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SMALL DATA BLOW-UP OF SOLUTIONS TO NONLINEAR SCHRÖDINGER EQUATIONS WITHOUT GAUGE INVARIANCE IN L^2

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Abstract. In this article we study the Cauchy problem of the nonlinear Schrödinger equations without gauge invariance

$$\begin{aligned} i\partial_t u + \Delta u &= \lambda(|u|^{p_1} + |v|^{p_2}), \quad (t,x) \in [0,T) \times \mathbb{R}^n, \\ i\partial_t v + \Delta v &= \lambda(|u|^{p_2} + |v|^{p_1}), \quad (t,x) \in [0,T) \times \mathbb{R}^n, \end{aligned}$$

where $1 < p_1, p_2 < 1 + 4/n$ and $\lambda \in \mathbb{C}\backslash\{0\}$. We first prove the existence of a local solution with initial data in $L^2(\mathbb{R}^n)$. Then under a suitable condition on the initial data, we show that the L^2-norm of the solution must blow up in finite time although the initial data are arbitrarily small. As a by-product, we also obtain an upper bound of the maximal existence time of the solution.

1. Introduction

In this article, we consider the Cauchy problem of the nonlinear Schrödinger equations without gauge invariance

$$\begin{aligned} i\partial_t u + \Delta u &= \lambda(|u|^{p_1} + |v|^{p_2}), \quad (t,x) \in [0,T) \times \mathbb{R}^n, \\ i\partial_t v + \Delta v &= \lambda(|u|^{p_2} + |v|^{p_1}), \quad (t,x) \in [0,T) \times \mathbb{R}^n, \\ u(0,x) &= \varepsilon f(x), \quad x \in \mathbb{R}^n, \\ v(0,x) &= \varepsilon g(x), \quad x \in \mathbb{R}^n, \end{aligned} \tag{1.1}$$

where $1 < p_1,\ p_2 < 1 + \frac{4}{n},\ T > 0$ and $\varepsilon > 0$ is a small parameter. $u = u(t,x)$ and $v = v(t,x)$ are complex-valued unknown functions, $f = f_1 + if_2$ and $g = g_1 + ig_2$ are prescribed complex-valued functions, and $\lambda = \lambda_1 + i\lambda_2 \in \mathbb{C}\backslash\{0\}$.

System (1.1) is a generalization of the Cauchy problem of the nonlinear equation

$$\begin{aligned} i\partial_t u + \Delta u &= F(u), \quad (t,x) \in [0,T) \times \mathbb{R}^n, \\ u(0,x) &= f(x), \quad x \in \mathbb{R}^n, \end{aligned} \tag{1.2}$$

where $F(u) = \lambda|u|^p$.

We know that a solution to (1.2) on $[0,T]$ gives rise to a family of solutions, i.e. for any $\gamma > 0$,

$$u_\gamma(t,x) := \gamma^{\frac{2}{p-1}} u(\gamma^2 t, \gamma x)$$

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is also a solution to (1.2) on $[0, T/\gamma^2]$. Moreover, a direct calculation gives

$$\|u_\gamma(t,\cdot)\|_{L^2(\mathbb{R}^n)} = \gamma^{\frac{2}{p-1}-\frac{n}{2}}\|u(t,\cdot)\|_{L^2(\mathbb{R}^n)}.$$

Thus if the order p satisfies

$$\frac{2}{p-1} - \frac{n}{2} = 0 \quad \text{i.e.} \quad p = p_0 := 1 + \frac{4}{n},$$

then the L^2-norm of the solution is also scale invariant. Therefore, the case $p = p_0$ is called L^2-critical case. The case of $p < p_0$ (resp. $p > p_0$) is called L^2-subcritical case (resp. L^2-supercritical case).

We say that a nonlinear function F satisfies the gauge invariance if $F(e^{i\theta}u) = e^{i\theta}F(u)$ for $\theta \in \mathbb{R}$. However, the nonlinear term in (1.2) $F(u) = \lambda|u|^p$ is not gauge invariant. This is different from $F(u) = \lambda|u|^{p-1}u$, which satisfies the gauge invariance and possesses the conservation of mass (and also energy for H^1-solution). However, in the case of non-gauge invariance, the conservation of mass (or energy for H^1-solution) fails (see [9]).

Equation (1.2) has various physical contexts and has been studied from the mathematical viewpoint in several papers. For example, it is related to the Gross-Pitaevskii equation, which describes the Bose-Einstein condensate in physics. The solution Φ of the Gross-Pitaevskii equation satisfies a non-zero constant boundary condition as $|x|$ tends to infinity. In that case, the nonlinearity $|u|^p$ appears if we introduce the new dynamical variable u by $\Phi = u + \text{constant}$ and expand the nonlinearity $|\Phi|^p\Phi$ in u (see [8, 17]). Thus, it is expected that the analysis of (1.2) may be helpful for the study of the Gross-Pitaevskii equation.

For (1.2), in the single equation case, when $1 < p < 1 + \frac{4}{n-2s}$ ($0 \le s < \frac{n}{2}$), it is well known that local well-posedness holds in Sobolev spaces H^s (see [3, 21] with the references therein). In one dimension, when $p = 2$, Kenig et al. [14] first proved the local well-posedness in $H^s(\mathbb{R})$ when $s > -\frac{1}{4}$. For general dimension, when p is sufficiently large, the small initial data L^2-solution exists globally. More precisely, for $L^2 \cap L^{1+\frac{1}{p}}$-data, when $p_S < p < p_0 = 1 + \frac{4}{n}$, where $p_S = \frac{n+2+\sqrt{n^2+4n+12}}{2n}$ is the Strauss exponent (see [18]), which is greater than $1 + \frac{2}{n}$ and less than $1 + \frac{4}{n}$, the global existence result for small initial data holds (see also [3]). When $1 < p \le 1 + \frac{2}{n}$, Ikeda and Wakasugi [11] showed that the L^2-norm of the solution for (1.2) blows up at finite time, provided that

$$\lambda_1 \operatorname{Im} \int_{\mathbb{R}^n} f(x)\,dx < 0, \quad \text{or} \quad \lambda_2 \cdot \operatorname{Re} \int_{\mathbb{R}^n} f(x)\,dx > 0.$$

In particular, this implies that there is no global well-posedness even for small initial data. Later, in [9] Ikeda and Inui proved a small initial data blow-up result of the L^2-solution for (1.2) in $1 < p < p_0$. Recently, Ikeda and Inui [10] proved the non-existence of the local weak-solution for (1.2) in the L^2-supercritical case $p > p_0$ for suitable L^2-data. To construct the blow up solution, the authors in [11, 9, 10] used a test-function method which heavily relies on the shape of the initial data, though their norms may be arbitrarily small.

