mathematical play. School can be a challenging place for students because, at times, they may not be motivated to learn what they are being taught. Because play is open-ended and pleasurable, by definition, it is only natural that play increases student engagement. Aesthetics is an important piece of play because aesthetics are the ways in which people enjoy sensing the world around them in multiple ways. In terms of engaging with ideas,"… this role of mathematical aesthetics involves the development of personal interests that support motivation to engage in particular ways" (Jasien & Horn, 2020, p. 43). Encouraging students to play with things that interest them and develop further ideas about what their interests are, specifically in terms of what they find pleasing to their senses, can help them to be motivated to continue to engage in scholastic pursuits, even mathematical academic pursuits. Being able to make choices and pursue new ideas can be very engaging for students because they are able to do what they want to do and in the way they choose to do it. It can thus be seen that "…rule-bound play has considerable motivational power," and is thus something valuable that should be used to create greater engagement in the classroom (Holton et al., 2001, p. 403).

In summary, there are many benefits to incorporating play in mathematics instruction. Using play with math in the classroom can help students to better understand and appreciate mathematics ideas for their practical use, to develop social and emotional skills alongside math learning, and to be more engaged with the content they are learning.

IV. DESIGNING PLAY-BASED ACTIVITIES

There are several factors essential to consider when using play-based activities in actual classrooms. When planning any activity for a group of students, the job of educators is to prepare in advance, consider potential problems, and ensure that the planned activities are beneficial for their students. In planning, a factor to consider is planning according to what is known about a specific group of students and their personal contexts. Every classroom is different in students' cultures, backgrounds, and personalities, so it is important that each teacher specifically considers how best to help their students to learn. An important part of any learning is that students are able to relate to the content they are learning and understand how it is connected to their lives. Using play in mathematics education has the potential to make the content more relatable, tangible, and realistic to students instead of abstract and foreign. Any play-based activities that are used in the classroom should ask students to play in ways that are familiar and relatable to their personal lives. For children, the very beginning of "mathematical thinking is 'projected' into the child's mind in the process of cultural interactions with others, wherein the mathematical meanings of children's actions are negotiated with the help of symbolic means" (van Oers, 2010, p. 26). Considering what kind of play students usually engage in and being mindful of this context in planning activities can be beneficial as students often have a foundational knowledge of some kinds of play and this background knowledge can be used to prepare relevant activities for students. These activities and explorations similar to the children's first engagements with mathematics ideas can then be built upon to give students more formal ways of solving and communicating about math.

Another important consideration is whether or not students will choose to engage with the activity provided. In order to be considered pure play, an activity must be chosen without the student having to be prompted or told to engage in it; however, in a classroom this becomes more complex. In order to encourage the purest form of play possible, students should be given as much choice as is possible. Some classrooms will be able to provide more choice and some will need to limit choice to ensure safety and productivity. It is important, if choice is being provided, that any activities available for students are interesting for students so that they will choose to engage in it as intended. Activities requiring students to complete challenging or frustrating tasks with no regard for their likes or dislikes is unlikely to yield true play because students will not choose to engage with these things.

Another important factor to consider in terms of attempting to maximize student engagement is aesthetics. "Aesthetic practices exist as a relation between sensory ways of knowing and making sense," which factors heavily into how and why students play (Jaisen & Horn, 2020, p. 41). The aesthetics of an activity will take into account the child's sensory ways of knowing including that which is perceived by the senses: sight, smell, taste, hearing, and touch. Sense making is defined as how an individual will understand something as both reasonable and desirable, or the opposite (Jaisen & Horn, 2020). These related features of sense-making and aesthetics are important to consider in play and in mathematics. Children are more likely to be intrigued by and engaged in an activity if the activity provided allows them to explore within those things which they can know and enjoy through their senses, or those things which are compelling in terms of aesthetics. It is impossible for an activity to plan for the specific practices which 20 different students find will find aesthetically pleasing before the activity itself, which is why any play-based activity must be flexible and open-ended, allowing for each individual student to explore their own personal aesthetics through engaging with it.

It is also essential to consider, beyond the pursuit of making an activity as play-based as possible, how to teach mathematics content using an activity. The foundation of any play-based mathematics activity should be a math-focused objective for students to learn. This can span many types of math, but there are some areas of math education which may be challenging to teach through play. Whether or not a given topic can be taught through mathematics play is a difficult question to consider. There are many factors influencing what is most helpful or engaging for a student. If the objective of a lesson is to have students explore a new idea, better understand something they have already been taught, or think about abstract concepts, then it may be helpful to use a play-based activity to accomplish this objective with students. However, if the goal of the lesson is to teach students how to computationally complete a task on an assessment, there is likely not much space for play. Engaging students initially in a concept or encouraging students to think more deeply are both helpful contexts for students to play and learn to enjoy mathematics through its practical applications. This is supported by the idea that "play aids in the development of creativity and aesthetic appreciation" and that "play in childhood allows the practice and mastery of activities which are later useful for serious endeavors" which is logical in considering the often-mimicking nature of play (Ruben, 1982, pp. 11-12). Thus, play should be used to develop more creative thinking and to practice thinking about serious concepts at early stages. Teaching students how to efficiently solve a given problem using pencil and paper only, however, is likely better taught through other forms of instruction for students to understand standardized ways of completing problems. Though students learn much through play, students also need to practice applying the concepts they learn to concrete examples of problems on tests and in projects for their future education, test-taking, and some career paths. In different play contexts, it is clear that "although [play] work provides an important context for developing mathematical ways of thinking and content knowledge… this sort of play is not the same as learning mathematics content." (Parks, 2015, p. 11).

