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OSCILLATION OF SECOND-ORDER NONLINEAR IMPULSIVE DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. In this article, we study the oscillation of second-order nonlinear impulsive dynamic equations on time scales. Using Riccati transformation techniques, we obtain sufficient conditions for oscillation of all solutions. An example is given to show that the impulses play a dominant part in oscillations of dynamic equations on time scales.

1. Introduction

This paper is concerned with the oscillations of second-order nonlinear impulsive dynamic equations on time scales. We consider the problem

$$\begin{gathered} y^{\Delta\Delta}(t) + f(t, y^{\sigma}(t)) = 0, \quad t \in \mathbb{J}_{\mathbb{T}} := [0, \infty) \cap \mathbb{T},\ t \neq t_k,\ k = 1, 2, \dots, \\ y(t_k^+) = g_k(y(t_k)), y^{\Delta}(t_k^+) = h_k(y^{\Delta}(t_k)), \quad k = 1, 2, \dots, \\ y(t_0^+) = y_0, \quad y^{\Delta}(t_0^+) = y_0^{\Delta}, \end{gathered} \tag{1.1}$$

where $\mathbb{T}$ is a time scale, unbounded-above, with $0 \in \mathbb{T}$, $t_k \in \mathbb{T}$, $0 \le t_0 < t_1 < t_2 < \dots < t_k < \dots$, $\lim_{k\to\infty} t_k = \infty$.

$$y(t_k^+) = \lim_{h\to 0^+} y(t_k + h), \quad y^{\Delta}(t_k^+) = \lim_{h\to 0^+} y^{\Delta}(t_k + h), \tag{1.2}$$

which represent right and left limits of $y(t)$ at $t = t_k$ in the sense of time scales, and in addition, if t_k is right scattered, then $y(t_k^+) = y(t_k), y^{\Delta}(t_k^+) = y^{\Delta}(t_k)$. We can defined $y(t_k^-), y^{\Delta}(t_k^-)$ similar to (1.2).

We always suppose that the following conditions hold:

- (H1) $f \in C_{rd}(\mathbb{T} \times \mathbb{R}, \mathbb{R})$, $xf(t, x) > 0$ $(x \neq 0)$ and $\frac{f(t,x)}{\varphi(x)} \ge p(t)$ $(x \neq 0)$, where $p(t) \in C_{rd}(\mathbb{T}, \mathbb{R}_+)$ and $x\varphi(x) > 0$ $(x \neq 0)$, $\varphi'(x) \ge 0$.
- (H2) $g_k, h_k \in C(\mathbb{R}, \mathbb{R})$ and there exist positive constants a_k, a_k^*, b_k, b_k^* such that
$$a_k^* \le \frac{g_k(x)}{x} \le a_k, \quad b_k^* \le \frac{h_k(x)}{x} \le b_k.$$

Throughout the remainder of the paper, we assume that, for each $k = 1, 2, \dots$, the points of impulses t_k are right dense (rd for short). In order to define the solutions

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of the problem (1.1), we introduce the space

$$AC^i = \big\{y : \mathbb{J}_{\mathbb{T}} \to \mathbb{R}\text{is } i\text{-times } \Delta\text{-differentiable, whose}$$
$$i\text{-th delta-derivative } y^{\Delta^{(i)}} \text{ is absolutely continuous}\big\}.$$

$$PC = \big\{y : \mathbb{J}_{\mathbb{T}} \to \mathbb{R} \text{ is right dense continuous at } t_k, k = 1, 2, \dots \text{ for which}$$
$$y(t_k^-), y(t_k^+), y^{\Delta}(t_k^-), y^{\Delta}(t_k^+) \text{ exist and } y(t_k^-) = y(t_k), y^{\Delta}(t_k^-) = y^{\Delta}(t_k)\big\}.$$

Definition 1.1. A function $y \in PC \bigcap AC^2(\mathbb{J}_{\mathbb{T}} \backslash \{t_1, \dots\}, \mathbb{R})$ is said to be a solution of (1.1), if it satisfies $y^{\Delta\Delta}(t) + f(t, y^{\sigma}(t)) = 0$ a.e. on $\mathbb{J}_{\mathbb{T}} \backslash \{t_k\}, k = 1, 2, \dots,$ and for each $k = 1, 2, \dots, y$ satisfies the impulsive condition $y(t_k^+) = g_k(y(t_k)), y^{\Delta}(t_k^+) = h_k(y^{\Delta}(t_k))$ and the initial condition $y(t_0^+) = y_0, y^{\Delta}(t_0^+) = y_0^{\Delta}$.

Definition 1.2. A solution y of (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. Equation (1.1) is called oscillatory if all solutions are oscillatory.

In recent years, the theory of dynamic equations on time scales, which provides powerful new tools for exploring connections between the traditionally separated fields, has been developing rapidly and has received much attention. We refer the reader to the book by Bohner and Peterson [4] and to the papers cited therein. The time scales calculus has a tremendous potential for applications in mathematical models of real processes, for instance, in biotechnology, chemical technology, economic, neural networks, physics, social sciences and so on, see the monographs of Aulbach and Hilger [2], Bohner and Perterson [4] and the references therein.

