MATHEMATICIANS AS PROFESSORS: NARRATIVE ACCOUNTS OF THEIR TEACHING EXPERIENCES AND CONCEPTIONS OF TEACHING

by

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DEDICATION

I am dedicating this work to those who have gone before me, my parents Beth and Jerry Jarrell for their love and encouragement when I was young. To my best friend, Katie Dickerson, who left this world too soon.

I am also dedicating this to my chosen family of friends, Caitlin, Diane, Gwynn, Frank, and many others who helped me on this six-year journey. To Josh, Katie's husband, and their three amazing children, Lexie, Zach, and Bea, for providing amazing encouragement to keep going even when dealing with their own tragedies. To my brother Matthew Jarrell, for being there through too many of life's insanities.

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TABLE OF CONTENTS

Page

LIST OF TABLES

Table **Page**

ABSTRACT

Mathematics has a reputation as a frustrating and complex field, and all undergraduates must take at least one mathematics course to graduate. Since mathematics is mandatory, the teaching involved needs to accommodate students with varying mathematical success. This study aims to document and analyze mathematicians recognized for their teaching as identified by awards at the university level or higher by exploring their journey toward recognition and their conceptions of teaching. To capture these professors' journeys and conceptions, a semi-structured interview solicited their definition of quality teaching and their self-identified significant events that helped form their conceptions of teaching. The five participants all had student centered definitions as well as a focus on selecting appropriate tasks and caring about students. They also focused on students' dispositions toward mathematics as well as their own dispositions impact on students. All the participants featured reflecting on past events as their most frequent feature of significant stories. They also noted the importance of mentoring or role models as examples to reflect on. These findings indicate a need to encourage faculty to reflect on their own teaching as a significant way to develop their conceptions of teaching. Bringing together experienced and recent professors to focus on reflecting about their teaching could help recent professors formalize their conceptions about teaching while also learning for the experiences of all present.

Keywords: higher education, mathematics professors, teaching, narratives

I. INTRODUCTION

In 2012, the President's Council of Advisors on Science and Technology (PCAST) called for universities to produce one million additional college graduates with degrees in science, technology, engineering, and mathematics (STEM) by 2022. The report cites the need to improve teaching methods, especially by including evidence-based teaching methods, to reach underrepresented students. The Mathematical Association of America's Common Vision, which calls for changes in how undergraduate mathematics is taught, describes the PCAST as reporting "dissatisfaction in how undergraduate mathematics is taught to students outside the mathematics major. Further, outdated course materials and teaching techniques have not provided students with the quantitative skills demanded for employment and good citizenship" (Saxe et al., 2015, p. v). This call to action has spurred research in many different directions.

Researchers have spent decades studying K-12 teaching and learning, yet only recently has that focus expanded to include post-secondary or higher education contexts. In one of the few earlier studies of higher education, Hativa (1997) reported results of a survey on university professors' conceptions, practices, and departmental differences in a research university. Administered in 1995, the paper survey clustered responses by general discipline categories, with math and natural sciences sharing a category and engineering as a separate category. The results suggested that the "very large majority of university professors, lacking any appropriate pedagogical preparation, learn to teach through trial-and-error using reflection" (Hativa, 1997, p. 13). Math, natural science, and engineering faculty tended to lecture and write on the chalkboard with little discussion with students. This study highlighted that university professors, at that time, had little to

no formal training in teaching, yet teaching is a skill they are required to use.

My motivation for this study comes from my personal experiences with a variety of different training and professional development opportunities. One of the most significant events occurred as I trained to be an instructor for Kaplan Test Preparation and Admissions. The instructor, and their skill in teaching, were the product that Kaplan sold to its customers. So, training included multiple tips that make differences in student reactions to class, like facing the class when talking and waiting at least seven seconds for an answer to a question (what researchers call wait time). At the time, I assumed there was some training that professors went through that would give them the access to the same information. I spent one-year teaching high school and often heard the requirement of five hundred hours of professional development in 5 years. Frustrated by the apparent lack of training requirements for any college or university professors, I wondered why university professors were excluded from this training requirement. I also investigated the requirements for being hired as a lecturer or professor at a few schools and found teaching, specifically training for learning about teaching, was surprisingly absent from the job requirements. If high school teachers must keep up to date on developments in their profession of teaching, I wondered why university professors did not also have to keep up to date on teaching, since it is a prominent part of their profession.

This was how I first encountered the "publish or perish" mantra and how it impacted teaching. If research is how you keep or advance your profession, then that is where people will put their time and effort. This tension between prioritizing research and teaching is referenced throughout higher education research literature (e.g., Austin, 2002; Brownell & Tanner, 2012; Shadle et al., 2017). This is one of the common reasons

assumed for why professors may not focus on teaching. Research also indicates that professors' professional beliefs are another contributing factor to this attitude towards teaching (e.g., Austin, 2002; Brownell & Tanner, 2012; Sturtevant & Wheeler, 2019). Throughout the enculturation process into academia, new faculty members learn about the beliefs and values of their chosen profession through the behaviors of their peers and mentors. Since some of the senior professors, as representative of their profession, do not appear to value teaching, new members may take this stance as common and acceptable. These learned behaviors and values become new faculty members' ideas of what a professional believes for their discipline.

Statement of the Problem

The PCAST report emphasizes how students within this last decade still struggle with math and find it a gateway course keeping them from STEM degrees. To increase STEM graduates, universities need to improve retention of STEM majors. An important factor in retaining students is instruction – in particular, improving instruction (Olson & Riordan, 2012; Saxe et al., 2015). To effect change in teaching, we first need to know where teaching is now. Current research tends to focus on how and why professors – and of particular interest for me, mathematics professors – act and speak and think the way they do (Dandy & Bendersky, 2014; Hativa et al., 2001; Kember, 1997). Moreover, most current studies focus on STEM as one entity without attending to the traditions and cultures of each subject and how the differences in philosophy impact teaching within each discipline. Given that, I propose a discipline specific study of mathematics professors, due to the inherent differences between mathematics and other STEM disciplines.

Mathematics is one of the oldest fields of study and thus, has a large set of traditions that are likely to influence teaching traditions. While overall, mathematics instruction is viewed as an area in need of improvement (Saxe et al., 2015), some mathematics professors are lauded by both students and colleagues as being excellent and effective teachers. Given society's portrayal of mathematics and the disposition of many mathematicians, the question is "How did these mathematics professors become these exemplary teachers?" I am less interested in what they do in the classroom, though that is important, and more interested in these professors' journeys, experiences, and the significant events that led them to become exemplary teachers. To do so, I have identified exemplary mathematics professors and interviewed them to understand their journeys toward becoming exemplary teachers. My study provides a better understanding of how mathematicians can become excellent teachers and their corresponding conceptions of teaching.

The PCAST report (2012) and MAA's Common Vision (2015) both call for an improvement in teaching and a move away from the current state of affairs. Current research has demonstrated the need to understand the differences between different disciplines and how they approach teaching. Studying these exemplary mathematics professors, and how they came to be exemplary, may uncover leverage points and critical experiences with implications for faculty development that could support the improvement of mathematics teaching more generally.

Purpose of the Study

The primary purposes of this study are (1) to document the self-identified significant events and journeys of mathematics-degreed faculty who are identified as

exemplary in teaching and (2) analyze the conceptions of teaching of these exemplary mathematics professors. As people journey through their lives, they form conceptions including conceptions of teaching, based on their experiences. People also reflect back on those journeys to discover different perspectives that enhance their conceptions. When about their beliefs and values, they often share a story that contextualizes and explains why they did what they did or choose their actions. The story selected is often a story the person has told to others and themselves many times. I am interested in the stories and corresponding conceptions of teaching that mathematics professors tell as they share their teaching journeys. To document and analyze these journeys, a case study approach provided an appropriate methodology for this study. After interviewing and documenting participants' stories, I used an open coding scheme to uncover trends within and across cases. Member checking with the participants ensured their stories have been represented faithfully in this study.

Research Questions

This study aims to answer the following research questions both:

1. How do mathematics professors become exemplary teachers?
    a. Specifically, what are the personal journeys and lived experiences for math professors recognized as exemplary teachers?
2. What are exemplary mathematics professors' conceptions of quality teaching?
3. How do their self-identified significant experiences and stories relate to their conceptions of teaching, and what factors have supported or hindered the participants' professional growth and change as teachers?

Summary

In this chapter, I proposed a need to document the journey of mathematicians who are recognized as exemplary teachers, explore their significant experiences related to teaching, investigate how and why these events are related to their conceptions of quality teaching, and identify possible commonalities across participants. In particular, this study focused on the formation of mathematics professors' teaching conceptions and the life events that influenced that formation. In the chapter that follows, I review literature on higher education with a focus on teaching and teachers. I then synthesize existing research on the constructs of affect, beliefs, conceptions, and identity. I continue by reviewing research on narratives, both as a methodological tool or form of data collection and relevant findings from narrative research focused on the teaching of higher education faculty. I close the chapter with the theoretical framework for my study.

II. REVIEW OF THE LITERATURE AND THEORETICAL FRAMING

This literature review begins with an overview of research on university faculty with a focus on teaching and professional development. I next provide a summary of research on affect, beliefs, conceptions, and identity. This is followed by a review of narrative research, both as a methodological tool or form of data collection, and relevant findings from narrative research focused on higher education faculties' teaching. Finally, I conclude by sharing the theoretical framework for this study.

Research about University Faculty

In some of the earliest work on the preparation of faculty, both Hativa (1997) and Austin (2002) noted a lack of professional development for professors across all disciplines. Austin describes "when teaching opportunities occur, they are often not organized systematically to ensure growth or appropriate preparation" (2002, p. 105). This suggests graduate students and early teachers are often slotted into courses without consideration for their development as teachers. Hativa documents that experienced professors, "lacking any appropriate pedagogical preparation, learn[ed] to teach through trial-and-error using reflection" (1997, p. 17), indicating that faculty do seek to improve their teaching skill but might benefit from training. In an investigation of biology faculty professional identity tensions, Brownell and Tanner noted "While some time as a teaching assistant may be required, in general, there is no requirement for evidence of developing competency in teaching"(2012, p. 341) in doctoral and postdoctoral training. These studies demonstrate that focused training or development of teaching did not routinely occur in the past.

But more recent research suggests that the state of pedagogical preparation for

professors may be changing. In a 2019 critical review of literature on STEM professors' knowledge of teaching, Winberg et al. noted that "The professional development of STEM university teachers has over-emphasized generic forms of teaching practice and neglected discipline-specific teaching practice." (p. 940). While this indicates teaching has received more attention recently, there is still considerable room for improvement. Austin (2002) points out that each discipline conducts research differently and socializes with their colleagues in unique ways, which supports a need to attend to discipline-specific instructional practices that take into account this uniqueness. Lund and Stains (2015) investigated departmental differences in awareness and adoption of evidence based instructional practices among biology, chemistry, and physics faculty and cautioned against overgeneralizing the results from one STEM field to all. In their work on professional identity, Brownell and Tanner (2012) also speak to the importance of one's academic discipline. They hypothesize that a shift in the discipline's view of teaching, and thus discipline members' professional identities, may be necessary to effect faculty change. These studies suggest that discipline specific research on professors and their teaching is needed. Thus, I argue that studies focused on *mathematics* professors and their instructional practices are necessary to document, understand, and eventually improve post-secondary mathematics instruction. However, much of the research I summarize below focuses on STEM as a single, monolithic entity without delving into discipline-specific differences or effects.

Several recent STEM studies have investigated evidence based instructional practices and their potential barriers and drivers for implementation. Brownell and Tanner (2012) list insufficient training, time, and incentives as the most cited barriers for

faculty change. More broadly, Lund and Stains (2015) focused on communication channels, contextual influences, and individual influences as factors affecting faculty implementation of evidence based instructional practices. Communication channels investigates whom professors use as their sources of educational knowledge, with lecturers and faculty conducting "bench" research as the most frequent choices. Contextual influences, for them, included the departmental norms and university classroom characteristics whereas individual influences included beliefs and knowledge about teaching and their role in those contexts. Shadle et al. (2017) also investigated STEM faculty perceptions of barriers and drivers toward faculty change via discussions and surveys clustered by department. Of the eighteen identified barriers, math faculty most frequently cited inadequate resources, time constraints, and loss of autonomy. The math department commented on time constraint less than the aggregate average (65% vs. 80%) and on inadequate resources more than the aggregate average (74% vs. 41%). Among the 15 drivers, over three fourths of the math faculty highlighted encouraging collaboration and shared objectives, expanding on current practices, and improving teaching and assessment (Shadle et al., 2017). The math department mentioned these three barriers on average more than the aggregate average, with encouraging collaborations and shared objectives as 94% vs. 70%, expanding on current practices at 78% vs. 70%, and improving teaching and assessment at 78% vs. 69%. Recently, researchers have developed instruments and measures to a) collect faculty members' knowledge and implementation of evidence based instructional practices and b) link evidence based instructional practice implementation with professional identity (see Sturtevant and Wheeler [2019] Faculty Instructional Barriers and Identity Survey).

Across these studies, common barriers include time and resources needed to implement evidence based instructional practices as well as beliefs about teaching student centered practices and insufficient training. While more recent work has focused on drivers for faculty change, especially the department norms, many of the previous studies only focused on barriers.

Several studies looked at the effect of a faculty member's social network and community and how these factors influence faculty views. Samaras et al. (2019) studied a small interdisciplinary STEM faculty group engaged in creating self-study research projects as an avenue to improve their teaching. The six faculty "reported that working with critical friends did help them reframe their thinking about teaching and understanding teaching as research" (Samaras et al., 2019, p. 205). Lane et al. (2019) also investigated social networks but did so through network mapping on a larger scale. To explore the role of faculty social networks across multiple institutions, they surveyed seven science departments and mapped out the social networks of each department as well as knowledge of evidence based instructional practices. They also statistically analyzed the networks for trends and central individuals to further research in idea propagation through these networks. Lane et al. (2019) found central figures, like mentors, may help propagate ideas through the social network, and peer influence showed an influence on implementation of evidence based instructional practices. They concluded that mentors and colleagues have an impact on instructional practice and thus represent a potential source of influence on professional identity formation.

Another theme in higher education research on teaching is studies investigating self-study or reflection on one's practice. Samaras et al. (2019) found that their faculty

participants shifted their focus from changing students, to changing their own actions and found "self-study as 'a mechanism for thinking carefully and reflectively' about their role as a teacher and with the group's support" (p. 202). Connolly et al. (2018) focused on the link between participants' belief in their own teaching effectiveness to participation in teaching development. They postulated that access to teaching development provides access to the four key sources of self-efficacy information: mastery experience, verbal persuasion, vicarious experience, and emotional or physiological arousal (Connolly et al., 2018, p. 9). These sources can help categorize different events in a faculty member's identity formation. Mastery experience and vicarious experience offer the opportunity to reflect on your own or your colleagues' performances. Reflection on practice is a key factor in changing teaching by changing beliefs, so I hypothesized that exemplary faculty members are more likely to think about their teaching and how to improve it.

Similar to Alba Thompson's (1984; see also Philipp, 2007) influential work in mathematics education in K-12 contexts, Kane et al. (2002) examined whether higher education studies studied only beliefs or studied both beliefs and practice. This lack of correlation between studying beliefs and practice may be due to perspective: an observer does not have access to thoughts and reasonings for instructional decisions. In other beliefs research in higher education, Hativa et al. (2001) qualitatively analyzed four exemplary, as measured by student reviews, university professors' beliefs and knowledge about effective teaching. The researchers then compared their stated beliefs and knowledge to what was enacted in the classroom. They determined that these exemplary professors held beliefs consistent with practice but that they did not excel in every category of effective teaching; yet they were still considered effective. This work seems

to indicate there are multiple pathways to becoming and being an effective university teacher, but that beliefs do play a role. Finally, Sturtevant and Wheeler (2019) investigated the tensions between teaching and research identities and corresponding faculty beliefs; the barriers identified by their study "point to deeply held beliefs founded on prior experiences with teaching and learning" (p. 17). Collectively, these studies show the complexity of beliefs and personal experiences related to teaching.

Conceptions, Beliefs and Affect

To study faculty conceptions of instructional practice and quality teaching, the term 'conceptions' and other similar constructs need to be defined and related to each other. Although I use the construct of conceptions in my research, much of the extant literature focuses on the related constructs of beliefs and (to a lesser extent) identity, thus the emphasis on these constructs in the following section.

In her synthesis of teachers' beliefs and conceptions, Alba G Thompson (1992) first makes the distinction between beliefs and knowledge. Beliefs can be held with varying degrees of conviction, they are disputable, and they do not meet the criteria to become knowledge. Thompson noted many researchers have pointed out that distinguishing teacher knowledge from teacher belief is immaterial to the true purpose of belief research, which is "to search for whether and how, if at all, teachers' beliefs – or what they may take to be knowledge – affect their experience" (1992, p. 129). She goes on to summarize Green's (1971) three dimensions of belief systems: (a) beliefs are interrelated with primary and derivative beliefs, (b) beliefs have a degree of conviction with central and peripheral beliefs, and (c) although beliefs cluster together, clusters are isolated from each other, allowing for conflicting beliefs in the same person. Thompson

also defines the term teachers' conceptions as "a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (Alba G Thompson, 1992, p. 130). Research tends to use multiple terms like beliefs, conceptions, and perceptions to refer to the same construct.

Though Alba Thompson is recognized as doing seminal work in mathematics education on beliefs in general, other scholars have considered more focused categories of beliefs. Ernest (1989), for example, distinguished three conceptions or views of mathematics: the problem-solving view, in which math is a process of enquiry; the Platonist view, in which math is a static but unified body of knowledge, and the instrumentalist view, in which math is a set of unrelated but utilitarian rules. Analyzing her 1984 work, Alba G Thompson (1992) used Ernest's (1989) categories to describe each teacher's conception of mathematics and noted that a view may simultaneously hold elements from all views, even potentially conflicting ones. Moreover, her work linked teachers' conceptions of mathematics to their instructional practice.

A large body of research exists on teachers' beliefs about mathematics teaching. After an extensive literature review, Kuhs and Ball (1986) identified four views of mathematics teaching: learner-focused, content-focused with an emphasis on conceptual understanding, content-focused with an emphasis on performance, and classroom-focused. Thompson links Kuhs and Ball's (1986) views of mathematics teaching to Ernest's (1989) views of mathematics and noted the relationship between instructional practice and one's view of mathematics is "a complex relationship with many sources of influence at work; one such source is the social context" (Alba G Thompson, 1992, p. 138). In other words, teachers' beliefs about teaching may influence practice, but perhaps

indirectly; this influence may be mediated by one's beliefs about mathematics itself as well as more localized factors relating to the social environment in which the teaching occurs. In particular, I use the idea of social context as mediator of practice to make connections that influence faculty' perceptions of teaching including departmental views, disciplinary views, and mentor's views.

Even broader than beliefs is the related construct of affect. In his 2007 Handbook chapter, Phillip reviews research on both beliefs and affect. He defines affect as "a disposition or tendency or an emotion or feeling attached to an idea or object. Affect is comprised of *emotions*, *attitudes*, and *beliefs*" (Philipp, 2007, p. 259, italics in original). Using Alba G Thomson's definition, Phillip defines conception as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Phillip, 2007, p. 259). Conception includes beliefs as well as various parts of knowledge. Phillip also discusses emotions, attitudes, and beliefs that exist on a spectrum of intensity and changeability, where emotions are experienced the most intently and change the quickest. Beliefs are felt with the least intense emotion and are the hardest of the three to change. Attitudes fall in the middle. A new contribution to belief research was the inclusion of values; "beliefs are true/false statements about constructs whereas the choice of the particular constructs one finds desirable or undesirable represents one's more context-independent values" (Philipp, 2007, p. 265). Philipp (2007) notes that other researchers view values as a subset of beliefs or as enduring beliefs; values could be placed closer to attitudes on the belief side of the emotional-changeability spectrum described above. He also points toward the distinction in how an idea is held as a key in differentiating between a belief and knowledge. If one

can respect another's viewpoint, then the corresponding idea or view is a belief; if one cannot respect another's viewpoint, the corresponding idea or view is knowledge. For example, a teacher may hold the belief that students learn best by repetition of procedures and a student may hold the same idea as knowledge. The teacher could consider other ways students might learn like developing their own algorithm for solving a problem, but the student would object since the teacher is not showing how to use the procedure first. Whether or not professors hold their views flexibly will impact how effective they are as teachers.

One of the more influential scholars for beliefs research in mathematics education is Thomas Cooney, whose work was conducted with preservice teachers. He described four characterizations of how preservice teachers held their beliefs: isolationist, naïve idealist, naïve connectionist, and reflective connectionist (1999). The isolationist holds their belief clusters separately and rejects the beliefs of others. The naïve idealist willingly listens to others but takes on the beliefs without consideration for the incorporation into their current belief system. The naïve connectionist cannot resolve conflicts that arise when they reflect on their experiences, but the reflective connectionist is able to resolve these conflicts or doubts. These four types vary in how they accommodate conflicting beliefs, which, he claims, relates to how teachers reflect on their own practice. Cooney (1999) found that the goal of teacher education is to move teachers toward being reflective connectionists, which supports teachers' ability to validate ideas themselves and incorporate them into their belief systems. This research on beliefs suggests that the ability to reflect on instructional practice and update belief systems is a laudable goal for any teacher, and likely a necessary one for excellent

teachers. Because this study focuses on professors' conceptions of quality teaching, following Philipp (2007), I define a conception as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (p. 259). Conceptions include beliefs but also include ideas related to knowledge and attitudes. Moreover, conceptions fall under the broader construct of affect because they include beliefs, knowledge, and attitudes, which are all part of affect. Conceptions are held more strongly than emotions, but parts of a conception can be more mutable than other parts in the same way belief is more flexible than knowledge. Conceptions are defined to be broader than beliefs and knowledge to avoid the entanglement of trying to distinguish between the two.

Most research on beliefs and affect is conducted in K-12 contexts. Studies of beliefs and affect at the undergraduate level, specifically, university faculty's beliefs or affect related to teaching, is less common. Few scholars frame their work around the construct of conceptions but instead, study, beliefs. I now turn to the belief literature situated in higher education. Norton et al. (2005) surveyed four U.K. universities to examine the link between teachers' beliefs of teaching and intentions when teaching to understand why different teachers adopted different approaches to teaching. Intention is explained as the enacted action in the classroom, that may or may not align with beliefs about teaching; intention is potentially influenced by the curriculum, the institution, or the students themselves. Norton et al. (2005) "were particularly interested in investigating whether teachers' beliefs and intentions were influenced by their institution, their academic discipline, their amount of teaching experience and their exposure to formal training in teaching in higher education" (p. 543). Based on earlier work by Kember

(1997), Norton et al. (2005) generally found consistency in beliefs and intentions. (Kember delineated five categories of teacher conceptions along a continuum with teacher-centered/concept-oriented on one end and student-centered/learning-oriented on the other. The five categories are imparting information, transmitting structured knowledge, student-teacher interaction, facilitating understanding, and conceptual change/intellectual development.) Faculty members' beliefs and intentions together tended toward either a learning-oriented conception or a knowledge transmission conception; however, they did find some evidence of intentions slipping towards knowledge transmission even though beliefs were learning-oriented. This misalignment between beliefs and intentions highlights the influence of other contextual factors like discipline or experience since beliefs did not always predict teachers' intentions.