When mathematics play is actively occurring, teachers often observe students engaging in mathematics ideas that were not planned for in the activity itself because "when children are allowed to make choices about how to spend their time, not only do they choose activities that they find pleasurable, but they also have opportunities to develop richer understandings over time" through a variety of means (Parks, 2015, p. 7). In my experiences working with children, they often have a way of using objects and opportunities in ways that adults often do not anticipate. This should be encouraged in play contexts since an important piece of pure play is that it is chosen by the child and the nature of the play is determined by the children engaged in it. As play-based activities are planned, we must remember that children will have ideas that we cannot anticipate. Being mindful of where there is freedom in the activity for new mathematical ideas is important to consider in planning play-based activities. Leaving space for students to do things that may not be directly related to the task given is essential. This spontaneity and unpredictability of student thinking during mathematical play is an important asset teachers can learn to harness in their classrooms. Students will have new and unique ideas that should be heard, acknowledged, and accepted by the teacher even if they cannot explore them more fully at that particular moment. Telling a student explicitly when you notice they have just had a wonderful mathematical idea is a powerful way to position students as mathematical doers and thinkers. Educators should be careful in using these activities to not only accept new ideas from students, but also encourage them. This can be another way for mathematics to be included in play-based activities. If students are trying something new or asking the teacher for suggestions in terms of what to engage with, the teacher can easily curate math thinking by guiding students towards explorations that relate to mathematics in some way without sacrificing the choice of the activity because the student asked for guidance. For example, if students are engaged in playing with blocks and building a tower, a teacher may ask students to consider which tower is taller or wider to teach different dimensional awareness as well as comparisons of distances. Students may then further engage in these ideas and attempt to build larger or taller towers, having been inspired by feedback from a teacher. Teachers should plan as much as possible in terms of potential suggestions to further mathematical thinking in play, but also be flexible by allowing many different ideas and pointing out math ideas when they are engaged in, regardless of whether they were planned beforehand.

There are also various contexts in which math play can be encouraged in the classroom. The kind of play-based activity that was explored in this project is through the use of stations. Stations are common in upper elementary school classrooms as a strategy for having students explore different related topics in small groups. Play-based stations can vary in content and amount of choice according to what is appropriate in a given classroom. To make stations resemble play, a recommendation is to allow students to choose which station they want to engage with on any given day. This may be impractical for some classrooms, but allowing students to choose their activity for the day increases the voluntary aspect of play and makes students more likely to be engaged in what they have chosen. If this is for any reason not practical to use in a particularly loud or challenging classroom, then required stations can still have aspects of play without being entirely voluntary. Other contexts for play-based activities to consider in planning for play include whole-group projects that may be curated to include play materials or open-ended prompts without being chosen freely by the students completing the project. This can be helpful in teaching some content in a less voluntary, and thus less pure play, way. Another potential way to incorporate play in planning for a classroom is through individual activities that can be chosen by students. This does still incorporate more choice, depending on the specifics of implementation, but can be more complicated to incorporate into the classroom due to different needs of students and a need for some amount of order during the day.

V. PLAY-BASED MATHEMATICS ACTIVITIES FOR A CLASSROOM

As part of my thesis, I designed several play-based mathematics activities for upper-elementary classrooms. Some of these activities were implemented for a week in a 4th grade classroom. Before implementation, the classroom teacher reviewed and slightly modified the activities to make them beneficial and realistic for her students in particular in terms of content covered and structure of the stations. The activities below are the finalized versions of the four play-based stations as implemented in Ms. H's fourth-grade classroom: tangrams, geoboards, business management, and designing a room. The original drafts for the stations are less refined and focus on a variety of grade level standards (See the Appendix for the original drafts of these play activities).

Tangrams

The first activity used in this classroom is titled Tangrams. Tangrams are well-known puzzles involving 7 shapes (triangles, a square, and a parallelogram) to create a larger shape or pattern. Many tangram patterns exist online for people to attempt to recreate. To make this station as open-ended as possible, no directions were given to the students, only the materials they needed and a few written suggestions for students who did not know where to start. Students were encouraged to create their own patterns after building an understanding of the puzzles and try to solve one another's patterns. This activity encouraged students to use their imaginations to create new puzzles. Elementary aged students are familiar with puzzles similar to the tangrams that have been used in other math classes or at home, making it far easier for them to engage with this station and begin to think about spatial reasoning through this activity.

In terms of content, this activity focuses on spatial awareness and reasoning as well as different shape types. In completing and considering puzzles, students need to rotate, flip, and shift pieces. These are all type of geometric transformations addressed in K-12 standards (translations, rotations, symmetry, etc.). Students also need to be able to identify shapes and work with area in different contexts. Thus, this activity can also be used to build student awareness about these different categories of shapes and their defining attributes as well how the area of these shapes is related to the area of larger composite shapes. This addresses ideas relating to the additivity of area and shape decomposition, which can help students in learning how to solve problems involving area and better understand where the formulas for areas of various shapes originate.

Geoboards

The second activity included in the stations for this 4th grade classroom was entitled Geoboards. This activity, again, aimed to primarily provide materials for math thinking rather than directions for students to follow. Geoboards are small boards with pegs placed in a 4x4 grid on them. The intent was for students to put rubber bands on these boards in order to produce shapes or patterns. There are many images that can be created using these boards and, in this context, students were encouraged to create whatever they would like. They could recreate a given pattern, create their own patterns, or do anything else they would like using these rubber bands and boards. Students were, again, encouraged to collaborate, but not required to, by creating patterns for a friend to replicate.

This activity emphasizes the importance of aesthetics in play. Students can create what they want to see and what they are proud to make. This activity allows for significant choice and can serve as an untapped medium for art. Geoboards may or may not be familiar to 4^{th} grade students, but the way in which the students use them in this activity is likely to be unique. Allowing students to express themselves by creating mathematical designs and objects can help students to see that math is practical and valuable to them in pursuing creative expression. In terms of mathematics content, this activity aims to teach students about perimeter and categories of shapes through their explorations using Geoboards. The rubber bands around the pegs can create a multitude of shapes for which the rubber bands represent the perimeter, even if this is not initially how students view the activity. This can be easily connected to the idea of perimeter and even area for some shapes. It can also teach concepts such as parallel or perpendicular lines since these are things students are likely to encounter in creating patterns and shapes.

