

OPTIMAL IMPULSIVE HARVEST POLICY FOR TIME-DEPENDENT LOGISTIC EQUATION WITH PERIODIC COEFFICIENTS

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ABSTRACT. We study a time-dependent logistic equation with periodic coefficients. First, we show that the impulsive harvest population equation has impulsive periodic solutions for constant effort harvest and for proportional harvest. Second, we investigate the optimal harvest effort that maximizes the sustainable yield per unit of time. Then we determine the corresponding optimal population levels. Our results generalize the results presented in [10].

1. INTRODUCTION

Most of the models for a single species dynamics have been derived from a differential equation of the form

$$\dot{x} = xf(x, t) - g(t, x), \quad (1.1)$$

where the solution $x = x(t)$ is the density (size, or biomass) of the resource population at time $t > 0$, the function $f = f(t, x)$ is characterized by the population change at the moment t , the function $g = g(t, x)$ describes the continuous influences of outside factors, such as hunting, cutting down the space available, etc.. Various choices of the functions f and g lead us to various models. When we only consider an isolated population without any perturbations, namely $g(t, x) = 0$, the classical model is the logistic equation

$$\begin{aligned} \dot{x} &= rx\left(1 - \frac{x}{K}\right) \\ x(0) &= x_0, \end{aligned} \quad (1.2)$$

or

$$\begin{aligned} \dot{x} &= r(t)x\left(1 - \frac{x}{K(t)}\right) \\ x(t_0) &= x_0, \end{aligned} \quad (1.3)$$

2000 *Mathematics Subject Classification.* 92D25, 34A37.

Key words and phrases. Impulsive harvest equation, global attractor, optimal impulsive harvest policy.

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Submitted March 4, 2003. Published December 9, 2003.

Project Supported by grants 10171010 and 10201005 from the National Science Foundation of China, and by grant 01061 from Major Project of Education Ministry of China.

where (1.2) is an autonomous evolutionary model, and (1.3) is treated as the non-autonomous evolutionary model because the coefficients of (1.3) are dependent on the time t .

In a real evolutionary processes of the population, the perturbation or the influence from outside occurs “immediately” as impulses, and not continuously. The duration of these perturbations is negligible compared to the duration of the whole process. For instance, as we know, fisherman can not fish day and night in 24 hours. Instead, they only fish in some time every day. Furthermore, the seasons also decide the fishing period. So the problem of impulsive harvest is more practical and realistic compared to the continuous harvest. However, to the best of our knowledge, there no results on impulsive harvest for renewable resources in the literature. In this paper, we research optimal impulsive harvest policy for a single population resource.

The organization of this article is as follows: In the next section, we establish the mathematical model for impulsive time harvest for the well known logistic equation. We also obtain the maximum of increasing density of population per unit time. In subsequent portions of this paper, the main results on the existence and the stability of impulsive periodic solution for the impulsive equation are proved. Then the optimal impulsive harvest policies are determined for both constant effort harvest and for harvest proportional to the size of the population.

2. THE IMPULSIVE HARVEST MODEL

Considering the feasible operation, we suppose that we harvest once every time T for the population X which obeys the logistic growth law. We shall establish the mathematical model of impulsive time harvest for the logistic equation:

$$\begin{aligned} \frac{dN}{dt} &= r(t)N\left(1 - \frac{N}{K(t)}\right) - \delta(s(t))Eh(N(t)) \\ N(t_0) &= N_0. \end{aligned} \quad (2.1)$$

Here, assume that r and K are both positive T -periodic functions with respect to t . $h(N(t))$ is the function of general harvest; δ is the Dirac impulsive function, which satisfies $\delta(0) = \infty$ and $\delta(s) = 0$ for $s \neq 0$ and $\int_{-\infty}^{\infty} \delta(s)ds = 1$, and

$$s(t) = \begin{cases} 0 & \text{if } t = nT, n \in N, \\ -1 & \text{otherwise.} \end{cases}$$