Other belief-related research in university settings explored faculty beliefs about the roles of ability and effort in learning (see Dweck, 2008). Aragón et al. (2018) surveyed science educators who attended a summer institute on evidence based instructional practices over a ten-year period. They examined whether professors had a fixed mindset, associated with believing people have an innate level of intelligence, or a growth mindset, which assumes individuals can increase their intelligence with effort. They then related participants' mindsets to their adoption and implementation of seven evidence based instructional practices. Participants progressed through five stages of adoption: exposure, persuasion, identification, commitment, and implementation. The survey results showed that having a fixed mindset had a negative influence on the persuasion step of the implementation model, indicating those who felt intelligence was fixed were not easily persuaded to adopt evidence based instructional practices. This

study focuses on only one belief, whether intelligence is fixed or can grow, but the impact of this belief was demonstrated to limit the adoption of evidence based instructional practices. Similarly, I argue that many such beliefs are likely to be included in and influence conceptions of teaching.

Narrative Research

To document and analyze mathematicians' journeys to who they are today, narrative research provides a foundation of techniques to explore a person's stories and how those stories influence who they are and how they behave. In the 1990s, narrative research emerged as a new approach to studying qualitative inquiries focused on collecting, constructing, or analyzing narrative data. After reviewing literature on mathematics teacher educators' use of narrative, Chapman (2008) states "narrative is viewed as a key form through which individuals come to know themselves, construct their lives, and make sense of their experiences" (p. 17). One of my assumptions in this work is that the stories professionals tell about themselves reveal how teaching factors into their views of themselves.

In 2005, Sfard and Prusak emphasized the significance of stories by positing the radical idea that identities are stories. They focused on the role of who was telling the stories and who was listening to the stories. They note "What a person endorses as true about herself may be not what others see enacted" (Sfard & Prusak, 2005, p. 17). For Sfard and Prusak (2005), "the focus is not on identities as such but rather on the complex dialectic between identity-building and other human activities" (p. 17). The focus is on the journey towards what they term a designated identity rather than acquiring an actual identity. Designated identities are stories about what a person wants to become rather

than an actual identity being what they already accept as a label. They distinguish two other elements which are critical stories, that make up core parts of our identity (similar to the core memories in the animated movie *Inside Out* (Docter et al., 2015)), and as significant narrators, whose voices carry more weight and have the potential for more impact on life decisions. Which stories become critical and which narrators become significant will help describe a person's journey toward their identity. Though my focus in this study is not on identity, I use Sfard and Prusak's idea of critical stories as a key part of my analysis to understand mathematicians' journeys as teachers.

In 2009, Clandinin et al. explored teachers' personal practical knowledge to develop narrative understandings of life in schools. Clandinin et al. (2009) explains that "a narrative way of thinking about teacher identity speaks to the nexus of teachers' personal practical knowledge and the landscapes, past and present, on which teachers live and work" (p. 141). Clandinin et al. (2009) further explains the "concept of 'stories to live by' as a way to speak of the stories that teachers live out in practice and tell of who they are, and are becoming, as teachers" (p. 141). This conception of narrative broadly includes more than what stories a teacher shares but also includes the context the teacher lives in and works from. These 'stories to live by' include not just the stories a teacher shares, but also the types of stories and how those stories are enacted in a teacher's practice and life.

In the book *Doing Narrative Research*, Squire (2013) examines how to study narratives as stories of experience, rather than events. Squire (2013) explains that experience-centered narrative research "rests on a phenomenological assumption that experience can, through stories, become part of consciousness" (p. 48). The act of

hearing makes the experience of that story become a part of the listener, even including the narrator as a listener. She continues noting experience-centered narrative research "assumes 'personal narrative' includes all sequential and meaningful stories of personal experience that people produce. Such stories may be an event narrative; but may also be more flexible about time and personal experience and defined by theme rather than structure" (p. 48). This definition of narrative research allows the focus to be on the experiences along the journey rather than the structure of each story individually.

Mueller (2019) describes a methodology for what she terms an episodic narrative interview. This methodology is a fusion of semi structured interview, narrative inquiry, and episodic interview. Mueller (2019) adopts a more flexible notion of interview to allow her participants more freedom to tell their stories. Narrative inquiry posits that "stories are construed as recollections of a person's life experience that are combined in a way to create a coherent narrative" (Mueller, 2019, p. 3). Episodic interview focuses on participants relating stories about requested episodes, so the participant has the freedom to answer in their own way, but the researcher can choose "the domains from which responses will be requested or required" (Mueller, 2019, p. 4). Mueller (2019) describes her method as "the aim of conducting an episodic narrative interview is to deeply understand a participant's experiences with a particular social phenomenon; consequently, the method requires that a phenomenon of interest be identified in advance" (p. 5). Mueller (2019) goes on to note that the researcher must choose participants carefully to ensure the participants have stories about the phenomenon of interest. The structure of the interview begins with participants defining the phenomenon of interest and then the researcher requests a story about an episode. Next, the researcher

requests that the participant share a story about the phenomenon within the context of the described episode. This focused approach allows the researcher and participant to target the phenomenon of interest and find stories that include the participant's understanding of this phenomenon. This type of funneled questioning also works to elicit stories about teaching from exemplary professors. In my study, I use Mueller's (2019) episodic narrative interview to balance both participant freedom in storytelling and my need to focus on the phenomenon of interest—their teaching journeys.

Use of Narratives in Research and Professional Development

In her review of the use of narratives in mathematics teacher education, Chapman (2008) notes that interest in narratives began in the early 1990s. Chapman explains "narrative can provide a basis for studying teacher knowledge and teaching and a basis for teacher professional development and education" (2008, p. 16). Based on her previous research, Chapman summarizes eight ways that she "engaged prospective secondary teachers in the process of narrative reflection/analysis/inquiry" (2008, p. 32). These eight ways of using stories are (1) initial self-reflection, (2) restorying, (3) unpacking teacher stories, (4) unpacking problem-solving stories, (5) narrative inquiry through peers, (6) comparing imagined versus actual actions, (7) comparing personal versus theoretical knowledge, and (8) identifying narrative themes. This list highlights a subset of the ways stories can be used to further teacher education.

As part of a larger study, Weston and McAlpine (1998) interviewed six exemplary professors trained as mathematicians about their teaching history and views about teaching and learning. Three of these professors were from the Faculties of Education, each with a pedagogical degree; the other three did not have a pedagogical

degree. After a thematic review of the interviews, nine codes emerged: Caring, Content, Learning Outcomes, Strategies, Evaluation, Passion, Research, Active Engagement, and Knowledge Construction (Weston and McAlpine, 1998). The authors noted "that our involvement in this research has had an impact on our own teaching and has moved us toward a more intentional caring perspective" (2008, p. 154). This research provides a starting point for thematic codes from analyzing professors' journeys as well as a note about the use of studying these journeys as potentially impacting peoples' perspectives.

Another study from Walker and Gleaves (2016) focused on defining the caring teacher in the context of higher education by interviewing six carefully selected 'caring' professors in the United Kingdom. The professors were selected through colleague recommendations from a reputational case selection procedure. The recommendations also contained reasoning for the selection of the professors, and the authors analyzed this data set to match pedagogical behavioral caring exemplifiers: "listening to students, showing empathy, supporting students, actively fostering learning in class, giving appropriate and encouraging feedback and praise, having high expectations in standards of work and behavior, and showing an active concern in students' personal lives" (Walker and Gleaves, 2016, p. 68). Each of these exemplifiers are a way teachers demonstrate their caring, which show up in the stories of exemplary professors.

This body of research on the use of narratives shows the potential of narratives as a methodological approach and as a way to support teacher learning and growth. Sfard and Prusak's significant stories, Clandinin et al.'s (2009) stories to live by, and Squire's (2013) experience centered narratives all provide different lenses to examine the significant stories from my participants. They point out the significance of mentoring

type figures, the context each person lives in, and the focus of stories is the experience. I focused on Mueller's (2019) description of episodic narrative interview as an in-depth methodological approach for my study. I also discussed several examples of narrative research highlighting potential themes for my research with exemplary mathematics professors. These themes include self-reflection, caring about students, and active engagement, all of which appear in my participants' profiles.

Theoretical Framework

In this section, I present the theoretical framework guiding this study. Because of the need for discipline-specific research on exemplary mathematics professors, I used an experience-centered narrative definition. To elicit the narratives of exemplary mathematics professors, I used a methodology similar to the episodic narrative interview method described by Mueller (2019). I first focused on the participants' definitions of quality teaching by asking for a definition and then probed for further details into different parts of their definition.

In her chapter of *Doing Narrative Research*, Squire (2013) explores experience-centered and culturally-oriented approaches to narrative. Squire (2013) explains that experience-centered approach "assumes that narratives are sequential and meaningful, are definitively human, 're-present' experience, reconstituting it, as well as expressing it, and display transformation or change" (p. 43). Experiential stories are defined by theme, like focusing on student experiences and teaching. By focusing on the experience rather than the event, these narratives attend to the narrator and their context. Finding narratives that display change can highlight events that influenced a change in conceptions about teaching.

Narratives identified as significant by the participants provided a window into the events that helped shape the participants. Their stories provided an account of these influential events but also a look into how the participants reflected back on these stories. By examining how the participants discussed these events, we gained insight into their beliefs about teaching as well as the types of events that are identified as significant. By examining these influences on beliefs, we highlight some of the influences on their conceptions of teaching.

I focused on uncovering the significant stories that motivate the development of professors' conceptions of quality teaching and how these stories shaped their views of teaching. I solicited stories about the following:

- Teaching and learning and themselves as an effective teacher
- Significant personal narratives related to teaching
- Significant personal narratives related to themselves as a student or learner

To investigate whether and how teaching is prevalent in professors' journeys, I investigated mathematics professors recognized as effective teachers to understand how and why their experiences are related to their conceptions of quality teaching.

Following Phillip's definition, I defined a conception as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Phillip, 2007, p. 259). This definition provided a broad array of events that can influence conceptions. By first asking for the participant's quality teaching definition, I provided the participant with a lens to view their experiences to help highlight the most influential events related to teaching. Then, I solicited stories of events related to teaching and asked for participants to link any ideas in their definition

with stories about their significant events. With these two pieces, I then looked for common themes to suggest links between their current conceptions of quality teaching and their significant stories of their past.

Consistent with the call to improve undergraduate instruction in mathematics as seen in both the PCAST's report and the MAA's Common Vision, I hope that by studying faculty who have been recognized for excellence in teaching, I can shed light on possible pathways for change and development within the profession by understanding how, why, and in what ways these mathematicians value teaching. Using exemplary mathematics professors allowed me to examine the journeys that professors took to develop their conceptions of teaching. By allowing the professors to identify and explain their significant stories, I found types of events that fostered changes in their conceptions of teaching. In the next chapter, I state the research questions this study will answer and discuss the methods by which I will answer these questions.

III. METHODOLOGY

The purpose of this study is to document and analyze the views of teaching of mathematics-degreed faculty who are identified as exemplary in teaching and the self-identified significant events that influenced those views. The research incorporated a multiple case study design using interviews as the primary data source. Yin defines case study as: "an empirical method that investigates a contemporary phenomenon (the "case") in depth and within its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident" (2017, p. 15). The phenomenon of interest for my study is the personal journeys of mathematics faculty who are recognized as exemplary teachers. Specifically, who and what are the significant events and people who shaped them into the teachers they are today and influenced them to see teaching as a core part of their profession. The research questions guiding this study are:

1. How do mathematics professors become exemplary teachers?
    a. Specifically, what are the personal journeys and lived experiences for math professors recognized as exemplary teachers?
2. What are exemplary mathematics professors' conceptions of quality teaching?
3. How do their self-identified significant experiences and stories relate to their conceptions of teaching and what factors have supported or hindered the participants' professional growth and change as teachers?

Since my research questions ask how and why this phenomenon occurs, the purpose of the study is both descriptive and explanatory. According to Yin, case studies are advantageous "when a 'how' or 'why' question is being asked about a contemporary

set of events over which a researcher has little or no control" (2018, p. 13). I am investigating how and why mathematicians form their conceptions of teaching. Since I am examining significant personal experiences and relating these experiences to each participant's conceptions of teaching, I can best describe this phenomenon with rich descriptions of professors who are available and willing to revisit their answers and clarify their meanings. Because this is a study of a current, personal, and highly contextualized phenomenon, I will use a case study to explore multiple mathematics professors' journeys as teachers. For this study, I created a case for each participant and then engaged in cross-case analysis to identify themes across cases (e.g., similarities in types of formative events or the timing of formative events that related to a specific belief about teaching). Below I explain how I analyzed my data, created individual cases, and conducted the cross-case analysis.

Assumptions

Recognizing that even when attending the same event, people will take different experiences and lessons from that event, I take a social constructivist perspective when interviewing my participants. Each person has their own perception of reality determined by their significant events and reflections; these experiences and reactions are what crystalize into beliefs and values. I recognize my own beliefs will influence my interpretation of my participants' narratives. Consequently, during interviews, my responses were limited to encouraging the participant to share more of their journey of developing their teaching but also included follow-up questions to ensure my own understanding was faithful. Additionally, I checked with my participants to ensure I understood and accurately represented their perceptions of experiences recounted during

the interview and as I coded data and wrote their cases. Member checking was a critical aspect of my study design (which I describe in more detail below).

Participant Selection

To study mathematics faculty's views of teaching, I identified mathematics professors who had integrated teaching as a prominent and valued feature of their professional identity. My rationale for participant selection was that mathematics faculty recognized as exemplary teachers were more likely to position teaching as a core aspect of their professional identity. Thus, I recruited potential participants who had earned a Doctorate in Mathematics and were exemplary teachers. Indicators of exemplary teaching included being a recipient of teaching awards at the university level or higher and endorsements from multiple colleagues regarding one's teaching.

After identifying twelve possible participants, I prioritized contacting and recruiting faculty with whom I could make a personal connection (e.g., someone who collaborates with a faculty member in my department or who knew my advisor). To explore different journeys that mathematics faculty took to develop their views on teaching, I worked with five exemplary mathematics professors, who had a range of personal experiences. Four participants were men, one was a woman; three were professors, one was a retired professor, and one was an assistant professor. The participants graduated with their degrees from the mid-1970s to the mid-2010s. Participants worked in a variety of institutions in the United States ranging from premier research institutions to community engaged universities. All of the participants are parents, and some participants also have grandchildren.

Data Collection

Pilot Study

I conducted a pilot study with one professor who had a PhD in mathematics and also had a National Science Foundation award to conduct research on teaching. The pilot study was conducted to refine the first iteration of interview protocol. After conducting and transcribing the pilot interview, I identified narratives, or stories, which appeared throughout the interview and began developing an initial set of codes for themes present within the narratives. The final analytic codes were developed over repeated rounds of data analysis that started with the pilot data (and are described later in this chapter).

Parts of my interview protocol were not eliciting the types of information I wanted. So, I conducted a follow up interview with the participant that was focused on the interview protocol itself and suggestions for improving the interview process. This resulted in a revised interview protocol that focused on narratives related to teaching and participants' personal definition of quality teaching.

Study Design

I conducted two interviews with each participant (I was unable to schedule a follow-up interview with one participant). I used a structure like Mueller's (2019) episodic narrative interview which started with a participant definition and then asked for a story of an event involving the phenomenon of interest followed by an explanation of the phenomenon in the story. Thus, my first interview was a semi-structured interview, and it consisted of two parts—the first part solicited the participant's definition of quality teaching, and the second part invited the participant to share narratives of their life related to teaching. I provided the participants with a partial version of the interview protocol

several days before the scheduled interview. This allowed participants to spend time reflecting on how to define and explain quality teaching. This also allowed participants to recall experiences from their lives that related to their definition of quality teaching.

The first interviews lasted from 39 minutes to 87 minutes with an average length of 66 minutes. Due to the pandemic, interviews were conducted and recorded over Zoom between the researcher and the participant. The purpose of the second, or follow-up interview, was to confirm my understanding of my participants' stories and to gather additional information about significant events. This also provided an instance for member checking. During the follow up interview, I would ask the participant to revisit a story and explain the relationship with their quality teaching definition. I also allowed for the participant to share any new stories that my follow-up questions prompted. The second interview lasted between 22 minutes to 35 minutes, with an average of 29 minutes. Below I share the interview protocol for the first, semi-structured interview.

Interview Protocol

Initial interview

I designed the interview protocol to first investigate the participant's conceptions of quality teaching and then to solicit personal narratives related to teaching. The complete interview protocol is included as Appendix A. The semi-structured interview was designed to learn more about participant's conceptions of quality teaching and their personal journeys and the significant stories that shaped them into the teachers they are today. I also asked questions about the ways their experiences may have influenced their conceptions of teaching. The purpose of the interview was to provide the primary source of data for creating a detailed case of each participant's personal narrative of their

teaching journey, how the participant conceptualized teaching, and the personal experiences that informed those conceptions.

The interview was split into two parts. First, the participant was asked to explain their definition of quality teaching by answering several questions including:

> 1. What do you think quality teaching is? I know this answer may change over time. As desired, you may add on to your definition as we continue the interview.
> 2. When you teach, what are your goals? What do you strive to embody?
> 3. Below are questions to determine both large picture and fine-grained aspects of your teaching.
>
> a. What would a successful course look like? What would you do? What would your students need to do?
> b. What would a successful lesson look like? What would you do? What would your students do?
> c. How do you handle student questions during class? During office hours?
> d. How do you handle a mistake made in class by yourself? By a student?

In the second part of the interview, the participant was asked to describe events from their past they felt influenced their concept of teaching or which were significant in their personal journey as a teacher. This question was left as open-ended as possible to allow the participant to relate their significant stories without any bias or suggestions of events to include from the questions themselves. Below is the prompt I used to elicit significant stories related to teaching.

> Now I'd like you to consider how your past has shaped you as the professor you are today. I'm interested in how you think about teaching and how that may have changed over time. What significant events from your past helped form your current views about teaching? How do these events relate back to your definition of quality teaching? Please explain the most influential experiences that influenced how you view teaching?

I also worked from a list of potentially influential events (see below) from Sturtevant and Wheeler's 2019 Faculty Instructional Barriers and Identity Survey. This list was not given to the participant but used as spontaneous suggestions of narrative topics after the

participant shared their initial narratives.

a. Experiences as an instructor
b. Experiences as a student taking classes
c. Experiences as a researcher
d. Graduate student experiences
e. Experiences from non-academic roles
f. Professional development/workshops/conferences about teaching
g. Formal teacher education
h. Influence of academic individuals (e.g., grad school advisor, dept chair)
i. Influence of your field's expectations/norms
j. Influence of non-academic individuals (e.g., mentor, parent, partner)
k. Conversations with educational professionals (e.g., faculty developers, instructional designers, education faculty)
l. Conversations with or observations of colleagues/other instructors
m. Reading research articles about teaching and learning
n. Personal passion for helping students learn and succeed

After the initial, open-ended prompts, I posed follow-up questions that specifically focused on the participant's stories and experiences in graduate school as well as the views of teaching of their primary research advisor or dissertation chair. For example, I asked participants, "What role did teaching play in your graduate student experience? How, if at all, did your perceptions about teaching change in graduate school?" I posed these questions because graduate school is typically the first-time graduate students are a teacher of record, even though that may not have been their first-time teaching in front of a class.

Follow-up Interview

The follow-up interview occurred between four and six months after the initial interview and was conducted after initial analysis of the first interview. Although I describe the data analysis process in more detail below, a key aspect of my analysis involved applying a set of codes for Quality Teaching to each participant's definition of quality teaching that they shared during the initial interview, and subsequently looking

for those Quality Teaching codes within the narratives themselves.

For each participant, I looked across their data for Quality Teaching codes the participant identified as part of their conception of quality teaching but were not coded for in the stories. I asked about this possible gap in the follow-up interview posing questions such as, "Was there an experience that influenced having mistakes as such a prominent part of your Quality Teaching definition?" Likewise, I identified significant stories without any Quality Teaching codes and probed about the meaning and significance of that story to the participants' teaching. For example, I asked, "You briefly mentioned founding your math camp and shared stories about that in our previous conversation. Could you expand on how that experience influenced your teaching?" I also realize that expressing how a significant story is linked to one's quality teaching definition can be difficult. People are the sum of their experiences and teasing out how one particular experience influenced one belief is an abstract and complex task. The follow-up interview helped me to fill in gaps, ask clarifying questions about particular stories and events, and in some cases allowed participants to share new stories. As a last step, I engaged in a second round of member checking by emailing the final version of the profile to the participant to ensure I accurately represented their beliefs and stories.

Data Analysis

After transcribing the interviews, I began analysis by separating the data into two parts: the answers to the quality teaching questions and the stories identified as significant. Next, I organized the stories chronologically into a timeline for the life events shared by the participant. Part of the data and findings presented in this study are the personal journeys of each participant reflected in the significant stories and events they

narrated and interpreted for me. As I condensed stories and selected excerpts to share when writing each case, I also considered *why* these stories were significant for each participant. In other words, I analyzed what features of these narratives may have influenced their understanding of teaching and their own journey as a teacher. After several rounds of open coding, I developed a set of codes to describe a salient commonality in significant stories across my participants—each story was a moment that potentially led to a change in teaching or adoption of a teaching practice for this person. I name this collection of thematic codes as Influential Change Agents and discuss these codes in subsequent sections. Additionally, I analyzed the participants' conceptions of quality teaching in more detail. Through several rounds of open coding, I developed a set of codes for Features of Quality Teaching, which I share below.

Analytic Codes

Features of Quality Teaching. The quality teaching codes describe features of instruction participants explicitly identified when explaining what quality teaching was to them. These codes were generated from the responses to interview questions specific to quality teaching. The Features of Quality Teaching codes are Appropriate Challenges, Caring About Students, Handling Mistakes, Supporting Independent Thinkers, Teaching More Than Math, Student Disposition Toward Math, and Teacher Affect. Several of these codes have subcategories. Definitions and descriptions of the codes and subcodes follow in Table 1 as well as examples for each code.

Table 1: Features of Quality Teaching Codes

Code Name	Definition	Example
Appropriate Challenges	Purposely challenging students to promote learning	Let them just work on something that maybe they don't entirely know how to do or that doesn't have a clean answer

Subcodes	Questioning to Support Student Learning	Interacting with or challenging students to promote their understanding through purposeful questions	The right question is “Can you explain how to do a, b, or c to me? Please do that.” Then you’ll find out they really understand.
	Selecting Appropriate Tasks	Discussing task selection for various purposes	I do some selection of some low floor, high ceiling tasks, where everyone can approach it and there should be some hard things in there as well just aspirational almost. Like not enough to demotivate anybody, I can star the hard ones.
Caring about Students		Expresses interest in their students beyond what math skills they acquire	One participant when discussing his own critical experiences said, “Where I think, because I’ve had these experiences, I try to engage the students a little bit more, see them as people first.”
Handling Mistakes		Explains their reaction to mistakes, both their own and student mistakes	I try to model the idea of being willing to hear – to make mistakes and to respond to the mistakes in a careful way.
Subcode	Safe Space for Mistakes	Comments about the social ramifications of making a mistake publicly and what is done to mitigate that situation	So, in my classes we celebrate mistakes as incredibly valuable part of the experience of either understanding something better or coming up with a new idea.
Supporting Independent Thinkers		Discussing students becoming more capable of thinking on their own	I view quality teaching as teaching that causes students to think for themselves and to develop their skills and abilities to think for themselves
Subcode	Supporting Students to Answer Their Own Questions	Discusses the student generating their own question and then being supported to solve it themselves	I try to figure out what they're asking and get them to figure out what the answer's themselves. So rather than just saying - they ask me how to do something - I might say "What do you think?" and "Are there simpler cases we could look at?"
Teaching More Than Math		Discussing what they do in their course, that goes beyond the minimum required	I think about what’s going to last for their whole lives and if their attitude is that they enjoy struggle and they enjoy working on things and they enjoy getting comments and feedback, then I think that’s a healthy attitude to embrace for life.
Student		Attending to student	So, when you talk about great teaching,

Disposition Toward Math		disposition and attitudes	you're talking about exciting your students and expanding their view of themselves and what they can do.
Subcode	Self as Learner	Discussing their own behaviors and attitudes as a student or learner	While reflecting on his disposition towards calculus, one participant recalls thinking "Surely, I'm smart enough to figure this out and if I can't, it's like I'm not trying hard enough or something."
Teacher Affect		Attitudes and behaviors a professor displays toward their students	So, I'm usually super excited - I guess I'm pretty much always super excited when there's a question
Subcode	Other Teacher's Affect	Discussing another teachers' or professors' behavior and attitude (usually a previous teacher of theirs)	My high school wrestling coach really believed in me when I was terrible. And didn't let the fact that I was terrible at wrestling influence the way he treated me or how much time he was willing to spend with me.

