Detailed numerical results for paper: A Set-Partitioning-Based Model for the Stochastic Vehicle Routing Problem

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Following are some tables with detailed results that complement the paper titled "A Set-Partitioning-Based Model for the Stochastic Vehicle Routing Problem".

1 Tables Related to Results in Section 3.3 of the Paper

Tables 1 and 2 present routes cost comparison between the set-partitioning model and SYMPHONY solution (Ralphs et al., 2003; Ralphs and Güzelsoy, 2008) for 106 instances of the deterministic vehicle routing problem (VRP). Instances were generated as described in Section 3.3 of the paper and they are grouped in two different sets (set 1 and set 2). The tables show routes cost, number of different routes generated for solving the set partitioning model and resulting number of vehicles k in the solution. Tables also include routes cost and number of vehicles k used for solving exactly the VRP with SYMPHONY.

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The last column in tables 1 and 2 is the percentage of difference in routes cost between the set-partitioning model and SYMPHONY. The number of instances is 58 for set 1 and 48 for set 2. An instance labeled as set1-5r1-9 corresponds to the first replication (r1) (i.e. a particular assignment of geographical locations and demand distributions for each customer) of an instance in set 1 with 5 customers and vehicle capacity of 9.

Table 1: Routes Cost Comparison between Set Partitioning Model and SYMPHONY - Instances Set 1

	Set Partitioning		SYMPHONY			
Instance name	Cost	No. Routes Generated	k	Cost	k	% Cost diff
set1-5r1-9	3.11	37	2	3.11	2	0.0%
set1-5r2-9	3.13	36	2	3.13	2	0.0%
set1-5r3-9	4.11	33	2	4.11	2	0.0%
set1-5r4-9	3.42	32	2	3.42	2	0.0%
set1-5r5-9	4.51	33	2	4.50	2	0.0%
set1-5r1-5	4.37	18	3	4.37	3	0.0%
set1-5r2-5	4.36	15	3	4.36	3	0.0%
set1-5r4-5	5.02	11	4	5.02	4	0.0%
set1-8r1-14	4.21	189	2	4.21	2	0.0%
set1-8r2-14	4.25	201	2	4.25	2	0.0%
set1-8r3-14	4.55	195	2	4.55	2	0.0%
set1-8r4-14	3.68	172	2	3.51	2	5.0%
set1-8r5-14	4.36	199	2	4.36	2	0.0%
set1-8r1-9	5.22	77	3	5.22	3	0.0%
set1-8r2-9	6.00	77	3	6.00	3	0.0%
set1-8r3-9	5.78	82	3	5.78	3	0.0%
set1-8r4-9	4.38	75	3	4.38	3	0.0%
set1-8r5-9	5.19	78	3	5.19	3	0.0%
set1-20r1-91	6.31	2543	2	6.14	2	2.7%
set1-20r2-91	5.92	3509	2	5.90	2	0.4%
set1-20r3-91	5.09	3699	2	5.10	2	0.0%
set1-20r4-91	5.20	3737	2	5.12	2	1.5%
set1-20r1-58	8.60	1347	4	8.56	4	0.6%
set1-20r2-58	7.17	2749	3	7.17	3	0.0%
set1-20r3-58	6.40	1610	3	6.36	3	0.7%
set1-20r4-58	6.48	2100	3	6.45	3	0.6%
set1-20r5-58	7.54	1209	4	7.54	4	0.0%
set1-30r1-137	5.51	6275	2	5.38	2	2.3%
set1-30r2-137	5.76	3861	2	5.37	2	7.3%
set1-30r3-137	6.48	5295	2	6.31	2	2.8%

Table 1: Routes Cost Comparison between Set Partitioning Model and SYMPHONY - Instances Set 1(continued)