From this explanation, it is obvious that the population X will increase according to logistic increasing curve without exploitation and the management of the resource will harvest $Eh(N(t))$ every time T . For explaining the latter, we discuss the impulse function δ deliberately. As is well known, the Heaviside function satisfies

$$\theta(t) = \begin{cases} 1 & \text{if } t \geq 0, \\ 0 & \text{if } t < 0. \end{cases}$$

Using generalized derivatives, $\theta' = \delta$. Thus, if $t \neq nT$, $s(t) = -1$ and $\theta(s(t)) = 0$, namely, the management does not harvest. If $t = nT$, $s(t) = 0$ and $\theta(s(t)) = 1$, namely, in nT , the management harvests $Q(nT)$, which satisfies

$$Q(nT) = \int_{-\infty}^{nT} \delta(s(t))Eh(x(t))dt - \int_{-\infty}^{(n-1)T} \delta(s(t))Eh(x(t))dt = Eh(x(nT)).$$

Clearly, the general solution of (1.3) may be written in the form

$$x(t, t_0, x_0) = \left(\frac{1}{x_0} \exp \left\{ - \int_{t_0}^t r(s) ds \right\} + \int_{t_0}^t \frac{r(s)}{K(s)} \exp \left\{ - \int_s^t r(\tau) d\tau \right\} ds \right)^{-1}.$$

For convenience, denote $x(t, t_0, x_0) = \frac{1}{\frac{1}{x_0} A(t) + B(t)} = \frac{x_0}{A(t) + B(t)x_0}$, where

$$A(t) = \exp \left\{ - \int_{t_0}^t r(s) ds \right\}, \quad B(t) = \int_{t_0}^t \frac{r(s)}{K(s)} \exp \left\{ - \int_s^t r(\tau) d\tau \right\} ds. \quad (2.2)$$

For biological considerations, we are interested only in positive solutions. In this paper, we always need $x_0 > 0$. After time T , the increase of population in (1.3) without harvest is $x(T, 0, x_0) - x_0 =: f(x_0)$. Then

$$f(x_0) = \frac{x_0}{A(T) + B(T)x_0} - x_0. \quad (2.3)$$

In the following, our objective is to find an x_0 such that $f(x_0)$ reaches its maximum at \bar{x}_0 . Let $f'(x_0) = 0$, so we have

$$x_0^1 = \frac{-A(T) + \sqrt{A(T)}}{B(T)} > 0, \quad x_0^2 = \frac{-A(T) - \sqrt{A(T)}}{B(T)} < 0.$$

Furthermore, $f''(x_0^1) < 0$, then $\bar{x}_0 = x_0^1$. Thus the maximum of increasing density of population is

$$\omega =: \max f(x_0) = f(\bar{x}_0) = \frac{(1 - \sqrt{A(T)})^2}{B(T)}, \quad (2.4)$$

and the maximum of increasing density of population per unit of time is

$$\max \frac{f(x_0)}{T} = \frac{f(\bar{x}_0)}{T} = \frac{(1 - \sqrt{A(T)})^2}{B(T)T}. \quad (2.5)$$

3. OPTIMAL IMPULSIVE HARVEST POLICY FOR CONSTANT EFFORT HARVEST

Now, we consider single population X of size $N(t)$, which obeys the logistic growth law, is impulsively harvested by means of a constant effort, $h(N) \equiv 1$, namely, every time T , the management harvest constant is E . Equation of the impulsively harvested population reads

$$\begin{aligned} \frac{dN}{dt} &= r(t)N \left(1 - \frac{N}{K(t)} \right) - \delta(s(t))E, \\ N(t_0) &= N_0. \end{aligned} \quad (3.1)$$

We always denote the solution of (3.1) by $N(t, t_0, N_0)$, while represent $x(t, t_0, x_0)$ as the solution of (1.3) without harvest. It is known that the solution of a nonautomated system with T -periodic coefficients has the property of periodic translation, we can denote $x(t, t_0, x_0)$ and $x(t-nT, t_0-nT, x_0)$ as the same solution of a system.