Appropriate Challenges codes for participants noting that providing challenges to students can promote learning. This includes two subcodes, which are *Questioning to Support Student Learning*, which codes for interacting with or challenging students to promote their understanding through the use of purposeful questions, and S*electing Appropriate Tasks*, which codes for the participant discussing task selection for various purposes. Finding an appropriate challenge to support student learning captures the idea of encouraging students to productively struggle which supports their understanding. This could appear as challenging a student to explain their thinking or as deliberately differentiating tasks so that students of all levels have questions with which they can mathematically struggle. The macro-level code captures any reference to challenging students to promote learning. The subcodes differentiate whether the instance was in a dialogue with a student or was the selection of tasks and the reasoning for that choice.

Caring about Students codes for instances where the participant expresses interest

in their students beyond what math skills they acquire. Instances of Caring about Students tend to contain more emotive words and are about responding to student feelings and well-being. This could be noting the students as having lives outside the class or empathy for the stress students experience. For example, one participant discussed his experiences by saying, “Where I think, because I’ve had these experiences, I try to engage the students a little bit more, see them as people first.” I coded this as Caring About Students because he mentioned seeing his students as people first.

Handling Mistakes codes for how the participant explained their reaction to mistakes, both their own mistakes and student mistakes. I posed specific questions about how mistakes were handled, so every participant discussed mistakes. The subcode, *Safe Space for Mistakes*, codes for when the participant comments about the social ramifications of making a mistake publicly and what is done to mitigate that situation. An example of the subcode *Safe Space for Mistakes* is seen by one participant recalling, “So in my classes we celebrate mistakes as incredibly valuable part of the experience of either understanding something better or coming up with a new idea.” I coded this with *Safe Space for Mistakes* because the participant is celebrating mistakes, which is counter to how many students react to making mistakes. The participant is emphasizing how mistakes help with learning, making the classroom a safe space for making mistakes.

Independent Effective Thinkers codes for the participant discussing students becoming more capable of thinking mathematically on their own. For example, a participant summed this up with “I view quality teaching as teaching that causes students to think for themselves and to develop their skills and abilities to think for themselves.” The subcode *Supporting Students to Answer Their Own Questions* codes for when the

participant discusses the student generating their own question and then being supported to solve or answer their question themselves. For example, I applied this subcode to "I try to get them to figure out what the answer is themselves. So, they ask me how to do something - I might say 'What do you think?' and 'Are there simpler cases we could look at?'" because the participant focused on the student answering their own question.

Teaching More Than Math codes for the participant discussing what they do in their course, which goes beyond teaching mathematical knowledge and skills. Teaching More Than Math codes could include discussing how to deal with stress or balancing life, work, and school commitments. In the table example above, one participant discussed students learning how to struggle and finished with "I think that's a healthy attitude to embrace for life." I coded this as Teaching More Than Math because the participant is discussing more than the math required to finish the course by focusing on attitudes for life.

Student Disposition Toward Math codes for the participant attending to student disposition and attitudes toward mathematics. This often includes emotive words like frustrated, excited, or bored. This ties in with student engagement because students' attitude and emotions impact their desire to engage with the course materials. But this code also includes statements in which participants attended to student attitudes and behaviors without tying those behaviors to student engagement. As an example, one participant explained, "So when you talk about great teaching, you're talking about exciting your students and expanding their view of themselves and what they can do." This quote received the Student Disposition Toward Math code because the participant is discussing the students' attitudes and pointing out that good teaching is not just about

improving math skills but also focusing on students' feelings and dispositions towards mathematics. The subcode *Self as Learner Perspective* captures instances when the participant discussed their own behaviors, dispositions, and attitudes as a student or learner. For example, one participant recalled how he felt and thought about a calculus class he was flunking saying, "If I can't do it, that's saying something about my own abilities. It's like I must not be – surely, I'm smart enough to figure this out and if I can't, it's like I'm not trying hard enough or something." This is coded as *Self as Learner Perspective* because the participant is directly discussing how he felt and how his thoughts about his own mathematical abilities impacted his persistence in completing the calculus course.

Teacher Affect codes for the emotions, attitudes, and behaviors a participant conveys to their students as their teacher. This could be references to a persona they put on for teaching or how they react to situations in class. For instance, one participant describes "So, I'm usually super excited - I guess I'm pretty much always super excited when there's a question." The emotive words being "super excited" and the reference to their own behavior and affect as a teacher are why this received the Teacher Affect code. The subcode *Other Teacher's Affect* codes for instances when the participant discussed another teachers' or professors' behavior and attitude, often in reference to a previous teacher or professor. One participant related the following example: "My high school wrestling coach really believed in me when I was terrible. And didn't let the fact that I was terrible at wrestling influence the way he treated me or how much time he was willing to spend with me." This quote highlights the behaviors of the coach (how he treated the participant) that showed the participant that the coach really believed in him;

as a result, I coded this quote as *Other Teacher's Affect* because the participant was discussing his coach's affect.

Influential Change Agents Codes. As mentioned above, I also developed a set of codes for influential change agents, which I applied to the stories the professor chose to share. The influential change agent codes describe how and why these stories were significant in terms of influencing a participant's actions or behaviors related to teaching. These categories all codify experiences that more than one participant experienced. The codes include Making a Broader Impact, Responding to Challenges, Reflecting on Parenting, Mentor or Role Model, Reflecting on Past Experiences, and Resourceful. In Table 2, I define and give examples of each code.

Table 2: Influential Change Agents

Code Name	Definition	Example
Making a Broader Impact	Discussing accomplishments that impacted more people than their own students (those enrolled in their courses).	Referring to the math camp he founded: "So, I wanted to make it not just learn some math; I wanted to make it change your life. So, this was what this program was about."
Responding to Challenges	Experienced and overcame a challenge or learned from their own or someone else's difficult experience	Turns out, I think this was helpful for me as a teacher, later, to flunk out. I had to go back, live at home with my parents, go to the community college for one or two quarters
Reflecting on Parenting	Noting the impact that having children has on them	When I had kids, I got very interested - I mean that's when I got interested in teacher education. That's when I got much more interested in math ed.
Mentor or Role Model	An influential person who provided an educational example, good or bad	I had a math teacher who was about - it's almost like you would say this is how you shouldn't teach
Reflecting on Past Experiences	When a participant is thinking about their experiences and explaining what they learned then. Also, when a participant	At university, I had wonderful teachers who really taught math at a high level, which worked great for me. But for many of the class,

	draws new conclusions from a recollection.	maybe they got lost. So, in retrospect, maybe some of the teaching wasn't as great as one would think because I'm not sure they were addressing the needs of everyone in the class.
Resourceful	Noting and using available resources to accomplish a goal	Those conversations and those experiences and chances to use intentional innovative curriculum that was informed by math ed researchers definitely forced me to think beyond just drill and kill

The Making a Broader Impact code accounts for instances when participants discussed how they tried to make a broader impact beyond just their classroom. In these stories the participants shared accomplishments, like authoring a book or founding a math camp, which impacted more people than their students. For example, one participant referred to the math camp he founded saying, “So, I wanted to make it not just learn some math; I wanted to make it change your life. So, this was what this program was about.” This experience received the Making a Broader Impact code because the participant discussed an accomplishment that made a lasting and wider impact than simply teaching math. The participant then discussed that individuals who attended this camp commented about how much they learned from camp and continued to support the camp after graduation.

Responding to Challenges codes for events where the participant experienced and overcame a challenge or learned from their own or someone else’s difficult experience. For example, one participant reflected on flunking out of school saying, “Turns out, I think this was helpful for me as a teacher, later, to flunk out. I had to go back, live at home with my parents, go to the community college for one or two quarters.” He went on

to discuss getting back into the same school and becoming a math major, showing how he bounced back from the set back of flunking out of college. Another participant discussed a difficult situation in a university course where everyone dropped except for her and one other student. She was successful in this course despite the bad teaching she described. Other participants discussed challenges related to the norms of mathematics, academia, or graduate school colleagues. For example, one participant recalled some of his fellow graduate students "would just kind of sit around and snarky laugh about how bad the students were." The participant took the higher road by striving to accomplish more for his students and not acting like these other graduate students.

Reflecting on Parenting codes for the participant noting the impact that having children has had on their teaching. It also codes for interactions with their own or other children that the participant noted as significant. For instance, a participant recalled, "When I had kids, I got very interested - I mean that's when I got interested in teacher education. That's when I got much more interested in math ed." She credits her children with helping her realize the importance of classes for prospective teachers because her children would be taught by those new teachers. Another participant recalled how he was taught to change a diaper as a new father and that trying to learn that by inquiry-based learning would have been a disaster. Both participants took their interactions with their children and applied those experiences to teaching.

The Mentor or Role Model code is applied when the participant discusses or shares a story about an influential person who acted as an educational example. This does not have to be a positive example. For example, one participant recounted a less positive example with:

> I had a math teacher who was about - it's almost like you would say this is how you shouldn't teach. One of them. But I've thought about this at various points during my teaching career. That wow that's this um - I kind of learned a lot from him and not just in a negative way.

While the professor provided a negative example of what not to do, the participant took that experience with her when considering her own teaching and learning.

Reflecting on Past Experiences codes for when a participant is thinking about their experiences and explaining what they learned. I also applied this code when a participant drew a later conclusion from a recollection, that often differed from their initial thoughts and conclusions. In the following example, the participant was sharing a past memory saying,

> At university, I had wonderful teachers who really taught math at a high level, which worked great for me, but for many of the class, maybe they got lost. So, in retrospect, maybe some of the teaching wasn't as great as one would think because I'm not sure they were addressing the needs of everyone in the class

While relating this story of his undergraduate professors, he realized that what he perceived as great teaching *then* may not have been so great given his present-day conceptions of teaching (as indicated by the phrase "So, in retrospect …"). When sharing his story about "wonderful" university teachers, he initially evaluates their teaching as effective for his own personal learning. From a different perspective that he now holds, maybe these professors were not so great since they did not "address the needs of everyone in the class."

The Resourceful code accounts for instances when the participant notes and *uses*

available resources to accomplish a goal, like a grant that allows one to focus on education and teaching. This code also captures instances when the participant noted educational research as being impactful (like reading math education articles or learning about a study). One participant recalled the impact of his mathematics education colleagues saying, "Those conversations and those experiences and chances to use intentional innovative curriculum that was informed by math ed researchers definitely forced me to think beyond just drill and kill, sort of what I'd been exposed to mostly as a student." This experience is coded as Resourceful because the participant is noting his colleagues and related curriculum materials as resources that impacted his teaching. He had to choose to have this interaction with math education researchers and take advantage of these resources.

Selection of Stories for Each Case

After coding all of the stories with the Influential Change Agents codes and coding the participants' definitions of quality teaching using the Quality Teaching codes, I then looked for instances of the Quality Teaching codes in the narratives themselves. As discussed in the description of the follow-up interview, when stories did not contain a Quality Teaching code or when a Quality Teaching code explicitly mentioned in the participant's definition of quality was not assigned to any story, I asked clarification questions in the follow-up interview to better understand the participant's perspective of the event and whether/how it influenced their teaching. The Co-occurrence of Quality Teaching codes in participants' stories and definitions of teaching was one factor I used to help select stories to include in each participant's case. However, determining the significance of a story is subjective. The fact that my participants selected and choose to

particular stories that were "significant events … informing their views of teaching" (as per the interview protocol prompt), indicates each of the shared stories is significant to them in their teaching journey. Due to length and space constraints, I could not include every story from each participant. I used several criteria to select stories for each participant's profile. I considered whether a given story was clearly and explicitly linked to their quality teaching definition. I also considered whether a story illustrated a shift in the participant's behavior or beliefs because events that participants themselves link to changes in beliefs or teaching behaviors are important to document. And finally, I included stories in profiles that the participants marked as particularly influential or significant saying things like "… there was still a core part of that [experience] that I found really valuable" or "I would say that course itself was very impactful."

Identifying Themes for Each Case

For each case, after sharing stories that reflect a participant's teaching journey, I then provide a thematic analysis using my codes for Influential Change Agents and Quality Teaching Conceptions. The first set of analytic themes discussed in each case is the influential change agent codes that I applied to the participants' stories. Using these coding categories, I documented the types of experiences that impacted a participant's teaching journey and their developing conceptions of teaching. I included any Influential Change Agent code that occurred at least once in the participant's narratives provided the story had sufficient detail to describe the significant event or was noted as influential by the participant. These codes helped me understand the types of events that may support teacher growth and change more broadly.

The second set of analytic themes focuses on features of quality teaching.

Specifically, I compared the Quality Teaching codes the participant explicitly identified in their quality teaching definition with occurrences of these codes in their stories. Recall that I used the Features of Quality Teaching codes to analyze the definition of quality teaching provided *and* also identified instances in the participant's stories when a given feature was present. After coding all data for a participant with the Quality Teaching codes, I identified the Co-occurring Quality Teaching codes and then confirmed with member checking that the quality teaching feature I attached to a story was valid. The Co-occurring, participant-validated Quality Teaching codes are included in each case study as the second thematic analysis.

Cross Case Analysis

After I finished compiling the profiles for each participant, I examined the overall frequency of each code as well as the co-occurrence of codes across participant quality teaching definitions and their stories. First, I looked for Quality Teaching Features codes that all the participants shared in their definitions. Since the interview protocol was the same for all participants, I had a structure to work from that allowed for an organized comparison of each participant's definition. Next, I compared the instances of co-occurring Quality Teaching Features codes across participants; that is, codes that occurred in a participant's quality teaching definition and their stories. Finally, I looked for similarities in the Influential Change Agent codes across participants. This provided me with further insights into the journeys of each of these exemplary mathematics professors.

IV. PARTICIPANT PROFILES

Case 1: Sylvia's Profile

Narrative Summary

Sylvia earned her PhD in mathematics, and she spent the early part of her career doing research in arithmetic geometry and teaching mathematics courses at a research university. After obtaining tenure, and with her own children nearing school age, she began to think more about the mathematical preparation of prospective teachers. She realized her department taught mathematics courses for elementary education majors, and she started investigating what these future teachers needed to learn mathematically. Through some university and state government support sources, Sylvia began investigating and, eventually, shifting her research to mathematics education. She learned firsthand how different mathematics research is from other fields and disciplines. Mathematicians have "uniform norms about how you create knowledge, how you investigate things," whereas nearly every other field, including mathematics education, must deal with the uncertainty of life. After training as a mathematician, Sylvia now identifies herself as both a mathematician and a mathematics education researcher though she has recently retired. Sylvia has been recognized for her teaching, receiving the Distinguished Teaching Professorship, which is the highest award for teaching offered at her institution, as well as the Regents' Teaching Award at her university. She has also received national awards from the Association for Women in Mathematics and the Mathematical Association of America.

Sylvia's Quality Teaching Conception

Sylvia's conception of quality teaching focuses on students. Her first statement about teaching is to "invite the students into the material that is being taught and hopefully get them interested." Also, quality teaching "should be an attempt to structure the material, organize it so that it becomes accessible to the students that are there." To achieve that accessibility, quality teaching necessitates knowing where your students are and negotiating "between where they are, what's interesting them, what's bringing them into it, and how can you actually genuinely make this material something that's interesting, exciting, inviting, and accessible." Another component is "students have to be actively engaging with and structuring for themselves or making sense of things for themselves in that process. And that can be helped along by discussions, not only with the course instructor, but also with other students."

When defining her goals for teaching, Sylvia initially states "the goal is, of course, for the students to learn the material … at some level hopefully a lot better than when they started." She strives to get the students as far along a certain path as possible, so her students will learn the specific concepts, ideas, approaches, methods, or solutions required in that course. She also strives to bring the students to a level of enthusiasm and interest in the material.

For a course to be successful, Sylvia again emphasizes that the students will have learned and understood the material. To Sylvia, understanding means "not just that you can do something, but you can also actually actively present it to somebody else." She measures her students' success indirectly through what they have written and said in class and directly through their answers to test problems showing they can "successfully give

explanations, do calculations, explain where they come from." Another part of a successful course is that the students feel the course was worthwhile and have "some level of enthusiasm or interest in the material."

To zoom in from a successful course, Sylvia describes a successful lesson as one where students have made progress on the course material. She explains this as,

> Some sense that the students have made an attempt on it, even if they haven't completely resolved everything. The ones I'm most happy with is honestly where they're just recognizing 'Oh this is - here's the tricky part and here's what I need to figure out or here's what I need to understand' and I'm thrilled with that. Even that amount of progress, just recognizing what the issue is. And then ideally making progress on it. And one where people have shared their thinking, listened to other peoples' thinking, expanded their view of how can you explain something or expanded their view of what constitutes a valid mathematical method for solving something

Her focus in a successful lesson is on the students expanding their understanding and view of the mathematics at hand. From Sylvia's perspective, a successful lesson will include students participating in class discussions and presenting material.

A final aspect of teaching for Sylvia is how student questions and mistakes are handled. Sylvia explains that as her career progressed, her "classes became more interactive." She goes on to explain that questions have always excited her, since questions indicate the students are thinking enough to generate a question. She often turns questions into a class discussion rather than give an immediate answer. Her focus here is on students engaging with and making sense of the material. When discussing mistakes,

Sylvia explains that she often turns her own mistakes into teachable moments. However, student mistakes require more consideration due to the social nature of a classroom. She sets the expectation in her class that "we are just putting our thinking out there and sometimes we're going to say stuff and then we're going to want to retract it later or modify and revise our thinking as we go along. And that should be totally okay." She recognizes the potential risk of making a public mistake and takes steps to make her classroom a safe space for sharing any thinking (correct or not), while allowing students to question each other and create a space for students to correct their mistake with grace and compassion. Her teaching conceptions describe a classroom focused on learning and understanding, where students collaborate and engage to better take ownership of the material.

Sylvia's Journey

After finishing high school in the mid-1970s, Sylvia started her undergraduate studies at an Ivy League University. She relates a story below about one professor who was influential in forming her teaching conceptions.

> I've thought about this at various points during my teaching career, that I kind of learned a lot from him and not just in a negative way. So, we started this course. It was like real analysis or something, and there was a whole pile of students in it because it was a required math major course. Well, a few weeks in, almost everybody had dropped out, and then by the middle of the semester it was down to like three of us. Then when we got to the final exam, one guy came in and said, "I just can't do it!" It was only two of us left at the end of the course. It was just like this can't possibly be good teaching.

> The thing was I realized he lectured of course - straight up. And he did everything in the ultimate generality, and the tests were all state and prove such and such theorem. But in fact, I learned a huge amount from doing that. I really learned the statements of theorems, what the statement meant, how you prove it. And I can't memorize things to save myself, so I really had to understand it and put it together. And there was something about really understanding a proof and really owning it to where you could present it again to somebody else. I found it really valuable, and it influenced - sometimes when I think about it, I've written this math for elementary teachers book and that's kind of embedded in there. It's like - the student, they're invited to come and bring it up, figure it out on their own, but at some point, there are explanations given for why we multiply fractions the way we do or other stuff like that. And those are things you can learn, like you can come to them yourself or you can read them and sort of incorporate it and kind of figure it out and make sense of it. And then be able to explain it to someone else. And I feel like maybe there is a little of that teaching that got incorporated into my process. That was not the way to do it in his case, but there was still a core part of that that I found really valuable in terms of how you handle learning the material that I found useful.

Sylvia took a difficult situation and learned some constructive lessons about her own learning which, in turn, informed how she taught. She highlighted the idea of owning knowledge through understanding as a valuable outcome from this experience, which shows up repeatedly throughout her definition of quality teaching and especially with how she defined understanding. Even though the initial experience was unpleasant, she

managed to distill valuable and influential lessons from that experience as she reflected on it over the years.

She started graduate school in the early 1980s, choosing a school with careful deliberation. She describes a visit to her graduate school's mathematics department as "inviting" and the professors as interested in talking to students, which solidified her decision to attend. Sylvia's graduate advisor influenced her views on teaching and interacting with students. She speaks of his demeanor around the department.

> My graduate advisor was really good at - of course he lectured. But he was really good at asking lots of questions and as grad students we kind of saw him around the department. So, he would just have lots of conversations about math with people. So, I think I was influenced by his style that was interactive already. And he had enthusiasm and really deep knowledge and this ability to kind of draw people into the subject. So, I would say he was influential because of that.

Having a mentor who was actively social with his intellectual conversations gave Sylvia an example of a more interactive teaching style. Her descriptions of quality teaching included engaging the entire class in discussions and getting students to share ideas. This mentor is an early influential example of the importance of interaction in engaging others mathematically. Sylvia also describes her graduate advisor as genuinely caring about teaching.

> My advisor I think was one of the people who was more on the spectrum who cares a lot about teaching versus not caring a lot about teaching. I think he cared a lot about teaching, and he thought that that was important. And you know he's a hot shot researcher too but even so, he always had a passion for teaching. I think I

appreciated that in him; I think that was a good trait.

A passionate and caring mentor on a receptive campus provided a fertile ground for Sylvia to develop her views on teaching and research. Sylvia's quality teaching definition also reflected care for her students, which focused on supporting her students to learn new material and find enjoyment in that learning.

Another notable influence on Sylvia's teaching came from her children. Her children and her interest in their (mathematical) education was the catalyst for shifting her focus towards teacher education.

> When I had kids, that's when I got interested in teacher education. That's when I got much more interested in math ed because I realized 'Oh my goodness - these kids are going to go to school, and their teachers take math courses for future teachers.' Like before that, I hadn't even realized that we were teaching courses like that at the university. I just genuinely just didn't know. I wasn't aware of them, or I hadn't really paid any attention to what all our courses were and that these were important ones we taught. So, once I had kids, I realized 'Oh wow this is actually really important.' So that changed my view on what one should attend to. I thought 'Wow I really want to get involved in these courses because this seems like something you shouldn't sort of just let fall by the wayside. It's something you really want to pay close attention to.'

Her concern for her children and wondering about their future teachers started Sylvia on a course to shift her research to mathematics education and, specifically, elementary preservice teacher education. During this same time around the year 2000, schools and states were investing heavily into education. This period of financial support gave Sylvia

a direction and network for her blooming research interest.

> The state [her home state] wanted to do revisions in their standards and then wanted to revise teaching - teacher preparation. Stuff like that. So at the state level, there was this flourishing time when there was greater interest in education and making sure that teachers are well prepared and then the Dean from the College of Arts and Sciences, which is where I am, and the College of Education kind of got together and produced this - they called it the dean's forum, where they tried to get people from arts and sciences and education together and it actually led to all sorts of - some big grants, I would say indirectly this whole flourishing - this whole time of increased attention certainly took me in a totally different direction than I chose. And it was something I wanted to do just of my own volition, but I think it supported me being able to get much more involved in teacher education and really ultimately switching my research to math education.