Business Management

The third activity, managing a business, used in this classroom was slightly altered for the students. Originally, the activity was very broad in nature, encompassing any business that students wanted to create and using any means for currency that the teacher had, though ideally it would be fake dollars and coins. In the original activity, students were asked to create a business of some kind that they were expected to grow, hire employees for, create products for, and manage the finances for. However, this activity was altered to suit the materials available, and it became specifically managing a restaurant. While this did remove some elements of pure play in that students were told what kind of business to manage, the activity was still open-ended in that students are able to choose how to run this business and what financial decisions to make. This activity was relevant to the students because it allowed them to explore the logistics of managing a grocery store or a restaurant, something that they all have encountered in some way in their lifetime.

Since most students are familiar with the idea of running a business, this activity helps them to see how complex it can be to manage money, stock, employees, and so much more. It also gives them the freedom to put any ideas they have had about businesses into practice to see how they might work in a future business career. In terms of mathematics content, every grade level must learn financial literacy skills. For 4^{th} graders, this includes recognizing coins and bills for their value and completing calculations with these values. In this activity, students practiced not only computational skills (i.e., counting, adding, and subtracting amounts of money), they also learned how to manage money through this experience. This activity in particular could be extended in many different directions depending on the instructional goals that the teacher is interested in addressing.

Designing a Room

The last activity used in the 4^{th} grade classroom was designing a room. This was, again, altered from the original packing plan in order to better suit the grade level. In this activity, students were given small boxes and doll house furniture to organize within these boxes to design a floor plan for a room in their house. In this exploration, they had graph paper available as well as a few examples of floor plans. Using these tools, students could design a floor plan for a particular room and then attempt to create it with the dollhouse materials or could create a room with physical materials and then attempt to map it on the two-dimensional graph paper, or explore either medium independently of the other. This activity was relevant to the students because they have seen many different rooms with furniture throughout their lives and maybe even spent time considering the layout of their own room at some point. They thus have a context to which they can connect this activity. Students could explore how a room is put together and how they could plan for more efficiency or space in a room by changing the locations or rotations of furniture.

In terms of the mathematical content, this activity aims to teach students how to move, identify, and manipulate shapes within a space in a practical context (i.e., geometric transformations). It also teaches foundational ideas about area. Rooms are excellent examples of how area appears in many students' lives because they can directly see how different objects take up different amounts of space. Through this activity, students become more aware of the implications of area and potentially of how to find the composite area of a space using smaller units or even many units of different sizes.

Why are these activities play-based?

We can identify these as play-based activities because they meet the discussed criteria for play. These activities are not what would be considered pure play by the definition given, nor were they intended to be. Rather, they are meant to attempt to mimic common play behaviors in an academic setting so that students are still engaged in thinking about practical/relatable applications of their learning, but they are not completely freely chosen activities so that learning is not impeded. Using materials that support play by being open-ended, providing suggestions rather than directions, and allowing many different ways to engage with the stations are all reasons these activities would be considered play-based.

Students were required to work on an assigned station each day and not permitted to move between stations, eliminating much of the choice that would have made their work pure play; however, there was some choice involved because students were not required to follow the provided prompts, as long as they were engaged with the materials provided. These activities were pleasurable for most students, but not as fully pleasurable as pure play would have been because choice was limited. Many students wrote that they enjoyed most of the stations, but some included in their descriptions of a few stations that they disliked them or became quickly bored by them. This is indicative of play-based practices because students generally enjoy what they are engaging with, but not universally due to a lack of complete choice. There was variation in these activities between days and between students, in how they interacted with the provided materials. Because each of the three play criteria (choice, pleasurable, varied) were partially addressed in the activities, I considered these activities to be play-based; but because none of the activities addressed all of the criteria to their fullest extent, these activities are not what I would consider pure play.

VI. REFLECTION ON ACTIVITIES

The four activities discussed above were implemented in a 4th grade classroom at Ruth Barron Elementary School in Pflugerville, Texas. These activities were implemented as stations, which students rotated through each day for a week. Each group of about 4 students engaged with the materials and prompts for each activity for about 30 minutes each throughout the week (See appendix). After each day of working with the stations, students were given a brief reflection prompt reading: Think about the math play in which you and your team just engaged. What math connections did you see and think about? How did the play deepen or support your learning of a math topic? The student responses to these prompts for each of these four activities as well as the cooperating teacher's reflections on the activities as a whole are included below. The effectiveness of these activities as well as potential future changes or applications for these stations are then discussed in light of these reflections.

Tangrams

Students who engaged in the tangrams activity most often noted in their reflections that they were noticing shapes—both in terms of identification of shapes and the composition of larger shapes with smaller shapes—and geometry more generally. Students identified some shape attributes such as right angles or parallel lines within the shapes, but a majority of students only had the language to say that they were doing geometry in general. There are some aesthetic practices in this station since students were able to create patterns that they found pleasing in sensory ways, but these practices were somewhat limited by the color choice of the shapes and the potential kinds of images that could be created by the shapes provided. Many students did not enjoy this station as much as the other ones, saying that they quickly got bored or did not enjoy the limited nature of the activity since a majority of the time was spent simply manipulating or tracing the shapes.

While some students did not enjoy this station and likely would not have chosen it on their own, they were able to manipulate and directly consider how shapes can work together to compose larger shapes or patterns. Many students spent their time with this station creating their own unique shapes by putting the smaller shapes together. Students needed some assistance from the teacher to trace the shapes they made, but enjoyed creating their composite shapes and challenging others to create the same pattern.