Theorem 3.1. (1) If $0 < E < \omega = \frac{(1-\sqrt{A(T)})^2}{B(T)}$, there exist two positive impulsive periodic solutions $\xi_1(t)$ and $\xi_2(t)$ of (3.1) with

$$\xi_1(nT) = \frac{1 - A(T) - EB(T) + \sqrt{(1 - A(T) - EB(T))^2 - 4EA(T)B(T)}}{2B(T)},$$

$$\xi_2(nT) = \frac{1 - A(T) - EB(T) - \sqrt{(1 - A(T) - EB(T))^2 - 4EA(T)B(T)}}{2B(T)},$$

for all $n \in N$.

(2) If $E = \omega = \frac{(1-\sqrt{A(T)})^2}{B(T)}$, there exists a unique positive impulsive periodic solution $\xi(t)$ of (3.1) with

$$\xi_3(nT) = \frac{1 - A(T) - EB(T)}{2B(T)}, \quad \forall n \in N.$$

Proof. Let

$$F(y) =: f(y) - E = \frac{y}{A(T) + B(T)y} - y - E,$$

where $f(y)$ is defined by (2.3). If $0 < E < \omega$, we know that

$$(1 - A(T) - EB(T))^2 - 4EA(T)B(T) > 0,$$

meanwhile, it is easy to see that the equation $F(y) = 0$ has two roots, that is

$$y_1 = \frac{1 - A(T) - EB(T) - \sqrt{(1 - A(T) - EB(T))^2 - 4EA(T)B(T)}}{2B(T)}, \quad n \in N,$$

$$y_2 = \frac{1 - A(T) - EB(T) + \sqrt{(1 - A(T) - EB(T))^2 - 4EA(T)B(T)}}{2B(T)}, \quad n \in N.$$

It follows that $y_2 > y_1 > 0$. Next, we prove that $N(t, 0, y_1)$ and $N(t, 0, y_2)$ are T -periodic solutions. It is obvious that

$$\begin{aligned} N(T, 0, y_1) &= x(T, 0, y_1) - E = x(T, 0, y_1) - y_1 - E + y_1 \\ &= f(y_1) - E + y_1 = F(y_1) + y_1 = y_1 = N(0, 0, y_1), \end{aligned}$$

and

$$N(2T, 0, y_1) = N(2T, T, N(T, 0, y_1)) = x(2T, T, y_1) - E = x(T, 0, y_1) - E = y_1.$$

Therefore, we obtain inductively

$$N(nT, 0, y_1) = y_1 \quad \text{for all } n \in N.$$

Similarly, we have

$$N(nT, 0, y_2) = y_2 = N(0, 0, y_2) \quad \text{for all } n \in N.$$

Let $N(t, 0, y_1) = \xi_1(t)$, $N(t, 0, y_2) = \xi_2(t)$. Then $\xi_1(t)$ and $\xi_2(t)$ are impulsive periodic solutions of (3.1) with $\xi_1(nT) = y_1$, $\xi_2(nT) = y_2$ for all $n \in N$.

If $E = \omega$, then $F(y) = 0$ has one and only one root with $y_3 = \frac{1 - A(T) - EB(T)}{2B(T)}$, so (3.1) has only one impulsive periodic solution $\xi_3(t)$ with

$$\xi_3(nT) = \frac{1 - A(T) - EB(T)}{2B(T)}, \quad \forall n \in N.$$

The proof is completed. \square

Theorem 3.2. (1) If $E < \omega$, then $N(t, 0, N_0) \rightarrow \xi_2(t)$ as $t \rightarrow +\infty$ for $N_0 > y_1$ and $N(t, 0, N_0) \rightarrow 0$ for $0 < N_0 < y_1$.

(2) If $E = \omega$, then $N(t, 0, N_0) \rightarrow \xi_3(t)$ as $t \rightarrow +\infty$ for $N_0 > y_3$ and $N(t, 0, N_0) \rightarrow 0$ for $0 < N_0 < y_3$.