Sylvia experienced a major shift in her research interests which was supported, in part, by her university and the state's financial investments to further those new interests. In addition to financial incentives and support, the increased prioritization of teacher preparation also supported an increased value in research in mathematics education.

Sylvia began teaching elementary preservice teacher courses in mathematics and soon found the textbook had not been updated to current ideas in research. This launched her on the process of writing her own textbook for elementary mathematics content courses. She describes the beginning of writing her textbook as:

> I realized that the book that we were using, it wasn't very activity oriented. And I had started reading some stuff from NCTM [The National Council of Teachers of

> Mathematics] and I had met people in math ed. And I was becoming aware of what the recommendations were for more active, engaged reasoning and thinking where you're actually developing ideas through activities. So, I thought there's nothing in my book that does that, so I'll write some activities. So, I started writing the activities for the students. And I just wrote stuff and that was pretty good. Then I realized – I thought we were having good discussions in class, and I thought "Oh wow, they've figured it out. They pulled this together." And then I realized "Wait a minute! The homework wasn't really supporting what we were doing in class. So, you'd get to the test, and I'd have the sense that all of stuff we did in class didn't gel. So, I thought "Well I'm going to have to start writing more support for this." So, it kind of evolved from just activities to activities plus a little bit of support text.

After she had refined these activities, a textbook editor approached her and recommended she submit the activities to the publishing company. At the same time, "the board of regents had decided they needed more courses for future elementary school teachers. So that was a great opportunity. That meant we were revising our courses and I thought 'Oh this is great! I'll develop these courses.'" Sylvia now had the school's support to enrich these courses with current research thinking. She also had the simultaneous process of writing a textbook for this course to help refine her ideas about learning and how to teach to encourage learning. Sylvia describes the confluence of all these events as "very much support[ing] exactly what I was working on. It was this perfect storm in a good way."

Prevailing Themes in Sylvia's Journey

After compiling her narrative, I analyzed Sylvia's interview with the two major

categories of themes discussed earlier: influential change agents and aspects from her quality teaching definition reflected in her narratives. Both categories are discussed below.

Influential Change Agents

Responding to Challenges. An important repeated theme that arises in the earliest narratives in her chronology is overcoming and learning from challenges. Challenging educational experiences are not just events in her life but opportunities to consider what was challenging about those events and whether the challenge can be resolved. For instance, Sylvia took an undergraduate math class with an uninspiring generalized lecture format and a high drop rate, ending the semester as one of two enrolled students. She overcame this difficulty and learned deep lessons about how she acquired knowledge. She also considered how few people were helped by his teaching style. When her kids began school, she researched how preservice teachers were trained to teach mathematics and investigated the path these preservice teachers take to become certified to teach. This personal inquiry became a professional challenge - how to teach this unique type of student (preservice teachers) and engage them with content that may not be directly mathematical (e.g., children's thinking) from a pure math standpoint. This challenge increased as she needed to supplement the book to incorporate current research and meet student learning needs. For Sylvia, challenges were important catalysts for change and growth.

Mentor or Role Model. Another theme repeated in her timeline is support from a mentor figure, in the form of her graduate advisor. He engaged students in insightful discussions through questions, which Sylvia reflected back on as an early model of a

more interactive teaching style. Her mentor allowed her the space to make her own decisions about teaching and research but provided a positive model for what teaching could look like. Her graduate school provided an environment where research was not prioritized to the detriment of teaching. Both her graduate school and her advisor acted as supports for her growth as a teacher.

Resourceful. A third theme is outside support and resources that created a space for change and Sylvia's use of those resources to further her teaching journey. In the eighties, Sylvia's department supported her pursuit in a tangential field of research (mathematics education) and promoted her to professor based on that research. This support and recognition can also be seen as a sign of the changing times and values as more emphasis was placed on education. In addition to departmental support, the state provided funding that allowed her to pursue math education as a research interest and to focus on pre-service elementary teachers. This combined support allowed her to shift her research focus in a quicker period than what she expected; her research shift also heavily influenced her teaching as seen by her inclusion of more active and engaging reasoning through activities, as recommended by current researchers.

Reflecting on Past Experiences. A final theme that runs throughout her narratives is reflecting on educational experiences. Sylvia takes an experience, good or bad, and considers what she can learn from that experience. Her initial reactions to her undergraduate professor were surprise at the dropout rate and determination to not teach like that. Thinking about how this class forced her to learn the material also provided insight when she was considering how to teach her own students and how she could support their learning. A larger cycle of reflection began with her children starting

school, prompting her to investigate the mathematical preparation of teachers. As she began to teach these courses, she continually reflected on the best way to support preservice teachers learning by aligning the in-class activities, homework, and exams with current research and her professional experiences, allowing her to update her textbook. Her ability to reflect back on any experience to learn new lessons allows for continual growth of Sylvia's conceptions of teaching.

Making a Broader Impact. Sylvia's foray into writing and publishing a textbook was initially motivated by her own needs as an instructor teaching prospective teachers. Although Sylvia did not initially set out to impact teacher education, her textbook has had that effect more broadly as one of the most widely adopted textbooks across the United States and now in its sixth edition. Sylvia's commitment to quality teaching extends beyond the walls of her own classroom to impact the teaching of preservice elementary and middle grades teachers more broadly through the use of student-centered, inquiry-based, mathematically-rigorous instruction reflected in her textbook and related resources.

Co-Occurring Features of Quality Teaching in Sylvia's Narratives

Questioning to Support Student Learning. Throughout her definition of quality teaching, Sylvia repeated the idea of asking questions to support student learning and helping her students take ownership of their own knowledge by helping students learn how to answer their own questions. Taking ownership is featured prominently in her undergraduate mathematics story. To face tests requiring proofs of specific theorems, Sylvia had to understand the material deeply enough to take ownership of it. She felt studying and presenting the material to a classmate gave her the necessary depth of

understanding. This ownership continued in her descriptions of teaching pre-service teachers, with her focus on the students being comfortable presenting and explaining the material, thereby supporting them to have ownership of the content.

Teaching More Than Math. Another prominent feature is her responsibility to her students to teach more than the minimum math required. Despite many people encouraging her to focus on research, Sylvia still felt she had a responsibility to her students to teach them well and help them take ownership of the material. She mentioned having discussions during office hours even before she found research supporting engaging students in mathematics. Her responsibility to her own children led her to investigate the preparation of teachers mathematically. Learning about these pre-service teacher courses and their potential impact provided a pivotal way to impact the future of more than her own students. Writing and keeping her textbook current with research shows another way she holds herself responsible to her students, by presenting them with the best information.

Caring About Students. A third conception woven throughout her quality teaching definition is how she cares about her students. Her concern is not simply coverage of the material but that her students engage with the material and take ownership of that knowledge. Her method of handling potential mistakes highlights how she considers her students' needs, by creating a classroom space where mistakes are permissible and a site for learning. She also attends to how students treat each other and how her own demeanor impacts her students' affects. Also, when describing her mentor, she lauds his caring about teaching and having a passion for teaching.

Summary

Collectively, these themes highlight a professor focused on her students' needs to learn the material in a way that is relevant to her students. She cares about her students and considers her responsibility to her students as important. While writing a textbook, Sylvia tackled the need for alignment among the lecture materials, the homework questions, and the classroom activities. Sylvia reflects on her past experiences to make decisions about teaching, showing a professor who is continually considering and modifying her conception of teaching and learning.

Case 2: Mitchell's Profile

Narrative Summary

Mitchell has been a professor of mathematics for over 40 years at the same R2 (doctoral university with high research activity) university. After attaining his undergraduate degree in mathematics, he immediately started work on his PhD in Mathematics in the field of algebra. Upon graduating in the late 1970s, Mitchell started his career and, over time, his interest in the teaching and learning of mathematics has increased. He founded a summer math camp program and is still actively involved in many aspects of his camps. Mitchell has been recognized for excellence in teaching, having received awards within his department, college, and at the university level. In 2008, he received the highest teaching honor his university awards. He also received a national award for excellence in Science, Mathematics, and Engineering Mentoring. Mitchell has earned many awards over the years for both teaching and mentoring and has a unique perspective as both a university professor and a math camp founder.

Mitchell's Quality Teaching Conception

Mitchell's conception of quality teaching focuses on the student's journey through mathematics. He thinks of teaching "as not so much telling students but as engaging them in the process of learning." He talks of developing a student "as an effective thinker and learner who can approach problem solving in a wide variety of settings." Mitchell sums up what he believes a professor's goal should be as a teacher: "To see our students become independent learners who themselves can make contributions in the future, either in math or whatever they do."

When describing the conditions for a successful course, Mitchell focuses on the

students' affect and supporting them to deepen their knowledge. He looks for "when the student becomes excited about what we're doing, they want to do more, they begin to ask questions themselves, and push what they can do to deeper levels." To support his students' growth, Mitchell discusses "finding the fine line to challenge them to figure out new things without overwhelming them from where they are so that they give up." His attention to his students' affect is highlighted by how he looks for a challenge of an appropriate level to get his students excited without overwhelming them.

A successful lesson for Mitchell is "engaging the student in doing the math themselves." In attending to his students, he looks for "where the students are engaged in the process of doing math, where they're learning new things, where they're asking good questions, and where they're becoming excited about what they're doing." In preparation for a lesson, Mitchell gives his students a preview of the lesson with questions to get "them in the mode of figuring out what's happening." Overall, the lesson will feature class discussions that "have the student engaged in the process of what we're doing." Mitchell describes the students' reaction to this approach and the end result with:

> It's a little bit of discomfort when you're asking them to figure out new things as opposed to just telling them how it works … So that is a fine line that they, by end of the process, that they're being asked to figure things out. But in the long run, I think they come to be able to do more themselves and less dependent on the teacher

Even though this type of student engagement can feel uncomfortable, Mitchell approaches a lesson as an opportunity for the students to grow and become more independent.

This discomfiting approach to lessons leads to student questions. Mitchell first focuses on what students are asking and also checks to see if the student understands by asking them "to articulate the problem or what are related definitions and have them engage in the process of figuring things out." He then gets them "to figure out what the answer is themselves to some extent. So, they ask me how to do something, I might say 'What do you think?' and 'Are there simpler cases we could look at?'" Mitchell sums up this idea well with "The easy thing to do is just tell them how to do the problem. The harder thing is to get them to work on solving the problem themselves."

By challenging the student with questions when they make inquiries, the opportunity for student errors and mistakes increases, as does the student anxiety about those mistakes. In responding to mistakes, Mitchell mentions the book *The 5 Elements of Effective Thinking* by Edward B. Burger and Michael Starbird, which has a chapter on the value of mistakes. Mitchell discusses the usefulness of mistakes.

> So, mistakes are opportunities for learning and if that occurs with me that's just fine. And in a way, it's probably better because it lets the student know it's perfectly okay. My teacher screwed up and I can screw up and not to worry about it. And a lot of times I've found students are afraid to try anything new because they're worried they'll make a mistake. And I think that's a good opportunity for learning and a welcome occurrence in class. So, if I make a mistake, it's fine. Not something that I fret over or worry about in the least.

Mitchell references both reassuring students and their apparent fear of mistakes, which again highlights an attention to his students' affect. His teaching conceptions describe a classroom focused on student engagement, activity, and learning, where students learn

how to do the math themselves and see the value in learning from their mistakes and false starts.

Mitchell's Journey

One of his earliest influential experiences is when Mitchell attended a summer math program during high school in the late 1960s. The focus of the program is summed up by Mitchell.

> So, when I was in high school, I went to this summer math program at State University, and I remember the program founder talking about doing math is to 'think deeply, simple things.' And that's really consistent with Starbird and Burger's book chapter 1 of learning to think deeply and develop a foundation based on simple principles and ideas.

Here, Mitchell is highlighting a connection between the summer math program he attended and a personally influential book he read later in life that highlights a foundational component of quality teaching for him—developing conceptual understanding of the content. The importance of "thinking deeply simple things" is a foundational goal of his teaching that he himself experienced as a student and has reflected on throughout his career.

After graduating high school in the late 1960s, Mitchell attended a midwestern American university to study mathematics. Mitchell recalled his time there with an interesting bit of retrospection.

> At the university, I had wonderful teachers who really taught math at a high level, which worked great for me. But for many of the class, maybe they got lost. So, in retrospect, maybe some of the teaching wasn't as great as one would think

because I'm not sure they were addressing the needs of everyone in the class. This ability to consider multiple perspectives of a past event, perspectives outside of his own, provided him with opportunities for reflection and continual learning. Additionally, the importance of considering *all* students' needs surfaces in this story.

Once Mitchell finished his undergraduate degree in Mathematics, he began studying for his PhD in mathematics at a southern American university. This also marks the first time Mitchell was in the role of lecturer for a college course or lab. Mitchell reflects on that time as a graduate student teacher.

> I'm sure at the time I was rather immature as a teacher. And in those kinds of courses, there was a more rigid syllabus than when you're more senior, and you have more freedom to do exactly what you want. As a graduate student, we had a syllabus, and we covered a certain amount. It was a group final that everybody gave the same final. It was much more structured and less engaged with what transpired.

In this story, he notes the restrictions on a graduate student teacher compared to the freedom he experienced later as a professor. When asked about formative teaching experiences in graduate school, Mitchell described his teaching as "more formulaic. Here's the material; you covered it." Additionally, Mitchell's assessment that he was immature as a teacher shows he believes his teaching abilities have changed and improved over time: his teaching is no longer "formulaic" or confined to a "rigid syllabus" which, presumably, did not allow for him to respond to the needs of his students in the past. This is in contrast to the next story, in which he highlights adjusting to students' prior knowledge as a feature of his teaching when recounting an early

teaching experience as a beginning assistant professor.

Once Mitchell finished his PhD, he began his career at the southern university where he would remain for his career thus far. When asked about an early teaching experience, Mitchell shared a story reflecting his ability to overcome challenges and respond to student needs.

> I was teaching my first topology course. And I went in, and it took me about three weeks to get through where I thought I would be at the end of one lecture. But it was just because it took quite some time to adjust to where the students were. And as I back tracked to figure out what I could do, it just took a while to adjust to teaching the student as opposed to - I mean it's easy enough to give a lecture but you have to see if anybody's understanding anything you're saying.

Mitchell realized his students were not understanding him and took the time to assess the level of his students' mathematical knowledge. Once he realized the gap between his students' understanding and the prerequisite knowledge for a topology course, he overcame this challenge by adjusting his lecture to meet the needs of his students. Again, we see Mitchell's attention to what the students are doing and understanding in this story of an early teaching experience.

Another significant event occurring during the first decade of his career was when Mitchell attended a professional development program designed to train mathematicians in computer science. Though he was trained as a mathematician, computer science was a burgeoning field in the eighties, and there was a lack of capable faculty to teach undergraduate computer science courses. When asked about how this experience was related to his understanding of teaching, Mitchell described his time in this program

saying, “When I went and worked in groups, I realized that I would be learning by explaining things to other people. And that's really an important aspect of teaching - building a social community.” While Mitchell attended this training to learn about programming computers, he also learned about how to learn.

> And over time I came to realize the importance of collaborating with others and one of the best ways to learn an idea is to explain it to someone else. So, I evolved more into a person who valued the social interactions more than I did back in those formative days, when I would just enjoy sitting down and doing things for myself.

His views on learning (and teaching) expanded on the basis of his own experiences as a learner during this professional development seminar. Instead of seeing himself as an individual learner, he experienced the power of collaboration and discussion as key components in his own learning. The importance of community and collaborative discussion became significant elements of his conception of teaching and were features he strove to incorporate into his own teaching.

After attaining tenure, Mitchell founded a summer math camp to help middle and high school aged students have meaningful mathematical experiences and to model his camp after his own experiences in high school. Mitchell took his earlier teaching and learning experiences with attending to student needs and affect and carried this knowledge into designing his camp. As his camp grew, he focused not only on the content and challenge level of the mathematics, but also what the students would experience in the program. Mitchell described how engaging students in research is a feature of his summer math camps.

> I think of teaching as leading to research. That we try to think of presenting problems to students that in a way are research questions to them. So, if we're covering a new theorem, for us it's maybe going over old things, but for the student, how do you guide them to figuring it out for themselves. So, in our summer math camps, students are working through old problems in number theory. And they're taking a journey along the same path that Gauss took when he was about 18. And it took him a year or so to give the first proof of quadratic reciprocity. The students do it a little quicker because in six weeks in the summer, they're getting to the same point. But for them, it's a research problem. And so that sort of shaped my idea of computing examples, looking for patterns, and then trying to give careful arguments why things work. So, the teaching leads to the research because you're preparing the students to become researchers themselves. So that's another reason that I don't think of teaching as just telling them theorems because then they're not going to be prepared to make new discoveries. But in teaching, you're trying to prepare that student to be able to do new problems themselves.

The camp is designed to provide an experience in doing research in mathematics that feels authentic to the attendees. Mitchell continues to consider his students' perspectives, even in interactions with middle school and high school age students. Additionally, he focuses on what the students will gain from his teaching and what he is preparing them for in the future.

Mitchell also focused on building a social community in the summer math program he founded, which he has continued to teach at for more than thirty years. He

notes that building a social community "occurred in my math camps, where I tried to develop that culture in everything we do in the program." One of the founding principles behind his math camp was "that you could have people from disadvantaged backgrounds still coming together, and although they may not have known as much, they could still do just as well as these people from these so-called magnet schools and rich schools." Mitchell reminisced about his students and outlined the interaction style of his camp as:

> I'm just as proud of the kid from a small place who goes on to make great progress, as somebody who comes from a magnet school and goes on to do great things. But I encouraged everyone. The idea was to raise the level for all the students, have them work together in the quote [collaborative] style of sharing ideas. Wasn't a competitive program, but everyone benefited from this environment that we created. And a lot of it came out of the expectation that little kids can do amazing things.

The social community Mitchell built in his camp came from his own experiences in a similar camp as a high school student.

A significant event in developing his teaching conceptions came from interacting with his own children. He remembers particularly well a story about his son learning algebra.

> He was in sixth grade, and I said to the teacher, "I want you to assign my son one section a day of problems to work." The teacher said, "Well I don't know how to work them." I said, "I didn't ask you to work them, I didn't ask you to grade them, I just want you to assign them." And my son came to me and said, "How do I do this?" And I said, "Well have you tried looking at the book." Because I wanted

> him to figure things out. And after two or three times, he decided he could figure things out himself, and he hardly ever came back. And later years he said I hadn't taught him any algebra at all. He said he had to teach himself algebra. And I thought good for you, you learned how to figure things out for yourself. So, I viewed that as a success, that he thought I was useless.

Instead of directly teaching his son algebra, Mitchell taught his son how to learn things for himself. In that process, Mitchell learned about the capabilities of children and how a child can learn higher level mathematics. His success as a teacher is based on what the student, his son in this case, learned and came away with, rather than his activities as a teacher.

Mitchell also notes what his math camp students take away from attending his math camp. He mentions graduated camp attendees.

> They would write these wonderful notes about how the program changed their lives and how they wanted now to provide that opportunity to future students. So, the students want to become counselors, they want to give back, and they feel a real commitment to what we're doing. So, I think one of the great, the most rewarding things as a teacher is to see your students doing incredible things.

This cycle of returning and giving back provided Mitchell with evidence of the impact his work with his math camp was having. Mitchell sums up another idea from working with students with how he defines teaching as "A lot of people think it's explaining some math concepts. That's, to me not, teaching. To me, it's changing a student's view of themselves and what they can do in life." His attention to the student experience and impact highlights how much he values and learns from his interactions with his students.

In considering significant events that influenced his teaching conceptions, Mitchell further discussed introducing his children to mathematics. Mitchell "was convinced that young kids could do math at a very high level if given the proper background and encouragement." He continues describing this process of mathematically encouraging his kids.

> So early on I was introducing them to variables and algebra and higher-level math, and I would say the lesson was that kids can do amazing things if challenged. So, it was really quite rewarding to see each of them become quite competent at it and do really exceptionally well in math. Now people would say 'Oh this is just because they were your kids.' But my view was not that they did well because they were my kids; they did well because they worked hard, because they were encouraged, and nurtured their abilities to do math at a high level. And it was very satisfying to see those theories that I had about learning and that all kids could do math at a high level if encouraged and nurtured. And I saw that with my young kids, and I've seen that in our math camps for many, many years. That challenging kids to do things they may have never thought they could possibly do, that they can make incredible progress and discoveries themselves.

Mitchell highlights how challenging kids and having high expectations for them can help them grow and that they will take that experience with them. He sees the similarities between his own children learning and all the kids at his math camps learning, showing his continuing interest in helping all of his students learn. Mitchell encourages kids and all his students to reach great heights in many of his stories.

Prevailing Themes in Mitchell's Journey

After compiling his narrative, I analyzed Mitchell's interview with the two major categories of themes discussed earlier: influential change agents and aspects from his quality teaching definition reflected in his narratives. Both categories are discussed below.

Influential Change Agents

Resourceful. Throughout Mitchell's narratives, he references the effect of atypical learning experiences like a summer math camp in high school or the professional development where he learned about programming computers. He took advantage of these opportunities as resources to further his own learning. And these experiences later affected his teaching conceptions. He notes the impact of the high school summer math camp he attended with the idea that "doing math is to 'think deeply simple things.'" He credits this summer camp experience as an early influence on his "philosophies about engaging students in doing math." Another experience influencing his teaching views is Mitchell's attendance at a professional development on computer science. Here, Mitchell learned "the importance of collaborating with others and one of the best ways to learn an idea is to explain it to someone else." This professional development also showed him "an important aspect of teaching is building a social community," which is highlighted in his math camps. Mitchell even borrowed the problem sets from the camp he attended in high school when starting his math camp. These external resources and programs allowed Mitchell to experience new ways to think and learn and to integrate those experiences into his teaching conceptions.

Reflecting on Past Experiences. Many of the stories Mitchell shared highlight his ability to reflect on his changing views of teaching over time. An example of the importance of reflection for Mitchell's conceptions of teaching is showcased by his quote about his undergraduate experiences in math classes. He first notes his professors as wonderful and teaching math at a high level, but then he continues reflecting on how his classmates' needs weren't being met. The first part of this quote is his memory of that time, with the second part highlighting Mitchell's reflections on that experience as an experienced professor. This juxtaposition shows multiple perspectives and interpretations of the memory—from his perspective as a student long ago, his perspective as a professor who cares about teaching, and his perspective of his classmates' experiences of that class. Additionally, when thinking back on his teaching experiences in graduate school, Mitchell recalls how formulaic and restricted teaching felt when he was part of a group teaching the same syllabus and giving the same final. He compares this to the freedom he experienced once he was more senior in his role as a professor. Mitchell's ability to reflect on personal experiences and apply new insights to present and future events highlight a central mechanism that influences his teaching conceptions.

Reflecting on Parenting. A significant event for any person, raising a family, also brought Mitchell the chance to reflect more on his teaching conceptions. When his son started taking algebra, he approached the teacher to give his son challenging mathematical problems. His story of his son tackling extra questions shows an example of a child rising to a challenge and overcoming it. By asking the teacher to assign problems, Mitchell is crafting an experience where his son "learned how to figure things out for [him]self. So, I viewed that as a success, that he thought I was useless" in helping

him learn algebra. By crafting this learning opportunity, Mitchell got to see some of his teaching conceptions be reaffirmed through the experience of his son. He emphasized the idea of "all kids could do math at a high level if encouraged and nurtured" in his stories about his children and continues this with the students in his math camps.