Encouraging students to work with shapes like these may be helpful in allowing children to use experiential learning in their exploration of shapes and ponder how different types of lines and angles may or may not fit together. These kinds of experiences can support students in (a) focusing on different attributes of shapes (e.g., side length, angle measure, number sides), (b) understanding the difference between some quadrilaterals which can be difficult to identify, and (c) considering relationships between shapes, such as noticing when right angles fit together or when multiple triangles make up one rectangle. This activity could also help students to visualize composite area problems in which they have to split an irregularly shaped object into simpler shapes and add their areas together in order to find the area of the total diagram. Tangram puzzles are a direct visualization of this process that students will be able to build upon in order to not just do, but also comprehend these problems.

While students were not able to clearly articulate the math that they were engaging with in this task, they were practicing geometry ideas in a non-standardized way. These fourth-grade students have already learned much about shape names, angle names, and types of line relationships, meaning that a majority of their learning was applying math knowledge they already had. This does not minimize the amount of math play occurring but may be a contributing factor to why the students were not as engaged with the task. Along with learning about geometry and shapes, students were able to practice manipulating shapes and thus develop spatial awareness. When students shared their patterns with one another, they also practiced social skills and considered the process of formalizing the work that they have done so others can imitate it.

Geoboards

Students engaged with the geoboard activity noted that they were thinking about ideas related to perimeter, angles, shapes, and area. There was much variation in the responses to this activity because the concepts the students explored depended largely on what kind of pattern they were attempting to create. Students saw how lines could be arranged in different ways relative to each other on a plane, how shapes could be created, and even where different types of angles were between some lines in their creations. A few students were even able to connect this activity with content they had learned previously. One student reflected on the parallel, perpendicular, and intersecting lines she saw as well as right angles appearing in her work. This is an impressive connection to make, and one that could be more directly illustrated for all students in a future lesson if so desired by the teacher. Students seemed to enjoy the aesthetic practices of this station, since each of them was able to make unique patterns using colors and a variety of rubber band lengths in whatever way they preferred to make pictures, patterns, or anything else. Students thus had a generally positive opinion of this station.

Students in this station were creating lines, angles, and shapes for themselves. The pegs on the geoboards make it very easy to visualize perimeter and length because they clearly show the distance from one peg to another. This station allows students to see a potentially new exploration of distance for themselves and consider how one rubber band can create many different shapes, with vertical or horizontal edges across the pegs or diagonal edges between diagonal pegs. Being able to create a geometric object themselves and determine how far it would be to ‘travel’ around the object (i.e., perimeter) can help students to visualize what perimeter means and how it relates to different types of shapes and area.

Similarly to the tangrams station, many of the ideas students were engaging with are ideas that have been previously explored and taught to them. Students did not engage in creating any formal definitions of new ideas, but did consider previously learned ideas about lines, angles, shapes, and perimeter. Students were able to work with one another to imitate different patterns provided that included repeated shapes or lines in particular directions. The previously learned content embedded in the activity along with the encouragement to imitate other provided patterns helped students to subtly engage with geometric ideas while still enjoying the process of creating. Allowing students to physically create and explore geometric ideas in this format before introducing standard names and processes for solving geometry problems is beneficial for students to connect different graphics that are used in math such as charts or coordinate planes with tangible experiences they have had in this context.

This activity, much like the tangrams, teaches not only geometry, but also spatial reasoning skills. Students are able to physically manipulate rubber bands to expand them and contract them as well as use them to copy a pattern. When students chose to copy a given pattern, they had to use problem solving skills to determine the best way of using their resources to create the pattern. I observed a student attempting to make a large shape with one rubber band, but he found he could not stretch it far enough to make the desired shape. The pattern required multiple bands being used in straight lines to make up the sides of the shape. Reasoning about ideas like this is an important skill for students to engage with whenever possible. Students also worked together and developed social skills through this station. Students asked one another questions about how they accomplished different compelling patterns and seemed engaged in one another’s creations.

Business Management

It was fascinating to read students' reflections on the business management, or restaurant, station. This was one of the students' favorite stations, as many pointed out in their responses. However, it was also a station in which a few students could not see math content at all. Most of the comments about the math being learned discussed the idea of 'money' as being the math content. This is a very vague way of describing the learning that took place. Some students mentioned that they engaged with ideas of pricing or making change in this station. The intent of this station to have students create a business with employees, prices, and profit was too complicated to fully incorporate into a 30-minute station activity. Students were, however, able to see that there would be math in adding or subtracting monetary amounts and engage with ideas of pricing and comparing price with the amount paid for a meal. Students were encouraged informally by their teacher to set a price for different meals and consider how much profit they might make if someone paid extra for a meal. One student wrote that she had to think critically about how much she needed for a meal to be sure that another student was not paying her less than the listed price.

This activity was a favorite of several students because there was much room for creativity in the implementation of the restaurant. Many students very much enjoyed dressing up with the apron and arranging the plastic foods on the plastic plates for the 'customer' to enjoy. Students enjoyed pretending to be different kinds of customers and servers, making different food recommendations, or pretending to be a demanding customer. This reflects social skills and social observations that students have made in the world around them about different ways that people may act in restaurants. This does not teach any cooking skills but may encourage students to think about the art of food preparation and the art of designing a business both in terms of customer service and in terms of how a restaurant might be organized.

In terms of the content, students may learn money management through hypothetical situations, word problems, or their own lived experiences. Encouraging students to manage money for a business or simply for themselves can give students practical knowledge for why money management is important and how to effectively save, set prices, and manage a business. Having students work with money and practice giving change is also beneficial because this is a skill they will likely need in their futures. Giving students a practical, hands-on experience in which they handle money and learn to quickly do mental math with coin values can be beneficial not just for money management skills, but also addition skills, subtraction skills, and rational numbers (specifically decimal numbers to the hundredths place).

