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EXISTENCE AND UNIQUENESS OF MILD AND CLASSICAL SOLUTIONS OF IMPULSIVE EVOLUTION EQUATIONS

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Abstract. We consider the non-linear impulsive evolution equation

$$u'(t) = Au(t) + f(t, u(t), Tu(t), Su(t)), \quad 0 < t < T_0,\ t \neq t_i,$$
$$u(0) = u_0,$$
$$\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, 3, \dots, p.$$

in a Banach space X, where A is the infinitesimal generator of a C_0 semigroup. We study the existence and uniqueness of the mild solutions of the evolution equation by using semigroup theory and then show that the mild solutions give rise to a classical solutions.

1. Introduction

The theory of impulsive differential equations is an important branch of differential equations, which has an extensive physical background. Impulsive differential equations arise frequently in the modelling many physical systems whose states are subjects to sudden change at certain moments, for example, in population biology, the diffusion of chemicals, the spread of heat, the radiation of electromagnetic waves and etc.,(see [1, 4, 11]).

Existence of solutions of impulsive differential equation of the form

$$u'(t) = f(t, u(t), Tu(t), Su(t)), \quad 0 < t < T_0,\ t \neq t_i, \tag{1.1}$$
$$u(0) = u_0, \tag{1.2}$$
$$\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, 3, \dots, p. \tag{1.3}$$

has been studied by many authors [2, 3, 12]. In the special case where f is uniformly continuous, Guo and Liu [2] established existence theorems of maximal and minimal solutions for (1.1)–(1.3) with strong conditions. Guo and Liu [7], Liu [8] obtained the same conclusion applying the monotone iterative technique when f does not contain integral operator S in (1.1). But they did not obtain a unique solution for (1.1)–(1.3). Recently, in the special case where (1.1)–(1.3) has no impulses, Liu [8] obtained a unique solution by the monotone iterative technique with coupled upper and lower quasi-solutions when $f = f(t, u, u, Tu, Su)$. A similar conclusion was obtained by Liu [6]. However, one of the require assumptions in [2, 7, 8] is that

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f satisfies some compactness-type conditions, which as we know is difficult and inconvenient to verify in abstract spaces. Rogovchenko [10], studied the existence and uniqueness of the classical solutions by the successive approximations for the evolution equation with an unbounded operator A; i.e, equations of the form

$$u'(t) = Au(t) + f(t, u(t)), \quad t > 0,\ t \neq t_i,$$

with impulsive condition in (1.2)-(1.3), where A is sectorial operator with some conditions given on the fractional operators $A^\alpha, \alpha \geq 0$.

Liu [5] studied the existence of mild solutions of the impulsive evolution equation

$$u'(t) = Au(t) + f(t, u(t)), \quad 0 < t < T_0,\ t \neq t_i,$$

where A is the infinitesimal generator of C_0 semigroup with the impulsive condition in (1.2)-(1.3) by using semigroup theory.

In this paper we study the existence and uniqueness of mild solutions for the nonlinear impulsive evolution equation

$$\begin{gathered} u'(t) = Au(t) + f(t, u(t), Tu(t), Su(t)), \quad 0 < t < T_0,\ t \neq t_i, \\ u(0) = u_0, \\ \Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, 3, \dots, p. \end{gathered}$$

in a Banach space X, where A is the infinitesimal generator of C_0 semigroup $\{G(t) | t \geq 0\}$. Then we prove that the existence and uniqueness of mild solutions give rise to the existence and uniqueness of classical solutions if f which is continuously differentiable.

2. Preliminaries and Hypotheses

Let X be a Banach space. Let $PC([0, T_0], X)$ consist of functions u that are a map from $[0, T_0]$ into X, such that $u(t)$ is continuous at $t \neq t_i$ and left continuous at $t = t_i$, and the right limit $u(t_i^+)$ exists for $i = 1, 2, 3, \dots p$. Evidently $PC([0, T_0], X)$ is a Banach space with the norm

$$\|u\|_{PC} = \sup_{t \in [0, T_0]} \|u(t)\| . \tag{2.1}$$

Consider the impulsive evolution equation of the form

$$u'(t) = Au(t) + f(t, u(t), Tu(t), Su(t)), \quad 0 < t < T_0,\ t \neq t_i, \tag{2.2}$$

$$u(0) = u_0, \tag{2.3}$$

$$\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, 3, \dots, p. \tag{2.4}$$