Very recently, impulsive dynamic equations on time scales have been investigate by Agarwal et al.[1], Belarbi et al.[5], Benchohra et al. $[6-9]$ and so forth. Benchohra et al [9]. considered the existence of extremal solutions for a class of second order impulsive dynamic equations on time scales, we can see that the existence of global solutions can be guaranted by some simple conditions. In [6], M.Benchohra et al. discuss the existence of oscillatory and nonoscillatory solutions for first order impulsive dynamic equations on time scales using lower and upper solutions method.

The oscillations of impulsive differential equations have been investigated by many authors and they gained many classical results. See Chen and Feng [10] and the papers cited therein. Using the method of Chen and Feng [10], the present paper is devoted to study the oscillations of a kind of very extensive second order impulsive nonlinear dynamic equations on time scales. An example is given to show that though a dynamic equations on time scales is nonoscillatory, it may become oscillatory if some impulses are added to it. That is, in some cases, impulses play a dominating part in oscillations of dynamic equations on time scales.

In the following, we always assume the solutions of (1.1) exist in $\mathbb{J}_{\mathbb{T}}$. Our attention is restricted to those solution y of (1.1) which exist on half line $\mathbb{J}_{\mathbb{T}}$ with $\sup\{|y(t)| : t \geq t_0\} \neq 0$ for any $t_0 \geq t_y$, where t_y is dependent on the solution y of (1.1). To the best of our knowledge, the question of the oscillations for second order nonlinear impulsive dynamic equations has not been yet considered. Hence, these results can be considered as a contribution to this field.

2. Main results

In this section, we give some new oscillation criteria for (1.1). In order to prove our main results, we need the following auxiliary result.

Lemma 2.1. *Suppose that* (H1)–(H2) *hold and $y(t) > 0$, $t \geq t_0' \geq t_0$ is a nonoscillatory solution of* (1.1). *If*

(H3)
$$(t_1 - t_0) + \frac{b_1^*}{a_1}(t_2 - t_1) + \frac{b_1^* b_2^*}{a_1 a_2}(t_3 - t_2) + \dots + \frac{b_1^* b_2^* \dots b_n^*}{a_1 a_2 \dots a_n}(t_{n+1} - t_n) + \dots = \infty,$$

then $y^\Delta(t_k^+) \geq 0$ and $y^\Delta(t) \geq 0$ for $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, where $t_k \geq t_0'$.

Proof. At first, we prove that $y^\Delta(t_k) \geq 0$ for $t_k \geq t_0'$, otherwise, there exists some j such that $t_j \geq t_0'$ and $y^\Delta(t_j) < 0$, hence

$$y^\Delta(t_j^+) = h_j(y^\Delta(t_j)) \leq b_j^* y^\Delta(t_j) < 0.$$

Let $y^\Delta(t_j^+) = -\alpha \quad (\alpha > 0)$. From (1.1), for $t \in (t_{j+i-1}, t_{j+i}]_{\mathbb{T}}, i = 1, 2, \dots,$ we obtain

$$y^{\Delta\Delta}(t) = -f(t, y^\sigma(t)) \leq -p(t)\varphi(y^\sigma(t)) \leq 0, \tag{2.1}$$

i.e. $y^\Delta(t)$ is nonincreasing in $(t_{j+i-1}, t_{j+i}]_{\mathbb{T}}, i = 1, 2, \dots,$ then

$$\begin{gathered} y^\Delta(t_{j+1}) \leq y^\Delta(t_j^+) = -\alpha < 0, \\ y^\Delta(t_{j+2}) \leq y^\Delta(t_{j+1}^+) = h_{j+1}(y^\Delta(t_{j+1})) \leq b_{j+1}^* y^\Delta(t_{j+1}) \leq -b_{j+1}^* \alpha < 0. \end{gathered} \tag{2.2}$$

It is easy to show that for any positive integer $n \geq 2$,

$$y^\Delta(t_{j+n}) \leq -b_{j+n-1}^* b_{j+n-2}^* \dots b_{j+1}^* \alpha < 0. \tag{2.3}$$

Now, we claim that for any positive integer $n \geq 2$,

$$\begin{aligned} y(t_{j+n}) \leq a_{j+n-1} a_{j+n-2} \dots a_{j+1} \Big[& y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}} \alpha(t_{j+2} - t_{j+1}) \\ & - \dots - \frac{b_{j+n-1}^* b_{j+n-2}^* \dots b_{j+1}^*}{a_{j+n-1} a_{j+n-2} \dots a_{j+1}} \alpha(t_{j+n} - t_{j+n-1}) \Big]. \end{aligned} \tag{2.4}$$

Since $y^\Delta(t)$ is nonincreasing in $(t_j, t_{j+1}]_{\mathbb{T}}$, hence

$$y^\Delta(t) \leq y^\Delta(t_j^+) \quad t \in (t_j, t_{j+1}]_{\mathbb{T}}. \tag{2.5}$$

Integrating (2.5) and using (2.2), we obtain

$$y(t_{j+1}) \leq y(t_j^+) + y^\Delta(t_j^+)(t_{j+1} - t_j) = y(t_j^+) - \alpha(t_{j+1} - t_j). \tag{2.6}$$

Similarly to (2.6) and using (H2), (2.2) and (2.6), we get

$$\begin{aligned} y(t_{j+2}) &\leq y(t_{j+1}^+) + y^\Delta(t_{j+1}^+)(t_{j+2} - t_{j+1}) \\ &= g_{j+1}(y(t_{j+1})) + h_{j+1}(y^\Delta(t_{j+1}))(t_{j+2} - t_{j+1}) \\ &\leq a_{j+1} y(t_{j+1}) + b_{j+1}^* y^\Delta(t_{j+1})(t_{j+2} - t_{j+1}) \\ &\leq a_{j+1} \big[y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}} \alpha(t_{j+2} - t_{j+1}) \big]. \end{aligned}$$