The coupled nonlinear Schrödinger equations

$$\begin{aligned} i\partial_t u + \Delta u &= \lambda(|u|^{p_1} + |v|^{p_2})u, \quad (t,x) \in [0,T) \times \mathbb{R}^n, \\ i\partial_t v + \Delta v &= \lambda(|v|^{p_1} + |u|^{p_2})v, \quad (t,x) \in [0,T) \times \mathbb{R}^n, \end{aligned} \tag{1.3}$$

where $1 < p_1,\, p_2 < 1 + \frac{4}{n}$, describe the minimum approximation of the transformation of light wave. For more details of the physical background, we refer the readers to [2, 1, 16, 20]. When u and v satisfy non-zero constant boundary condition as $|x|$ tends to infinity, the same analysis as for (1.2) leads to (1.1).

In this article, our main aim is to prove a small initial data blow-up result of L^2-solution for (1.1) in the subcritical case $1 < p_1,\ p_2 < p_0$. We also obtain an upper bound of the lifespan for (1.1) when $1 < p_1, p_2 < p_0$.

For the rest of this article, we let $p := \min\{p_1, p_2\}$. Since $1 < p_1, p_2 < p_0$, we have $p \in (1, p_0)$. We impose the additional assumption on the initial data,

$$\begin{gathered}\lambda_2(f_1(x)+g_1(x)) \geq \begin{cases}|x|^{-k}, & \text{if } |x|\geq 1,\\ 0, & \text{if } |x|<1,\end{cases} \quad \text{or} \\ -\lambda_1(f_2(x)+g_2(x)) \geq \begin{cases}|x|^{-k}, & \text{if } |x|\geq 1,\\ 0, & \text{if } |x|<1,\end{cases}\end{gathered} \tag{1.4}$$

where $n/2 < k < 2/(p-1)$. Note that such k exists if and only if $1 < p < p_0$. Now, we can state our main result.

Theorem 1.1. *Let $1 < p_1, p_2 < 1 + \frac{4}{n}$, $\lambda = \lambda_1 + i\lambda_2 \in \mathbb{C}\backslash\{0\}$ and $f, g \in L^2(\mathbb{R}^n)$. If f and g satisfy initial data condition* (1.4)*, then there exist $\varepsilon_0 > 0$ and a constant $C = C(k, p_1, p_2, \lambda) > 0$ such that for any $\varepsilon \in (0, \varepsilon_0)$,*

$$T_\varepsilon \leq C\varepsilon^{-1/\theta},$$

where $\theta = \frac{1}{p-1} - \frac{k}{2}$. Moreover, the L^2-norm of the local solution blows up in finite time,

$$\lim_{t\to T_\varepsilon^-} (\|u(t)\|_{L^2} + \|v(t)\|_{L^2}) = \infty. \tag{1.5}$$

The definition of T_ε can be found in (2.6) below. This theorem gives an upper bound of the local existence time to the Cauchy problem (1.1) in $L^2(\mathbb{R}^n)$. At the same time, we note that (1.5) means that the conservation law of mass does not hold for equation (1.1).

The rest of this paper is arranged as follows. In Section 2, we prove the local well-posedness for (1.1) with initial data in $L^2(\mathbb{R}^n)$ and give the definition of L^2-solution. In Section 3, we show that an L^2-solution of (2.1) on $[0, T)$ is a weak solution of (1.1). In Section 4, we give the proof of Theorem 1.1.

We concluding this section, by introducing some notation. For $1 \leq r \leq \infty$, let $L^r = L^r(\mathbb{R}^n)$ denote the usual Lebesgue space. For a time interval I, we use a time-space Lebesgue space $L^q(I; L^r(\mathbb{R}^n))$, with the norm $\|u\|_{L^q(I;L^r(\mathbb{R}^n))} := \|\|u(t)\|_{L^r(\mathbb{R}^n)}\|_{L^q(I)}$. We often omit the time interval I and $\mathbb{R}^n$ and denote simply $L^q(I; L^r(\mathbb{R}^n))$ as L^qL^r, when no confusion may occur. We write $A \lesssim B$ if there exists a constant $C > 0$ such that $A \leq CB$.

2. Local well-posedness

Firstly, by the Duhamel formula, we consider the integral equations

$$\begin{gathered}u(t) = \varepsilon S(t)f - i\lambda\int_0^t S(t-\tau)\left(|u|^{p_1} + |v|^{p_2}\right)\, d\tau \\ v(t) = \varepsilon S(t)g - i\lambda\int_0^t S(t-\tau)(|u|^{p_2} + |v|^{p_1})\, d\tau,\end{gathered} \tag{2.1}$$

as the integral version of the Cauchy problem (1.1), where $S(t)=e^{it\Delta}$ is the free evolution group of the linear Schrödinger equation in $H^s(\mathbb{R}^n)$.

Definition 2.1 ([3, 19])**.** The pair (q,r) of real numbers is said to be admissible if $\frac{2}{q}=\frac{n}{2}-\frac{n}{r}$ and

$$2\le r<\frac{2n}{n-2}\quad (2\le r\le\infty \text{ if } n=1;\ 2\le r<\infty \text{ if } n=2).$$

Next, we define the function space

$$X_T=C([0,T);L^2(\mathbb{R}^n))\cap L^{q_1}((0,T);L^{r_1}(\mathbb{R}^n))\cap L^{q_2}((0,T);L^{r_2}(\mathbb{R}^n)),$$

where (q_j,r_j) is an admissible pair defined by $r_j=p_j+1$, $j=1,2$.