	Set Partitioning		SYMPHONY			
Instance name	Cost	No. Routes Generated	k	Cost	k	% Cost diff
set1-30r4-137	6.00	6812	2	5.99	2	0.1%
set1-30r5-137	6.11	6071	2	6.00	2	1.7%
set1-30r1-87	7.04	4125	3	7.02	3	0.2%
set1-30r2-87	7.39	2534	3	7.19	3	2.7%
set1-30r3-87	7.94	4423	3	7.69	3	3.3%
set1-30r4-87	7.46	4338	3	7.16	3	4.2%
set1-30r5-87	7.17	4880	3	7.10	3	0.9%
set1-40r1-183	6.95	6574	2	6.09	2	14.2%
set1-40r3-183	6.40	7843	2	6.16	2	4.0%
set1-40r4-183	6.86	7832	2	6.47	2	6.0%
set1-40r5-183	7.06	7417	2	6.94	2	1.7%
set1-40r1-116	7.43	5871	3	6.96	3	6.7%
set1-40r2-116	8.36	4378	4	7.96	4	5.0%
set1-40r3-116	7.77	6735	3	7.62	3	2.0%
set1-40r4-116	7.90	7231	3	7.75	3	1.9%
set1-40r5-116	8.32	5253	3	8.08	3	2.9%
set1-60r1-274	7.51	16475	2	6.71	2	11.9%
set1-60r2-274	8.83	6523	3	8.12	3	8.8%
set1-60r1-175	8.40	7797	3	7.81	3	7.6%
set1-60r2-175	10.18	6356	4	9.72	4	4.7%
set1-60r3-175	8.77	7053	3	8.65	3	1.4%
set1-60r5-175	8.65	6109	3	8.38	3	3.2%
set1-20r5-91	5.21	2352	2	5.14	2	1.3%
set1-40r2-183	6.90	5481	2	6.40	2	7.8%
set1-60r3-274	8.16	7415	3	7.46	2	9.4%
set1-60r4-274	7.56	17292	3	6.87	2	10.0%
set1-60r5-274	8.28	13897	3	6.95	2	19.2%
set1-60r4-175	8.38	7687	3	8.19	3	2.3%

Table 2: Routes Cost Comparison between Set Partitioning Model and SYMPHONY - Instances Set 2

	Set Partitioning		SYMPHONY			
Instance name	Cost	No. Routes Generated	k	Cost	k	% Cost diff
set2-5r1-9	4.37	20	3	4.37	3	0.0%
set2-5r2-9	4.36	14	3	4.36	3	0.0%
set2-5r3-9	6.67	9	4	6.67	4	0.0%
set2-5r4-9	5.02	8	4	5.02	4	0.0%

Table 2: Routes Cost Comparison between Set Partitioning Model and SYMPHONY - Instances Set 2 (continued)

	Set Partitioning		SYMPHONY			
Instance name	Cost	No. Routes Generated	k	Cost	k	% Cost diff
set2-5r5-9	7.26	8	4	7.26	4	0.0%
set2-8r1-13	6.43	40	4	6.43	4	0.0%
set2-8r2-13	7.46	42	4	7.46	4	0.0%
set2-8r3-13	6.59	45	4	6.59	4	0.0%
set2-8r4-13	5.06	45	4	5.06	4	0.0%
set2-8r5-13	6.49	37	4	6.50	4	0.0%
set2-8r1-10	6.75	29	4	6.75	4	0.0%
set2-8r2-10	8.86	28	5	8.86	5	0.0%
set2-20r1-60	9.32	1259	4	9.25	4	0.7%
set2-20r2-60	7.29	2201	3	7.29	3	0.0%
set2-20r3-60	6.36	1352	3	6.36	3	0.0%
set2-20r4-60	6.78	1659	4	6.78	4	0.0%
set2-20r5-60	8.01	1079	4	7.88	4	1.7%
set2-20r1-45	10.92	635	5	10.89	5	0.3%
set2-20r2-45	8.95	1379	4	8.95	4	0.0%
set2-20r3-45	7.68	775	4	7.68	4	0.0%
set2-20r5-45	9.85	544	5	9.81	5	0.4%
set2-30r1-90	7.62	3832	4	7.44	4	2.4%
set2-30r3-90	7.97	4274	3	7.73	3	3.1%
set2-30r5-90	7.38	4103	3	7.31	3	1.1%
set2-30r1-68	9.11	2286	5	8.84	5	3.0%
set2-30r2-68	9.04	2294	4	9.04	4	0.1%
set2-30r3-68	9.41	2972	4	9.29	4	1.3%
set2-30r5-68	8.81	3249	4	8.52	4	3.4%
set2-40r1-120	7.74	2921	3	7.23	3	7.0%
set2-40r2-120	8.90	3612	4	8.54	4	4.3%
set2-40r3-120	7.92	6197	3	7.66	3	3.4%
set2-40r5-120	9.59	2829	4	9.24	4	3.8%
set2-40r1-90	8.95	4432	4	8.36	4	7.0%
set2-40r2-90	10.43	1497	5	10.09	5	3.3%
set2-40r3-90	9.18	4639	4	9.18	4	0.1%
set2-40r5-90	11.49	3823	5	10.96	5	4.9%
set2-60r1-180	9.24	6664	3	7.91	3	16.9%
set2-60r5-180	9.34	5319	4	8.47	3	10.3%
set2-60r2-135	11.89	5811	5	11.73	6	1.3%
set2-8r3-10	8.28	29	5	-	-	-
set2-8r4-10	5.32	29	5	-	-	-