Then (2.4) holds for $n=2$. Now we suppose that (2.4) holds for $n=m$, i.e.

$$\begin{aligned} y(t_{j+m}) \le a_{j+1}a_{j+2}\dots a_{j+m-1}\Big[y(t_j^+) - \alpha(t_{j+1}-t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2}-t_{j+1}) \\ - \dots - \frac{b_{j+1}^* b_{j+2}^* \dots b_{j+m-1}^*}{a_{j+1}a_{j+2}\dots a_{j+m-1}}\alpha(t_{j+m}-t_{j+m-1})\Big], \end{aligned} \tag{2.7}$$

now we prove that (2.4) holds for $n=m+1$. Since $y^\Delta(t)$ is nonincreasing in $(t_{j+m}, t_{j+m+1}]_{\mathbb{T}}$, we have

$$y^\Delta(t) \le y^\Delta(t_{j+m}^+) \quad t \in (t_{j+m}, t_{j+m+1}]_{\mathbb{T}}.$$

Integrating and using (H2), (2.2), (2.3) and (2.7), we obtain

$$\begin{aligned} y(t_{j+m+1}) &\le y(t_{j+m}^+) + y^\Delta(t_{j+m}^+)(t_{j+m+1}-t_{j+m}) \\ &\le a_{j+m}y(t_{j+m}) + b_{j+m}^* y^\Delta(t_{j+m})(t_{j+m+1}-t_{j+m}) \\ &\le a_{j+1}a_{j+2}\dots a_{j+m}\Big[y(t_j^+) - \alpha(t_{j+1}-t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2}-t_{j+1}) \\ &\quad - \dots - \frac{b_{j+1}^* b_{j+2}^* \dots b_{j+m-1}^*}{a_{j+1}a_{j+2}\dots a_{j+m-1}}\alpha(t_{j+m}-t_{j+m-1})\Big] \\ &\quad - b_{j+1}^* b_{j+2}^* \dots b_{j+m}^* \alpha(t_{j+m+1}-t_{j+m}) \\ &= a_{j+1}a_{j+2}\dots a_{j+m}\Big[y(t_j^+) - \alpha(t_{j+1}-t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2}-t_{j+1}) \\ &\quad - \dots - \frac{b_{j+1}^* b_{j+2}^* \dots b_{j+m-1}^*}{a_{j+1}a_{j+2}\dots a_{j+m-1}}\alpha(t_{j+m}-t_{j+m-1}) \\ &\quad - \frac{b_{j+1}^* b_{j+2}^* \dots b_{j+m}^*}{a_{j+1}a_{j+2}\dots a_{j+m}}\alpha(t_{j+m+1}-t_{j+m})\Big]. \end{aligned}$$

Then (2.4) holds for $n=m+1$. By induction, (2.4) holds for any positive integer $n \ge 2$. (2.4) and (H3) is contrary to $y(t) > 0$. Therefore, $y^\Delta(t_k) \ge 0$ $(t_k \ge t_0')$. From (H2), we get for any $t_k \ge t_0', y^\Delta(t_k^+) \ge b_k^* y^\Delta(t_k) \ge 0$. Since $y^\Delta(t)$ is nonincreasing in $(t_k, t_{k+1}]_{\mathbb{T}}$, we know $y^\Delta(t) \ge y^\Delta(t_{k+1}) \ge 0, t \in (t_k, t_{k+1}]_{\mathbb{T}}$. The proof of Lemma 2.1 is complete. □

Remark 2.2. In the case of $y(t)$ is eventually negative, under the hypothesis (H1)–(H3), it can be proved similarly that $y^\Delta(t_k^+) \le 0$ and for $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, $y^\Delta(t) \le 0$ for $t_k \ge T$.

Theorem 2.3. *Suppose that* (H1)–(H3) *hold and there exists a positive integer* k_0 *such that* $a_k^* \ge 1$ *for* $k \ge k_0$. *If*

$$\begin{aligned} &\int_{t_0}^{t_1} p(t)\Delta t + \frac{1}{b_1}\int_{t_1}^{t_2} p(t)\Delta t + \frac{1}{b_1 b_2}\int_{t_2}^{t_3} p(t)\Delta t \\ &+ \dots + \frac{1}{b_1 b_2 \dots b_n}\int_{t_n}^{t_{n+1}} p(t)\Delta t + \dots = \infty, \end{aligned} \tag{2.8}$$

then (1.1) *is oscillatory.*

Proof. Suppose to the contrary that (1.1) has a nonoscillatory solution $y(t)$, without loss of generality, we may assume that $y(t)$ is eventually positive solution of (1.1), i.e. $y(t) > 0, t \geq t_0$ and $k_0 = 1$. From lemma 2.1, we have $y^{\Delta}(t) \geq 0$, $t \in (t_k, t_{k+1}]_{\mathbb{T}}$, $k = 1, 2, \dots$. Let

$$w(t) = \frac{y^{\Delta}(t)}{\varphi(y(t))}, \tag{2.9}$$

then $w(t_k^+) \geq 0$, $k = 1, 2, \dots$ and $w(t) > 0, t \geq t_0$. Using (H1) and (1.1), we get when $t \neq t_k$,