Lemma 2.2 ([3, 19])**.** *Let (q,r) and (γ,ρ) be any admissible pairs. For any time interval I, we have the estimates*

$$\|S(\cdot)\varphi\|_{L^q(\mathbb{R},L^r(\mathbb{R}^n))}\le C\|\varphi\|_{L^2},$$
$$\|\int_0^t S(t-s)F(s)\,ds\|_{L^q(I,L^r(\mathbb{R}^n))}\le C\|F\|_{L^{\gamma'}(I,L^{\rho'}(\mathbb{R}^n))}.$$

Theorem 2.3. *Let $1<p_1,p_2<1+\frac{4}{n}$, $\lambda\in\mathbb{C},\varepsilon>0$ and $f,g\in L^2(\mathbb{R}^n)$. Then there exist a positive time $T=T(\varepsilon,\|f\|_{L^2},\|g\|_{L^2})$ and a unique solution $(u,v)\in X_T\times X_T$ of* (2.1).

The proof of this theorem is based on contractive mapping principle. See [21, 4, 13, 7] for the gauge invariance case. For the convenience of the reader, we give a brief proof.

Proof. Let $R>0$ and $B(R)=\{(u,v)|u,v\in X_T,\|u\|_{X_T}\le R,\|v\|_{X_T}\le R\}$, where

$$\|u\|_{X_T}=\|u\|_{L^\infty L^2}+\|u\|_{L^{q_1}L^{r_1}}+\|u\|_{L^{q_2}L^{r_2}}. \tag{2.2}$$

Endowed with the metric

$$d((u_1,v_1),(u_2,v_2))=\|u_1-u_2\|_{X_T}+\|v_1-v_2\|_{X_T},$$

It is easy to see that, $B(R)$ is a complete metric space.

We expect to find the proper conditions of T and R, which imply that $\Gamma:(u,v)\mapsto(\Gamma_1u,\Gamma_2v)$, given by

$$\Gamma_1u(t)=\varepsilon S(t)f-i\lambda\int_0^t S(t-\tau)(|u|^{p_1}+|v|^{p_2})\,d\tau$$
$$\Gamma_2v(t)=\varepsilon S(t)g-i\lambda\int_0^t S(t-\tau)(|u|^{p_2}+|v|^{p_1})\,d\tau,$$

is a strict contraction on $B(R)$.

For $(u_1,v_1),(u_2,v_2)\in B(R)$, we have

$$\begin{aligned}&\|\Gamma_1u_1-\Gamma_1u_2\|_{X_T}\\&\le|\lambda|\|\int_0^t S(t-\tau)(|u_1|^{p_1}-|u_2|^{p_1})\,d\tau\|_{X_T}+|\lambda|\|\int_0^t S(t-\tau)(|v_1|^{p_2}-|v_2|^{p_2})\,d\tau\|_{X_T}\\&:=I+II.\end{aligned}$$

By Lemma 2.2 and Hölder's inequality, we obtain

$$I\lesssim\|\,|u_1|^{p_1}-|u_2|^{p_1}\|_{L^{q_1'}L^{r_1'}}$$

$$
\begin{aligned}
&\lesssim T^{\alpha}\Big(\|u_1\|_{L^{q_1}L^{r_1}}^{p_1-1}+\|u_2\|_{L^{q_1}L^{r_1}}^{p_1-1}\Big)\|u_1-u_2\|_{L^{q_1}L^{r_1}}\\
&\lesssim T^{\alpha}R^{p_1-1}\|u_1-u_2\|_{L^{q_1}L^{r_1}},
\end{aligned}
$$

where $\frac{1}{r_1'}=\frac{p_1}{p_1+1}$, $\frac{1}{q_1'}=\frac{p_1}{q_1}+\alpha$, $\alpha=\frac{n}{4}(1+\frac{4}{n}-p_1)>0$, and

$$
II\lesssim T^{\beta}(\|v_1\|_{L^{q_2}L^{r_2}}^{p_2-1}+\|v_2\|_{L^{q_2}L^{r_2}}^{p_2-1})\|v_1-v_2\|_{L^{q_2}L^{r_2}}\quad\lesssim T^{\beta}R^{p_2-1}\|v_1-v_2\|_{L^{q_2}L^{r_2}},
$$

where $\frac{1}{r_2'}=\frac{p_2}{p_2+1}$, $\frac{1}{q_2'}=\frac{p_2}{q_2}+\beta$, $\beta=\frac{n}{4}(1+\frac{4}{n}-p_2)>0$. So, we have

$$
\|\Gamma_1u_1-\Gamma_1u_2\|_{X_T}\lesssim T^{\alpha}R^{p_1-1}\|u_1-u_2\|_{L^{q_1}L^{r_1}}+T^{\beta}R^{p_2-1}\|v_1-v_2\|_{L^{q_2}L^{r_2}}. \tag{2.3}
$$

Similarly, we obtain

$$
\|\Gamma_2v_1-\Gamma_2v_2\|_{X_T}\lesssim T^{\alpha}R^{p_1-1}\|v_1-v_2\|_{L^{q_1}L^{r_1}}+T^{\beta}R^{p_2-1}\|u_1-u_2\|_{L^{q_2}L^{r_2}}. \tag{2.4}
$$