Table 2: Routes Cost Comparison between Set Partitioning Model and SYMPHONY - Instances Set 2 (continued)

	Set Partitioning		SYMPHONY			
Instance name	Cost	No. Routes Generated	k	Cost	k	% Cost diff
set2-8r5-10	7.39	27	5	-	-	-
set2-20r4-45	8.19	974	5	-	-	-
set2-30r2-90	7.30	2804	3	7.28	3	0.2%
set2-30r4-90	7.88	3658	3	7.60	3	3.7%
set2-30r4-68	9.28	1606	4	9.14	4	1.5%
set2-40r4-120	8.40	5649	3	7.82	3	7.5%
set2-40r4-90	9.72	5145	4	9.44	4	3.0%
set2-60r2-180	10.10	6151	4	9.90	4	2.0%
set2-60r3-180	9.10	6392	3	8.67	3	4.9%
set2-60r4-180	8.73	6822	4	8.19	3	6.7%
set2-60r4-135	9.89	6090	4	9.98	5	-0.9%
set2-60r1-135	9.79	6556	4	-	-	-
set2-60r3-135	11.10	5795	4	-	-	-
set2-60r5-135	11.69	5053	6	-	-	-

2 Tables Related to Section 4.2 of the Paper

Table 3 summarizes statistical results from an Analysis of Variance (ANOVA) procedure for comparing the *implementation cost* of the route plans generated by the stochastic set-partitioning based model with extended recourse (SP-ER), the stochastic set-partitioning based model with traditional recourse (SP-TR), and a deterministic VRP model that employs the expected customer demands (VRP). More details about the comparison and the two experiments performed are in Section 4.2 of the paper. Again, the solution to the VRP model is produced by the SYMPHONY branch and cut implementation described in Ralphs et al. (2003).

The models comparison is in terms of total cost (i.e.first plus second stage costs). To estimate the cost of a route plan from any of the models, 1000 demand scenarios are randomly generated for each instance. The base effect selected for the ANOVA

comparison is the deterministic VRP solved with SYMPHONY and it is notated as (-). The relative percentages of change in cost vs. SYMPHONY presented in Table 3 result from dividing the absolute effects over the average cost computed over all the 36 instances studied. In experiment 1 (23 instances), SP-ER performs about the same as SP-TR and better than SYMPHONY. Instances in experiment 1 leave several customers unserved per route so completions occur sparingly while extra-trips are the preferred recourse actions. In experiment 2 (13 instances), SP-ER performs better than both SP-TR and SYMPHONY. In these instances, routes complete successfully the few unserved customers left avoiding extra-trips. Results from both experiments encourage the use of SP-ER over deterministic models for solving VRP's with stochastic demands.