$$\begin{aligned} w^{\Delta}(t) &= -\frac{f(t, y^{\sigma}(t))}{\varphi(y^{\sigma}(t))} - \frac{y^{\Delta}(t)}{\varphi(y(t))\varphi(y^{\sigma}(t))} \int_0^1 \varphi'(y(t) + h\mu(t)y^{\Delta}(t))dhy^{\Delta}(t) \\ &\leq -p(t) - \frac{\varphi(y(t))}{\varphi(y^{\sigma}(t))}\Big(\frac{y^{\Delta}(t)}{\varphi(y(t))}\Big)^2 \int_0^1 \varphi'(y(t) + h\mu(t)y^{\Delta}(t))dh \\ &\leq -p(t), \end{aligned} \tag{2.10}$$

since $\varphi'(y(t)) \geq 0$ and $\varphi(y(t)) > 0$. From (H2) and $a_k^* \geq 1$, we obtain

$$w(t_k^+) = \frac{y^{\Delta}(t_k^+)}{\varphi(y(t_k^+))} \leq \frac{b_k y^{\Delta}(t_k)}{\varphi(a_k^* y(t_k))} \leq \frac{b_k y^{\Delta}(t_k)}{\varphi(y(t_k))} = b_k w(t_k), \quad k = 1, 2, \dots. \tag{2.11}$$

Integrating (2.10), we have

$$w(t_1) \leq w(t_0^+) - \int_{t_0}^{t_1} p(t)\Delta t. \tag{2.12}$$

Using (2.11) and (2.12), we obtain

$$w(t_1^+) \leq b_1 w(t_1) \leq b_1 w(t_0^+) - b_1 \int_{t_0}^{t_1} p(t)\Delta t.$$

Similarly, we get

$$\begin{aligned} w(t_2^+) &\leq b_2 w(t_2) \leq b_2\Big[w(t_1^+) - \int_{t_1}^{t_2} p(t)\Delta t\Big] \\ &\leq b_1 b_2 w(t_0^+) - b_1 b_2 \int_{t_0}^{t_1} p(t)\Delta t - b_2 \int_{t_1}^{t_2} p(t)\Delta t. \end{aligned} \tag{2.13}$$

By induction, for any positive integer n, we have

$$\begin{aligned} w(t_n^+) &\leq b_1 b_2 \dots b_n w(t_0^+) - b_1 b_2 \dots b_n \int_{t_0}^{t_1} p(t)\Delta t - b_2 \dots b_n \int_{t_1}^{t_2} p(t)\Delta t \\ &\quad - \dots - b_{n-1} b_n \int_{t_{n-2}}^{t_{n-1}} p(t)\Delta t - b_n \int_{t_{n-1}}^{t_n} p(t)\Delta t \\ &= b_1 b_2 \dots b_n \Big[w(t_0^+) - \int_{t_0}^{t_1} p(t)\Delta t - \frac{1}{b_1} \int_{t_1}^{t_2} p(t)\Delta t - \dots \\ &\quad - \frac{1}{b_1 b_2 \dots b_{n-2}} \int_{t_{n-2}}^{t_{n-1}} p(t)\Delta t - \frac{1}{b_1 b_2 \dots b_{n-1}} \int_{t_{n-1}}^{t_n} p(t)\Delta t\Big]. \end{aligned} \tag{2.14}$$

Using (2.8) and $b_k > 0$, $k = 1, 2, \dots$, we obtain $w(t_n^+) < 0$, $n \to \infty$, which contradicts to $w(t_n^+) \geq 0$. $\square$

Theorem 2.4. *Assume that* (H1)–(H3) *hold and* $\varphi(ab) \geq \varphi(a)\varphi(b)$ *for any* $ab > 0$. *If*

$$\begin{aligned} &\int_{t_0}^{t_1} p(t)\Delta t + \frac{\varphi(a_1^*)}{b_1}\int_{t_1}^{t_2} p(t)\Delta t + \frac{\varphi(a_1^*)\varphi(a_2^*)}{b_1b_2}\int_{t_2}^{t_3} p(t)\Delta t \\ &+ \dots + \frac{\varphi(a_1^*)\varphi(a_2^*)\dots\varphi(a_n^*)}{b_1b_2\dots b_n}\int_{t_n}^{t_{n+1}} p(t)\Delta t + \dots = \infty, \end{aligned} \tag{2.15}$$

then (1.1) *is oscillatory.*

Proof. As before, we may suppose $y(t) > 0$, $t \geq t_0$ be a nonoscillatory solution of (1.1), Lemma 2.1 yields $y^{\Delta}(t) \geq 0, t \geq t_0$, define $w(t)$ as in (2.9) and we get $w(t_k) \geq 0$, $t \geq t_0$, $w(t_k^+) \geq 0$, $k = 1, 2, \dots$ and (2.10) holds for $t \neq t_k$ and

$$w(t_k^+) = \frac{y^{\Delta}(t_k^+)}{\varphi(y(t_k^+))} \leq \frac{b_k y^{\Delta}(t_k)}{\varphi(a_k^* y(t_k))} \leq \frac{b_k y^{\Delta}(t_k)}{\varphi(a_k^*)\varphi(y(t_k))} = \frac{b_k}{\varphi(a_k^*)} w(t_k). \tag{2.16}$$