in a Banach space X, where $f \in C([0, T_0] \times X \times X \times X, X)$,

$$Tu(t) = \int_0^t K(t, s) u(s) \mathrm{d}s,\ K \in C[D, R^+], \tag{2.5}$$

$$Su(t) = \int_0^{T_0} H(t, s) u(s) \mathrm{d}s,\ H \in C[D_0, R^+], \tag{2.6}$$

where $D = \{(t, s) \in R^2 : 0 \leq s \leq t \leq T_0\}$, $D_0 = \{(t, s) \in R^2 : 0 \leq t, s \leq T_0\}$ and $0 < t_1 < t_2 < t_3 < \dots < t_i < \dots < t_p < T_0$, $\Delta u(t_i) = u(t_i^+) - u(t_i^-)$.

We assume the following hypotheses:

(H1) $f : [0, T_0] \times X \times X \times X \to X$, and $I_i : X \to X$, $i = 1, 2, \dots, p$. are continuous and there exists constants $L_1, L_2, L_3 > 0$, $h_i > 0$, $i = 1, 2, 3, \dots, p$. such that

$$\begin{aligned} &\|f(t, u_1, u_2, u_3) - f(t, v_1, v_2, v_3)\| \\ &\le L_1\|u_1 - v_1\| + L_2\|u_2 - v_2\| + L_3\|u_3 - v_3\|, \quad t \in [0, T_0],\ u, v \in X; \end{aligned} \tag{2.7}$$

$$\|I_i(u) - I_i(v)\| \le h_i\|u - v\|, \quad u, v \in X. \tag{2.8}$$

Let $G(\cdot)$ be the C_0 semigroup generated by the unbounded operator A. Let $B(X)$ be the Banach space of all linear and bounded operators on X. Let

$$M = \max_{t\in[0,T_0]} \|G(t)\|_{B(X)}, \quad L = \max\{L_1, L_2, L_3\}\,.$$

$$K^* = \sup_{t\in[0,T_0]} \int_0^t |K(s, t)|\mathrm{d}t < \infty, \quad H^* = \sup_{t\in[0,T_0]} \int_0^{T_0} |H(s, t)|\mathrm{d}t < \infty$$

(H2) The constants $L, L_1, L_2, L_3, K^*, H^*$ satisfy the inequality

$$M\Big[LT_0(1 + K^* + H^*) + \sum_{i=1}^{p} h_i\Big] < 1$$

3. Existence Theorems

3.1. **Mild solution.** A function $u(\cdot) \in PC([0, T_0], X)$ is a mild solution of equations (2.2)–(2.4) if it satisfies

$$\begin{aligned} u(t) = G(t)u_0 &+ \int_0^t G(t - s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s \\ &+ \sum_{0<t_i<t} G(t - t_i)I_i(u(t_i)),\ 0 \le t \le T_0. \end{aligned} \tag{3.1}$$

Theorem 3.1. *Assume that (H1)-(H2) are satisfied. Then for every $u_0 \in X$, for $t \in [0, T_0]$ the equation*

$$u(t) = G(t)u_0 + \int_0^t G(t - s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s + \sum_{0<t_i<t} G(t - t_i)I_i(u(t_i)), \tag{3.2}$$

has a unique solution.

Proof. Let $u_0 \in X$ be fixed. Define an operator F on $PC([0, T_0], X)$ by

$$\begin{aligned} (Fu)(t) = G(t)u_0 &+ \int_0^t G(t - s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s \\ &+ \sum_{0<t_i<t} G(t - t_i)I_i(u(t_i)),\ 0 \le t \le T_0 \end{aligned} \tag{3.3}$$

Then it is clear that $F : PC([0, T_0], X) \to PC([0, T_0], X)$. Now we show that F is contraction. For any $u, v \in PC([0, T_0], X)$,

$$\begin{aligned} \|(Fu)(t) - (Fv)(t)\| \le &\int_0^t \|G(t - s)\|_{B(X)}\|f(s, u(s), Tu(s), Su(s)) \\ &- f(s, v(s), Tv(s), Sv(s))\|\mathrm{d}s \\ &+ \sum_{0<t_i<t} \|G(t - t_i)\|_{B(X)}\|I_i(u(t_i)) - I_i(v(t_i))\| \end{aligned} \tag{3.4}$$