In this specific context, students had trouble practicing these math skills because the materials provided did not have pre-set prices listed for different foods nor was there a fixed amount of money provided to the students who were pretending to be customers. This led to less engagement with the money itself and further engagement with the fake foods and acting pieces of this activity. Students were, however, able to engage with the conceptual idea of running a business in which prices must be larger than costs and change must be given for payments over the stated cost. These are valuable lessons to learn before being introduced to formal processes for these skills. These students still have not begun their financial literacy unit, so this activity likely introduced some students to the idea of price and payment. These ideas can be built on when the teacher introduces this content and the specific curriculum standards that students are required to know more formally. These activities lay a foundation in reality and lived experience for students to draw upon as they learn about the mathematics content.

Floor Planning

Students, in reflecting on the floor planning activity, were very adept at recognizing the ideas of perimeter, area, and measurement in the course of their exploration. This was the most successful station in terms of children enjoying the play they were engaged with while still recognizing the specific mathematics skills and concepts they were using to do so. Students also made comments that indicated their developing skills in spatial awareness. Some students tried and succeeded in fitting all of the provided objects in the room or reflected on how they had to move and adjust furniture to help more to fit.

This activity gives students a practical application for area and perimeter knowledge. Students are able to see why the area of different objects is important to consider and begin to better understand that a total area can also be described by a set of smaller objects which take up the entire space when combined. The ability for students to move the objects around themselves and truly think about how objects can fit in the room in different arrangements gives students experiences with multiple scenarios in which they have to consider area or perimeter. Students could also choose to work with grid paper, an important mathematical tool, particularly for finding area and perimeter. This activity, much like the others discussed, could be used as an introductory activity for students learning area and perimeter for the first time. However, since these students already had some foundational understanding of area and perimeter ideas, this activity did not teach new knowledge. It did, however, provide a safe and engaging space for children to apply their math knowledge in practical ways. Word problems about similar scenarios become easier to comprehend when you have interacted with the world in the way being described. In addition to spatial awareness, this activity provided opportunities for practicing social skills as several students worked together to design a given room. This play-based activity allowed students to see how these mathematical concepts might be seen or used practically in their own lives.

Feedback from Classroom Teacher

Mrs. H, the cooperating teacher who agreed to implement these activities in her 4th grade classroom for a week, was incredibly helpful in revising and making the above activities practical for an actual classroom. In asking for her feedback on how well these activities went in her classroom and whether she would consider using similar activities again, she reflected on the experience:

The students loved playing! They enjoyed exercising their imaginations and working together. In the future, I would use these math play activities to pre-teach/prep my students for a unit. Since I already do math stations daily, I can imagine having one station of my four per week be for this purpose next year. I think having a station where students build background knowledge, have common experiences from which we can draw, and explore/review prior knowledge, and build excitement for the upcoming unit would be so valuable! I would definitely mathematize the stations more, however. I found that, for the most part, the students were not having mathematical conversations, and seemed unaware of the math they were doing/thinking about. I would have a journal entry, key vocabulary words, or a task to frame the experience and elicit more mathematical thinking/work. It could be looser than what I require for my other stations though since I would have slightly different goals for these introductory stations.

Coming from a PK-2 background, and as a teacher who believes that student agency and engagement in mathematics is essential, I was not a hard sell on your activities! I do think this might be a large shift for many upper elementary teachers. Just the idea of offering stations/centers as part of a math block, and providing students with choice, and opportunities to explore and play with math concepts might be a hard pill to swallow. Helping teachers see how to include centers routinely in their math block and helping them think through the practical management piece is essential. We bypassed that in my room because I already have that structure in place, but I think that is often one of the biggest hurdles for teachers. However, this type of learning is so valuable and motivating!

Overall great ideas! My students and I benefitted from this experience… You also helped me revisit my thinking on the place of play in my classroom, and I now have a new idea for a great way to build background knowledge and common experiences for my students prior to starting new units. This is especially important now as many of our students do not have the depth and breadth of experiences, they would typically have had due to changes caused by virtual learning (or just missed days).

In reflecting on the experience this way, Mrs. H brings up many valuable points to be considered in future implementation. The balance between the amount of choice offered and the amount of mathematics content covered is important to emphasize in the planning of these activities. It is also important to note that many upper elementary teachers, in her experiences, may have a challenging time finding both the time for and value in implementing these kinds of activities.

Ideas for Improvement of Play-based Activities

The tangrams activity was mathematical in nature. Students engaged in turning shapes, flipping shapes, looking at composite areas, and identifying shapes. It could be improved, however, if students are more engaged in completing it. Providing easier patterns to follow, with the image of where each shape goes, may benefit students since some patterns provided for this implementation were challenging. Providing smaller or simpler patterns would be ideal so that students can accomplish some goals in imitating designs and these simpler patterns would ensure access points for all students. Students should also be encouraged to build on top of a piece of paper so their designs can be more easily traced to make patterns for others to try. I also think it could be helpful to have colorful sets of tangrams, where each shape is a different color. This may leave more space for creative expression, as happened with the geoboards, as well as making the experience more enjoyable in terms of aesthetics.

The geoboard activity was largely successful in terms of math content and engagement. Students were interested in creating pictures or patterns on the board according to what they find to be aesthetically pleasing, or what they find sensorily pleasing. The main change would be using standardized rubber band sizes. If there are only small, medium, and large rubber bands, this may help students to better comprehend that perimeter affects how much space the shape can take up. Another improvement would be to know the interests of the students so some of the prepared patterns could be selected purposefully to be more interesting or appropriate for them.

An example of such an improvement may be if a given group of students is particularly interested in science, there may be some patterns that could be followed to make an image of a plant or animal, or if a student enjoys sports there could be patterns which look like various sports equipment or professional team logos.