As in the proof of (2.14), by induction, for any positive integer n,

$$\begin{aligned} &w(t_n^+) \\ &\leq \frac{b_1b_2\dots b_n}{\varphi(a_1^*)\varphi(a_2^*)\dots\varphi(a_n^*)}\Big[w(t_0^+) - \int_{t_0}^{t_1} p(t)\Delta t - \frac{\varphi(a_1^*)}{b_1}\int_{t_1}^{t_2} p(t)\Delta t - \dots \\ &\quad - \frac{\varphi(a_1^*)\varphi(a_2^*)\dots\varphi(a_{n-2}^*)}{b_1b_2\dots b_{n-2}}\int_{t_{n-2}}^{t_{n-1}} p(t)\Delta t - \frac{\varphi(a_1^*)\varphi(a_2^*)\dots\varphi(a_{n-1}^*)}{b_1b_2\dots b_{n-1}}\int_{t_{n-1}}^{t_n} p(t)\Delta t\Big]. \end{aligned}$$

Let $n \to \infty$ and use (2.15), we obtain the desired contradiction. $\square$

In the following , we will use the hypothesis

(H4) $\int_{\pm\epsilon}^{\pm\infty} \frac{\Delta u}{\varphi(u)} < \infty$, for any $\epsilon > 0$, where $\int_{\pm\epsilon}^{\pm\infty} \frac{\Delta u}{\varphi(u)} < \infty$ denotes $\int_{\epsilon}^{\infty} \frac{\Delta u}{\varphi(u)} < \infty$ and $\int_{-\epsilon}^{-\infty} \frac{\Delta u}{\varphi(u)} < \infty$.

Theorem 2.5. *Assume that* (H1)–(H4) *hold and there exists a positive integer* k_0 *such that* $a_k^* \geq 1$ *for* $k \geq k_0$. *If*

$$\begin{aligned} &\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\Big[\int_{s}^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t \\ &+ \dots + \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots\Big]\Delta s = \infty, \end{aligned} \tag{2.17}$$

then (1.1) *is oscillatory.*

Proof. As before, we may assume $y(t) > 0, t \geq t_0$ be a nonoscillatory solution of (1.1) and $k_0 = 1$, Lemma 2.1 shows that $y^{\Delta}(t_k^+) \geq 0$, $k = 1, 2, \dots$ and $y^{\Delta}(t) \geq 0, t \geq t_0$. Since $a_k^* \geq 1$, $k = 1, 2, \dots$, we get

$$y(t_0^+) \leq y(t_1) \leq y(t_1^+) \leq y(t_2) \leq y(t_2^+) \leq \dots, \tag{2.18}$$

its easy to see that $y(t)$ is nondecreasing in $[t_0, \infty)$, hence (1.1) yields

$$y^{\Delta\Delta}(t) = -f(t, y^{\sigma}(t)) \leq -p(t)\varphi(y^{\sigma}(t)), \quad t \neq t_k; \tag{2.19}$$

hence, $y^{\Delta}(t_1) - y^{\Delta}(t_0^+) \leq -\int_{t_0}^{t_1} p(t)\varphi(y^{\sigma}(t))\Delta t$. Using (H2), we obtain

$$y^{\Delta}(t_0^+) \geq y^{\Delta}(t_1) + \int_{t_0}^{t_1} p(t)\varphi(y^{\sigma}(t))\Delta t \geq \frac{y^{\Delta}(t_1^+)}{b_1} + \int_{t_0}^{t_1} p(t)\varphi(y^{\sigma}(t))\Delta t.$$

Similarly,

$$y^{\Delta}(t_1^+) \geq \frac{y^{\Delta}(t_2^+)}{b_2} + \int_{t_1}^{t_2} p(t)\varphi(y^{\sigma}(t))\Delta t.$$

Generally, for any positive integer n, we get

$$y^{\Delta}(t_n^+) \geq y^{\Delta}(t_{n+1}) + \int_{t_n}^{t_{n+1}} p(t)\varphi(y^{\sigma}(t))\Delta t \geq \frac{y^{\Delta}(t_{n+1}^+)}{b_{n+1}} + \int_{t_n}^{t_{n+1}} p(t)\varphi(y^{\sigma}(t))\Delta t.$$

From this inequality and (2.19), noting that $y^{\Delta}(t_k^+) \geq 0$, $k = 1, 2, \dots$, we have for $s \in (t_k, t_{k+1}]_{\mathbb{T}}$,

$$\begin{aligned}
y^{\Delta}(s) &\geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + y^{\Delta}(t_{k+1}) \\
&\geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{y^{\Delta}(t_{k+1}^+)}{b_{k+1}} \\
&\geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{1}{b_{k+1}}\Big[\int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{y^{\Delta}(t_{k+2}^+)}{b_{k+2}}\Big] \\
&\geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y^{\sigma}(t))\Delta t \\
&\quad + \frac{1}{b_{k+1}b_{k+2}}\int_{t_{k+2}}^{t_{k+3}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{y^{\Delta}(t_{k+3}^+)}{b_{k+1}b_{k+2}b_{k+3}} \\
&\geq \dots \geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y^{\sigma}(t))\Delta t + \dots \\
&\quad + \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{y^{\Delta}(t_{k+n+1}^+)}{b_{k+1}b_{k+2}\dots b_{k+n+1}}.
\end{aligned}$$