Using Assumption (H1) and definition of M, we have

$$
\begin{aligned}
&\|(Fu)(t)-(Fv)(t)\| \\
&\le M\int_0^t \|f(s,u(s),Tu(s),Su(s)-f(s,v(s),Tv(s),Sv(s)\|\mathrm{d}s \\
&\quad + M\sum_{0<t_i<t}\|I_i(u(t_i))-I_i(v(t_i))\| \\
&\le M\Big[\int_0^t L_1\|u-v\|+L_2\|Tu-Tv\|+L_3\|Su-Sv\|\Big]\mathrm{d}s \\
&\quad + M\|u-v\|_{PC}\sum_{i=1}^p h_i
\end{aligned}
\tag{3.5}
$$

Now,

$$
\begin{aligned}
\int_0^t L_2\|Tu-Tv\|\mathrm{d}s &\le L_2\int_0^t\int_0^s \|K(s,\tau)\|\|u(\tau)-v(\tau)\|\mathrm{d}\tau\mathrm{d}s \\
&\le L_2\int_0^t \|u(s)-v(s)\|\int_0^s \|K(s,\tau)\|\mathrm{d}\tau\mathrm{d}s \\
&\le L_2\|u(t)-v(t)\|\int_0^t K^*\mathrm{d}s \\
&\le L_2\|u-v\|_{PC}K^*T_0
\end{aligned}
\tag{3.6}
$$

Similarly,

$$
\int_0^t L_3\|Su-Sv\|\mathrm{d}s \le L_3\|u-v\|_{PC}H^*T_0 \tag{3.7}
$$

Substitute (3.6), (3.7) in (3.5), we have

$$
\begin{aligned}
&\|(Fu)(t)-(Fv)(t)\| \\
&\le M\Big[L_1T_0\|u-v\|_{PC}+L_2T_0\|u-v\|_{PC}K^*+L_3T_0\|u-v\|_{PC}H^* \\
&\quad + M\|u-v\|_{PC}\sum_{i=1}^p h_i\Big] \\
&\le M\Big[L_1T_0+L_2T_0K^*+L_3T_0H^*+\sum_{i=1}^p h_i\Big]\|u-v\|_{PC}\,.
\end{aligned}
\tag{3.8}
$$

Using the definition of L, we have

$$
\begin{aligned}
\|Fu-Fv\|_{PC} &= \max_{t\in[0,T_0]}\|Fu(t)-Fv(t)\| \\
&\le M\Big[LT_0(1+K^*+H^*)+\sum_{i=1}^p h_i\Big]\|u-v\|_{PC}.
\end{aligned}
\tag{3.9}
$$

Now from Assumption (H2), we have

$$
\|(Fu)(t)-(Fv)(t)\| \le \|u-v\|_{PC}, \quad u,v\in PC([0,T_0],X). \tag{3.10}
$$

Therefore, F is a contraction operator on $PC([0,T_0],X)$. This completes the proof. ☐

Next we study the classical solutions. First we give the definition.

3.2. **Classical solutions.** A classical solution of Equations (2.2)–(2.4) is a function $u(\cdot)$ in $PC([0,T_0],X)\cap C^1((0,T_0)\backslash\{t_1,t_2,\dots,t_p\},X)$, $u(t)\in D(A)$ for $t\in(0,T_0)\backslash\{t_1,t_2,\dots,t_p\}$, which satisfies equations (2.2)–(2.4) on $[0,T_0]$.

To prove the main theorem we need the following two Lemmas.

Lemma 3.2. *Consider the evolution equation*

$$u'(t)=Au(t)+f(t,u(t),Tu(t),Su(t)),\quad t_0<t<T_0, \tag{3.11}$$

$$u(0)=u_0, \tag{3.12}$$

If $u_0\in D(A)$, and $f\in C^1((0,T_0)\times X\times X\times X,X)$, then it has a unique classical solution, which satisfies

$$u(t)=G(t-t_0)u_0+\int_{t_0}^{t}G(t-s)f(s,u(s),Tu(s),Su(s)\mathrm{d}s,\quad t\in[t_0,T_0). \tag{3.13}$$

The above lemma can be proved easily using the [9, Theorem 6.1.5].