(3) If $E > \omega$, then $N(t, 0, N_0) \rightarrow 0$ as $t \rightarrow +\infty$ for all $N_0 > 0$.

Proof. First, we know that $F(y) > 0$ for $y_1 < y < y_2$ and $F(y) < 0$ for $y < y_1$ or $y > y_2$. Suppose $E < \omega$ and $N_0 > y_2$. For convenience, denote $N_n = N(nT, 0, N_0)$. We can write

$$N_1 = N(T, 0, N_0) = x(T, 0, N_0) - E = f(N_0) + N_0 - E = F(N_0) + N_0 < N_0.$$

On the other hand, $N_0 > y_2$ implies

$$N_1 = x(T, 0, N_0) - E > x(T, 0, y_2) - E = N(T, 0, y_2) = \xi_2(T) = y_2.$$

Similarly, we have

$$\begin{aligned} N_2 &= N(2T, 0, N_0) = N(2T, T, N_1) \\ &= x(2T, T, N_1) - E = x(T, 0, N_1) - E \\ &= f(N_1) + N_1 - E = F(N_1) + N_1 < N_1 \end{aligned}$$

and

$$N_2 = x(T, 0, N_1) - E > x(T, 0, y_2) - E = \xi_2(T) = y_2.$$

Therefore, by the same arguments we can obtain a monotone decreasing sequence $\{N_n\}$ with a lower bound y_2 . It is obvious that the sequence $\{N_n\}$ has a limit, suppose it is β , then $\beta \geq y_2$.

If $\beta > y_2$, then

$$\begin{aligned} N_{n+1} - N_n &= N((n+1)T, 0, N_0) - N_n = N((n+1)T, nT, N_n) - N_n \\ &= x((n+1)T, nT, N_n) - N_n = x(T, 0, N_n) - E - N_n = F(N_n). \end{aligned}$$

Therefore, $F(\beta) = 0$ as $n \rightarrow \infty$. Because $F(y) = 0$ has only two roots y_1 or y_2 , we get a contradiction. Thus $\beta = y_2$, that is $\lim_{n \rightarrow \infty} N_n = \beta = y_2$. According to the continuous dependence of solution on initial value in finite time, for any given $\epsilon > 0$ there is a $\delta \in (0, \epsilon)$, such that $|x_0 - y_2| < \delta$ implies $|x(t, 0, x_0) - x(t, 0, y_2)| < \epsilon$ for $t \in [0, T]$. In addition, we know that $\lim_{n \rightarrow \infty} N_n = \beta$, for the previous δ , there exists a natural number \bar{N} such that $n \geq \bar{N}$ implies that $0 < N_n - y_2 < \delta$, and then for any $n \geq \bar{N}$ and $t \in [nT, (n+1)T)$, we have

$$\begin{aligned} |N(t, 0, N_0) - \xi_2(t)| &= |N(t, 0, N_0) - N(t, 0, y_2)| \\ &= |N(t, nT, N_n) - N(t, nT, y_2)| \\ &= |x(t, nT, N_n) - x(t, nT, y_2)| \\ &= |x(t - nT, 0, N_n) - x(t - nT, 0, y_2)| < \epsilon \end{aligned}$$

for $t \in [nT, (n+1)T)$, which implies

$$|N(t, 0, N_0) - \xi_2(t)| < \epsilon \quad \text{for } t \geq \bar{N}T.$$

It is proved that if $E < \omega$, $N(t, 0, N_0) \rightarrow \xi_2(t)$ as $t \rightarrow \infty$ for $N_0 > y_2$. If $y_1 < N_0 < y_2$, we can get a monotone increasing sequence $\{N_n\}$ with upper bound y_2 ; furthermore, $\lim_{t \rightarrow \infty} N_n = y_2$. The other argument is the same as the previous, so $E < \omega$, $N(t, 0, N_0) \rightarrow \xi_2(t)$ as $t \rightarrow \infty$ for $N_0 > y_2$ or $y_1 < N_0 < y_2$. The other conclusions of Theorem 3.2 can be proved by similar methods, we omit them here. The proof is complete. \square