Noting that $b_k > 0$ and $y^{\Delta}(t_k^+) \geq 0$, $k = 1, 2, \dots$, the above inequality yields

$$\begin{aligned}
y^{\Delta}(s) &\geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y^{\sigma}(t))\Delta t + \dots \\
&\quad + \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y^{\sigma}(t))\Delta t,
\end{aligned} \tag{2.20}$$

holds for any positive integer n, then

$$\begin{aligned}
y^{\Delta}(s) &\geq \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y^{\sigma}(t))\Delta t + \dots \\
&\quad + \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \dots.
\end{aligned} \tag{2.21}$$

Using (H1) and the above inequality, we obtain that for $s \in (t_k, t_{k+1}]_{\mathbb{T}}$,

$$\begin{aligned}
\frac{y^\Delta(s)}{\varphi(y(s))} &\geq \int_s^{t_{k+1}} p(t)\frac{\varphi(y^\sigma(t))}{\varphi(y(s))}\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\frac{\varphi(y^\sigma(t))}{\varphi(y(s))}\Delta t + \dots \\
&\quad + \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\frac{\varphi(y^\sigma(t))}{\varphi(y(s))}\Delta t + \dots \\
&\geq \int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t \\
&\quad + \dots + \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots .
\end{aligned}$$

Integrating it from t_k to t_{k+1} and using (2.3), we have

$$\begin{aligned}
&\int_{t_k}^{t_{k+1}} \Big[\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \dots \\
&+ \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots\Big]\Delta s \\
&\leq \int_{t_k}^{t_{k+1}} \frac{y^\Delta(s)}{\varphi(y(s))}\Delta s \\
&= \int_{y(t_k^+)}^{y(t_{k+1})} \frac{1}{\varphi(u)}\Delta u.
\end{aligned}$$

Using (2.18), and (H4), the above inequality yields

$$\begin{aligned}
&\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}} \Big[\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \dots \\
&+ \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots\Big] \\
&\leq \sum_{k=0}^{\infty}\int_{y(t_k^+)}^{y(t_{k+1})} \frac{1}{\varphi(u)}\Delta u \\
&\leq \int_{y(t_0^+)}^{\infty} \frac{1}{\varphi(u)}\Delta u < \infty,
\end{aligned}$$

which contradicts (2.17). $\square$

Theorem 2.6. *Suppose that* (H1)–(H4) *hold and there exists a positive integer* k_0 *such that* $a_k^* \geq 1$ *for* $k \geq k_0$. *Assume, furthermore, that* $\varphi(ab) \geq \varphi(a)\varphi(b)$ *for any* $ab > 0$ *and*

$$\begin{aligned}
&\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}} \Big[\int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t \\
&+ \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)}{b_{k+1}b_{k+2}}\int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t + \dots \\
&+ \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\dots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots\Big]\Delta s = \infty.
\end{aligned} \tag{2.22}$$

Then (1.1) *is oscillatory.*

Proof. As before, we may assume that $y(t) > 0$, $t \geq t_0$, is a nonoscillatory solution of (1.1) and $k_0 = 1$. According to the proof of Theorem 2.5, (2.18) and (2.21) hold. Furthermore, from $(H1)$ and Lemma 2.1, we obtain $\varphi(y)$ is nondecreasing and $y(t)$ is also nondecreasing in $(t_k, t_{k+1}]_{\mathbb{T}}$, $k = 0, 1, 2, \dots$. Therefore, $\varphi(y(t))$ is nondecreasing in $(t_k, t_{k+1}]_{\mathbb{T}}$. Hence,

$$\varphi(y(t_{k+1}^+)) \geq \varphi(a_{k+1}^* y(t_{k+1})) \geq \varphi(a_{k+1}^*)\varphi(y(t_{k+1})),$$

and

$$\varphi(y(t_{k+2}^+)) \geq \varphi(a_{k+2}^* y(t_{k+2})) \geq \varphi(a_{k+2}^*)\varphi(y(t_{k+1}^+)) \geq \varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\varphi(y(t_{k+1})).$$

By induction, it can be proved that for any positive integer n,

$$\varphi(y(t_{k+n}^+)) \geq \varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\dots\varphi(a_{k+n}^*)\varphi(y(t_{k+1})). \tag{2.23}$$

From this inequality, (2.21), and using the fact that $\varphi(y(t))$ is nondecreasing, we obtain, for $s \in (t_k, t_{k+1}]_{\mathbb{T}}$,

$$\begin{aligned}
y^{\Delta}(s) \geq& \int_s^{t_{k+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \frac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y^{\sigma}(t))\Delta t + \dots \\
&+ \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y^{\sigma}(t))\Delta t + \dots \\
\geq& \varphi(y(s)) \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(y(t_{k+1}^+))}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \dots \\
&+ \frac{\varphi(y(t_{k+n}^+))}{b_{k+1}b_{k+2}\dots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots \\
\geq& \varphi(y(s)) \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)\varphi(y(t_{k+1}))}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \dots \\
&+ \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\dots\varphi(a_{k+n}^*)\varphi(y(t_{k+1}))}{b_{k+1}b_{k+2}\dots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots .
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{y^{\Delta}(s)}{\varphi(y(s))} \geq& \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)}{b_{k+1}} \frac{\varphi(y(t_{k+1}))}{\varphi(y(s))} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \dots \\
&+ \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\dots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\dots b_{k+n}} \frac{\varphi(y(t_{k+1}))}{\varphi(y(s))} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots \\
\geq& \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \dots \\
&+ \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\dots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\dots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots .
\end{aligned}$$