Lemma 3.3. *Assume hypotheses (H1)-(H2) are satisfied. Then for the unique classical solution $u(\cdot)=u(\cdot,u_0)$ on $[0,t_1)$ of equations (2.2)-(2.3) without the impulsive conditions (guaranteed by Lemma 3.2), one can define $u(t_1)$ in such a way that $u(\cdot)$ is left continuous at t_1 and $u(t_1)\in D(A)$.*

Proof. Consider the following evolution equation without the impulsive condition on $(0,T_0)$,

$$w'(t)=Aw(t)+f(t,w(t),Tw(t),Sw(t)),\qquad 0<t<T_0,$$
$$w(0)=w_0,$$

From Lemma 3.2, there is a classical solution given by

$$w(t)=G(t)u_0+\int_0^t G(t-s)f(s,w(s),Tw(s),Sw(s))\mathrm{d}s,\quad t\in[0,T_0). \tag{3.14}$$

and $w(t)\in D(A)$ for $t\in[0,T_0)$. Next, applying Lemma 3.2 to the function $u(\cdot)$, one has, for $t\in[0,t_1)\subset[0,T_0)$,

$$u(t)=G(t)u_0+\int_0^t G(t-s)f(s,u(s),Tu(s),Su(s))\mathrm{d}s,\quad t\in[0,t_1), \tag{3.15}$$

Now, we can define

$$u(t_1)=G(t_1)u_0+\int_0^{t_1} G(t_1-s)f(s,u(s),Tu(s),Su(s))\mathrm{d}s, \tag{3.16}$$

So that $u(\cdot)$ is left continuous at t_1. Then apply Lemma 3.2 on $[0,t_1]$ to get

$$u(t)=w(t),\quad t\in[0,t_1]. \tag{3.17}$$

Thus we have, $u(t_1)=w(t_1)\in D(A)$ which completes the proof. □

Before proving the main theorem, we prove the following theorem.

Theorem 3.4. *Assume that $u_0\in D(A)$, $q_i\in D(A)$, $i=1,2,\dots,p$. and that $f\in C^1((0,T_0)\times X\times X\times X,X)$. Then the impulsive equation*

$$u'(t)=Au(t)+f(t,u(t),Tu(t),Su(t)),\quad 0<t<T_0,\ t\neq t_i, \tag{3.18}$$

$$u(0)=u_0, \tag{3.19}$$

$$\Delta u(t_i)=q_i,\quad i=1,2,3,\dots,p. \tag{3.20}$$

has a unique classical solution $u(\cdot)$ which, for $t \in [0, T_0)$, satisfies

$$u(t) = G(t)u_0 + \int_0^t G(t-s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s + \sum_{0<t_i<t} G(t-t_i)q_i. \quad (3.21)$$

Proof. Consider the interval on $J_1 = [0, t_1)$, apply Lemma 3.2 to the equation

$$u'(t) = Au(t) + f(t, u(t), Tu(t), Su(t)), \quad 0 < t < t_1, \quad (3.22)$$

$$u(0) = u_0, \quad (3.23)$$

has a unique classical solution $u_1(\cdot)$ which satisfies

$$u_1(t) = G(t)u_0 + \int_0^t G(t-s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s, \quad t \in [0, t_1), \quad (3.24)$$

Now, define

$$u_1(t_1) = G(t_1)u_0 + \int_0^{t_1} G(t_1-s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s, \quad (3.25)$$

Applying Lemma 3.3, we see that $u_1(\cdot)$ is left continuous at t_1, and $u_1(t_1) \in D(A)$. Next on $J_2 = [t_1, t_2)$, consider the equation

$$u'(t) = Au(t) + f(t, u(t), Tu(t), Su(t)), \quad t_1 < t < t_2, \quad (3.26)$$

$$u(t_1) = u_1(t_1) + q_1, \quad (3.27)$$

Since $u_1(t_1) + q_1 \in D(A)$, once again we can use Lemma 3.2 again to get a unique classical solution $u_2(\cdot)$ which satisfies

$$u_2(t) = G(t-t_1)[u(t_1)] + \int_{t_1}^t G(t-s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s. \quad (3.28)$$

Now, define

$$u_2(t_2) = G(t_2-t_1)[u_1(t_1) + q_1] + \int_{t_1}^{t_2} G(t_2-s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s. \quad (3.29)$$

Therefore, $u_2(\cdot)$ is left continuous at t_2 and $u_2(t_2) \in D(A)$ using Lemma 3.3. Continuous in this process on $J_k = [t_{k-1}, t_k)$, $(k = 3, 4, 5, \dots, p+1)$ to get the classical solutions

$$\begin{aligned} u_k(t) &= G(t-t_{k-1})[u_{k-1}(t_{k-1}) + q_{k-1}] \\ &\quad + \int_{t_{k-1}}^t G(t-s)f(s, u(s), Tu(s), Su(s))\mathrm{d}s. \end{aligned} \quad (3.30)$$

for $t \in [t_{k-1}, t_k)$, with $u_i(\cdot)$ left continuous at t_i and $u_i(t_i) \in D(A)$, $i = 1, 2, \dots, p$. Now, define

$$u(t) = \begin{cases} u_1(t) & 0 \le t \le t_1, \\ u_k(t) & t_k - 1 < t \le t_k, k = 2, 3, \dots, p. \\ u_{k+1}(t) & t_p < t < t_{p+1} = T_0. \end{cases} \quad (3.31)$$