Combining (2.3) with (2.4), we have

$$
\begin{aligned}
d\big(\Gamma(u_1,v_1),\Gamma(u_2,v_2)\big)=&\|\Gamma_1u_1-\Gamma_1u_2\|_{X_T}+\|\Gamma_2v_1-\Gamma_2v_2\|_{X_T}\\
&\lesssim T^{\alpha}R^{p_1-1}(\|u_1-u_2\|_{L^{q_1}L^{r_1}}+\|v_1-v_2\|_{L^{q_1}L^{r_1}})\\
&\quad+T^{\beta}R^{p_2-1}(\|u_1-u_2\|_{L^{q_2}L^{r_2}}+\|v_1-v_2\|_{L^{q_2}L^{r_2}}),
\end{aligned}
$$

Let

$$
T\le\min\{(4R^{p_1-1})^{-1/\alpha},(4R^{p_2-1})^{-1/\beta}\}. \tag{2.5}
$$

Then there exists a constant $\delta\in(0,1)$ such that

$$
d(\Gamma(u_1,v_1),\Gamma(u_2,v_2))<\delta(\|u_1-u_2\|_{X_T}+\|v_1-v_2\|_{X_T}).
$$

It follows that Γ is a strict contraction on $B(R)$, and thus has a unique fixed point (u,v). This completes the proof. □

The above solution (u,v) is called an "L^2-solution". Let T_ε be the maximal existence time of the local L^2-solution,

$$
\begin{aligned}
T_\varepsilon=\sup\Big\{&T\in(0,\infty]:\ \text{a unique solution }(u,v)\text{ to (2.1) exists and}\\
&\text{belongs to }X_T\times X_T\Big\}.
\end{aligned} \tag{2.6}
$$

Then (2.5) provides lower bound of lifespan.

Corollary 2.4. *Under the the assumptionsin Theorem 2.3, we have the estimate*

$$
T_\varepsilon\ge C\min(\varepsilon^{-1/\theta_1},\varepsilon^{-1/\theta_2}),
$$

where $\theta_j=\frac{1}{p_j-1}-\frac{n}{4}>0$, $j=1,2$ *and* $C=C(n,p_1,p_2,\|f\|_{L^2},\|g\|_{L^2})>0$ *is a constant.*

Combining Theorem 1.1 with Corollary 2.4, we obtain the estimate of the lifespan

$$
\min(\varepsilon^{-1/\theta_1},\varepsilon^{-1/\theta_2})\lesssim T_\varepsilon\lesssim\varepsilon^{-1/\theta}.
$$

However, it is not optimal. Actually, to the best of our knowledge, if $p_1<p_2$, we have $p=\min\{p_1,p_2\}=p_1$, then the following estimate holds for sufficiently small $\varepsilon>0$,

$$
\varepsilon^{-1/\theta_1}\lesssim T_\varepsilon\lesssim\varepsilon^{-1/\theta}.
$$

But we know that

$$
\theta-\theta_1=\frac{n}{4}-\frac{k}{2}<0.
$$

Similarly, if $p_2 < p_1$, then $\varepsilon^{-1/\theta_2} \lesssim T_\varepsilon \lesssim \varepsilon^{-1/\theta}$ holds. However, this is also not optimal. For the time being, to our knowledge, the optimal order of the lifespan is an open question.

3. Weak solutions

To obtain our main results, we first define a weak solution of (1.1).

Definition 3.1. Let $T > 0$. (u, v) is a weak solution of (1.1) on $[0, T)$, if $(u, v) \in L^{p_1}_{\rm loc}([0, T) \times \mathbb{R}^n) \cap L^{p_2}_{\rm loc}([0, T) \times \mathbb{R}^n)$ and satisfies

$$\begin{aligned} &\int_{[0,T)\times\mathbb{R}^n} u(-i\partial_t\psi + \Delta\psi)\,dx\,dt \\ &= i\varepsilon\int_{\mathbb{R}^n} f(x)\psi(0,x)\,dx + \lambda\int_{[0,T)\times\mathbb{R}^n} (|u|^{p_1} + |v|^{p_2})\psi\,dx\,dt, \end{aligned} \tag{3.1}$$

$$\begin{aligned} &\int_{[0,T)\times\mathbb{R}^n} v(-i\partial_t\psi + \Delta\psi)\,dx\,dt \\ &= i\varepsilon\int_{\mathbb{R}^n} g(x)\psi(0,x)\,dx + \lambda\int_{[0,T)\times\mathbb{R}^n} (|u|^{p_2} + |v|^{p_1})\psi\,dx\,dt \end{aligned} \tag{3.2}$$

for any $\psi \in C_0^2([0, T) \times \mathbb{R}^n)$. Moreover, if T can be chosen arbitrary large, then we say that (u, v) is a global weak solution of (1.1).

We note that an L^2-solution as in Theorem 2.3 is always a weak solution in the sense of Definition 3.1. Then, we have the following proposition.

Proposition 3.2. *Let $T > 0$. If (u, v) is an L^2-solution of (2.1) on $[0, T)$, then (u, v) is also a weak solution on $[0, T)$ in the sense of Definition 3.1.*

Proof. Let $T > 0$ and (q_j, r_j) be admissible pairs, where $r_j = p_j + 1$, $j = 1, 2$. Let (u, v) be an L^2-solution to (2.1) on $[0, T)$ and $\psi \in C_0^2([0, T) \times \mathbb{R}^n)$. It is easy to see that

$$u, v \in L^{p_1}_{\rm loc}([0, T) \times \mathbb{R}^n) \cap L^{p_2}_{\rm loc}([0, T) \times \mathbb{R}^n).$$

Let $u = U_1 + U_2$, where

$$U_1 = \varepsilon S(t)f, \quad U_2 = -i\lambda\int_0^t S(t-\tau)(|u|^{p_1} + |v|^{p_2})\,d\tau.$$

By a standard density argument and integration by parts, we can obtain, for any $\psi \in C_0^2([0, T) \times \mathbb{R}^n)$,

$$\int_{[0,T)\times\mathbb{R}^n} U_1(-i\partial_t\psi + \Delta\psi)\,dx\,dt = i\int_{\mathbb{R}^n} \varepsilon f(x)\psi(0,x)\,dx.$$

Thus, it suffices to prove that

$$\int_{[0,T)\times\mathbb{R}^n} U_2(-i\partial_t\psi + \Delta\psi)\,dx\,dt = \lambda\int_{[0,T)\times\mathbb{R}^n} (|u|^{p_1} + |v|^{p_2})\psi\,dx\,dt. \tag{3.3}$$

Let

$$\begin{gathered} K_1 = \int_{[0,T)\times\mathbb{R}^n} U_2\Delta\psi\,dx\,dt, \quad K_2 = -i\int_{[0,T)\times\mathbb{R}^n} U_2\partial_t\psi\,dx\,dt, \\ K = \lambda\int_{[0,T)\times\mathbb{R}^n} (|u|^{p_1} + |v|^{p_2})\psi\,dx\,dt. \end{gathered} \tag{3.4}$$

So, it is sufficiently to prove that $K = K_1 + K_2$.