Making a Broader Impact. One of the foundational principles of Mitchell's math camp was including kids from not only magnet schools but also kids from disadvantaged backgrounds. He saw many of his students "go on to do great things" and he also noted "the program would have more impact on these [disadvantaged] kids than the other ones." So, his camp continues to reinforce his idea that kids can do amazing things. His camp graduates also remind him that adults can do amazing things by how highly they spoke of the program and how committed they are to the math camp's success. Mitchell instructed his counselors with, "All you do when a student asks you a question is you say, 'What do you know, what are you trying to show?' And let them explain their ideas to you." Even if the counselor didn't understand the math, getting the students to explain their thinking allowed students to discover and do amazing things. Mitchell describes a key principle of his summer math camps as "kids can do much more than you ever think is possible."

Co-Occurring Features of Quality Teaching in Mitchell's Narratives

Supporting Students to Answer Their Own Questions. Woven throughout Mitchell's definition of quality teaching is a focus on the students becoming independent learners who do not depend on the teacher. He references wanting students to become effective thinkers and learners, which also shows up when he talks about Starbird and Burger's book *The 5 Elements of Effective Thinking.* His description of his son teaching

himself Algebra highlights the process of becoming an independent learner. Mitchell's style of answering questions with questions is designed to help students learn how "to be able to do new problems themselves." Supporting his students to answer their own questions shows an attention or care to more than the students' mathematical skills.

Caring About Students. Another feature across Mitchell's quality teaching definition is "how important it is to care about your students and what they're thinking." His attention to the students' needs is highlighted in his story about teaching an upper-level mathematics course (topology) and how he had to restructure his course scope and sequence "because it took quite some time to adjust to where the students were." When asked what a successful course would look like, Mitchell discusses "finding the fine line between - to challenge them to figure out new things without overwhelming them from where they are so that they give up." His thought to not overwhelm them demonstrates his attention to more than what the students are learning, but also their general affect in doing math. Mitchell summarizes his stance with "if you're providing a setting that encourages and nurtures the student, then you can be successful in seeing their growth."

Appropriate Challenges. Mitchell's focus on finding the "fine line" between challenging and overwhelming his students not only reflects his concern with student affect but also shows the importance of providing appropriate challenges for his students. In the case of the story, he shared about the topology course he taught, he realized the content and tasks were too challenging. So, he adjusted his expectations and instruction to meet the students where they were at and provide appropriate challenges. He convinced his son's algebra teacher to assign extra questions to challenge his son mathematically because his son was not being sufficiently challenged. He continued to

challenge his son by encouraging him to use the book as a resource. When discussing crafting the problem sets for his summer math camps, Mitchell considers “if we're covering a new theorem, for us it's maybe going over old things, but for the student, how do you guide them to figuring it out for themselves.” Although the novelty of the problem and solution is gone for himself as a learner, his students have yet to reach an understanding of the solution. This attention to the appropriateness of the mathematical challenge of problems and tasks is a key feature of his conception of quality teaching.

Summary

Overall, Mitchell is a professor who focuses on the students’ experiences. He pays attention to not only what they will learn, but also how they will learn and how they will feel as they learn. He thinks of the larger picture with what his students could use as future skills and as thinkers and learners overall. This focus on the students’ prior knowledge, affect, and perspective shows a professor focused on helping his students become better versions of themselves.

Case 3: Jared's Profile

Narrative Summary

Jared's mathematical journey began with a scattering of entertaining teachers in middle and high school with whom he experienced success in learning mathematics followed by a contrasting experience in university. After flunking out as a freshman in college, Jared attended a community college to pick up his grades and then reapplied to the same university to finish his degree. After Jared graduated with a PhD in Mathematics in the late 1990s, he became a college professor, which he has been for over twenty years. While his own personal experience as an undergraduate and graduate student featured large lecture classes with recitation sections, most of his years teaching have been in small classes. Jared has earned nine awards for teaching from three different institutions and strives to make authentic connections with his students, so they seem him as a resource for learning while still challenging them mathematically.

Jared's Quality Teaching Conception

Jared's conception of quality teaching focuses on "how it impacts the student." His intention is to convey mathematical ideas in such a way, so these ideas become simple to the students. He believes "this is something that students are capable of, if they work at it, that they can also come to appreciate it to be simple." He explains this understanding "shouldn't feel like a burden to try to remember what you've learned if you get to the point where it seems simple." He also shares that quality teaching should make the material seem interesting to the student, so they are "empowered by what they learn in class." He sums up his conception of quality teaching as "ultimately be trying to make things simpler [and] I think ultimately it should try to convey an interest in the

subject."

Jared states his goal for his teaching as trying "to make the students feel that they could have asked the questions that the content is designed to answer." He further explains this idea with:

> So, my goal as a teacher is to bring the question alive that the math is trying to answer and to make the students think "Oh yeah, I could have asked that question, like that's a question that seems worth asking." And then also, "Oh if you look at it the right way, here's a way of trying to answer that." So, for me that's my goal as a communicator.

Even though typical math questions are often hundreds of years old, Jared strives to keep the math practical and worthy of learning.

When asked what he strives to embody when teaching, Jared referenced his time as one of several teaching assistants for a large lecture class. He described his role in that power dynamic as

> it was structurally easy to be kind of the "good guy" in the lecture recitation set up, because I wasn't the one necessarily writing the exam questions. I wasn't setting the grade cut offs, and I was more just in the classroom to help the students clear up their confusions. I'm not pitting myself against the instructor necessarily or the book or the math. Or maybe I guess somehow all three at different times, but to the extent that I felt myself to be there on the side of the student, or there as a resource for them, felt very natural, and I liked it.

While he does not typically teach large lecture classes, he still presents himself as being on the student's team or "as a resource" to support students and help clear up any

confusions.

When asked to take a step back and define a successful course, Jared describes two qualifications for successful courses. He first explains that every course has a list of topics or targeted learning to try and get through. The first qualification is "every student should find some topics in that list, that kind of resonate, that they are curious about, that they maybe weren't curious to begin with, but they became curious about it because of the course." He does not feel his students need to resonate with all the topics in a course, but that success is defined by the students finding some material that piques their interest. From a course designer perspective, Jared's second qualification dovetails with the first as "I want every topic to have at least a few students that resonate with it." Every student resonates with at least one topic and every topic resonates with at least one student. These qualifications highlight Jared's desire to engage his students with interesting course topics.

When asked about his expectations for his own behavior and that of his students, Jared wants to communicate his enthusiasm for a new topic while also ensuring he provides the best mathematical tasks and examples. One of the tricky parts to balance when selecting tasks is first "that they be the most natural, obvious thing you would wonder about for that topic." These kinds of tasks provide a floor-level task or entry point for any student. Another part to balance, Jared highlights, is finding tasks with "entry points at different levels – that's really important at this stage – and that's up to me to do, is to carefully select stuff like that." This is important for students who are more curious, so they have tasks to investigate further. He explains that he wants upper-level tasks to be almost aspirational, but not enough to demotivate anybody; so, he finds identifying the

hard ones helps set expectations, while also providing a challenge for the students who want a little bit more.

Another component in Jared's successful course is providing opportunities to do corrections to the homework. This aspect of his courses is due to personal experience with "topics where it takes me a while to get the hang of it and I need to get it wrong and then get it right." From Jared's perspective, "it should always be worth it to look back especially at stuff that they've gotten wrong. And so therefore I think that there should be points and incentive tied to that." He supports students to revise their understandings over time by allowing homework corrections. Jared's emphasis on understanding content is seen in his selection of tasks, differentiating instruction, and allowing corrections.

Shifting his focus to a successful lesson, Jared highlights the importance of student collaboration. He recognizes his students might struggle with "math anxiety or imposter syndrome or anything students might be struggling with in the class. There's a lot of things that just a support network of fellow students." Jared believes that providing a "support network" in class will help combat those feelings. Another feature of a successful lesson is having content "approached almost like a puzzle. If I could make every problem seem like a sudoku puzzle, … that would be the mindset I would want a student to be in when they're approaching it." With students in this mindset of solving a puzzle, Jared finds,

> It's a lot easier to make recommendations and have them taken in the right way as suggestions for solving this trick puzzle as opposed to, "I'm evaluating your smartness every time I say something about what you do." It's more like "Oh no it's you and me against this hard puzzle." It feels more natural to me to be on the

> side of a student solving a hard puzzle rather than for me to just feel like a judge at all times or something.

Jared again emphasizes that he wants to be a resource for students and is "on their side." He is aware that his actions, including task selection, influence student engagement, motivation, and their perception of him as an ally in their learning. He also notes that part of learning the material is "letting them try and mess things up and hopefully having them not feel crushed. I mean – they wouldn't be learning a lot if the first thing they tried always solved the problem." He views this as the process of doing math, which he teaches along with the content.

When students ask questions, Jared gets excited because this indicates they are paying attention and comfortable enough to speak up. To reward the student publicly and encourage future questions, "I will essentially do anything in my human power to make it be a positive contribution to what we're doing collectively. Like there are some laughable examples of to what length I will go." Jared will consider why a student asks a question and where they are conceptually, so he can help move the class conversation forward. In office hours, he has the luxury of exploring why a student is asking that question, where "it's more like, "Oh I really want to help *You* now take the broader picture and see similarities between what you're doing now and how you approach these." Jared compares answering questions in class versus in office hours with

> I have different goals in the two different settings. And one [in-class questions] is a lot more about rewarding and preserving status and making it be a valuable contribution and have everyone see the value of this contribution. And in the one-on-one [office hours] setting, it can be like "Oh okay yes. I see why this is

difficult." It's more individualized.

Jared shows his consideration of more than the conceptual understanding of his students by attending to their status in class and their motivation with their own learning.

Jared's approach to mistakes also considers how his actions will impact his students. First, he actively encourages his students to "call me out. And then I reward them when they do." He also notes that it is human nature to be embarrassed by mistakes, but he feels he has outgrown that feeling, which shows the student a different way to react to making a mistake. He sums up his reactions to his own mistakes with

> I think as long as you can take a moment and explain why you fell into that trap, then students will be like it's not that bad. Especially if it happens a bunch of times, then it's like, "Hey we're all just trying to get through this,"

When thinking about student mistakes, he strives to make being wrong the norm rather than an experience to be feared. Jared wants his students to be able to analyze their mistakes and figure out what led to the mistake. He notes students "learn more from that kind of exchange than if everything just goes up there perfectly every time." He notes that the more honest you are with your students over the course of the term, the better the relationship with them.

Jared's Journey

Jared's first memorable event related to teaching recalls his middle and high school math teachers. He describes the most memorable teachers as playful entertainers. He "just had exposure early on to some very playful, very engaging teachers, which was fortunate for me. But I feel like that set a little bit my expectation" for class being something fun or interesting. He reflected that the math of those years came pretty easy

for him, and “I left high school with a very firmly fixed mindset. And I just thought I’m good at math, like that’s one of my strengths.” (Here Jared is referencing a term, fixed mindset, from educational literature.) A fixed mindset is described by Jared as “I’m smart enough to figure this out and if I can’t, it’s like I’m not trying hard enough or something.” Carol Dweck (2015) defines the two mindsets for learning as fixed and growth mindsets. A fixed mindset is “the belief that intelligence is fixed and unchangeable” whereas a growth mindset is “the belief that intelligence can be developed” for example, through effort, good strategies, and input and mentoring from others. Jared thought enough work would produce a solution to any math problem eventually. He said, “That was just my pride in my math skills thanks to some supportive high school teachers.”

Jared began his university journey at an R1 midwestern university majoring in Engineering. His first math class was in a large lecture hall, and the professor was “not an entertainer. Not someone who knew my name and would engage me and explain how they were thinking about things.” This led Jared to an unproductive situation.

> I didn’t have the right kind of support group to explain to me how school and learning actually works. Because I’ve never really had to work to learn stuff before. So, I assumed if I just kept retrying to read the book and do stuff, that I could do it. And then when I felt “Oh, I’m not being successful,” it would be weirdly not even all the way through the semester. I’d be halfway through the semester. I’m starting to get lower grades on some of the quizzes or whatever. And I’d be like this is not working. I just judged the whole course to be not working. I’m just going to fail it and retake it.

Jared ended up failing calculus twice, so he attributes his flunking out of university "to calculus and a couple of other physics courses and stuff, but really because I had no idea how to study." Reflecting back on the experience, Jared notes "I think this was helpful for me as a teacher, later, to flunk out."

Jared ended up moving back home with his parents and attending a community college. While his professor was "not a super entertaining calc teacher," he was "a teacher that learned my name and helped me through stuff." He remembers he "learned quite a bit of math on the way – just things I didn't know" like prerequisite trigonometry concepts. After being accepted back to the same university on academic probation, Jared considered what he needed to do differently to increase his chances to succeed. He deliberately picked a new roommate who was the type to work hard and do their homework. Jared recalls "I just basically like imitated – every time he was studying, I'd be studying. And so, my grades were much better. In fact, it started to be like can I get a perfect score on the exam?" This represents a marked shift in his thinking and his work ethic.

Jared's new work ethic paid off, and he successfully completed the calculus sequence and made it into the upper-level math courses. "Somehow going through that to get through calculus and then hitting the reward, like the heaven of the upper-level math courses, with a smaller and better community of math majors that I knew suddenly by name. That turned things around." He graduated with a bachelor's degree in mathematics and went to graduate school.

His first assignment as a teaching assistant in calculus was to be "the person that I was afraid to ask for help when I was the student." He considered, "If I can reach my

students as their TA, I can help people like me who were not studying," which reflects how his experiences as a student—particularly a struggling student—informed his teaching decisions.

> Just being able to be on the flip side of something, that was such a profound change, that I had gone through a few years earlier was really cool. I felt like, "Oh yeah this is really interesting to think about – how students either engage or don't engage with this material." And there could be some really bright students here who just don't want to do the work. Or they don't really know what they want to do. Or they don't know how to study. Or a thousand different reasons why someone might not like resonate with the material.

Being able to reflect on how his students' experiences could mirror or differ from his own experiences as a learner indicates he sees his students as more than names on a roster. Jared's experiences of flunking out and then returning as the teaching assistant for the course he flunked "made a deep impression on me."

Jared also identified a professional development experience from his time as a teaching assistant for algebra as eye opening. Every week, one of the class sessions for the college algebra class he was assigned as the TA was a problem-solving session instead of normal (recitation-style) class. The problems were "very carefully designed, and they were meant to be worked on in groups in class. And they were hard in kind of weird ways. There could be different ways to approach the problem." As a teaching assistant, he was forbidden from giving the answer, even after students turned in their write up. Jared remembers learning "to appreciate the idea of 'Don't insult your students by giving them necessarily problems that have a clean answer.' Go ahead and with

warning, let them work on something that maybe they don't entirely know how to do." Jared saw firsthand how students could rise to a challenge and learn concepts, even if they did not 'solve' the question.

When reflecting on expectations and norms, Jared contrasted his own behavior as a teaching assistant with that of some of his fellow graduate students. He remembers wanting to fit in, "But having been on both sides of teaching, I feel like the bar was kind of high in my own mind. That I really do need to connect with students, maybe more than some of my other fellow grad students." He recalled some of the graduate students "would just kind of sit around and snarky laugh about how bad the students were." He connected their behavior to his past experiences as a struggling learner.

> I didn't feel comfortable there though because of my own personal experience. I always wanted to get inside – like what is this student doing, It's so weird. I don't understand what they're thinking. They're not just making up random stuff; they're trying to solve this problem but they're doing this weird thing. I felt like that was an engaging and interesting puzzle to try to solve. And it helps the student if you can figure it out.

Jared's thoughts highlight his interest in his students' thought processes and his desire to help his students by solving the "puzzles" their work presented.

Jared's professors in graduate school presented another interesting comparison related to teaching. As a graduate student Jared was in the position of a learner and teacher of mathematics; he had the simultaneous experience of being in front of and behind the lectern.

> The people who were teaching me the advanced math, were just kind of lecturing

> and giving us homework problems and exams and stuff. Very uninspiring examples for how to teach in an engaging manner. But pair that at the same time with this kind of innovative problem-solving method that I was doing as a TA, and there was good stuff happening there. So, I had a kind of a clear contrast with what I was getting as a consumer of their product for graduate students versus what they were trying to do for the undergraduates and what I was a part of on that side.

The contrast between fostering engagement in an entry level math class versus the lack of engagement in graduate level classes highlighted two differing ways of teaching. Jared reflected on those contrasting experiences and came away with insights into many styles of teaching.

Another significant experience that influenced Jared's teaching was starting a family the same year he began graduate school. Thinking back on raising his kids, Jared relates "even having small children when I was a grad student, I do kind of think – I'd be hard pressed to even put words to what it is, but I think it made me a better teacher." He continued saying, "maybe [having kids] made me a little bit more caring about the next generation, the future of our planet." But being a parent also forced him to mature because he needed a career to support his growing family. Having grown children now provides Jared with "even more perspective now on kids and learning than I did [know] then." Raising kids while teaching adults provided fertile ground for Jared to consider how people learn and to empathize with any difficulties.

After attaining a position as a professor, Jared continued to reflect on his teaching and was receptive to trying different teaching curricula. As he worked with mathematics

education colleagues, he was recruited into their mathematics education grants. He reflects that "those conversations and experiences and chances to use intentional, innovative curriculum that was informed by math ed researchers definitely forced me to think beyond just drill and kill. Sort of what I'd been exposed to mostly myself as a student." These experiences with mathematics education research provided resources, support, and direction to further develop his teaching abilities. After pondering the type of professor he may have been without these education-oriented colleagues, Jared reflects

> Because I've had these experiences, I try to engage the students a little bit more, see them as people first. Part of it is getting older too where I feel like I'm not quite as hung up on the content in terms of "Am I going to blow it or am I going to totally humiliate myself if I teach it? There's a combination of "Well I've taught it a bunch now, so I feel pretty comfortable explaining the material." And just over time, I think that takes a little of the edge off. But also, I think my approach to teaching makes me less worried about making a mistake.

Having mathematics education colleagues gave Jared an outside source of knowledge and innovations to continue to enhance his teaching abilities over the years.

Upon further reflection on the impact of mathematics and mathematics education research on his teaching, Jared realizes

> I actually think both of them [math and math education research] have an impact on the way I teach. In other words, when I think about math, things tend to get simpler over the years in my mind because I start to realize better ways to think about it. And often times through teaching, I'll get posed questions that make me think "Oh I haven't really thought about why that doesn't work." And I have to

> end up thinking about it and then "Oh yeah." Things fall into place over time, and I think it makes me a better teacher as well, that I have a more robust understanding of all that. But also, when I do see math ed research or papers explaining how students approach problems or what kind of supports work well or don't work well, that also – I think both of them have legitimate ways that they over time can be very helpful in helping someone to teach better and reach students better.

Jared's thoughtful explanation displays a professor who is considering all the resources around him that can help him to improve his teaching.

Prevailing Themes in Jared's Journey

Jared's journey features obvious high points (enthusiastic and charismatic teachers in middle and high school) and obvious low points (flunking out of university) that he had to navigate across his academic journey. After compiling his narrative, I analyzed Jared's interview with the two major categories of themes discussed earlier: influential change agents and aspects from his quality teaching definition reflected in his narratives. Both categories are discussed below.

Influential Change Agents

Responding to Challenges. Jared identified two life events as influencing his view of teaching. He had to overcome the challenges of flunking out of school and learning how to teach as a TA. Failing a course twice leaves an impact on any person, but Jared was able to regroup and bounce back from this setback. He took courses at a community college, which allowed him the time to learn from his mistakes and figure out what concepts he was missing. After successfully completing calculus, Jared enrolled in

"the heaven of the upper-level math courses, with smaller and like a better community of math majors that I knew suddenly by name." This momentum carried him into graduate school, where he began teaching for the first time as a teaching assistant for the same course he flunked. His position as a teaching assistant provided the chance to realize "If I can reach my students as their TA, I can help people like me who were not studying. I can show them that this stuff is kind of fun and interesting." This was both an opportunity and a challenge. During this time, his fellow graduate students created another challenging context from which he reflected on the social norms of graduate mathematics and teaching. He noted that other TAs would "snarky laugh about how bad the students were" which made Jared uncomfortable due to his personal experience in flunking out. Those challenging experiences helped him to see things from the student perspective and reflect on what the student was thinking; they also seemed to be a catalyst for holding himself to a higher standard for his teaching interactions.

Resourceful. Another type of experience that influenced Jared's personal journey and his teaching conceptions was participating in professional development when he was teaching algebra. His ability to interact with students was purposefully structured to be "as Socratic as possible" to keep the students thinking. This allowed him to observe how students can struggle with a problem and come away with useful knowledge even if the solution is not technically 'correct.' Jared did not simply attend the problem-solving days for his algebra class; he observed how the students were thinking and in what directions they explored to give himself insight into their learning. He later built on this knowledge when selecting tasks for his classes emphasizing the importance of carefully selecting tasks with "entry points at different levels."

When contemplating the impact of mathematics or mathematics education research, Jared notes that "both of them have an impact on the way I teach." As he learned more and taught more mathematics, Jared said, "Things fall into place over time and I think it makes me a better teacher as well, that I have a more robust understanding of all that." He also points out the benefit of math education research as well because it explains "how students approach problems or what kind of supports work well or don't work well." Both doing mathematics research and reading about (and participating in) mathematics education research were resources Jared took advantage of to deepen his knowledge of teaching mathematics.

Mentor or Role Model. Over Jared's journey, he referenced times when he either had excellent examples or lacked guidance to help him overcome the challenges of school. His early memories of enthusiastic middle school and high school teachers gave him a working image of a math teacher as entertaining. While this vision was lost in his freshman classes, Jared still considers his own enthusiasm for teaching mathematics and how that may impact his students. Before returning to university classes the second time, Jared noticed his lack of knowledge in how to study for classes. He deliberately sought out a more studious roommate on which he modeled his study habits. In graduate school, Jared noted that few of his graduate professors focused on crafting engaging lectures despite having a professional development devoted to engaging undergraduate students. Despite this lack of teaching role models in graduate school, his early charismatic teachers, and the professional development he participated in provided models for teaching he wished to emulate.

Reflecting on Past Experiences. Throughout his teaching journey, Jared consistently reflected on past experiences, sharing his perspectives and how he thought at that time. While he was unaware of the definition of a fixed mindset as an undergraduate, he could look back on his past self and clearly define his mindset before flunking out of university as "I couldn't have been more fixed mindset if I had tried." Now that he knows what fixed and growth mindsets are, Jared realizes this current information is influential "now in the way I reflect and talk back about things." When reflecting on past experiences, he is able to apply new knowledge and ideas and reframe those experiences from a different perspective. When reflecting on the impact of math and math education research on his teaching style, Jared notes "Often times through teaching, I'll get posed questions that make me think 'Oh I haven't really thought about why that doesn't work.' And I have to end up thinking about it."

When recounting his experience as a new teaching assistant (TA), Jared commented on the peculiarity of the situation saying, "I literally failed the course that I'm now teaching." This realization was a powerful a tool he used to relate to his students. His reflection on a challenging personal experience of failing a course and flunking out of college supported him to consider how his students might receive and engage with material he presented. Even grading tests as a TA provided opportunities to reflect. He remembers thinking "They're not just making up random stuff- they're trying to solve this problem but they're doing this weird thing." His interest in this unique puzzle of student learning led him to position himself differently from his fellow graduate students.