Table 3: ANOVA results

Feature	Experiment 1	Experiment 2
Number of Observations	69000	39000
Average Total Cost	8.18	23.59
Absolute Effects:		
SP-ER	-0.2507	-0.5176
SP-TR	-0.2237	0.6469
SYMPHONY	(-)	(-)
Relative % of change in cost:		
% SP-ER vs. SYMPHONY	-3.06%	-2.19%
% SP-ER vs. SP-TR	-0.33%	-4.94%
% SP-TR vs. SYMPHONY	-2.73%	2.74%

Tables 4 and 5 show detailed results for each model and instance. The results are the average *implementation* costs (fist stage plus second stage cost), number of routes in the first-stage solution, and filling coefficient $f = \frac{\sum_{i=1}^{n} \mathbb{E}[d_i]}{kQ}$. In the previous formula, k represents number of vehicles in the solution, d_i is the demand for customer i, n is the total number of customers in the instance, and Q is the vehicle capacity. The filling coefficient represents the amount of expected demand relative to total vehicle capacity. Values of f

near to one indicate that the instance has a considerable probability of failure. The last two columns in the tables present the percentage difference in average total cost for SP-ER vs. SYMPHONY and for SP-ER vs. SP-TR. We recall that the average cost is taken over 1000 simulation replicates by instance and model type and that an instance labeled as exper1-8r2-9 corresponds to the second replication (r2) (i.e. a particular assignment of geographical locations and demand distributions for each customer) of an instance in experiment 1 with 8 customers and vehicle capacity of 9.

The results in Table 4 show that SP-ER ranks as the model with the lowest total cost in 16 out of the 23 instances tested (69.6%) in experiment 1. The results in Table 5 show that SP-ER ranks as the model with the lowest total cost in 12 out of the 13 instances tested (92.3%) in experiment 2.

Table 4: Detailed Numerical Results for Experiment 1

				SP-ER vs.	
Instance	SP-TR	SP-ER	SYMPHONY	SYMPHONY	SP-TR
exper1-5r3-5	7.14	7.21	7.16	0.68%	0.92%
	4	4	4		
	0.80	0.80	0.80		
exper1-5r4-5	5.32	5.42	5.32	1.84%	1.87%
	4	4	4		
	0.75	0.75	0.75		
exper1-5r5-5	7.75	7.75	7.75	0.00%	-0.01%
	4	4	4		
	0.80	0.80	0.80		
exper1-8r2-9	6.86	6.65	6.78	-1.94%	-2.98%
	3	3	3		
	0.89	0.89	0.89		
exper2-8r3-10	9.56	9.32	10.29	-9.48%	-2.53%
	5	5	4		
	0.78	0.78	0.98		
exper2-8r3-13	7.33	7.16	7.21	-0.66%	-2.23%
	4	4	4		
	0.75	0.75	0.75		
exper2-8r4-10	6.58	6.59	7.48	-11.96%	0.04%
	5	5	4		

Table 4: Detailed Numerical Results for Experiment 1 (continuation)