Since $u, v \in L^{q_1}L^{r_1} \cap L^{q_2}L^{r_2}$, and $C_0^\infty([0,T) \times \mathbb{R}^n)$ is dense in $L^{q_1}L^{r_1} \cap L^{q_2}L^{r_2}$, there exist two sequences $\{u_k\}_{k\in\mathbb{N}}$, $\{v_k\}_{k\in\mathbb{N}}$ in $C_0^\infty([0,T) \times \mathbb{R}^n)$, such that

$$\lim_{k\to\infty} \|u_k - u\|_{L^{q_1}L^{r_1}\cap L^{q_2}L^{r_2}} = 0, \quad \lim_{k\to\infty} \|v_k - v\|_{L^{q_1}L^{r_1}\cap L^{q_2}L^{r_2}} = 0.$$

We also introduce an approximate sequence $\{U_{2,k}\}_{k\in\mathbb{N}}$ to U_2,

$$U_{2,k} = -i\lambda \int_0^t S(t-\tau)(|u_k|^{p_1} + |v_k|^{p_2})\, d\tau.$$

By Lemma 2.2 and Hölder's inequality with $\frac{1}{r_j'} = \frac{1}{r_j} + \frac{p_j-1}{p_j+1}$ and $\frac{1}{q_j'} = \frac{1}{q_j} + \frac{p_j-1}{q_j} + \alpha_j$, where $\alpha_j = \frac{n}{4}(1 + \frac{4}{n} - p_j) > 0,\ j = 1, 2$, we obtain

$$\begin{aligned}
&\|U_2 - U_{2,k}\|_{L^\infty L^2} \\
&\lesssim \|\int_0^t S(t-\tau)(|u|^{p_1} - |u_k|^{p_1})\, d\tau\|_{L^\infty L^2} + \|\int_0^t S(t-\tau)(|v|^{p_2} - |v_k|^{p_2})\, d\tau\|_{L^\infty L^2} \\
&\lesssim \|(|u|^{p_1} - |u_k|^{p_1})\|_{L^{q_1'}L^{r_1'}} + \|(|v|^{p_2} - |v_k|^{p_2})\|_{L^{q_2'}L^{r_2'}} \\
&\lesssim T^{\alpha_1}\|u - u_k\|_{L^{q_1}L^{r_1}}(\|u\|^{p_1-1}_{L^{q_1}L^{r_1}} + \|u_k\|^{p_1-1}_{L^{q_1}L^{r_1}}) \\
&\quad + T^{\alpha_2}\|v - v_k\|_{L^{q_2}L^{r_2}}(\|v\|^{p_2-1}_{L^{q_2}L^{r_2}} + \|v_k\|^{p_2-1}_{L^{q_2}L^{r_2}}).
\end{aligned} \tag{3.5}$$

Noting $U_{2,k}(0,x) = 0$, by (3.5) and integration by parts, we have

$$K_2 = -i \lim_{k\to\infty} \int_{[0,T)\times\mathbb{R}^n} U_{2,k}\partial_t\psi\, dx\, dt = i \lim_{k\to\infty} \int_{[0,T)\times\mathbb{R}^n} \partial_t U_{2,k}\psi\, dx\, dt. \tag{3.6}$$

By almost the same argument as in (3.5), we find that $U_{2,k} \in C([0,T); H^1)$ and the time derivative $\partial_t U_{2,k} \in C([0,T); H^{-1})$ satisfy

$$i\partial_t U_{2,k} + \Delta U_{2,k} = \lambda(|u_k|^{p_1} + |v_k|^{p_2}). \tag{3.7}$$

By changing variables with $t - \tau = \tau'$, we have

$$\begin{aligned}
\partial_t U_{2,k} &= -i\partial_t \int_0^t \lambda S(t-\tau)(|u_k|^{p_1} + |v_k|^{p_2})(\tau)\, d\tau \\
&= -i\partial_t \int_0^t \lambda S(\tau')(|u_k|^{p_1} + |v_k|^{p_2})(t-\tau')\, d\tau' \\
&= -i \int_0^t \lambda S(t-\tau)\partial_t(|u_k|^{p_1} + |v_k|^{p_2})(\tau)\, d\tau - i\lambda S(t)(|u_k|^{p_1} + |v_k|^{p_2})(0).
\end{aligned}$$

Applying Lemma 2.2, we have

$$\begin{aligned}
\|\partial_t U_{2,k}\|_{L^2} &\lesssim \|\partial_t(|u_k|^{p_1})\|_{L^{q_1'}L^{r_1'}} + \|\partial_t(|v_k|^{p_2})\|_{L^{q_2'}L^{r_2'}} + \|u_k(0)\|^{p_1}_{L^{2p_1}} + \|v_k(0)\|^{p_2}_{L^{2p_2}} \\
&\lesssim T^{\alpha_1}\|u_k\|^{p_1-1}_{L^{q_1}L^{r_1}}\|\partial_t u_k\|_{L^{q_1}L^{r_1}} + T^{\alpha_2}\|v_k\|^{p_2-1}_{L^{q_2}L^{r_2}}\|\partial_t v_k\|_{L^{q_2}L^{r_2}} \\
&\quad + \|u_k\|^{p_1}_{L^\infty L^{2p_1}} + \|v_k\|^{p_1}_{L^\infty L^{2p_1}} < +\infty,
\end{aligned}$$

for any $k \in \mathbb{N}$. Thus we obtain $\partial_t U_{2,k} \in C([0,T); L^2)$. Therefore from the identity (3.7), we can find $U_{2,k} \in C([0,T); H^2)$. Then we have

$$(\Delta U_{2,k},\ \psi)_{L^2} = (U_{2,k},\ \Delta\psi)_{L^2}, \quad \forall k \in \mathbb{N}. \tag{3.8}$$