The restaurant activity needed more direction in many ways. Upon further reflection, I believe this would have worked better as a long-term project. I also think that the smaller scale and compressed time frame needed more specific guidance in terms of how much money the customers started with and how much each meal costs. An easy way to implement this would have been to include a fake menu with prices listed on it so that students have a model to follow in running their restaurant. Having this might have encouraged students to play with the money in a way that is conducive to math learning because they were being forced to pay for things according to what was printed on the menu and thus make change in order to properly engage with the materials. For future implementation of this play-based activity, I would change the station to a whole-class activity in which everyone has the opportunity, without being forced, to take out a 'loan' of fake money from the teacher and invest in business supplies to make goods or provide services (e.g., making a craft or Lucy's counseling services in Peanuts). I think this would be a more effective way for students to consider the costs of a business and why prices must be set. These changes could encourage students to find a business that they are passionate about and truly consider what it takes to engage in necessary planning for prices and expenses. Having a system for loans, in which students must pay back any money they want to borrow but do not have, may be helpful as a foundation for their learning about banking systems and the idea of interest.

The floor plan activity was greatly appreciated by these students because they were able to design whatever they wanted using the materials provided. To improve this activity, the cooperating teacher suggested that multiple boxes be provided instead of just one to design with the provided furniture. This would allow more varied spaces for students to explore multiple designs and could be used with prompts that asked students to use the same furniture in multiple spaces to encourage further thinking about how area and non-standard units for area work. Another potential change for the floor plan activity would be to give more direction for what the room is supposed to look like. Providing premade blueprints for students to attempt to follow or add to may also be helpful for students who do not have specific design ideas from the start. This would still include blank floorplans for students to create themselves, but the more different arrangements that students engage with, the more they are engaging with ideas relating to area. This activity could also be expanded into a larger project that builds up to formalized mathematics problem solving if desired. Students could be asked to design an entire house that they would like to live in, and then asked to calculate different areas to consider if more objects could potentially fit or to find where a particular object may fit. This opens many possibilities for different tasks that could be assigned for more formal math knowledge using this activity as an introductory anchoring activity.

VII. REALISTIC IMPLEMENTATION

The practical application of these ideas is the most challenging aspect of considering math play. Play is beneficial for students of all ages in terms of engagement and motivation to complete tasks but balancing the amount of time students spend playing with the amount of time that they spend explicitly engaging with mathematical concepts without choice is essential. As much as students would likely enjoy playing with various play materials throughout their days at school, the fact remains that there are still standards to be met and required tests to be taken. So the question remaining is where this balance is—how can we implement play in functional classrooms while still ensuring students receive the instruction they need?

Stations

The primary way that play in the classroom has been explored throughout this paper is in stations prepared for small groups of students. Different stations with materials intentionally chosen to produce mathematical thinking for specific content areas can be implemented in many various ways. Having free choice station time in which the available stations are all mathematics related would be the closest implementation to pure play. If this would be too chaotic or challenging for a specific group of students in a class, then structure could be helpful, as was practiced in the stations at Ruth Barron Elementary. This takes away some of the choice of play, but is still play-based in nature because the stations use typical play materials, are generally enjoyable for students, and vary from day to day.

Regardless of how they are implemented, having stations like these can provide promising introductions to new content. For example, if a class was about to begin a unit on area, the teacher could choose to let students play with tangrams as one of their stations the week before, taking pictures of interesting ideas and noting how students engaged with them. In particular, the teacher might focus on different ways students combined smaller shapes to create larger geometric objects to illustrate the important mathematical concept that area is additive. That is, the total area of an object is the same as the sum of the areas of the smaller shapes. When the idea of area is later introduced, the teacher could begin the lesson with images of student work and a discussion focused on discoveries and observations students made the week before. This can help students to make connections to their own lives and be more engaged with the more formalized work of calculating area using composite units and actual numbers to express this area because they have seen for themselves how it could be practically seen and used in their lives through manipulating and creating patterns with the tangram tiles.

Another use for stations could be as engaging activities to apply and practice math ideas during or after lessons. In a similar example, if finding area of complex shapes (shapes made up of multiple smaller shapes) has been taught, students could then be given tangrams with which they create their own pattern or shape that they will later practice finding the total area of themselves by adding up the smaller areas of the individual shapes to find the total, additive area of this non-standard shape. This, again, can increase engagement and help students to visualize the content they have learned.

Individual

Students could also engage with play math activities individually. One way to implement this would be to have individually focused activities available at the back of the room for students to engage with if they finish other classwork early. This is commonly seen in classrooms today but having stations that are play-based and math-focused can help students to both look forward to finishing quickly, but also still engage with important ideas and continue learning content. Any of the above stations, other than the restaurant/business management activity, could be easily converted into a smaller-scale, independent activity that one student could play with at their desks during free time.

Whole Class

Whole class applications of math play are likely the furthest application from pure play. Because the entire class will be required to work on a certain objective, much choice is removed from the play, though ideally some aspects of variety and pleasurable play can still be implemented. Whole class play activities largely occur as projects that must be worked on collaboratively. Some potential projects may include building a bridge for something to roll across, designing a slingshot or catapult to launch an object, or creating and running a business. These projects would have students working with play materials such as craft supplies or play money to engage in activities that are often chosen by students in pure play, making them play-based activities. The removal of some choice for these activities does make space for more focused math learning. In these activities teachers could require students to complete a variety of tasks in a variety of ways, which may help teachers to implement play-based practices while still teaching important formalized content. For example, having all students engage in creating businesses at the same time would make it far easier for the teacher to run a 'bank' for the classroom which issues loans and sells materials. In this format, students can choose to buy any materials they want to create any kind of business they want, but to do so they must calculate the price necessary for them to make a profit off of the business. This can still be engaging for students as they sell to and buy from their friends, even if some choice is removed for the purposes of including more formal math content in the project.