Instance	SP-TR	SP-ER	SYMPHONY	SPER vs. SYMPHONY	SPER vs. SP-TR
	0.80	0.80	1.00		
exper2-8r5-10	8.93	8.82	9.85	-10.44%	-1.14%
	5	5	4		
	0.80	0.80	1.00		
exper1-20r1-58	9.10	9.61	9.26	3.73%	5.55%
	4	4	4		
	0.75	0.75	0.75		
exper1-20r1-91	6.74	6.72	6.72	-0.08%	-0.42%
	2	3	2		
	0.96	0.64	0.96		
exper1-20r2-58	7.49	7.43	7.58	-1.98%	-0.85%
	3	3	3		
	0.72	0.72	0.72		
exper1-20r4-58	6.74	6.72	7.24	-7.26%	-0.43%
	3	3	3		
	0.93	0.93	0.93		
exper1-20r5-58	8.14	8.49	8.00	6.12%	4.30%
	4	4	4		
	0.78	0.78	0.78		
exper2-20r1-45	12.09	11.74	11.86	-0.97%	-2.87%
	5	5	5		
	0.87	0.87	0.87		
exper2-20r1-60	9.70	9.62	10.23	-5.94%	-0.78%
	4	4	4		
	0.81	0.81	0.81		
exper2-20r2-60	7.63	7.78	8.12	-4.18%	2.03%
	3	3	3		
	0.81	0.81	0.81		
exper2-20r3-45	8.34	8.44	8.48	-0.46%	1.19%
	4	4	4		
	0.92	0.92	0.92		
exper2-20r3-60	6.95	6.77	6.94	-2.42%	-2.52%
	3	3	3		
	0.92	0.92	0.92		
exper2-20r4-45	8.90	8.85	10.02	-11.70%	-0.58%
	5	5	4		
	0.80	0.80	1.00		
exper2-20r5-45	10.59	10.51	10.90	-3.54%	-0.70%

Table 4: Detailed Numerical Results for Experiment 1 (continuation)

Instance	SP-TR	SP-ER	SYMPHONY	SPER vs. SYMPHONY	SPER vs. SP-TR
	5	5	5		
	0.89	0.89	0.89		
exper2-20r5-60	8.56	8.36	8.56	-2.31%	-2.37%
	4	4	4		
	0.83	0.83	0.83		
exper1-30r3-87	8.20	8.26	8.13	1.59%	0.69%
	3	3	3		
	0.86	0.86	0.86		
exper2-30r5-90	7.89	7.69	7.79	-1.21%	-2.51%
	3	3	3		
	0.83	0.83	0.83		

Table 5: Detailed Numerical Results for Experiment 2

Instance	SP-TR	SP-ER	SYMPHONY	SP-ER vs. SYMPHONY	SP-ER vs. SP-TR
exper1-8r1-6	8.43	7.93	8.27	-4.12%	-5.85%
	4	5	4		
	1.00	0.80	1.00		
exper1-8r5-6	8.77	8.66	8.43	2.79%	-1.20%
	5	5	5		
	0.80	0.80	0.80		
exper1-20r1-16	27.59	26.70	26.84	-0.51%	-3.23%
	13	13	13		
	0.84	0.84	0.84		
exper1-20r3-16	17.02	16.34	16.67	-1.98%	-4.00%
	10	10	11		
	0.91	0.91	0.82		
exper1-20r4-16	19.94	18.73	18.86	-0.69%	-6.04%
	12	11	11		
	0.84	0.92	0.92		
exper1-20r5-16	24.68	23.56	24.03	-1.94%	-4.52%
	13	12	13		
	0.87	0.94	0.87		
exper2-20r1-18	27.86	27.03	27.26	-0.86%	-2.99%
	13	13	14		
	0.83	0.83	0.77		
exper2-20r3-18	17.20	16.67	17.19	-3.04%	-3.09%

Table 5: Detailed Numerical Results for Experiment 2 (continuation)

Instance	SP-TR	SP-ER	SYMPHONY	SP-ER vs. SYMPHONY	SP-ER vs. SP-TR
	11	10	11		
	0.83	0.92	0.83		
exper2-20r4-18	20.10	18.91	19.15	-1.26%	-5.90%
	12	12	11		
	0.83	0.83	0.91		
exper1-30r4-16	31.39	30.45	30.88	-1.41%	-2.98%
	17	17	18		
	0.88	0.88	0.83		
exper2-30r1-18	31.28	29.53	30.38	-2.81%	-5.62%
	19	17	18		
	0.82	0.92	0.86		
exper2-30r4-18	32.90	30.78	31.21	-1.37%	-6.44%
	18	18	17		
	0.83	0.88	0.83		
exper2-40r5-18	47.38	44.09	46.94	-6.07%	-6.94%
	26	24	25		
	0.81	0.88	0.84		

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