Integrating the above inequality and using (2.18), (2.8), we obtain

$$\begin{aligned}
&\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\Big[\int_s^{t_{k+1}} p(t)\Delta t+\frac{\varphi(a_{k+1}^*)}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t+\dots\\
&\quad+\frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\dots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t+\dots\Big]\\
&\le\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\frac{y^{\Delta}(s)}{\varphi(y(s))}\\
&=\sum_{k=0}^{\infty}\int_{y(t_k^+)}^{y(t_{k+1})}\frac{1}{\varphi(u)}\Delta u\\
&\le\int_{y(t_0^+)}^{\infty}\frac{1}{\varphi(u)}\Delta u<\infty,
\end{aligned}$$

□

which contradicts (2.22).

From Theorems 2.3–2.6, we have the following corollaries.

Corollary 2.7. *Suppose that* (H1)–(H3) *hold and there exists a positive integer* k_0 *such that* $a_k^*\ge 1$, $b_k\le 1$ *for* $k\ge k_0$. *If* $\int^{\infty} p(t)\Delta t=\infty$, *then* (1.1) *is oscillatory.*

Proof. Without loss of generality, let $k_0=1$. By $b_k\le 1$, we get

$$\begin{aligned}
&\int_{t_0}^{t_1} p(t)\Delta t+\frac{1}{b_1}\int_{t_1}^{t_2} p(t)\Delta t+\frac{1}{b_1b_2}\int_{t_2}^{t_3} p(t)\Delta t+\dots+\frac{1}{b_1b_2\dots b_n}\int_{t_n}^{t_{n+1}} p(t)\Delta t\\
&\ge\int_{t_0}^{t_1} p(t)\Delta t+\int_{t_1}^{t_2} p(t)\Delta t+\int_{t_2}^{t_3} p(t)\Delta t+\dots+\int_{t_n}^{t_{n+1}} p(t)\Delta t\\
&=\int_{t_0}^{t_{n+1}} p(t)\Delta t.
\end{aligned}$$

Let $n\to\infty$, from $\int^{\infty} p(t)\Delta t=\infty$, the above inequality yields (2.8). By Theorem 2.3, we conclude that (1.1) is oscillatory. □

Corollary 2.8. *Assume that* (H1)–(H4) *hold and there exists a positive integer* k_0 *such that* $a_k^*\ge 1$, $b_k\le 1$ *for* $k\ge k_0$. *If* $\int^{\infty}\int_s^{\infty} p(t)\Delta t\Delta s=\infty$, *then* (1.1) *is oscillatory.*

The proof of the above result is similar to the proof of Corollary 2.7 and using Theorem 2.5.

Corollary 2.9. *Suppose that* (H1)–(H3) *hold and there exist a positive integer* k_0 *and a constant* $\alpha>0$ *such that*

$$a_k^*\ge 1,\quad \frac{1}{b_k}\ge\Big(\frac{t_{k+1}}{t_k}\Big)^{\alpha},\quad \text{for } k\ge k_0. \tag{2.24}$$

If

$$\int^{\infty} t^{\alpha}p(t)\Delta t=\infty. \tag{2.25}$$

Then (1.1) *is oscillatory.*

Proof. As before, let $k_0 = 1$. Then (2.24) yields

$$
\begin{aligned}
&\int_{t_0}^{t_1} p(t)\Delta t + \frac{1}{b_1}\int_{t_1}^{t_2} p(t)\Delta t + \frac{1}{b_1 b_2}\int_{t_2}^{t_3} p(t)\Delta t \dots + \frac{1}{b_1 b_2 \dots b_n}\int_{t_n}^{t_{n+1}} p(t)\Delta t\\
&\geq \frac{1}{t_1^\alpha}\Big[\int_{t_1}^{t_2} t_2^\alpha p(t)\Delta t + \int_{t_2}^{t_3} t_3^\alpha p(t)\Delta t + \dots + \int_{t_n}^{t_{n+1}} t_{n+1}^\alpha p(t)\Delta t\Big]\\
&\geq \frac{1}{t_1^\alpha}\Big[\int_{t_1}^{t_2} t^\alpha p(t)\Delta t + \int_{t_2}^{t_3} t^\alpha p(t)\Delta t + \dots + \int_{t_n}^{t_{n+1}} t^\alpha p(t)\Delta t\Big]\\
&= \frac{1}{t_1^\alpha}\int_{t_1}^{t_{n+1}} t^\alpha p(t)\Delta t.
\end{aligned}
$$

Let $n \to \infty$. Then using (2.25), the above inequality yields (2.8). By Theorem 2.3, we obtain that (1.1) is oscillatory. $\square$

Corollary 2.10. *Assume that* (H1)–(H3) *hold and* $\varphi(ab) \geq \varphi(a)\varphi(b)$ *for any* $ab > 0$. *Suppose there exist a positive integer* k_0 *and a constant* $\alpha > 0$ *such that*

$$
\frac{\varphi(a_k^*)}{b_k} \geq (\frac{t_{k+1}}{t_k})^\alpha, \quad \textit{for } k \geq k_0.
$$

If $\int^\infty t^\alpha p(t)\Delta t = \infty$, *then* (1.1) *is oscillatory.*

The above corollary follows from Theorem 2.4, and its proof is similar to that of Corollary 2.9.