Co-Occurring Features of Quality Teaching in Jared's Narratives

Handling Mistakes. One of the first Co-occurring links between Jared's quality teaching definition and his narratives comes with his explanation of his homework policy and how he handles mistakes. His goal is to allow the student to make mistakes and then grow from them, which is paralleled in his story of flunking and then returning to university. Before flunking out, Jared recalls, "I didn't trust myself to go to a professor in office hours and ask them for help. I didn't have friends that were good students. I didn't know how to study." He made several mistakes, some of which were not academic in nature, but persevered and learned from those mistakes. His homework policy of allowing students to submit work after the deadline and to correct homework grants students that same grace that he might have benefitted from in his first undergraduate experience. In the classroom, Jared handles mistakes as an opportunity for learning and growth, while still maintaining the student's status within the class.

Selecting Appropriate Tasks. Another feature of Jared's quality teaching definition that is reflected in his stories is selecting appropriate tasks for his students. Observing the curriculum used for the algebra problem-solving days as part of the professional development he participated in showed Jared the value of selecting appropriate problems and questioning to promote student learning. Jared learned to appreciate the idea of, "Don't insult your students by giving them necessarily problems that have a clean answer." These problem-solving days allowed students to grapple with complex mathematical concepts from class and try to apply those concepts to non-standard questions. Similarly, Jared outlined his expectations for students when they tackle initial homework tasks. Jared noted the tricky part of assigning tasks "is hitting the

right level of something that they can now do that's going to be meaningful." He continued, "From the point of view of a person who's taught these classes for a lot, I do actually have a good sense in my mind of things that students cognitively maybe need to sort out when they're doing these problems." The importance of selecting appropriately challenging tasks, which is a cornerstone of his teaching, is seen in formative stories and experience about his time as a teaching assistant.

Teacher Affect. Another Co-occurring theme in Jared's quality teaching definition is his focus on teacher affect, or the attitudes and behaviors the teacher purposefully displays to their students. When describing a successful lesson, Jared notes, "You want to both be presenting the challenge but also you want to be the resource there, right. I feel like that's actually the most important role I feel – I'm there to actually help them learn the material." He takes on the role of a helpful resource to his students which highlights the importance, to Jared, of establishing a relationship of trust with his students. In his stories recalling his middle and high school teachers he shares, "The ones that I remember most and sort of formed my concept image of what a math teacher was back in those days were – they were like playful entertainers." This set his expectations for a more engaging professor, which were unmet at university in a large lecture math class with a professor who was "not someone who knew my name and would engage me and explain how they were thinking about things." The contrast of his experiences with an engaging, enthusiastic teacher and an impersonal, distant, and unengaging professor supported him to consider his own affect and how "I need to help the students get into it by sharing my own enthusiasm." He also dealt with a contrasting affect with his fellow graduate students and their 'behind the door' treatment of student work. He recalls "I feel

like the bar was kind of high in my own mind, that I really do need to connect with students, maybe more than some of my other fellow grad students." His need to connect with his students comes across with his enthusiasm for teaching and the trusting relationship he strives to establish with his students.

Student Disposition Toward Math. Another theme related to quality teaching that occurs in his narratives is his interest in both his own disposition toward math as a student and his own students' affect. When thinking about student activity in a successful lesson, Jared feels the need for students to "mess things up and hopefully having them not feel crushed. They wouldn't be learning a lot if the first thing they tried always solved the problem." He attends to a tasks' ability to have a student feel crushed and contrasts that crushed feeling with supporting students to persist in their work, even after making a mistake. When recalling his return to university and his unproductive study skills, Jared deliberately sought out a studious roommate to learn from. He noted his own attitude impacted his ability to succeed, describing himself as "having a fixed mindset." Similarly, as a teacher, he now attends to his own students' attitudes to help them persevere in the face of challenges. He clearly notes how his fixed mindset from high school contributed to his flunking out of university. After reading a book on growth mindset, he pointed out how the book "so perfectly captures the fixed mindset, so perfectly captures exactly how I felt about myself, when I came out of high school and started those courses." This internal reflection on his own disposition, beliefs, and affect as a student supported him to gain insight into how and why students may struggle and can be seen in the value he places on student affect, motivation, and engagement in his definition of quality teaching.

Summary

Altogether, Jared is a professor who empathizes with his students and has the lived experiences to support that empathy. He focuses on bringing a sense of enthusiasm to his students while keeping his explanations straightforward and simple. His ability to reflect back on his experiences as a student, both good and bad, allows him to gain further insights into student learning. Jared considers how his decisions as a teacher such as task choice, framing mistakes, his enthusiasm and approachability, and so on will impact student affect and student learning. He carefully attends to these features to create the best learning opportunities for his students.

Case 4: Stephen's Profile

Narrative Summary

After being raised in a family that instilled a lifelong love of learning, Stephen studied mathematics and graduated with his PhD in topology in the mid-1970s. Eventually his research interests shifted towards education and learning. He notes that "if you can get a person to just have as a practice of life that they are curious and enjoy thinking and want to learn more," then those skills will serve that person far more than "any amount of specific knowledge you can teach in a semester." The impact of these skills allows Stephen to teach "the whole person about their whole intellectual life." As he progressed through his career, his university awarded him a Regents' award, which is the highest honor granted for teaching. He also received a national award in teaching from the Mathematical Association of America.

Stephen's Quality Teaching Conception

Stephen defines quality teaching with a focus on students. He says, "One of my teaching mantras is 'It's what the student does that counts.'" His view of quality teaching is any teaching that "causes students to think for themselves and to develop their skills and abilities to think for themselves and also their attitudes toward clear thinking." This definition applies to more than simply mathematical thinking; it encompasses his students' critical thinking skills.

Stephen's goal when teaching is also student focused: "My goal is to have students develop an attitude toward thinking for themselves, for being able to apply good reasoning to everything they do." He is interested in how his students take on and use strategies for critical thinking. He states his aim as "getting people to become

independent thinkers and reasoners well beyond the constraints of the course." To accomplish this, Stephen tries "to model the idea of being willing to hear – to make mistakes and to respond to the mistakes in a careful way." More than this, he stresses, "I want people to be comfortable learning from mistakes, making mistakes, and learning from them and not having that be a crushing blow to a person, realizing that's part of the trajectory of learning." He also strives to model "the enjoyment of thinking and of struggle. And not thinking that struggle is a negative, but in fact that it is a positive, even if you can't solve something." His focus goes beyond improving how students *think* about mistakes towards how students *feel* about mistakes.

When defining a successful course, Stephen differentiates based on the level of the course. For upper division math courses, he teaches with an inquiry-based learning method. The students do the bulk of the work and also present their work to the whole class. Stephen describes this set up as "I always give challenge questions that are sufficiently difficult that every single student is strongly challenged and yet some that are easy enough that every student is having success at the same time." He also notes a development of a community within the class as successful, saying "so that people in the course feel that they are joyfully exploring the challenge and having it be a fun kind of a thing." Another course Stephen teaches is an introductory level course for liberal arts majors. Focusing on a successful course by having students learn how to question, he describes this with

> State exactly what the question is, being able to then say that, adapt and adopt practices looking at simpler versions of the question, trying to formulate simpler versions. And then can they take questions in the world and formulate them

clearly.

He does this by presenting the material as puzzles, and one of his first questions to them is "now put it away and just tell me what the puzzle is." Stephen has also taught math courses for students whose mathematical preparation is weak. These courses are successful if he can shift the students' view of mathematics and themselves. He states his goal is "to convince students: (1) that they can personally learn and understand and do mathematics well but (2) that it requires a significant effort to do that and what kind of effort is valued." Since these are students who have not experienced success in mathematics, both of these are key pieces to the success of the course. Overall, Stephen notes that he wants "that every student feels that they're changing their relationship to mathematics to become producers of mathematics rather than merely consumers of somebody else's knowledge of mathematics."

When shifting his focus to a successful lesson, Stephen still focuses on the student.

> A successful lesson is, one, posing a challenge or challenges, having the students engage with the challenge at a meaningful level. Meaning it's a challenge that is posed at a level where it causes them to have to employ the strategies of thinking and knowledge that they have before. And then extend it further, coming up with a new idea, a new way of looking at things, a new wrinkle on that. And then having by virtue of that struggle and having them having insight into the issue at hand.

A successful lesson will tap into the previous knowledge and skills students have but also focus on the students persevering through a challenge or struggle to arrive at some new

insight or idea at the end.

Questions are not simply an occurrence in Stephen's classes but a skill he fosters in his classes. He will ask his students to generate a question and then talk to their neighbor and write down two questions from their discussion. Then Stephen will pick a group to share their questions. Students are expected to explain their question and make progress on answering their questions. But not all questions or prompts are student-generated. Students will end up having to think through the problem with the goal being "to have them see [the work] as a logical consequence of what they know." On the rare occurrence when students ask Stephen questions in office hours and he asks them to summarize the question, he notes, "If they [students] have to look in a book or something, then I'll tell them, 'Well look you haven't begun thinking about the question if you can't even state the question without a prompt.'" Overall, his goal with supporting students to ask questions and the questions he himself poses is to show students "they have the agency to actually resolve these questions."

Stephen treats mistakes as a valuable part of the learning process. His focus with mistakes is understanding the error and learning to see mistakes as indicators of learning.

> The value is not in the making of the mistakes, but the value is in looking at the mistakes and saying what was wrong. Because then the mistake was directing you to turn your mind toward what would have been different. So, in my classes we celebrate mistakes as incredibly valuable part of the experience of either understanding something better or coming up with a new idea. So, I work very hard on creating an atmosphere in which people do not feel bad for making a mistake.

For Stephen mistakes are not simply a learning tool but something to be celebrated. Stephen also promotes the idea of understanding as more than a yes or no, but rather understanding is a continuum. “As you move along the continuum, you see relationships, all of which are helpful in giving a more robust understanding so that every piece is part of a bigger whole.” This robust understanding and seeing how each piece fits into that is what he finds helps students avoid certain kinds of mistakes. Overall, his focus is for students so “they don’t worry so much if you’ve created an atmosphere in which mistakes are just a norm.”

Stephen’s Journey

One of his early influential memories related to teaching centers around the dinner table with his family. Both of Stephen’s parents taught. His father was a community college math and science professor, and his mother taught remedial reading at a junior high school. He shares,

> At the dinner table, very often my dad would bring things like a math puzzle to my brother and me. We would talk about it, and we would try to figure things out. Or he would have read an article in Scientific American, and he would say “I read about this dolphin, and they were trying to teach it to do things. Here was the experiment. What do you think about that?” And then we would proceed to discuss that. That was the norm for the discussions at dinner time.

Stephen credits these dinner time discussions with instilling a sense of curiosity and lifelong learning. Instead of focusing on getting more knowledge into their heads, Stephen states “If you can promote these attitudes of wonder and joy and learning, those things over a lifetime can dominate and can really make a huge difference.” Instilling this

sense of wonder is what quality teaching is about for Stephen.

Another influential event for Stephen happened in graduate school in the early 1970s. He took a topology class that was taught in a style he had never experienced before.

> [The professor] just handed out these notes that consisted of theorem statements, definitions, and theorem statements without proofs. And then he said, "Okay your job is to prove them." And his method of doing this – this technique was that he said, "Just prove them." And then he would come into class, and he would say, "Okay does anybody have number 12?" And then somebody would volunteer, and then they'd go up to the board, and they would present their own personal attempt at proving number 12.

Then, one day, there was a problem no one could do. So, the professor just told them to keep working and moved on to the next one. After a few days, Stephen's classmate volunteered and did this theorem that no one could do. Stephen pondered this, "And I said to myself, 'Huh I could have done that!' And then it occurred to me, 'Well maybe I should do that.' And so, then I started to actually work on these theorems. And it was just great fun." By the end of the course, Stephen could reproduce every theorem and proof "because I had personally worked on every single one of them." He reflected on this experience and realized two things.

> One is the idea that I could produce mathematics on my own. Second that the level of understanding that I had attained in that class was so vastly different in both quantity and quality. It wasn't that I understood 70% of what the teacher said. It was that I understood 120% of what the teacher said. Because I not only

> understood everything that was done, I could see all the things that went wrong as I was thinking about things and other variations that weren't talked about as well. So, it was a whole different relationship to mathematics that I had experienced during that course. So that really had a very strong influence on my teaching and the specifics of the teaching. Namely really focusing on giving challenges as opposed to giving clear explanations.

His experience with this style of teaching (what was then called the Moore method and is called Inquiry Based Learning) led him to teach topology in this inquiry style. He refined his teaching style over the years, but still focuses on "posing challenges as opposed to just explaining things."

Stephen only taught one semester in graduate school--a beginning calculus course. He recalls, "I actually understood what calculus was about when I was teaching the course." That experience of learning through teaching led him to create a type of assignment that he still uses in his current classes. "We have assignments called calc-xplanation, which tells people to explain the ideas that they're trying to learn. So, we actually have them produce videos and write essays to explain things as they're learning, as a method to learning." Because he had done such an excellent job teaching, his professor wrote an unsolicited letter of recommendation where this professor said, "Yeah the students learn more from him then from me." Even in his early teaching, Stephen applied the techniques he learned as a student in his undergraduate topology class that used the Moore method of instruction.

After publishing a number of papers in topology, Stephen was promoted to full professor. Yet, he reflected on the impact of his research:

> I realized I knew every single human being on the planet who could understand what I was writing. Mathematics is very esoteric; it's broken up into many different fields and I was doing abstract topology research. And the number of people who could understand, even theoretically could understand that kind of work was a tiny handful of people. And if I wrote a paper, the number of people who would actually read it was very small. And I'm talking about a handful, literally certainly less than twenty, maybe ten or less.

Although he lauds researchers and mathematics research as being important and should "be supported because of societal value overall," his interests have "shifted toward education things" He continued explaining, "I've produced books and DVD courses and stuff that now close to a million people have actually seen and read and engaged with." Stephen notes "the order of magnitude of interaction is just entirely different."

One of the harder influences for Stephen to quantify was the influence of his field's norms and expectations. He recalls that at "universities like this one [the R1 institution at which he is employed] is that people are more or less happy to have people do what they value. The teaching is not valued as research. But on the other hand, they've been tolerant of my eccentricities." He also notes that many of his colleagues "had the idea that the profession includes all sides, and that different people can emphasize different things." So, some of his colleagues emphasized teaching more, and others emphasized research more.

Stephen shared a couple of stories about interacting with children around mathematical ideas. He recounted a recent interaction with a grandnephew, who was having trouble in geometry. Stephen commented on his grandnephew's attitude.

> What was interesting was that, it's typical I guess, [he] just didn't have the idea that he could think through anything on his own. And so, to me, the strategy was I'd just give him a thing. If I give you a straight edge and a compass, can you bisect an angle? And he said no I don't know how. He hasn't been taught how to do it. And so, I'd say well okay, try it. Try stuff and see how it goes. And sure enough, he was a perfectly bright kid, and he could figure things out. And then I'm hoping that part of what he learned is to have a different view of his ability to actually figure things out on his own.

In this story we can see that Stephen's focus was not just on his grandnephew learning geometry, but more about the importance of learning how to think and analyze critically.

Prevailing Themes in Stephen's Journey

Stephen's quality teaching definition focused on the student experience among other themes which show in his journey. Stephen was the only participant who did not have a follow up interview. After compiling his narrative, I analyzed Stephen's interview with the two major categories of themes discussed earlier: influential change agents and aspects from his quality teaching definition reflected in his narratives. Both categories are discussed below.

Influential Change Opportunities

Mentor or Role Model. Stephen credits his parents as a significant influence on his learning style and attitude towards learning. He recalls "they had a lifelong curiosity, a lifelong learning that they themselves absolutely embraced and demonstrated on a daily basis." Their influence appears in Stephen's attention to more than the knowledge students acquire, but rather seeing them take on "attitudes of wonder and joy and

learning." Another role model emerged in his inquiry style topology class when his classmate answered the question that no one else had managed. This event caused Stephen to reflect and realize he could do exactly what his classmate did and began working in earnest on his mathematical work, which brought him joy and satisfaction. Both his parents and his classmate served as role models that influenced his understanding of teaching and learning.

Responding to Challenges. Stephen recalled a turning point in his mathematical journey during his topology class, "'Huh I could have done that!' And then it occurred to me, 'Well maybe I should do that.'" Stephen was challenged in that topology class in a way he never experienced before. The end result of that experience is summarized with, "It was a whole different relationship to mathematics that I experienced during that course. So that really had a very strong influence on my teaching … Namely really focusing on giving challenges as opposed to giving clear explanations." His response to this challenge gave him a different and influential experience about learning mathematics, which, in turn, influenced his teaching.

Reflecting on Past Experiences. Stephen's ability to consider his own learning experiences appeared across his teaching journey. His experience with his classmate solving the topology question showed an ability to reflect on and change his actions. Additionally, when recounting his experiences in topology, "this idea of seeing the struggle required to clarify even rather fundamental ideas, was something that I learned in that class." He also emphasizes "Those ideas were not just an add on but were actually part of me and my knowledge." He reflected on how he learned via an inquiry-based style of teaching and incorporated those lessons into his own teaching. He also shared how his

teaching evolved over time explaining, “When I was a younger teacher, I thought about teaching math. And over time it has become more general. So that I’m teaching the whole person about their whole intellectual life.” His focus now, compared to his focus earlier in his career, has changed. His conception of teaching and his goals have expanded to develop the whole student and not just ensure the student has learned the requisite knowledge. And finally, later in his career, Stephen reflected on the impact his research was having on society. This reflection combined with his personal values resulted in a shift in his professional focus toward education so he could have a bigger impact.

Making a Broader Impact. As Stephen produced topology research papers, he realized, “if I wrote a paper, the number of people who would actually read it was very small.” This led to a shift toward education research and presentations. Stephen has “produced books and DVD courses that now close to a million people have actually seen and read and engaged with.” He also “personally presented on many workshops on teaching. Particularly inquiry kind of teaching, but also teaching not for mathematics necessarily, general teaching strategies overall.” Stephen summarized his reasoning for his research shift, “So that the order of magnitude of interaction is just entirely different. So, I enjoy thinking about these [topology] puzzles, but my interest shifted more toward the human side.” Stephen chose to work towards impacting more people than the handful who could actually understand his topology research.

Co-Occurring Features of Quality Teaching in Stephen’s Narratives

Supporting Students to Answer Their Own Questions. In his quality teaching definition, Stephen emphasizes the ability to ask questions as its own skill. Likewise, he emphasized the importance of students learning how to answer those questions. We see

this feature of teaching in Stephen's first significant story which features his parents fostering a sense of curiosity paired with an assurance that he could answer any question. The importance of supporting students to ask and answer their own questions also appears in his story where he asked his students to submit 'calc-xplanation' solutions explaining how they solved their homework questions, which reinforced their belief in their own ability to answer these questions. In his own classes, he will often turn a question back to the class, "To illustrate that they have the agency to actually resolve these questions." Stephen also noted that he hoped he fostered that same ability in his grandnephew.

Selecting Appropriate Tasks. Another feature of Stephen's teaching definition is selecting the right tasks for the ability levels of all students in a class. An early example in his life of appropriate task selection was recounted with "my dad would bring things like a math puzzle to my brother and me and we would talk about it." His dad had to select puzzles and topics that would interest two young boys but still provide room for learning. As a professor, Stephen describes his choice of questions in his topology classes as, "I always give challenge questions that are sufficiently difficult that every single student is strongly challenged and yet some that are easy enough that every student is having success at the same time." This same idea is reflected in his experience with inquiry learning as a student in a topology class. The course was structured as "notes that consisted of theorem statements, definitions, and theorem statements without proofs." To promote learning a new subject, the theorem statements must be sequenced to build off previous theorems, making task selection a significant decision in this style of teaching. Since his professor chose the appropriate tasks that challenged him, he experienced

mathematics at a deeper level of understanding. His work through the sequence of topology questions prompted this thought, "I could see all the things that went wrong as I was thinking about things and other variations that weren't talked about as well."

Student Disposition Toward Math. In his quality teaching definition, Stephen notes, "So to me, a successful lesson is one where all the students are engaged in it and that they are meaningfully developing an increase in their skill, in their persistence, in their attitudes." He also attends to student disposition in his discussion of mistakes as "I try to teach people to not feel this sense of defeat and cringing and embarrassment and all those things from mistakes." In his experience learning topology and inquiry style learning, Stephen "said to myself, 'Huh I could have done that!' And then it occurred to me, 'Well maybe I should do that.'" His ability to reflect on his experiences brought about a change in his sense of self-efficacy, or his affect. When tutoring his grandnephew in geometry, Stephen specifically notes how he hopes his grandnephew figured out that "part of what he learned is to have a different view of his ability to actually figure things out on his own." His journey highlights his attention to his students' affect and how his actions can craft a meaningful learning experience for them.

Summary

Stephen's quality teaching definition focused on what his students do, learn, feel, and gain from his class. His attention to the student as a whole person highlights an attention to details that surpass simply what a student knows mathematically. His ability to reflect has allowed him to refine his teaching skills and to impact far more students directly than his abstract topology research did. His early experiences with learning illustrated the significance of not simply what you learn but how you learn it. These

experiences have helped shape Stephen into the highly recognized professor he is today.

Case 5: Anthony's Case

Narrative Summary

Anthony's journey towards becoming a mathematics professor started with two parents who were teachers. With the added encouragement of other mentors, Anthony decided to pursue a degree to become a secondary mathematics teacher in the mid 2000's. He discovered a love for math that led him to earn a Bachelor's in Math simultaneously with a Bachelor's in secondary math education, then a Master's in applied math, and eventually a PhD in mathematics, with a focus in group theory. He was recently awarded a national award for beginning university faculty who demonstrate a teaching ability that carries beyond the classroom.

Anthony's Quality Teaching Conception

Anthony defines quality teaching in terms of productive struggle, saying teaching is "the productive struggle towards the development of skills and increasing knowledge." He also introduces the idea that "part of that productive struggle is making sure that everyone is included and that they have an opportunity to struggle in a way that feels comfortable to them." He expanded, "if you give someone a problem that's way too easy or way too hard, it just destroys the experience." He is noting the importance of the student experience and how choosing the correct task or set of tasks makes a difference in that experience.

Anthony uses the metaphor of a doctor in discussing his teaching goals. "My first goal is almost like a doctor: do no harm. Because there's so much math trauma and so many negative experiences people have." He recognizes that people may come into his classroom with emotional trauma associated with math and actively tries to ensure their

experience in his classroom does not add to those negative experiences. His second goal is, "To make sure there are no barriers. So, anyone who wants to be part of the class has some opportunity – there's something they can catch that allows them an on ramp." While attending to the students' past experiences and previous knowledge, Anthony tries to ensure his students have multiple access points which can provide more equitable opportunities for learning in his classroom. His third goal is for his students to practice the skills of being a mathematician at an appropriate level and in content-specific ways.

> So, for calculus one, [the content] that's different from an upper-level class, but the skills are the same things. They're still asking questions, formulating conjectures, seeking evidence in favor of or against those conjectures, thinking critically about statements, representing knowledge in different ways. So, I think if the class is done well, they should get practice with all of those things around whatever the content knowledge of the class is.

Anthony focuses on the students' experience doing mathematics by highlighting specific skills or mathematical practices they will develop in his classroom. His goals show a focus on broadened student access to mathematics and developing key skills or practices for doing mathematics.