VIII. IMPLICATIONS

Math play is an important instructional practice that uses the basic tenants of play to either teach, introduce, or help students explore and apply specific math content. Math play includes activities that are voluntary, pleasurable, and varied, and that involve some kind of mathematics learning. Math play can be used and identified in a wide variety of contexts. As shared in this thesis, there are many ways to use math play in classrooms to engage students in play. The activity itself can range from pure play, if there is time available for this in the classroom, to play-based, which is easier to implement yet still motivating for students. Students need to learn mathematics in the context of their lives and through personal experience so that they understand why mathematical knowledge is valuable, useful, and practical and so that they are more motivated to engage in both formal and informal mathematics learning.

Understanding more about math play and how it can change the way students learn math is valuable for educators to consider and use in designing activities for classrooms. Using play-based activities as stations, projects, or individual work can be beneficial for students as seen in the reflection on implemented activities above as well as other research and empirical studies. In order to teach mathematics our goal most effectively should be to help students to make connections between the formal math processes that they must learn and the things that they enjoy doing and understand more experientially. Play-based methods in elementary school classrooms are one way to we can achieve this goal by making math more practical for students and providing a path toward understanding formal math content that goes beyond pencil and paper.

APPENDIX

Play-Based Mathematics Activities

These activities are not intended to teach students formal mathematics content. These activities are meant to help students make connections between mathematics content and their own lives and interests. By allowing students to explore independently within the bounds of different math-focused activities, the hope is that students become engaged with mathematics ideas before the teacher introduces the formalized methods for solving and using these ideas in problem solving.

Geoboards Activity

Materials-

- Geoboards
- Colored rubber bands
- Patterns as samples for students to copy if desired

TEKS Explored-

4^{th} Grade Mathematics- Knowledge and skills.

5: Algebraic reasoning. The student is expected to…

(C) – Use models to determine the formulas for the perimeter of a rectangles (l+w+l+w or 2l+2w), including the special form for perimeter of a square (4s) and the area of a rectangle (l x w); and

(D) – Solve problems related to the perimeter and area of rectangles where dimensions are whole numbers.

6: Geometry and measurement. The student is expected to…

(A) – Identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;

(B).- Identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;

(C) – Apply knowledge of right angles to identify acute, right, and obtuse triangles, and;

(D) – Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Directions for Students-

Students will use the geoboards in whatever manner they would like. Students may want to create patterns either by following the provided patterns or creating their own.

Students can also play a game with the geoboards, either a suggested game provided with the station, or one they create themselves.

Suggestions for Teacher Involvement-

If a student creates a pattern that correlates to current or future course content, the teacher should ask students for permission to take pictures of their work for future use. If students agree, these images can be used to introduce or explore further different geometric ideas in the classroom.

If a student is unsure of what to do or create, the teacher can make suggestions such as:

- Attempting to copy a provided pattern
- Attempting to copy a classmate's creation
- Trying to create a given shape or object

Teachers are also encouraged to participate in the activity if time permits. This may mean encouraging student thinking, assisting students by sorting to find different colors or patterns, or even making suggestions for things to add or notice about their geoboards.

Games-

- Have a partner add rubber bands to the board. The other partner tries to write out the steps the student followed (whether through pictures or writing). This creates a new pattern which other students can choose to try and imitate.
- Students can try to make a shape or object using the geoboards and have a friend guess what they made (Pictionary).
- Pull a prompt and attempt it using the smallest number of rubber bands possible. Try to make the same pattern using the largest number of rubber bands possible. See if a friend can do it with more or less rubber bands than you.

Running a Business Activity

Materials-

- Fake money (dollars, coins)
- Play cash register (optional)
- Materials for creating goods/providing services (craft supplies)
- Prompts for business suggestions

TEKS Explored-

4th Grade Mathematics- Knowledge and skills.

2: Number and operations. The student is expected to…

(E) – Represent decimals, including tenths and hundredths, using concrete and visual models and money;

(F) – Compare and order decimals using concrete and visual models to the hundredths

4: Number and operations. The student is expected to…

(A)- Add and subtract whole numbers and decimals to the hundredths place using the standard algorithm

8: Geometry and measurement. The student is expected to…

(C) – Solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.

10: Personal financial literacy. The student is expected to…

(A)- Distinguish between fixed and variable expenses;

(B) – Calculate profit in a given situation;

(C) – Compare the advantages and disadvantages of various savings options;

(E) – Describe the basic purpose of financial institutions, including keeping money safe, borrowing money, and lending.

Directions for Students-

Students are encouraged to explore many aspects of creating their own business. They can choose a name, logo, and product for a business either independently or working in groups. Students have much freedom after this point. The teacher can loan students money to purchase different goods and materials they can use for their business. Students are encouraged to purchase from one another's shops or choose to be employed at various businesses. Students can agree on a set price for a product, wages for employees, or even put items on sale.

Suggestions for Teacher Involvement-

The teacher should act as the bank for students if possible. Students can loan money, but teachers are encouraged to charge a simple interest rate per week or month. If students do not pay this money back, inform them that they will be able to draw progressively less from the bank. If students choose to, they can dissolve a given business and begin a new one at any time.

The teacher should also engage with students in terms of their business by buying goods, becoming a temporary employee, and prompting students to buy further advertising. If students are not sure what to do, the teacher can help brainstorm or encourage students to form groups to create a business.

If a student creates a business choice that correlates to current or future course content, the teacher should ask students for permission to take pictures of their work for future use. If students agree, these images can be used to introduce or explore further different ideas about financial literacy in the classroom.

Prompts-

- Try to create a business that has a profit of at least $15.
- Create a business in which you sell one of your favorite things.
- Pay for advertising for your business.

Tangrams Activity

Materials-

- Tangram tiles
- Tangram patterns

TEKS Explored-

5th Grade Mathematics- Knowledge and skills.

5: Geometry and measurement. The student applies mathematical process standards to classify two-dimensional figures by attributes and properties. The student is expected to classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties.

6th Grade Mathematics- Knowledge and skills.