Corollary 2.11. *Suppose that* (H1)–(H4) *hold and there exist a positive integer* k_0 *and a constant* $\alpha > 0$ *such that*

$$
a_k^* \geq 1, \quad \frac{1}{b_k} \geq t_{k+1}^\alpha, \quad \textit{for} \quad k \geq k_0. \tag{2.26}
$$

If

$$
\sum_{k=0}^{\infty}(t_{k+1} - t_k)\int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t = \infty. \tag{2.27}
$$

Then (1.1) *is oscillatory.*

Proof. As before, we assume $k_0 = 1$, $t_1 \geq 1$. From (2.26), we get

$$
\begin{aligned}
\frac{1}{b_{k+1}} &\geq t_{k+2}^\alpha, \frac{1}{b_{k+1}b_{k+2}}\\
&\geq t_{k+2}^\alpha t_{k+3}^\alpha, \dots, \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\\
&\geq t_{k+2}^\alpha t_{k+3}^\alpha \dots t_{k+n+1}^\alpha, \dots.
\end{aligned}
$$

Similar to the proof of Corollary 2.9, we have

$$
\begin{aligned}
&\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \frac{1}{b_{k+1}b_{k+2}}\int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t + \dots\\
&+ \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t\\
&\geq \int_{t_{k+1}}^{t_{k+n+1}} t^\alpha p(t)\Delta t.
\end{aligned}
$$

Let $n \to \infty$, we have

$$
\begin{aligned}
&\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \frac{1}{b_{k+1}b_{k+2}} \int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t + \dots \\
&+ \frac{1}{b_{k+1}b_{k+2}\dots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \dots \\
&\ge \int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t.
\end{aligned}
$$

Using (2.27) and the above inequality, we get

$$
\begin{aligned}
&\sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \Big[\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \frac{1}{b_{k+1}b_{k+2}} \int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t + \dots \Big] \Delta s \\
&\ge \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t \Delta s \\
&= \sum_{k=0}^{\infty} (t_{k+1} - t_k) \int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t = \infty.
\end{aligned}
$$

By Theorem 2.5, we obtain that (1.1) is oscillatory. $\square$

Corollary 2.12. *Suppose that* (H1)–(H4) *hold and there exists a positive integer* k_0 *and a constant* $\alpha > 0$ *such that*

$$
a_k^* \ge 1, \quad \frac{\varphi(a_k^*)}{b_k} \ge t_{k+1}^\alpha, \quad \text{for } k \ge k_0.
$$

Suppose that $\varphi(ab) \ge \varphi(a)\varphi(b)$ *for any* $ab > 0$ *and*

$$
\sum_{k=0}^{\infty} (t_{k+1} - t_k) \int_{t_{k+1}}^{\infty} p(t)\Delta t = \infty.
$$

Then (1.1) *is oscillatory.*

The proof of the above corollary is similar to that of Corollary 2.11, so we omit it.

3. Example

Consider the the second-order impulsive dynamic equation

$$
\begin{gathered}
y^{\Delta\Delta}(t) + \frac{1}{t\sigma^2(t)} y^\gamma(\sigma(t)) = 0, \quad t \ge 1,\ t \ne k,\ k = 1, 2, \dots, \\
y(k^+) = \frac{k+1}{k} y(k), \quad y^\Delta(k^+) = y^\Delta(k), \quad k = 1, 2, \dots, \\
y(1) = y_0, \quad y^\Delta(1) = y_0^\Delta.
\end{gathered}
\tag{3.1}
$$

where $\gamma \ge 3$ and $\mu(t) \le Kt$, and K is a positive constant. Since $a_k = a_k^* = \frac{k+1}{k}$, $b_k = b_k^* = 1$, $p(t) = \frac{1}{t\sigma^2(t)}$, $t_k = k$ and $\varphi(y) = y^\gamma$. it is easy to see that (H1)–(H3) hold. Let $k_0 = 1$, $\alpha = 3$, hence

$$\frac{\varphi(a_k^*)}{b_k} = (\frac{k+1}{k})^\gamma = (\frac{t_{k+1}}{t_k})^\gamma \geq (\frac{t_{k+1}}{t_k})^3,$$
$$\int^\infty t^\alpha p(t)\Delta t = \int^\infty t^3 \frac{1}{t\sigma^2(t)}\Delta t = \int^\infty (\frac{t}{\sigma(t)})^2 \Delta t.$$

Since $\mu(t) \leq Kt$, we get

$$\frac{t}{\sigma(t)} = \frac{t}{t+\mu(t)} \geq \frac{1}{1+K},$$

hence

$$\int^\infty (\frac{t}{\sigma(t)})^2 \Delta t \geq \frac{1}{(1+K)^2} \int^\infty \Delta t = \infty.$$

By Corollary 2.9, we obtain that (3.1) is oscillatory. But by [3] we know that the dynamic equation $y^{\Delta\Delta}(t) + \frac{1}{t\sigma^2(t)} y^\gamma(\sigma(t)) = 0$ is nonoscillatory.

Note that in the above example, the dynamic equation without impulses is nonoscillatory. However, when some impulses are added, it become oscillatory. Therefore, this example shows that impulses play an important part in oscillations of dynamic equations on time scales.

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