For Anthony, a successful course is highlighted in both his actions and his students' experiences. He first defines a course as successful if "I've seen growth. And that's not an easy thing to quantify or objectively measure, but I think we know it when we see it." His second qualification for a successful course is "when students have deepened their relationship with the subject matter." Anthony also notes he considers whether "the students were fairly assessed in what the topics were, and then do I think

that the subject was fairly presented and in a way that made it alive and interesting." From the student perspective, he defines a successful course as one in which the students experience "productive struggle, with emphasis on both: productive and struggle. And if they can look back on some experiences that they've had that they can say 'I feel more like a mathematician than I did when this class started.'" Overall, Anthony defines a successful course by what the students experience and know leaving the course.

When defining a successful lesson, he focuses on day-to-day interactions. Anthony defines a successful lesson as "one in which students are engaged." He discusses two different kinds of talking present in a lesson: his talk and student talk. "My talking, that's presenting. There's my talking that is questioning or opening. And then there's their talking. So, I try to make it so my presenting talk is no more than about a third." He also recognizes that "skill transfer is important, so a successful lesson is one where they have an opportunity to practice what the skills are." Anthony works to engage students in the lesson by making the space for them to talk, giving them the opportunity to practice skills, and deliberately limiting his own talk and presentation of content.

Given the large amount of student talk in Anthony's classroom, student questions are a common occurrence. When a student stops and voices a question, Anthony focuses on why a student asks a question before he answers it. A simple clarification question about written text on the board is answered quickly, but "a question that could productively lead to another question" is handled with more finesse. He also recognizes that sometimes students ask questions because they are bored, but "I try to treat every question with respect and meet it where it is in a way that helps the student advance." He also notes that when his strongest students come to his office hours, "it ends up being like

a precursor to research, where they're asking questions that are more advanced than what we really need to answer in the class." Anthony is willing to pursue knowledge the students find interesting, but also recognizes that asking questions is another skill students need to practice and develop. Early in a semester, he will "create situations where students have no choice but to ask questions. So, email me a question; it's an assignment, you have to email me a question. And then also, you can ask what's a question a [fellow] student might have." The second situation allows students to ask a question that a peer might have, but "they're not forced to own their question. If they feel they have a good understanding, they can try to practice empathy and thinking about it based on what they see their classmates doing." He also occasionally tasks students with writing potential exam and quiz questions because "that's a valuable experience for the students, to think about what different types of questions look like." Not only does Anthony take student questions seriously and respectfully, but he also tries to foster their ability to formulate questions.

When asked about mistakes, Anthony discusses the feeling of uncertainty and whether something is a mistake. He first explains "as a working mathematician, I spend very little time being certain of things. So, students can't always handle the level of uncertainty that we [mathematicians] are comfortable with as people who are more experienced." He also notes "it's important to recognize that we don't always know when we've made a mistake, and that mistakes are inevitable." Early in a class, Anthony tries to reframe success, failure, and mistakes for his students.

> One thing I like to ask students is "What do you think is the top percentage of success you could expect on these skills?" So, like for hitting a baseball at the

> major league level is like 30%. The best hitters in the world struggle to hit 33%. For a concert pianist, it's like 95%, 99%, somewhere in there. But just thinking about that question, that perfection is impossible, so what do we think it looks like? What are reasonable expectations in terms of how often we can do this and get the exact right answer?

Through these conversations he tries to provide students with a different perspective on making mistakes and on focusing on what is reasonable rather than treating mistakes as a failure.

Anthony also shared two major research ideas that have shaped his teaching: Armstrong's 2010 revision of Bloom's taxonomy (Bloom, 1956) and the Rule of Four (1992). Anthony explained "Bloom's taxonomy is a framework for understanding different levels of questions." Questions are categorized on the basis of the underlying cognitive processes required. These categories apply across content and disciplines moving from the more basic questions involving knowledge retrieval to knowledge application to analyzing and synthesizing knowledge. The rule of four, however, is specific to mathematics because it refers to "four different ways of representing anything – symbolic, numerical, verbal, and visual." The Rule of Three is credited to the Harvard Calculus Consortium in their first newsletter issue in Spring of 1992 as the "belief that these three aspects of calculus – graphical, numerical, and analytical – should all be emphasized." Researchers later included 'verbal' in the list, creating the Rule of Four that Anthony referenced.

In circling back to the beginning of his interview, Anthony expanded on why he considers doing no harm to be a goal for his teaching. He first discussed that his

interactions with students have showed him "that their teachers were like prison guards, with regard to certain topics and certain ways of doing things." These experiences can be traumatic for students in a way not seen in other fields of learning. By keeping this idea of math trauma in mind, "I try to be very aware of that. And I try to be very aware of not belittling students in any way, not putting them in high pressure situations, where they haven't had a chance to prepare." He sums up this stance with "I want to create an environment where anyone who is there, gets something out of it. But if they don't, I at least want them to leave no worse off." Anthony attends to the student experience beyond the mathematical knowledge and skills they are learning and shows empathy for their experiences with starting a college life.

Anthony's Journey

Anthony's first memorable teachers are his parents. Both were teachers, though not in a typical K-12 context. His parents were supportive and encouraging. Thinking back on their influence on his teaching, he recalls

> My mom was a much better teacher than I am. She was a speech pathologist, and her breadth of skill and approaches were incredible. My dad was just much the same. He was more scholarly than she was. He was a pastor, and he spent a lot of time studying. But they were both excellent teachers.

His dad taught him how to drive, which required patience to help Anthony get past a traumatic car accident. Anthony explains his father's great attributes with

> He broke everything down into very simple component steps and did a great job of pacing and staggering. He was always very very patient with people and very very understanding of their limitations. He was also very scholarly in his work.

> He was a pastor, and he liked to get out the original Greek and really dig deeply into it. So, I think those two things together – the fact that he really liked to learn for himself, and he was also very understanding of what the people that he was working with might need – were his great attributes.

Anthony, too, demonstrates patience in how he treats student questions with respect and views mistakes as inevitable indicators of learning instead of things to avoid. His mother was "incredibly creative" in using multiple techniques to solve client problems as a speech pathologist. Anthony notes "she could pinpoint the issue and then come up with the precise solution. I think often she came up with a different solution for each client." Her ability to creatively solve individualized problems shows in Anthony's attention to the different ways students learn and a desire to differentiate his instruction to support all students. Growing up with two people working as educators gave Anthony an inside view of being a teacher and the commitment and dedication required in that calling.

Another striking teacher and role model in his past is Anthony's high school wrestling coach. When describing his coach and his teaching, Anthony remembers that "My high school wrestling coach really believed in me when I was terrible. And didn't let that influence the way he treated me or how much time he was willing to spend with me." This encouragement and positive influence led Anthony to "eventually become a decent wrestler, and that was unexpected to everyone. But he never quit on me, he never gave me any negative. He was always encouraging." While reflecting on experiences that influenced his teaching, Anthony notes

> So, I wrested in high school, and I was a wrestling coach for a while. And that was a big influence on my teaching and my math experience because there, there

> is no expectation that you can follow the procedure and get the right answer. You learn the techniques and you practice them. But you know that there's another level where there will be a situation where you can do it perfectly, and it still won't get the result you're looking for. And that also helps me see both the value and the limitation of drill. What drill was good for.

Anthony noted the purpose of drilling or practicing a technique, but also noted that drilling does not guarantee success. Anthony spent time as an assistant wrestling coach at a local high school before and during some of his undergraduate college years. He carried this knowledge and experience into his teaching.

Another memorable experience about the benefit of asking questions came during Anthony's time as a journalist. Before attending college, Anthony worked as a journalist which "gave me a real perspective on the power of questions, asking questions, and that you can't just ask a question and expect – and also about searching for information. So, I brought those as sort of non-standard perspectives on math." Anthony highlighted the importance of questions in his quality teaching definition when he discussed the difficulty of crafting and asking a question. He approaches questions as a skill to be learned, practiced, and tries to foster this skill in his students.

After working as a journalist, Anthony went to school starting at a community college. He recalls one of his math professors as encouraging and that this professor found a perfect question to pique his interest in math.

> The first time I remember being interested in math was when my precalculus instructor at the community college gave me a problem that I did not know how to solve, but I found really interesting. And it had to do with having a counterfeit

> coin and a balance, and then determining how many times you have to weigh the coins to determine which one is counterfeit. And I don't know why that particular – I think that problem was approachable, it wasn't obvious, and it had no baggage in terms of notation or symbolism or anything. So, I could just approach it like any other thing I thought about. And that was really the moment that I decided maybe I was wrong about certain aspects of math. And then he told me, "Well if you've taken pre-calculus, you might as well take calculus." So, and I viewed that at the time like a challenge.

Anthony rose to this challenge posed by this mentoring professor and went on to eventually earn his doctorate in mathematics. After years of less engaging mathematics experiences, his pre-calculus professor found the right question to engage Anthony in doing mathematics.

Anthony had several formal opportunities to reflect on teaching. While Anthony earned a bachelor's in secondary mathematics education, he did a semester of student teaching as well as observations. One interactive observation placed him in a combined first and second grade special education class. He remembers contrasting his attempt at explaining the concepts of 'more than' and 'less than' in this classroom with the teacher's lesson.

> I remember very clearly; the teacher did an excellent job of involving me and letting me try things and trying to teach the students about more or less. And how much some of them struggled with it. You know like putting six balls on one side and putting eight balls on the other. They couldn't really reliably tell me which was bigger, so seeing that and then when the teacher took over, as we had

planned. When she did it, she started with 1 and 9 to make it really extreme. Anthony reflected on the differences and relative success of each of their lessons and highlighted the teacher's knowledge in her selection of number choice to support student learning. Later, he was part of the Mathematics Association of America's project New Experiences in Teaching (NExT), which is a professional development program for new or recent doctoral graduates that addresses all aspects of an academic career including improving the teaching and learning of mathematics. He also attended conferences about math circles and State Council of Teachers of Mathematics conferences. Anthony later was awarded a "Council Research in Undergraduate Mathematics grant that included a three-day conference on how to mentor undergraduate research. And then I also just get to listen to the people in my department teaching. And they're all wonderful." Not only were these experiences an early exposure to teaching, but they also provided a formal opportunity to refine his thinking about teaching.

His experiences with Math Circles had a large impact on his teaching. He reflected that, "I tend to make a lot of my classes be like math circles." He describes math circles as:

> A math circle is a meeting where people come together to work on math. And it's different than a class. There's no expectation that you will leave with any particular objective. It's just like here are some problems, here are some things you can do to play with them, and let's see what happens from there.

He worked with elementary and middle school students as well as in-service teachers in his time as a math circle facilitator. Similar to math circles, Anthony also participated in a math puzzle program. He described the program as "there are right or wrong answers in

the math puzzle program, but it's so open ended" that students can solve the puzzle multiple ways. In these experiences, students can experience a success or failure different from the typical mathematics classroom. Both of these experiences show the value of posing a good question that allows students to explore math and where they "see the value in collaborative open-ended problems that require creativity." Anthony added that students are not always successful. In his words, "Sometimes it's a flaming disaster." Anthony's experiences provided models for alternative ways a math classroom can look and how students can interact with and think about math.

During graduate school, Anthony worked as a personal tutor outside of his time as a teaching or research assistant. He explains what he learned from tutoring individuals.

> When I did a lot of tutoring, I would go from client to client. And be doing the same class. I'd give an explanation to one client; it would work perfectly. Next client, completely missed. It's like I was having to come up with all these different ad hoc things, and I think that was really crucial. I got to see, one on one, in a way I wouldn't have seen in a classroom, just how different people's brains can be.

Anthony's creativity in attending to his students' individual needs was reminiscent of his mother's creativity in finding the right solution for each of her students. He also took the experience of working individually with students to ponder the differences in how people think about math. Anthony concluded that there could be many ways to solve a problem, and those solution methods will not work for everyone in the same way.

When asked about the influence of the norms of his field or his professors in graduate school, Anthony recalls his advisor "did not denigrate" teaching. While

contemplating other professors' views on teaching, he notes

> There are people who thought teaching was important, who just weren't very good at it. Because when they encountered a failure, they did not reflect on what they could do differently; they only blamed the students' lack of effort or lack of preparation.

This story shows Anthony's ability to consider others' perspectives when reflecting on his professors' teaching decisions and motivations. He also notes that a professor's lack in teaching ability does not necessarily equate to valuing teaching less than other pursuits.

Anthony even found insight into teaching when reflecting on events as a new dad. He recounts one of his first experiences with his newborn child.

> And finally, after I started teaching, I had children for the first time. And that helped me to understand what it felt like to be overwhelmed by something. Even if intellectually, you understood it, how it could still be really, really difficult to enact. And I had really gotten into inquiry-based learning, and then, when I went to change the diaper at the hospital, this wonderful nurse patiently walked me through it. And I thought what a disaster it would be to teach changing diapers through inquiry-based learning. Like you just need it step by step by step, patiently explained. So that sort of counterbalanced and helped me see the value in some of the traditional approaches that are about keeping students comfortable and not necessarily challenging them as much."

He took feeling overwhelmed as a new parent and considered that his students likely feel overwhelmed as well, possibly due to math and the open-ended nature of tasks in inquiry-based instruction. He notes that traditional approaches to teaching, especially for

introducing new concepts, are familiar to students and can reduce the possibility of a negative experience in his classroom. Being a father has also influenced his perspectives on other aspects of teaching. “I’ve had times, when my kids were sick, and I couldn’t come to class. And nobody demanded that I show proof and I think they deserve the same respect.” He now tries to extend the same respect to his students. His attention to student needs, worries, and perspectives shows a highly reflective professor who extends his students respect and courtesy.

Prevailing Themes in Anthony’s Journey

Anthony’s quality teaching definition focused on the student experience and the stories he shared to describe his teaching journey which highlighted themes of empathy for the student experience. After compiling his narrative, I analyzed Anthony’s interview with the two major categories of themes discussed earlier: influential change agents and aspects from his quality teaching definition reflected in his narratives. Both categories are discussed below.

Influential Change Opportunities

Mentor or Role Model. Anthony grew up with both parents having teaching as a significant part of their job. This experience gave Anthony a view of teaching from behind the scenes and a different perspective. He noted his dad’s patience with explaining concepts and his mom’s creative ability to solve problems. Another role model in his life is Anthony’s wrestling coach from high school. His coach believed in him and encouraged him even though he “was terrible at wrestling.” Anthony notes that he was not treated differently for being terrible or that the coach did not mind spending time helping him. Anthony’s coach provided a role model for teaching by exemplifying

someone who responded to his student's needs.

Reflecting on Past Experience. Throughout his interview, Anthony demonstrated the ability to reflect on his past and to find lessons about teaching from unexpected sources. When describing wrestling, he noted that athletes practice drills to improve skills, but they do not expect those skills to lead them to a win every time. He notes "that also helps me see both the value and the limitation of drill" in terms of his mathematics teaching. When he taught and observed in an elementary classroom, he critically contrasted his lesson and the elementary teacher's lesson and this reflection led him to notice a key difference in the task itself (i.e., number choice). Additionally, his time as a tutor provided an opportunity to notice and reflect on the differences in students and student learning, which he continues to attend to as a professor.

Resourceful. Anthony sought out professional development beginning in his years as an undergraduate student. He ended up "going to different schools to do observations, seeing different high school teachers, going to the local conferences and local teacher professional developments." He was part of the MAA's project NExT as well as attending State Council of Teachers of Mathematics conferences. He took part in Math Circles and became a facilitator and attended conferences for Math Circles. These opportunities provided the material and support for thoughtful reflection on teaching, that Anthony took advantage of.

Responding to Challenges. When Anthony took calculus, he viewed it as a challenge. "I thought running a half marathon and taking calculus were like the same thing." He also saw the counterfeit coin problem in pre-calculus as "really the moment that I decided maybe I was wrong about certain aspects of math." These challenging

experiences served to motivate him. When describing his experience as a graduate student, he recalls "And there were times where I just wrote, I was unable to solve this problem … I was okay if I didn't get it perfect." In these stories, Anthony demonstrates a willingness to meet and respond to challenges.

Reflecting on Parenting. Another eye-opening moment for Anthony was changing his child's first diaper at the hospital. He reflected on his learning how to successfully change a diaper and realized "what a disaster it would be to teach changing diapers through inquiry-based learning." Anthony shares that through this experience he can "see the value in some of the traditional approaches that are about keeping students comfortable and not necessarily challenging them as much." He also notes that being a new parent gave him an appreciation for being overwhelmed and what that feels like. And he realizes that students sometimes experience similar feelings of being overwhelmed. His experiences as a father have given him some additional empathy for students.

Co-Occurring Features of Quality Teaching in Anthony's Narratives

Questioning to Support Student Learning. Anthony learned firsthand the importance of questions when he worked as a journalist. Before earning his degree in math, Anthony's work gave him "a real perspective on the power of questions, asking questions, and also about searching for information." His community college pre-calculus professor found the right question to spark his interest in math, which led Anthony to pursue mathematics. Anthony also uses Bloom's Taxonomy to craft questions to support multiple levels of learning.

Teacher Affect. Anthony's definition of quality teaching featured different aspects of teacher affect (i.e., the behaviors and dispositions he wants to share with his students). In his stories, he recalls his graduate school professors as "very effective and well-prepared. Their style is very different than mine. I will use almost anything to get students engaged, including things that I know are cheap gimmicks." Anthony is noting that a teacher can present different personas to a class and still accomplish teaching the class well. Anthony continued talking about teacher affect, "I realized the importance of charisma. So, when I'm not teaching, I'm pretty introverted, but I realized that you're fighting for the students' attention. And you really have to develop a persona that works for you." He is reflecting on how his students will perceive his behaviors and what he can do to keep his students engaged in the mathematics.

Student Disposition Toward Math. In his discussion of quality teaching Anthony indicates a desire for students to experience "productive struggle, with emphasis on both: productive and struggle." In his community college pre-calculus class, Anthony recalled solving the counterfeit coin problem and "that was really the moment that I decided maybe I was wrong about certain aspects of math." His success at solving an interesting problem provided a reason to reflect on his own disposition toward math. Later on in graduate school, Anthony recalls, "understanding that even if I didn't complete an exercise, I might still be making progress." His attitude highlighted that simply making progress was a success.

Caring about his students. Throughout his stories, Anthony shows careful thought to the student experience and the impact of past experiences on their ability to learn. When discussing his class policies about attendance, Anthony noted that when he

was absent due to his kids being sick, “nobody demanded that I show proof and I think [the students] deserve the same respect.” He chooses to treat his students with respect and as people with various life commitments. Anthony also strives to “really allow students to make choices without fear of unnecessarily harsh repercussions.” He notes that the fear of failure can lead students to experience extra anxiety and tries to mitigate that with his patient and caring attitude. Anthony sums up this idea with “I try to be very aware of not belittling students in any way, not putting them in high pressure situations, where they haven’t had a chance to prepare.” His attention to their needs and how his own actions could impact those needs depicts a professor who considers the entire student when teaching.

Summary

Anthony’s profile describes a patient and caring professor with a creative flexibility to adapt to students’ needs and responsibilities. His reflecting on his experiences and consideration of other viewpoints depicts a professor who attends to all of a student’s potential challenges and difficulties with mathematics and with life. His attention to student understanding allows him to ask the right questions for improved student learning. His life experiences provided pivotal moments in his journey for considering the best step forward as a mathematics professor.

Cross-Case Analysis

In this section, I identify some general trends and summarize key similarities and differences across the participants. First, I examine similarities and differences in the participants' quality teaching definitions by comparing the relative frequency of the different codes for Quality Teaching Features across participants. Second, I consider the Quality Teaching codes that co-occur in a given participant's definition and stories to examine whether and in what ways participants directly connect their lived experiences with their conceptions of teaching. Finally, I explore the similarities and differences of the participants' journeys to becoming exemplary mathematics professors by examining the relative frequency of the Influential Change Agent codes across participants.

Commonalities Across Participants in Quality Teaching Definitions

The frequency of the Quality Teaching Feature codes in the participants' definitions of quality teaching provides a different perspective on the data by looking across cases to identify commonalities and basic trends. Table 2 highlights the frequency of the Quality Teaching codes across all participants.

Table 3: Frequency of quality teaching codes across participant definitions

Quality Teaching Definitions	Sylvia	Mitchell	Jared	Stephen	Anthony	Total	Mean
Appropriate Challenges	0	0	1	0	2	3	0.5
Questioning to Support Student Learning	1	1	4	6	3	15	3.0
Selecting Appropriate Tasks	1	1	3	2	2	9	1.8
Caring about Students	1	0	5	3	4	13	2.6

Handling Mistakes	1	1	2	5	1	10	2.0
Safe Space for Mistakes	1	1	10	5	3	20	4.0
Supporting Independent Thinkers	4	4	3	6	4	21	4.2
Supporting Students to Answer Their Own Questions	1	4	6	9	2	22	4.4
Teaching More Than Math	0	1	1	2	0	4	0.8
Student Disposition Toward Math	5	5	9	8	14	41	8.2
Teacher Affect	3	0	10	5	0	18	3.6

(Note that the codes for Self as Learner and Other Teachers' Affect do not appear in the table because these codes did not occur in the part of the interview when participants were explicitly defining and explaining what quality teaching was to them.)

The Student Disposition toward Math is the most frequent code across participants occurring forty-one times across all cases and ranging from five to fourteen distinct instances per participant. Every participant received the code Student Disposition Toward Math at least five times which indicates that attending to student disposition is an important part of all the participants' definitions of quality teaching. All of the participants focused on what the student needs to *do* to learn and not simply what topics need to be covered. The focus on what the student is feeling shows an attention to the student experience, both past and present.

Additionally, the Supporting Independent Thinkers code occurred relatively

frequently within participants: this code was mentioned at least three times per participant (with an overall total of twenty-one instances). This pattern suggests that a second common goal across these teachers was for their students to take responsibility for their own learning which depends on the students believing they are capable of doing so (reflected in the high frequency of the Student Disposition code). Independent thinking is the stated end goal for their students, to be able to handle new questions and figure out how to start answering those questions.

As a point of contrast, codes for Supporting Students to Answer Their Own Questions and Safe Space for Mistakes occur, overall, at about the same frequency as the Supporting Independent Thinkers code (roughly twenty total instances). However, the frequency per participant varies more for these codes—from a low of one instance (Sylvia) to a high of ten instances (Jared) in the Safe Space for Mistakes coding category. Although there is a shared view that supporting safe spaces to make mistakes and revise thinking is part of quality teaching and that a worthy goal of instruction should be to support students to ask and answer their own questions, these conceptions are more variable across participants than the first two conceptions.

I also find it noteworthy that the three most frequent codes for quality teaching have an explicit focus on the student and consider both cognitive as well as affective factors. These exemplary professors view teaching not just as something they (i.e., teachers) do, but recognize that this is an interactive and relational activity, the success of which depends, in part, on the students themselves. Additionally, I also see the Safe Space for Mistakes code as related to the Student Disposition code since making a mistake publicly impacts a student's affective and emotional experience in the

mathematics classroom. Supporting students to become independent thinkers and ask their own questions also parallels the Student Disposition code, since encouraging students to engage with mathematics involves attending to students' dispositions and supporting them to see themselves as capable to doing mathematics.

Commonalities Across Participants in Co-Occurring Quality Teaching Codes

The frequency of Quality Teaching Feature codes that occur in the participants' definitions and stories is given in Table 3. The two columns of numbers within the table are the frequency of the code in the quality teaching definition (from Table 2) and the frequency of that same code across all of the participants' stories. For example, for the Questioning to Support Student Learning for Sylvia, there was one instance of this code in her definition of quality teaching and two instances of this code across the stories she shared. Note that the subcodes Self as Learner and Other Teacher's Affect have been aggregated to the macro-level codes of Student Disposition and Teacher Affect, respectively. I used yellow squares to visually indicate when a given code co-occurs at roughly the same frequency which I operationalized as a difference of at most two when comparing the two different counts.