From Theorem 3.1, we know that if $0 < E < \omega := \frac{(1-\sqrt{A(T)})^2}{B(T)} = \max f(y)$, there exist two positive impulsive periodic solutions $\xi_1(t)$ and $\xi_2(t)$ of (3.1). From Theorem 3.2, it follows that $\xi_2(t)$ is stable and that $\xi_1(t)$ is unstable. In the case of $0 < E < \omega$, if the initial population is $N_0 > y_2$ or $y_1 < N_0 < y_2$, then $N(t, 0, N_0)$ will converge to $\xi_2(t)$ asymptotically under constant harvest. But if the initial population N_0 is less than y_1 , then $N(t, 0, N_0)$ will approach 0 as time tends to infinity.

If $E > \omega$, the population approaches 0 for any initial level N_0 in a finite time.

If $E = \omega$, there exists a unique positive impulsive periodic solution $\xi_3(t)$ of (3.1) with $\xi_3(nT) = \frac{1-A(T)-EB(T)}{2B(T)}$, which is “semi-stable” in the sense that $N(t, 0, N_0)$ approaches $\xi_3(t)$ if $N_0 > y_3 = \xi_3(T)$, but $N(t, 0, N_0)$ approaches 0 if $N_0 < y_3$.

4. OPTIMAL IMPULSIVE HARVEST POLICY FOR PROPORTIONAL HARVEST

The assumption in section 3 that the harvesting effort is a constant implies that we cannot control exploitation for dangerous region. In this section, we will use the phrase *catch-per-unit-effort hypothesis* to describe an assumption that catch-per-unit-effort is proportional to the stock level, or that $h(x) = x$, where E denotes effort and satisfies $0 \leq E < 1$. In other words, the management harvests $Q(nT) = Ex(nT)$ in nT . Equation of the impulsively harvested population reads

$$\begin{aligned} \frac{dN}{dt} &= r(t)N\left(1 - \frac{N}{K(t)}\right) - \delta(s(t))EN, \\ N(t_0) &= N_0. \end{aligned} \quad (4.1)$$

In this section, the solution of (4.1) is still denoted by $N(t, t_0, N_0)$.

Now we investigate the optimal impulsive harvest policy, namely, the optimal harvesting effort, the maximum sustainable yield and the corresponding optimal population level.

Definition [9]. A solution $\xi(t)$ of (4.1) is globally attractive for positive initial value if any other solution of (4.1) $N(t, 0, N_0)$ with $N_0 > 0$ satisfies:

$$\lim_{t \rightarrow +\infty} |N(t, 0, N_0) - \xi(t)| = 0.$$

Theorem 4.1. If $0 < E < \frac{1-A(T)}{B(T)}$, there exists a unique positive impulsive periodic solution $\xi(t)$ of (4.1), which satisfies $\xi(nT) = \frac{1-E-A(T)}{B(T)}$. In addition, $\xi(t)$ is globally attractive for positive initial value.

Proof. Let

$$\begin{aligned} G(y) &= (1-E)x(T, 0, y) - y = (1-E)(f(y) + y) - y \\ &= (1-E)f(y) - Ey \\ &= (1-E)\left(\frac{y}{A(T) + B(T)y} - y\right) - Ey. \end{aligned}$$

It is easy to prove that when $0 < E < 1 - A(T)$, the unique positive root for $G(y) = 0$ is

$$\tilde{y} = \frac{1-E-A(T)}{B(T)}. \quad (4.2)$$

We have also $G(y) > 0$ for $0 < y < \tilde{y}$, and $G(y) < 0$ for $y > \tilde{y}$. Next we prove that $N(t, 0, \tilde{y})$ is impulsive periodic solution of (4.1). It is easy to see that

$$N(T, 0, \tilde{y}) = (1 - E)x(T, 0, \tilde{y}) = G(\tilde{y}) + \tilde{y} = \tilde{y}$$

and

$$\begin{aligned} N(2T, 0, \tilde{y}) &= N(2T, T, N(T, 0, \tilde{y})) = N(2T, T, \tilde{y}) \\ &= (1 - E)x(2T, T, \tilde{y}) = (1 - E)x(T, 0, \tilde{y}) = \tilde{y}. \end{aligned}$$