3: Geometry and measurement. The student will demonstrate an understanding of how to represent and apply geometry and measurement concepts.

(6.8A) - extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;

(6.8B) - model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes;

(6.8D) - determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

8th Grade Mathematics- Knowledge and skills.

3: Geometry and measurement. The student will demonstrate an understanding of how to represent and apply geometry and measurement concepts.

(8.10A) – generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane;

Directions for Students-

Students will play with the tangram tiles in any way they desire. Patterns are made available for students who want to try and put together different shapes, the solutions to creating these patterns are on the back of the pattern cards. Students are also encouraged to make their own patterns and draw them for other students to attempt.

Suggestions for Teacher Involvement-

Show students how you accomplish different puzzles and encourage them to make unique things. If a student creates a pattern that correlates to current or future course content, the teacher should ask students for permission to take pictures of their work for future use. If students agree, these images can be used to introduce or explore further different geometric ideas in the classroom.

Floor Plan Activity

Materials-

- Open-topped boxes appropriately sized to be model rooms
- Doll house furniture that fits in the boxes
- Graph paper

TEKS Explored-

4th Grade Mathematics- Knowledge and skills.

5: Algebraic reasoning. The student is expected to…

(C) – Use models to determine the formulas for the perimeter of a rectangles (l+w+l+w or 2l+2w), including the special form for perimeter of a square (4s) and the area of a rectangle (l x w); and

(D) – Solve problems related to the perimeter and area of rectangles where dimensions are whole numbers.

3: Geometry and measurement. The student will demonstrate an understanding of how to represent and apply geometry and measurement concepts.

(6.8A) - extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;

(6.8B) - model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes;

(6.8D) - determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

Directions for Students-

Students are asked to arrange the furniture in their houses as they would like. Then to see if they can draw a floor plan to show their design. Students are encouraged to try and follow other blueprints for floor plans, including those made by other students.

Suggestions for Teacher Involvement-

Teachers can show students how to arrange furniture in a particular way. They can also encourage students to try and fit as many possible pieces of furniture in a room as possible to try and elicit ideas about area. Suggestions for where to put objects or what kinds of rooms to design may also be helpful.

Packing Activity

Materials-

- Storage bins or shoeboxes
- Small briefcases
- Various small objects (blocks, cups, school supplies, etc.)

TEKS Explored-

5th Grade Mathematics- Knowledge and skills.

4: Algebraic reasoning.

(G) – use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube;

(H) – represent and solve problems related to perimeter and/or area and related to volume.

6th Grade Mathematics- Knowledge and skills.

3: Geometry and measurement.

(6.8C) - write equations that represent problems related to the… volume of right rectangular prisms where dimensions are positive rational numbers;

(6.8D) - determine solutions for problems involving the… volume of right rectangular prisms where dimensions are positive rational numbers.

8th Grade Mathematics- Knowledge and skills.

3: Geometry and measurement.

(8.7A) – solve problems involving the volume of cylinders, cones, and spheres;

Directions for Students-

Students will use the provided materials to pack the available boxes and briefcases with as many or as few objects as possible. They are allowed to do so however they would like. Prompts are available as challenges for students to strive to complete, as listed below, if they would like to.

Suggestions for Teacher Involvement-

Teachers can help students choose items to include in their boxes or make suggestions for different objects to attempt to bring along in the box. The teacher can also point out things they notice about the skills students are engaging in, such as what does and does not fit in the box and whether different orientations might help.

If a student engages with the objects in a way that correlates to current or future course content, the teacher should ask students for permission to take pictures of their work for future use. If students agree, these images can be used to introduce or explore further different geometric ideas in the classroom.

Prompts-

- If you were traveling to a deserted island with nothing else, what would you attempt to bring in your box?
- If you were trying to bring as many objects as possible and keep the weight as light as possible, what would you do?
- Compete with a friend to see who can fit the most objects into a single box?
- How many cups could you fit in one box?

Designing Golf Club/Golf Course Activity

Materials-

- Graphing paper
- Pencils
- Colored pencils
- Images of golf club heads
- Craft materials
- Popsicle sticks
- Tape
- Foam pieces

TEKS Explored-

4th Grade Mathematics- Knowledge and skills.

7: Geometry and measurement. The student is expected to…

(A) – illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;

(B) – illustrate degrees as the units used to measure an angle, where 1/360 of any circle is one degree and an angle that "cuts" n/360 out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;

(C) – determine the approximate measures of angles in degrees to the nearest whole number using a protractor;

(D) – draw an angle with a given measure; and

(E) – determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.

Directions for Students-

Students are encouraged to design golf clubs, golf courses, or both using graphing paper to do so. Prototypes may also be made using smaller craft materials if desired to create different designs and test them on a small scale. If enough students have a final product that they would like to test out, the teacher may choose to provide larger materials for students to use to create full-sized golf clubs or courses to be set up outside for a special activity.

Suggestions for Teacher Involvement-

Show students how others have made golf clubs in the past and encourage them to consider how and why they would choose to do so. Encourage students to practice with their various creations to see how their various clubs do in terms of hitting a small ball high into the air, hitting it far distances, and hitting it accurately. Teachers will help students to practice playing with golf clubs and encourage them to consider how changing the angles of the heads may help them improve distance, height, etc.

If a student engages with the materials in a way that correlates to current or future course content, the teacher should ask students for permission to take pictures of their work for future use. If students agree, these images can be used to introduce or explore further different geometric ideas in the classroom.

Prompts-

- Build a golf club that will result in the ball going the highest possible
- Build a golf club that will result in the ball going the furthest possible
- Build a golf club that is incredibly accurate and can hit a target from a distance
- Build a golf club that is light weight, but still powerful
- Build a heavy golf club and compare it to a lighter one- which do you prefer?
- Design a golf course and have a friend try and map the fastest way to complete it from the start to the end.

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