Therefore, $u(\cdot)$ is the unique classical solution of equations (3.18)–(3.20). Using induction method we show that (3.21) is satisfied on $[0, T_0)$. First (3.21) is satisfied on $[0, t_1]$. If (3.21) is satisfied on $(t_{k-1}, t_k]$, then for $t \in (t_k, t_{k+1}]$,

$$
\begin{aligned}
&u(t) \\
&= u_{k+1}(t) = G(t-t_k)[u_k(t_k) + q_k] \\
&\quad + \int_{t_k}^{t} G(t-s) f(s, u_{k+1}(s), Tu_{k+1}(s), Su_{k+1}(s))\mathrm{d}s \\
&= G(t-t_k)[G(t_k)u_0 + \int_0^{t_k} G(t_k - s) f(s, u(s), Tu(s), Su(s))\mathrm{d}s \\
&\quad + \sum_{0<t_i<t_k} G(t_k - t_i)q_i + q_k] \\
&\quad + \int_{t_k}^{t} G(t-s) f(s, u_{k+1}(s), Tu_{k+1}(s), Su_{k+1}(s))\mathrm{d}s \\
&= G(t-t_k)G(t_k)u_0 + \int_0^{t_k} G(t-s) f(s, u(s), Tu(s), Su(s))\mathrm{d}s \\
&\quad + \sum_{0<t_i<t_k} G(t-t_i)q_i + G(t-t_k)q_k + \int_{t_k}^{t} G(t-s) f(s, u(s), Tu(s), Su(s))\mathrm{d}s \\
&= G(t)u_0 + \int_0^t G(t-s) f(s, u(s), Tu(s).Su(s))\mathrm{d}s + \sum_{0<t_i<t} G(t-t_i)q_i.
\end{aligned}
$$

Thus (3.21) is also true on $(t_k, t_{k+1}]$. Therefore, (3.21) is true on $[0, T_0)$, which completes the proof. □

Next Theorem gives the proof of the main theorem.

Theorem 3.5. *Assume the hypotheses (H1)-(H2) are satisfied. Let $u(\cdot) = u(\cdot, u_0)$ be the unique mild solution of (2.2)–(2.4) obtained in Theorem 3.1. Assume that $u_0 \in D(A)$, $I_i(u(t_i)) \in D(A)$, $i = 1, 2, \dots, p$, and that $f \in C^1((0, T_0) \times X \times X \times X, X)$. Then $u(\cdot)$ gives rise to a unique classical solution of (2.2)–(2.4).*

Proof. . Let $u(\cdot)$ be the mild solution. Let $q_i = I_i(u(t_i))$, $i = 1, 2, \dots, p$. Then from Theorem 3.4, equation (3.18)-(3.20) has a unique classical solution $w(\cdot)$ which satisfies for $t \in [0, T_0)$

$$
\begin{aligned}
w(t) = G(t)u_0 &+ \int_0^t G(t-s) f(s, w(s), Tw(s), Sw(s))\mathrm{d}s \\
&+ \sum_{0<t_i<t} G(t-t_i) I_i(w(t_i))
\end{aligned}
$$

Since $u(\cdot)$ is the mild solution of (2.2)–(2.4), for $t \in [0, T_0]$,

$$
\begin{aligned}
u(t) = G(t)u_0 &+ \int_0^t G(t-s) f(s, u(s), Tu(s), Su(s))\mathrm{d}s \\
&+ \sum_{0<t_i<t} G(t-t_i) I_i(u(t_i))
\end{aligned}
$$

Suppose $u(t)$ and $w(t)$ are two mild solutions, then

$$
w(t) - u(t) = \int_0^t G(t-s)[f(s, w(s), Tw(s), Sw(s)) - f(s, u(s), Tu(s), Su(s))]\mathrm{d}s.
$$

Since by Theorem 3.1, the mild solutions is unique, it follows that $w(t) - u(t) = 0$. This implies that $w(\cdot) = u(\cdot)$. This shows that $u(\cdot)$ is also a classical solution. This completes the proof. □

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