Inductively, we prove that $N(nT, 0, \tilde{y}) = \tilde{y}$ for all $n \in \mathbb{N}$. Therefore, (4.1) has unique impulsive periodic solution $N(t, 0, \tilde{y}) := \xi(t)$ with $\xi(nT) = \tilde{y}$ for $\forall n \in \mathbb{N}$.

Next, we prove the global attractiveness of $\xi(t)$. Suppose that $N_0 > \tilde{y}$, and $N_n := N(nT, 0, N_0)$, $n \in \mathbb{N}$. We have

$$N_1 = N(T, 0, N_0) = (1 - E)x(T, 0, N_0) = G(N_0) + N_0 < N_0$$

and

$$N_1 = N(T, 0, N_0) = (1 - E)x(T, 0, N_0) > (1 - E)x(T, 0, \tilde{y}) = N(T, 0, \tilde{y}) = \tilde{y}.$$

Similarly, we can prove that $\tilde{y} < N_2 < N_1$. Thus we get a monotone decreasing sequence $\{N_n\}$ with a lower bound \tilde{y} . Assume that the sequence $\{N_n\}$ has a limit $\tilde{\beta}$, it is obvious $\tilde{\beta} \geq \tilde{y}$. Using the similar method with the section 3, suppose $\tilde{\beta} > \tilde{y}$, then

$$\begin{aligned} N_{n+1} - N_n &= N((n+1)T, 0, N_0) - N_n \\ &= N((n+1)T, nT, N_n) - N_n \\ &= (1 - E)x((n+1)T, nT, N_n) - N_n \\ &= (1 - E)x(T, 0, N_n) - N_n = G(N_n), \end{aligned}$$

which implies that $G(\tilde{\beta}) = 0$, this contradicts with the fact that the equation $G(y) = 0$ has a unique root \tilde{y} . Thus $\tilde{\beta} = \tilde{y}$ and we have proved that

$$\lim_{n \rightarrow +\infty} N_n = \tilde{\beta} = \tilde{y}.$$

Therefore, for any given $\epsilon > 0$, there is a $\delta \in (0, \epsilon)$ such that $n > \tilde{N}$ implies $0 < N_n - \tilde{y} < \delta$, then according to continuous dependence of solution to initial value, we have $|x(t, 0, N_n) - x(t, 0, \tilde{y})| < \epsilon$ for $t \in [0, T)$. Then notice $1 - E < 1$, for $n \geq \tilde{N}$ and $t \in [nT, (n+1)T)$,

$$\begin{aligned} |N(t, 0, N_0) - \xi(t)| &= |N(t, nT, N_n) - N(t, nT, \tilde{y})| \\ &= |1 - E||x(t, nT, N_n) - x(t, nT, \tilde{y})| \\ &< |x(t - nT, 0, N_n) - x(t - nT, 0, \tilde{y})| < \epsilon. \end{aligned}$$

That is,

$$\lim_{t \rightarrow \infty} |N(t, 0, N_0) - \xi(t)| = 0 \quad \text{for } N_0 > \tilde{y}.$$

By a similar argument, we can prove

$$\lim_{t \rightarrow \infty} |N(t, 0, N_0) - \xi(t)| = 0 \quad \text{for } 0 < N_0 < \tilde{y}.$$

Therefore, we have proved that the impulsive periodic solution $\xi(t)$ is globally attractive for positive initial value. The proof is complete. \square

Note that if $E = 1 - A(T)$, we obtain $\xi(nT) = \frac{1 - A(T) - E}{B(T)} = 0$. So the following statement is valid.