By the same way as for (3.5), we obtain

$$
\begin{aligned}
&\Big|\int_{[0,T)\times\mathbb{R}^n}\lambda(|u|^{p_1}+|v|^{p_2})\psi\,dx\,dt-\int_{[0,T)\times\mathbb{R}^n}\lambda(|u_k|^{p_1}+|v_k|^{p_2})\psi\,dx\,dt\Big|\\
&\lesssim \Big|\int_{[0,T)\times\mathbb{R}^n}(|u|^{p_1}-|u_k|^{p_1})\psi\,dx\,dt\Big|+\Big|\int_{[0,T)\times\mathbb{R}^n}(|v|^{p_2}-|v_k|^{p_2})\psi\,dx\,dt\Big|\\
&\lesssim T^{\alpha_1}\|u-u_k\|_{L^{q_1}L^{r_1}}\Big(\|u\|_{L^{q_1}L^{r_1}}^{p_1-1}+\|u_k\|_{L^{q_1}L^{r_1}}^{p_1-1}\Big)\|\psi\|_{L^{q_1}L^{r_1}}\\
&\quad+T^{\alpha_2}\|v-v_k\|_{L^{q_2}L^{r_2}}\big(\|v\|_{L^{q_2}L^{r_2}}^{p_2-1}+\|v_k\|_{L^{q_2}L^{r_2}}^{p_2-1}\big)\|\psi\|_{L^{q_2}L^{r_2}},
\end{aligned}
\tag{3.9}
$$

and

$$
\Big|\int_{[0,T)\times R^n}(U_{2,k}-U_2)\Delta\psi\Big|\lesssim T\|U_{2,k}-U_2\|_{L^\infty L^2}\|\Delta\psi\|_{L^\infty L^2}. \tag{3.10}
$$

Thus, combining (3.6)-(3.7) with (3.8)-(3.10), we obtain

$$
\begin{aligned}
K_2&=\lim_{k\to\infty}\Big(\int_{[0,T)\times\mathbb{R}^n}\lambda(|u_k|^{p_1}+|v_k|^{p_2})\psi\,dx\,dt-\int_{[0,T)\times\mathbb{R}^n}\psi\Delta U_{2,k}\,dx\,dt\Big)\\
&=K-\lim_{k\to\infty}\int_{[0,T)\times\mathbb{R}^n}U_{2,k}\Delta\psi\,dx\,dt\\
&=K-K_1.
\end{aligned}
\tag{3.11}
$$

Combining (3.4) with (3.11), we obtain (3.3), thus (3.1) is valid. Similarly, (3.2) is also valid. The proof is complete. □

4. Proof of main result

We first obtain an upper bound of lifespan via a test function method, inspired by [15, 9]. For $1<p_1,p_2<1+\frac{4}{n}$, to use this method, we take the intermediate variable $p=\min\{p_1,p_2\}$. Then we give the proof of Theorem 1.1. Without loss of generality, we assume that $\lambda_1>0$. The other cases in (1.4) can be treated in the almost same way.

We introduce the non-negative smooth radial bump function $\phi\in C_0^2(\mathbb{R}^n)$ as follows (see [5, 6, 9]),

$$
\phi(0)=1,\quad 0<\phi(x)\le 1,\quad \text{for } |x|>0,
$$

where $\phi(x)$ is decreasing with respect to $|x|$ and $\phi(x)\to 0$ as $|x|\to\infty$ sufficiently fast. Moreover, there exists $\mu>0$ such that

$$
|\Delta\phi|\le\mu\phi,\quad x\in\mathbb{R}^n, \tag{4.1}
$$

and $\|\phi\|_{L^1}=1$. For sufficiently large θ, we set

$$
\eta(t)=\begin{cases}(1-t/T)^\theta, & \text{if } 0\le t\le T,\\ 0, & \text{if } t>T,\end{cases}
$$

where $T>0$. Furthermore, for $R>0$, we set

$$
\eta_R(t)=\eta(t/R^2),\quad \phi_R(x)=\phi(x/R),\quad \psi_R(t,x)=\eta_R(t)\phi_R(x).
$$

Next, we introduce some notation. Let T_ε be the maximal existence time. For $T,R>0$ with $TR^2<T_\varepsilon$, define

$$
I_R^1(T)=\int_{[0,TR^2)\times\mathbb{R}^n}(|u|^{p_1}+|v|^{p_2})\psi_R(t,x)\,dx\,dt,
$$

$$
\begin{gathered}
I_R^2(T)=\int_{[0,TR^2)\times\mathbb{R}^n}(|u|^{p_2}+|v|^{p_1})\psi_R(t,x)\,dx\,dt,\\
J_R=-\varepsilon\int_{\mathbb{R}^n}(f_2(x)+g_2(x))\phi_R(x)\,dx,\\
H_1(T)=\mu\Big(\int_{[0,T)\times\mathbb{R}^n}\eta(t)\phi(x)\,dx\,dt\Big)^{1/q},\\
H_2(T)=\Big(\int_{[0,T)\times\mathbb{R}^n}|\partial_t\eta(t)|^q\eta(t)^{-q/p}\phi(x)\,dx\,dt\Big)^{1/q},
\end{gathered}
$$

where $p=\min\{p_1,p_2\}$ and q satisfy $\frac{1}{p}+\frac{1}{q}=1$. By direct computations, we have

$$
\begin{gathered}
H_1(T)=\mu(\theta+1)^{-1/q}T^{1/q}:=bT^{1/q},\\
H_2(T)=\theta(\theta-1/(p-1))^{-1/q}T^{-1/p}:=aT^{-1/p}.
\end{gathered}
$$

We also denote $H(T)=H_1(T)+H_2(T)$ and $I_R(T)=I_R^1(T)+I_R^2(T)$.