Table 4: Frequency of quality teaching features in participants' definitions and stories

Codes in both	Sylvia		Mitchell		Jared		Stephen		Anthony		Total
Definitions & **S**tories	D	S	D	S	D	S	D	S	D	S	
Questioning to Support Student Learning	1	2	1	8	4	2	6	2	3	6	35
Selecting Appropriate Tasks	1	2	1	2	3	2	2	2	2	3	20
Caring about Students	1	1	0	6	5	5	3	2	4	7	34
Handling Mistakes	1	0	1	0	2	4	5	2	1	3	19
Safe Space for Mistakes	1	0	1	0	10	0	5	0	3	0	20

Supporting Independent Thinkers	4	2	4	2	3	0	6	1	4	1	27
Supporting Students to Answer Their Own Questions	1	2	4	2	6	1	9	4	2	1	32
Teaching More Than Math	0	1	1	4	1	0	2	2	0	1	12
Student Disposition Toward Math + Self as Learner	5	1+2	5	6+3	9	6+8	8	3+3	14	10+7	90
Teacher Affect + Other Teacher's Affect	3	3+1	0	4+3	10	11+8	5	0+2	0	8+7	65

Red indicates a feature of quality teaching that occurred more frequently in the participant's definition than stories (a difference greater than two when comparing counts). Blue shading indicates a feature of quality teaching that occurred more frequently in the participant's stories than definition (a difference greater than two). The Student Disposition Toward Math row is colored grey since determining the count frequencies is variable due to the subcode counts, but all of the codes have at least one instance in each participant.

Student Disposition Toward Math along with the subcode Self as Learner co-occur for each participant (ranging from eight instances for Sylvia to thirty-one for Anthony). Recall, the Student Disposition Toward Math codes for the participant attending to student disposition and attitudes toward mathematics, and the Self as Learner subcode is for the same attention to disposition, but the subject of the story is the participant. If aggregated, these two codes total to ninety instances (across participants) in which they discussed either their own or others' dispositions toward math and experiences that influenced student disposition. Student Disposition Toward Math is the most frequent code in both quality teaching definitions and participants' stories. This frequency suggests a relationship between memorable stories and experiences related to

one's own disposition and affect when learning mathematics and considering the importance of Student Disposition as an important aspect of quality teaching.

The two subcodes for Appropriate Challenges—Questioning to Support Student Learning and Selecting Appropriate Tasks—co-occurred at least once in quality teaching definitions and memorable stories for every participant. These subcodes of Appropriate Challenges were also among the most frequent across all Features of Quality Teaching with an overall total of thirty-five instances (Questioning subcode) and twenty instances (Selecting Tasks subcode.) All of the participants recognized the importance of appropriately challenging their students, whether that be differentiating student tasks within class or posing purposeful questions to students as part of their real-time instruction. Additionally, each participant also had memorable experiences with appropriate (or inappropriate) challenges reflected in their stories. This could suggest that having experienced either poor or excellent task selection may lead to this feature's inclusion in quality teaching definitions.

Similarly, the codes for Teacher Affect and Other Teacher's Affect appeared in stories 26 and 21 times, respectively. This highlights the participants' frequency of discussing their own affect and the affect of their teachers, both of which impact student learning. Mitchell and Anthony did not mention teacher affect in their quality teaching definitions, but every participant had memorable stories of other teachers' affect. This frequency of teacher affect is contrasted by the disproportionate and lessened frequency total of eighteen in the quality teaching definitions. This variation could indicate that stories with poor or excellent teacher affect are more memorable for those who have reflected on quality teaching. Likewise, the Caring about Students code appeared twenty-

one times across the participants' stories but was only mentioned in quality teaching definitions thirteen times. This is likely due to the emotive nature of this code since people tend to remember their emotions when retelling a story as compared to defining a complex concept.

Also, the Supporting Independent thinkers and its subcode, Supporting Students to Answer Their Own Questions, appears more in participants' definitions rather than their stories, although every participant did have at least one story with this code. This further ties in with the focus on Student Disposition. Since learning to think independently and answer their own questions is a great step towards succeeding in math, this success should improve student dispositions toward math.

Commonalities Across Participants in Influential Change Agent Codes

In order to better understand the teaching journeys of these exemplary mathematics professors, I considered similarities in their stories. Since the specifics of their stories were different, I used my Influential Change Agent codes to identify why and in what ways particular events influenced my participants' understanding of teaching more generally. One of my goals was to identify commonalities that might have implications for designing learning experiences for mathematics professors to support their professional growth in teaching. Table 4 shows the total of each Influential Change Agent code for each participant across all of their stories. The greyed and hashed squares are lower frequencies. The squares left white represent the two highest frequency codes for each participant; for example, Sylvia has Responding to Challenges as her most frequent code with eight instances and a tie between Resourceful and Reflective, both with six codes each.

Table 5: Influential Change Agent Code Frequency

	Sylvia	Mitchell	Jared	Stephen	Anthony	Total
Responding to Challenges	8	7	3	5	7	30
Role Models	5	1	10	5	8	29
Resourceful	6	5	5	3	11	30
Major Contribution	2	6	0	2	0	10
Reflective	6	8	15	10	20	59
Reflecting on Parenting	1	2	1	3	1	8

The Reflective code is the most frequent for each participant except Sylvia, who has it as her second most frequent. All of the participants were deliberate in considering their teaching and the impact it has on others as well as teaching they experienced as learners and how that informs their conceptions of teaching. Being a reflective professor indicates a willingness to consider other ways of teaching with the goal of improving one's own ability to teach. Also, this receptiveness can help professors analyze their own past mistakes and motivations with an eye toward improving their own teaching in the future. Being reflective indicates a growth mindset towards teaching rather than a fixed mindset. Teaching is not seen as a characteristic that one possesses like being tall or having green eyes. Rather, teaching is treated as a collection of different features that can be focused on in a variety of different ways that can still lead to quality teaching.

The Responding to Challenges code indicated instances when participants responded to difficult situations. The counts ranged from 3 to 8 counts with an average of

six. All participants mentioned challenging situations and how they met those challenges. This shows determination and resiliency in the participants. Overcoming a challenge, or deciding to bypass it, is an emotional ride from the frustration of experiencing the difficulties to the triumph of resolving those difficulties. Highly emotional events are likely to leave a more memorable impression. The participants not only responded to their challenges, but then reflected back on what made those events challenging and used those challenges related to teaching and learning as a catalyst for positive growth in their own lives.

The Resourceful code indicated when participants noted and made use of helpful resources, like a new funding opportunity related to teaching or colleagues' advice. The Resourceful code ranged from 3 to 11 counts with an average of six counts. While there is a certain degree of luck in finding resources, the participants paid attention and figured out how to use the resources around them to their benefit. These participants also used available resources to educate themselves and further their teaching.

The Role Models code was also frequent, and this corroborates the research literature about the importance of mentors. Both Samaras et al. (2019) and Lane et al. (2019) reported the importance of critical friends or mentor figures in helping faculty reframe their thinking. The frequency ranged from 1 to 10 counts with an average of 5.8. While some of the role models were not the best, the participants viewed those experiences as learning moments and drew on them when reflecting on their own teaching. This ties in with being reflective and resourceful. The participants not only thought about their experiences, but also found a way to use those experiences to improve their own teaching abilities.

In my study, my participants' lived experiences have impacted their conceptions of teaching. Dealing with the ups and downs or resources and challenges of life can support thoughtful reflection on what quality teaching is. These participants continually considered the examples of those around them, whether those examples were good or bad, and whether the example was a person in an authority position, a colleague, or a student. While each of the participants is a unique person, the shared experiences and definitions show interesting commonalities. All the participants' quality teaching definitions focused on the student experience by attending to student disposition, ensuring a safe space for mistakes, and students becoming independent thinkers. Their definitions of quality teaching were grounded in their own experiences and significant stories about teaching. The participants' stories focused on their own and their students' disposition toward math while also noting the importance of appropriate challenges. The stories attending to crafting appropriate challenges showed an attention to not only a single student's experience, but also how the whole classroom can engage with the material. Broadly, the most common change agent that supported the participants' continued growth across their career was the ability to reflect back on their own teaching and experiences to arrive at new conclusions. Participants also utilized their surroundings by being resourceful and responding to challenges as well as making use of their role models, either good or bad. The combined data shows all five participants' focus on their students' experiences while reflectively attending to the influences of their past.

V. DISCUSSION AND CONCLUSION

In this study, I investigated the personal teaching journeys of exemplary mathematics faculty and their conceptions of quality teaching. The participants in this study were recognized as exemplary teachers of mathematics who earned their doctorate in mathematics. The research questions guiding this study were:

1. How do mathematics professors become exemplary teachers?
    a. Specifically, what are the personal journeys and lived experiences for math professors recognized as exemplary teachers?
2. What are exemplary mathematics professors' conceptions of quality teaching?
3. How do their self-identified significant experiences and stories relate to their conceptions of teaching and what factors have supported or hindered the participants' professional growth and change as teachers?

In the previous chapter, I shared profiles for each of my exemplary mathematics professors—Sylvia, Mitchell, Jared, Stephen, and Anthony—that described their personal teaching journeys. I documented stories they identified as significant to their understanding of teaching. These profiles answer my first research question. My study also involved understanding how my participants defined quality teaching (research question 2 above). Certain features of quality teaching were consistent across my participants. For example, they all focused on Student Disposition Toward Math and Teacher's Affect, showing a student-focused and affective perspective toward teaching. The participants all noted the importance of Questioning to Support Student Learning and Selecting Appropriate Tasks, which are both related to student disposition and engagement. All the participants mentioned Caring About Students, which again links

with student disposition. I also found that for my participants, the majority of their explicitly-stated features of quality teaching were reflected in the memorable stories and events they shared suggesting a connection between their lived experiences and conceptions. And finally, although the details of my participants' experiences and stories varied, there were some underlying commonalities across their stories. Each participant was able to respond productively to challenges and use them as a catalyst for future growth. They also took advantage of resources to improve their teaching and had role models (both good and bad) that informed their views on teaching. But the most important change agent across my participants was thoughtful reflection—considering what went well or poorly about an experience or event and why, and then making changes based on that evaluation. I now situate these findings more broadly in the existing literature by sharing how this study is related to and builds on existing research on undergraduate teaching and professional learning opportunities.

Existing research showed that professional development opportunities for professors of any discipline is usually non-existent (Hativa, 1997, Austin, 2002, Brownell and Tanner, 2012). The only participant with any professional development was Anthony, the youngest participant and the only participant who earned a degree in education. This may suggest that some current programs include professional development. Several recent STEM studies have investigated faculty use of evidence based instructional practices and their potential barriers and drivers for implementation (Brownell and Tanner, 2012, Lund and Stains, 2015, Shadle et al., 2017). These studies commonly noted time and resources as barriers to implementation as well as beliefs about teaching student centered practices. However, all of my participants found ways around

barriers by capitalizing on university initiatives and persevering through challenges. These professors also all shared strong beliefs about teaching being student centered.

Several existing studies looked at the effect of a faculty member's social network and community and how these factors influence faculty views (Samaras et al., 2019, Lane et al., 2019). These studies found critical colleagues or mentor figures helped faculty to reframe their thinking about teaching or helped faculty consider evidence based instructional practices. Mentors was one of the influential change agents identified in my study. Each participant had mentoring figures and each thought critically about teaching scenarios from their pasts. Other studies investigated the importance of self-study or reflection on one's practice (Samaras et al., 2019, Connolly et al., 2018, Cooney, 1999). Similarly, the most frequent change agent across all participants in my study was reflecting thoughtfully on events and considering their own teaching in relation to their reflections. Cooney (1999) found that the goal of teacher education is to move teachers toward being reflective connectionists, which my participants all moved toward this or arrived at this goal. Also note that these studies were on STEM professors and my study replicated the results with exemplary mathematics professors.

To explore themes about university and learning, Weston and McAlpine (1998) interviewed six mathematicians who were also identified as exemplary. Three of these professors were from the education faculty, and those three had pedagogical degrees. The analysis was on each professor's teaching history interview. The results from this mid 1990's study identified nine thematic codes including Caring, Course Design, Passion, Research, and Active Engagement. Now, over 20 years later, my participant's journeys share similar themes, while also attending to student disposition, and teacher affect. My

study further focused on the significant stories of their past that influenced shifts in their views on teaching.

Clandinin et al. (2009) investigated teachers' stories to live by as a way to understand how a teacher sees themselves in their environment at work and at home. Teachers also use these stories to identify who they are becoming. Their research was conducted with preservice teachers or teachers within 5 years of starting their career. Since my participants were university professors and more experienced than new teachers, they had different experiences to craft their stories to live by and had more experiences within their profession.

Walker and Gleaves (2016) defined the caring teacher framework based on six carefully selected professors. The behaviors mentioned most that signified a caring teacher included "listening to students, showing empathy, supporting students, actively fostering learning in class, giving appropriate and encouraging feedback and praise, having high expectations in standards of work and behavior, and showing an active concern in students' personal lives" (p. 68). Similarly, my professors focused on actively engaging their students and providing feedback to encourage independent thinking and learning. They also listened to their students to support active learning within the classroom.

In a focus group study on Scholarship of Teaching and Learning (SoTL) researchers, Mathany et al. (2017) investigated the factors that inhibited these faculty members from identifying publicly as a SoTL researcher – that is a researcher who has shifted their research area to education rather than discipline-focused research. They found the faculty were motivated by a sense of professional responsibility to ensure that

they are providing students with the best possible learning experience. Sylvia spoke of a responsibility to her students which took time away from what other professors thought research required. Whenever the norms of mathematics and university professors were brought up, my participants' response was to still ensure their students received quality teaching.

In a study of science professors, Aragón et al. (2018) examined whether professors had a fixed or growth mindset and how that mindset related to their adoption and implementation of evidence based instructional practices. All of my participants demonstrated an ability to reflect on experiences and learn from them. Jared even identified his mindset in his freshman year of college as fixed and lamented not learning about these mindsets earlier. Being a reflective thinker and willing to reconsider past events sets all of my participants now as having a growth mindset.

Implications

The PCAST report (2012) and MAA's Common Vision (2015) both call for an improvement in teaching and a move away from the current state of affairs. Current research has demonstrated the need to understand the differences between different disciplines and how they approach teaching. Studying these exemplary mathematics professors, and how they came to be exemplary, uncovered leverage points and critical experiences with implications for faculty development that could support the improvement of mathematics teaching more generally. Anthony summed up this future direction with "Many professors think about teaching, but in my opinion, many of them don't know what to do or where to go with questions about teaching."

To better understand the experiences of my participants with an eye toward

designing future learning opportunities for other mathematics faculty, I present an organization of the Quality Teaching Feature codes into three macro categories: Caring, Engagement, and Sensemaking. The Caring macro category includes Caring About Students, Student Disposition Toward Math, Teacher Affect, Handling Mistakes with the Safe Space subcode, and Teaching More Than Math. The overarching idea is attention to students as people with varying histories, both with math and with life. The Engagement macro category includes Student Disposition Toward Math, Teacher Affect, Handling Mistakes with the Safe Space subcode, the Appropriate Tasks subcode, and Teaching More Than Math. The focus is on students engaging with mathematics in a productive way and attending to students' struggles with encouragement. The Sensemaking macro category includes Supporting Independent Thinkers with the Answering Their Own Questions subcode, Handling Mistakes with the Safe Space subcode, and Appropriate Challenges with the Questioning to Support Student Learning and Appropriate Tasks subcodes. This category deals with students understanding the math from the careful structuring of the course and the provided space to flourish and succeed with mathematics. Because the macro-level categories of Caring, Engagement, and Sensemaking were common features of quality teaching across my participants, I suggest that these features of teaching mathematics may be important staring points when designing professional learning experiences for mathematics faculty. I discuss this in more detail as one of the implications of this study.

Implications for Professional Development

One implication from my research is the creation of a professional development (PD) to help new and current professors that incorporates critical leverage points

identified by my participants. The first design element for PD that I recommend, based on my findings, is supporting faculty to become reflective practitioners. In their 2010 article on developing a model of reflection for professional development, Black and Plowright defined reflection as follows.

> Reflection is the process of engaging with learning and/or professional practice that provides an opportunity to critically analyze and evaluate that learning or practice. The purpose is to develop professional knowledge, understanding and practice that incorporates a deeper form of learning which is transformational in nature and is empowering, enlightening and ultimately emancipatory (p. 246).

Reflection is an iterative process that focuses on learning and practice as the source, target, and purpose of reflection. In PD sessions, professors will focus on their own learning experiences and consider how that informs their instruction. They will also consider experiences from the classroom and ponder alternative actions and reasonings for both their own and student behavior. These experiences don't have to be personal; professors could learn from others' stories, including the stories from my research participants, or created teaching vignettes.

Another implication for the design of professional development from my research is providing mentoring to new faculty and communities of support for current faculty. In a study on Early Career Teachers (ECTs) who shifted careers to become secondary math and science teachers, Surrette (2020) investigated the influence of mentoring and professional communities and found "The results of this study point to the ability of mentors and professional communities from outside the school to influence ECTs who are career changers to adopt learner-centered approaches to teaching to increase student

engagement during classroom instruction." (p. 187). The study noted the support from public universities as helpful to these secondary school teachers. Likewise, mentors at the university would provide support for new and current faculty.

A third implication based on my findings is to foster an attitude of resiliency in the face of challenges and to foster that same attitude in students. Jared and other participants discussed a fixed vs a growth mindset (Dweck 2008). Jared even discussed wanting that knowledge when he was an undergraduate student. This is the type of knowledge that would benefit both faculty and students. Part of a growth mindset is understanding that learning takes work and effort. Many everyday people seem to feel that math is an innate ability rather than a skill that can be learned.

A final implication for designing professional development for mathematics faculty comes from the quality teaching macro categories I described earlier: Caring, Encouragement, and Sensemaking. Specifically, I suggest designing professional development to engaging faculty in considering the benefits of these three features of teaching and how demonstrating caring and encouraging student participation and sensemaking could impact students. First, faculty will reflect on a caring teacher from their past and how that teacher's attitude benefitted them. Then, faculty can extend this same idea to their current practice. How can they, as teachers, demonstrate a caring attitude to their students? How might this affect their students? Why is this important? Second, faculty will consider how they craft their classroom environment and that impact on their students and the progression of that class over the semester. Then, faculty can consider others' classrooms and how their own classroom environment could shift to better encourage and support student engagement. Third, faculty will ponder how to

support students to learn themselves and make sense of the mathematics. This will require faculty to remember what learning felt like to consider the student's perspective. I conjecture that supporting faculty to become more caring, more encouraging, and support sensemaking in their classrooms will help current and future students.

Implication for Higher Education

Within my study, I identified exemplary professors through their receipt of university level or higher awards for teaching. This is a quantifiable metric and easily verified. However, in practice, defining and identifying exemplary professors and what that entails is a much more complex task. Based on my findings, exemplary teachers demonstrate caring for their students in more than the classroom, encourage students to persevere in their mathematical journeys, and support students to make sense of mathematics. These three qualities are not easily quantifiable or determined. As a field, we need to continue to consider how to measure an educator's progress in learning to teach and what it means to be exemplary.

Limitations

This study has several limitations. First, these findings are not intended to be representative of all exemplary mathematics professors or all mathematics professors. The participants self-selected into participation in the study, which further limits the generalizability of the findings. By using a convenience sampling from my network of mathematics and mathematics education professors, this may have caused my participants to be more exposed to mathematics education conceptions over other exemplary mathematics professors. Another limitation is in the diversity of my participants. I only had one female participant and no persons of color among my

participants. This may be due to the type and thus age of the professors I selected. Sylvia was the first female professor promoted to tenure at her university. Receiving an award for exemplary teaching generally occurs later in a professor's career. This means most of my participants were attending public school and university through the civil rights movement and women's liberation. The historical opportunities available to women and persons of color would have been severely limited, which means they are underrepresented in the population from which I selected my participants.

Next, the design of this study created a few limitations. I did suggest various aspects of quality teaching with my interview questions. The participants may have defined quality teaching in directions other than those described in the questions had my interview been completely unstructured. Specifically, I asked about how mistakes are handled and how questions are attended to, but in an unstructured interview these may not have come up as part of a natural flow of conversation. Moreover, the participants may have discussed different stories without those initial interview questions influencing where their attention was focused. Also, by only interviewing exemplary professors, this study lacks the ability to compare these professors to their colleagues. Other professors may hold similar conceptions of teaching but have not been recognized for their teaching at a level outside their department. Another limitation of my study design was not observing my participants' teaching practice. I cannot link conceptions of quality teaching and significant stories with classroom behaviors and instructional practices.

Overall, these five professors' journeys document different events that influenced their thoughts about quality teaching. Their definitions of quality teaching featured the students' learning and experience with attention to the impact of teacher affect—that is

how their affect might influence their students' experiences in class. They attended to the importance of selecting appropriate tasks that differentiated for student ability while still providing an appropriate challenge. The importance of asking and answering questions featured in all of their stories, as well as attention to the social ramifications of making a public mistake. All five professors demonstrated caring for their students, which supports student engagement. An important finding from this study is how reflective each of these professors are about their students and their teaching. Every professor found resources to support them to improve their teaching. They all reflected on mentoring figures and what their observations meant for their quality teaching definitions. By reflecting on these experiences, mathematics professors of the future may overcome some of the challenges in learning to teach more quickly to the benefit of all their future students.

A major contribution of this study is these portraits of exemplary teaching and the experiences that contributed to the formation of those exemplary professors. These portraits suggest that how we define excellence in teaching in higher education needs to be updated. We should consider teachers that are reflective, seeking to learn more about teaching and continually improve their craft. We should consider caring teachers, who attend to more than their students' mathematical abilities. We should consider teachers who promote sensemaking in their classrooms, even though it often frustrates their students. We should consider teachers who keep their classroom as a safe space for learning. Current K12 teachers are required to maintain their professional development of teaching. Universities need to raise the standards of teaching for the betterment of all.

APPENDIX SECTION

Appendix A

Interview Protocol IRB #6953

Interview Introduction:
I am researching how mathematicians' form their views on teaching. I am looking for foundational experiences that influenced how you view teaching and how those views have shifted throughout your life.

1. What do you think quality teaching is? I know this answer may change over time. As desired, you may add on to your definition as we continue the interview.

2. When you teach, what are your goals? What do you strive to embody?

3. Below are questions to determine both large picture and fine-grained aspects of your teaching.

a. What would a successful course look like? What would you do? What would your students need to do?

b. What would a successful lesson look like? What would you do? What would your students do?

c. How do you handle student questions during class? During office hours?

d. How do you handle a mistake made in class by yourself? By a student?

Now I'd like you to consider how your past has shaped you as the professor you are today. I'm interested in how you think about teaching and how that may have changed over time.

4. What significant events from your past helped form your current views about teaching? How do these events relate back to your definition of quality teaching?

5. Please explain the most influential experiences that influenced how you view teaching.

6. What role did teaching play in your graduate student experience? How, if at all, did your perceptions about teaching change in graduate school?

7. How did your primary research advisor/dissertation chair view teaching?

8. What prior experiences have shaped your beliefs about research (e.g., approach to running a research lab, emphasis on publications and grants)?

Thank you for your time and insight into your evolution of your views on teaching as a Professor of Mathematics. Once I have analyzed the video and audio recordings, I would like to contact you to confirm my understandings and make sure I am accurately reflecting your beliefs and ideas. I may also ask to conduct a follow up interview as my research develops. Again, thank you for your valuable time!

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