Theorem 4.2. *If $E \geq 1 - A(T) > 0$, the size of population X tends to extinction.*

In real life, fishers would like to make a decision how to obtain maximum harvest. From Theorem 4.1, when T is a fixed constant, the sustainable yield per unit time is

$$Y(E) = E \frac{1 - E - A(T)}{B(T)T(1 - E)}. \quad (4.3)$$

Our objective is to find an E^* such that $Y(E)$ reaches its maximum at $E = E^*$. This is the optimization of a function. The derivative of $Y(E)$ is written as

$$Y'(E) = \frac{E^2 - 2E + 1 - A(T)}{TB(T)(1 - E)^2},$$

then the equation $E^2 - 2E - A(T) + 1 = 0$ has two roots, which are $E_1 = 1 + \sqrt{A(T)} > 1$ and $E_2 = 1 - \sqrt{A(T)} < 1$. Furthermore, we can obtain

$$Y''(E) = \frac{2A(T)}{TB(T)(-1 + E)^3} < 0, \quad \forall 0 < E < 1.$$

So we have

$$E^* = E_2 = 1 - \sqrt{A(T)}. \quad (4.4)$$

Then $Y(E)$ reaches its maximum at $E = E^*$. Substituting (4.4) into (4.2), we have

$$x^*(T) = \frac{\sqrt{A(T)}(1 - \sqrt{A(T)})}{B(T)}. \quad (4.5)$$

Substituting (4.4) into (4.3), we can get the maximum sustainable yield per unit time $Y(E^*)$:

$$Y(E^*) = \frac{(1 - \sqrt{A(T)})^2}{TB(T)}. \quad (4.6)$$

So we obtain the optimal harvest effort E^* that maximizes the sustainable yield per unit time $Y(E^*)$, the corresponding optimal population level $x^*(T)$.

At last, we want to point out that our results are compatible to the conclusion by Clark. As is well known, the Logistic equation which is subjected to continuous exploitation reads

$$\begin{aligned} \dot{x} &= rx\left(1 - \frac{x}{K}\right) - Ex, \\ x(0) &= x_0 \end{aligned} \quad (4.7)$$

The maximum sustainable yield is $Y = Kr/4$ corresponding to the optimal harvest effort $E^* = r/2$ and the optimal population level $x^* = K/2$. If the coefficients of (4.1) become constant K and r , we will consider the following impulsive equation [10].

$$\begin{aligned} \dot{N} &= rN\left(1 - \frac{N}{K}\right) - \delta(s(t))EN, \\ N(0) &= N_0 \end{aligned} \quad (4.8)$$

Obviously, (2.2) becomes

$$\begin{aligned} A(T) &= \exp\left\{-\int_0^T r ds\right\} = e^{-rT}, \\ B(T) &= \int_0^T \frac{r}{K} \exp\left\{-\int_s^T r(\tau) d\tau\right\} ds = \frac{1 - e^{-rT}}{K}. \end{aligned} \quad (4.9)$$

Substitute (4.9) into (4.2), we obtain the result in [10]:

$$\hat{y} = \frac{(e^{rT}(1-E) - 1)K}{e^{rT} - 1}$$

is a global attractive impulsive periodic solution. Using the same technique, (4.4)-(4.6) also are the corresponding results in [10]: When T is fixed value, the optimal harvest effort $\hat{E}^* = 1 - e^{-rT/2}$, the optimal population level $\hat{x}^*(T) = \frac{K}{e^{rT/2}} + 1$, the maximum sustainable yield per unit time

$$\hat{Y}(E^*) = \frac{K(e^{rT/2} - 1)^2}{T(e^{rT} - 1)}.$$

T is harvesting time interval in (4.8), if $T \rightarrow 0$, $\hat{Y}(E^*) \rightarrow Kr/4$, which implies that the less is time interval T of impulsive harvest, the nearer is the maximum yield (4.7) and (4.8), namely, the optimal impulsive harvesting policy is continuous harvest.

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