Let $\sigma>0$ and $0<\omega<1$. We introduce the function

$$
\Psi(\sigma,\omega)\equiv\max_{x\ge 0}(\sigma x^\omega-x)=(1-\omega)\omega^{\frac{\omega}{1-\omega}}\sigma^{\frac{1}{1-\omega}}. \tag{4.2}
$$

Now, we give an upper bound of J_R as an integral inequality that plays an important role in the proof of Theorem 1.1.

Lemma 4.1. *Let (u,v) be an L^2-solution of (2.1) on $[0,T_\varepsilon)$. Then we have the inequality*

$$
J_R\le C_1R^{sq}H(T)^q \tag{4.3}
$$

for any $T,R>0$ with $TR^2<T_\varepsilon$, where $s=\frac{2+n}{q}-2$ and $C_1=\lambda_1^{1-q}(p-1)(2/p)^q$.

Proof. Since (u,v) is an L^2-solution on $[0,T_\varepsilon)$ and $\psi_R\in C_0^2([0,T)\times\mathbb{R}^n)$, according to Proposition 3.2 and $TR^2<T_\varepsilon$, by substituting the test function in Definition 3.1 into ψ_R, we have

$$
\begin{aligned}
&\lambda\int_{[0,TR^2)\times\mathbb{R}^n}(|u|^{p_1}+|v|^{p_2})\psi_R(t,x)\,dx\,dt+i\varepsilon\int_{\mathbb{R}^n}f(x)\psi_R(0,x)\,dx\\
&=\int_{[0,TR^2)\times\mathbb{R}^n}u(-i\partial_t\psi_R+\Delta\psi_R)\,dx\,dt,
\end{aligned} \tag{4.4}
$$

and

$$
\begin{aligned}
&\lambda\int_{[0,TR^2)\times\mathbb{R}^n}(|u|^{p_2}+|v|^{p_1})\psi_R(t,x)\,dx\,dt+i\varepsilon\int_{\mathbb{R}^n}g(x)\psi_R(0,x)\,dx\\
&=\int_{[0,TR^2)\times\mathbb{R}^n}v(-i\partial_t\psi_R+\Delta\psi_R)\,dx\,dt.
\end{aligned} \tag{4.5}
$$

Taking the real part in (4.4) and (4.5) respectively, we obtain

$$
\begin{aligned}
\lambda_1I_R^1(T)-\varepsilon\int_{\mathbb{R}^n}f_2(x)\phi_R(x)\,dx&=\operatorname{Re}\int_{[0,TR^2)\times\mathbb{R}^n}u(-i\partial_t\psi_R+\Delta\psi_R)\,dx\,dt\\
&\le\int_{[0,TR^2)\times\mathbb{R}^n}(|u||\partial_t\psi_R|+|u||\Delta\psi_R|)\,dx\,dt\\
&:=K_R^1+K_R^2,
\end{aligned} \tag{4.6}
$$

and

$$\begin{aligned} \lambda_1 I_R^2(T) - \varepsilon \int_{\mathbb{R}^n} g_2(x)\phi_R(x)\,dx &= \operatorname{Re}\int_{[0,TR^2)\times\mathbb{R}^n} v(-i\partial_t\psi_R + \Delta\psi_R)\,dx\,dt \\ &\le \int_{[0,TR^2)\times\mathbb{R}^n} (|v||\partial_t\psi_R| + |v||\Delta\psi_R|)\,dx\,dt \\ &:= K_R^3 + K_R^4. \end{aligned} \tag{4.7}$$

From these two inequalities we obtain

$$\lambda_1 I_R(T) + J_R \le \sum_{j=1}^4 K_R^j.$$

Now, estimate the terms K_R^j, $j = 1,2,3,4$. A direct calculation yields

$$\begin{gathered} \Delta\phi_R = R^{-2}(\Delta\phi)(x/R), \\ \partial_t\psi_R(t,x) = R^{-2}\phi_R(x)(\partial_t\eta)(t/R^2). \end{gathered}$$

By the above equality, Hölder's inequality, and noting that $p = \min\{p_1, p_2\}$, we obtain

$$\begin{aligned} K_R^1 &= \frac{1}{R^2}\int_{[0,TR^2)\times\mathbb{R}^n} |u|\psi_R^{1/p}\eta_R^{-1/p}\phi_R^{1/q}|(\partial_t\eta)(t/R^2)|\,dx\,dt \\ &\le \frac{1}{R^2}\Big(\int_{[0,TR^2)\times\mathbb{R}^n} |u|^p\psi_R\,dx\,dt\Big)^{1/p} \\ &\quad \times \Big(\int_{[0,TR^2)\times\mathbb{R}^n} \eta_R^{-q/p}\phi_R|(\partial_t\eta)(t/R^2)|^q\,dx\,dt\Big)^{1/q} \\ &\le \Big(\int_{[0,TR^2)\times\mathbb{R}^n} (|u|^{p_1} + |u|^{p_2})\psi_R\,dx\,dt\Big)^{1/p} R^{-2}H_2(T)R^{\frac{2+n}{q}} \\ &\le (I_R^1(T) + I_R^2(T))^{1/p}H_2(T)R^s \\ &\le I_R(T)^{1/p}H_2(T)R^s. \end{aligned} \tag{4.8}$$

By (4.1) and Hölder's inequality, we have

$$\begin{aligned} K_R^2 &= \frac{1}{R^2}\int_{[0,TR^2)\times\mathbb{R}^n} |u||\Delta\phi_R|\eta_R(t)\,dx\,dt \\ &\le \mu\frac{1}{R^2}\int_{[0,TR^2)\times\mathbb{R}^n} |u|\psi_R\,dx\,dt \\ &\le \mu\frac{1}{R^2}\Big(\int_{[0,TR^2)\times\mathbb{R}^n} |u|^p\psi_R\,dx\,dt\Big)^{1/p}\Big(\int_{[0,TR^2)\times\mathbb{R}^n} \psi_R\,dx\,dt\Big)^{1/q} \\ &= I_R(T)^{1/p}H_1(T)R^s. \end{aligned}$$

Similarly, we can obtain

$$K_R^3 \le I_R(T)^{1/p}H_2(T)R^s, \quad K_R^4 \le I_R(T)^{1/p}H_1(T)R^s. \tag{4.9}$$

Putting (4.8)-(4.9) together, we obtain

$$\lambda_1 I_R(T) + J_R \le 2R^s I_R(T)^{1/p}H(T).$$

Thus, combining the above inequality with (4.2), noting that $\lambda_1 > 0$, we have

$$J_R \le 2R^s H(T)I_R(T)^{1/p} - \lambda_1 I_R(T)$$

$$
\begin{aligned}
&\le \lambda_1 \Psi(2H(T)R^s/\lambda_1, 1/p) \\
&= \lambda_1^{1-q}(p-1)(2/p)^q R^{sq} H(T)^q.
\end{aligned}
$$

This completes the proof. □

Proof of Theorem 1.1. By changing variables and applying (1.4), we obtain that

$$
\begin{aligned}
J_R &= -\varepsilon \int_{\mathbb{R}^n} (f_2(x) + g_2(x))\phi_R(x)\,dx \\
&= \varepsilon R^n \int_{\mathbb{R}^n} -(f_2(Rx) + g_2(Rx))\phi(x)\,dx \\
&\ge \varepsilon R^{n-k}\lambda_1^{-1} \int_{|x|\ge 1/R} |x|^{-k}\phi(x)\,dx \\
&\ge \varepsilon R^{n-k}\lambda_1^{-1} \int_{|x|\ge 1/R_0} |x|^{-k}\phi(x)\,dx \\
&= C_k \varepsilon R^{n-k},
\end{aligned}
\tag{4.10}
$$

for any $R > R_0$, where $0 < R_0 < (b/a)^{1/2}$ is a constant and

$$
C_k = \lambda_1^{-1} \int_{|x|\ge 1/R_0} |x|^{-k}\phi(x)\,dx \le \lambda_1^{-1} \int_{|x|\ge 1/R_0} R_0^k \phi(x)\,dx \le \lambda_1^{-1} R_0^k < \infty.
$$

Next, by Corollary 2.4, there exists $\varepsilon_0 > 0$ such that $T_\varepsilon > 1$ for any $\varepsilon \in (0, \varepsilon_0)$. Let $\tau \in (1, T_\varepsilon)$ and $R > R_0$. By using (4.3) with $T = \tau R^{-2}$, from (4.10) we deduce that

$$
\begin{aligned}
\varepsilon &\le C_k^{-1} C_1 R^{sq} H(T)^q R^{k-n} \\
&= C_k^{-1} C_1 (a\tau^{-1/p} R^{k/q} + b\tau^{1/q} R^{-2+k/q})^q.
\end{aligned}
\tag{4.11}
$$

For each $\tau \in (1, T_\varepsilon)$, setting $R_\tau = (\tau b/a)^{1/2} > R_0$, by substituting R in (4.11) into R_τ, we have

$$
\begin{aligned}
\varepsilon &\le C_k^{-1} C_1 \Big(a\tau^{-1/p} (\tau b/a)^{k/2q} + b\tau^{1/q} (\tau b/a)^{-1+k/2q} \Big)^q \\
&= C_k^{-1} C_1 2^q a^{q-k/2} b^{k/2} \tau^{k/2 - 1/(p-1)} = C_2 \tau^{-\theta},
\end{aligned}
\tag{4.12}
$$

where $\theta = \frac{1}{p-1} - \frac{k}{2} > 0$ and $C_2 = C_k^{-1} C_1 2^q a^{q-k/2} b^{k/2}$. Since $\theta > 0$, (4.12) yields $\tau \le C\varepsilon^{-1/\theta}$ for arbitrary $\tau \in (1, T_\varepsilon)$, with some constant $C > 0$. Because τ is arbitrary in $(1, T_\varepsilon)$, this implies $T_\varepsilon \le C\varepsilon^{-1/\theta}$.

Next, we prove (1.5). We suppose that

$$
\liminf_{t\to T_\varepsilon^-} (\|u(t)\|_{L^2} + \|v(t)\|_{L^2}) < +\infty.
$$

Then there exist a sequence $\{t_k\}_{k\in\mathbb{N}} \subset [0, T_\varepsilon)$ and a positive constant $M > 0$ such that

$$
\lim_{k\to\infty} t_k = T_\varepsilon, \tag{4.13}
$$

$$
\sup_{k\in\mathbb{N}} (\|u(t_k)\|_{L^2} + \|v(t_k)\|_{L^2}) \le M. \tag{4.14}
$$

On the one hand, by (4.14) and $T_\varepsilon < \infty$, there exists a positive constant $T(M)$ such that we can construct a solution (u, v) of (2.1) that satisfies

$$
u, v \in C([t_k, t_k + T(M)); L^2) \cap L^{q_1}([t_k, t_k + T(M)); L^{r_1}) \cap L^{q_2}([t_k, t_k + T(M)); L^{r_2})
$$

for all $k \in \mathbb{N}$. On the other hand, by (4.13), when k is sufficiently large, the inequality $t_k + T(M) > T_\varepsilon$ holds, which contradicts the definition of T_ε. Therefore,

$$\liminf_{t \to T_\varepsilon^-} (\|u(t)\|_{L^2} + \|v(t)\|_{L^2}) = +\infty.$$

This completes the proof. $\square$

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