

IS TUTORING TEACHING? EXPLORING TUTORING'S POTENTIAL TO
IMPROVE MATHEMATICS TEACHER EDUCATION

by

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LIST OF ABBREVIATIONS

Abbreviation	Description
NCTM	National Council of Teachers of Mathematics
PST	Pre-Service Teacher
PSMT	Pre-Service Mathematics Teacher

ABSTRACT

This study investigated the tutoring practices of mathematics tutors working in one university tutoring center and the corresponding rationale exhibited by the mathematics tutors. This study illustrates how the tutoring practices of mathematics tutors align with the Eight Mathematics Teaching Practices outlined in NCTM's recent publication *Principles to actions: Ensuring mathematical success for all*. The three participants were chosen from among five mathematics tutors who volunteered for this study, and were selected because they each had a different amount of tutoring experience and because they each had no prior teaching experiences outside of tutoring nor had they taken any courses about teaching. Data were collected by means of observations of the participants' tutoring and interviews focused on asking the participants to explain various decisions they were observed to make while tutoring. The observations and interviews were analyzed using qualitative coding. The significance of this study lies in its contribution to mathematics education research focused on how to utilize tutoring experiences as a means of providing pre-service mathematics teachers with authentic opportunities to practice the work of teaching mathematics.

Keywords: experiential learning, situated cognition, teacher education, mathematics teaching education, tutoring experiences

I. INTRODUCTION

The design of many teacher education programs in the United States, consisting of methods courses, in which pre-service mathematics teachers (PSMTs) learn about teaching theory and discuss various teaching practices, and practical experience, in the form of student teaching and other in-school experiences, contributes to a fragmented and disconnected understanding of the theory and practice of teaching (Hibert, Morris, & Glass, 2003; Smagorinsky, Cook, & Johnson, 2003; Grossman, Hammerness, & McDonald, 2009). Teacher education programs need to aid future teachers in learning how to learn from their teaching experiences as a means of improving their teaching ability (Hibert et al., 2003). These programs must place profound emphasis on the interconnection between the learning of teaching theory and the practice of teaching (Smagorinsky et al. 2003). Additionally, prospective teachers need to be able to attend to both the complex theoretical aspects of teaching as well as their practical implementations within the classroom (Grossman et al., 2009). In order to address these concerns, teacher education programs must include numerous opportunities for PSTs to engage in high quality teaching experiences to reflect upon and learn from the implementation of teaching theory (Hibert et al., 2003; Smagorinsky et al., 2003; Grossman et al., 2009).

Literature demonstrates that many teacher education programs are insufficiently preparing their students to implement the theoretical teaching perspectives discussed during the program and attributes some portion of this inadequacy to be the result of a lack of useful teaching experience incorporated into teacher education programs (Hibert et al., 2003; Smagorinsky et al., 2003; Grossman et al., 2009). Moreover, the

recommendations presented by Hibert et al. (2003), Smagorinsky et al. (2003), and Grossman et al. (2009) to address these concerns are based on the belief that future mathematics teachers need to learn to teach mathematics through opportunities which permit them to practice implementing teaching theories and reflect upon these implementations. Kolb's (2015) experiential learning theory explains that learning is an ongoing process grounded in experiences, and requires that learners transform wisdom gained from reflection upon experiences into abstract concepts which can later be applied to other experiences, leading to either an affirmation that the abstraction is still valid and useful or to the learner making modifications to his/her abstraction in light of the new experiences. This process is an ongoing cycle of engaging in a concrete experience, reflecting on that experience to create or evaluate abstractions, then testing these abstractions during other concrete experiences upon which learners reflect, and so on. Situated cognition posits that learning is a process of enculturation facilitated by novices participating in activities which authentically reflect problems that masters would engage with in a manner that is consistent with how masters would endeavor to solve these problems (Browns, Collins, Duguid 1989). Applied to the education of PSMTs, situated cognition suggests that in order for these learners to develop the skills and knowledge required to become effect teachers, their teacher education programs need to provide them with opportunities to actively participate in the culture of professional mathematics teachers by engaging with mathematics learners in a context that allows the PSMTs to struggle with and solve problems using the tools of mathematics teaching, under the guidance of professional mathematics teachers.

Thus, one path to begin resolving the issues prevalent in many teacher education programs is to have them incorporate more opportunities to actually practice teaching, but what is not so clear is how to achieve this. Implementing these supplementary experiences into teacher education programs will require time to incorporate proposed alterations into program curricula and will necessitate that programs have access to additional resources, which will likely result in an increased need for monetary resources, an obstacle facing most if not all education reform efforts. Thus, in order for teacher education programs to be able implement the necessary improvements to the quality of PSMT education in a timely manner proposed changes need to be sensitive to the limited availability of financial resources as well as the ease with which these changes may be integrated into existing programs.

This study will focus on how one tutoring center, a resource common among many universities, might be used as a medium through which teacher education programs may incorporate the practical experiences research has identified as necessary for the improvement of PSMT education. The experiences of mathematics tutors will be examined to determine whether or not such tutoring experiences have the capability of serving as a medium for PSMTs to authentically practice the work of teaching mathematics.

Purpose

The purpose of this embedded case study will be to investigate the activities of mathematics tutors working in one university tutoring center. Developing an understanding of tutors' work and experiences will help to examine the extent to which the mathematics tutoring culture and activities authentically approximate those of those

of professional mathematics teachers. This understanding will be used to as a basis for discussing how, if at all, mathematics tutoring experiences might serve as a means for addressing recommendations for the preparation of effective mathematics instructors.

Epoche

In research that examines experiences that are similar to those that the researcher has personally participated in, it is necessary for the researcher to be open and explicit regarding his/her personal connection to the phenomenon being investigated. To this end, the researcher will provide an explanation of his prior experiences as a tutor employed by the Learning Center, the tutoring center that will serve as the site of this study. In order to better convey the researcher's story, the following section will be presented in first person to avoid indirect and unclear wording.

My journey as an educator began long before I ever wanted to be a teacher. During all four years of my undergraduate education, I worked as a Learning Center mathematics tutor. Reflecting upon this experience, I am able to watch myself grow from having a moderate understanding of mathematical content and the abysmal pedagogical strategy of just telling my students what to do, to spending my last two years as widely known and sought after tutor due to my then robust and deep understanding of the content as well as personally developed pedagogical strategies that were targeted at improving student understanding. While my improvement in content knowledge is due, in large part, to my work towards a B.S. in Mathematics, my desire to teach my tutees in a manner that facilitated their individual understanding developed as a result of my ever growing sense of responsibility to share my knowledge of and passion for mathematics in a manner that best served my students. As a result, when I began to study literature in

teacher education programs and was confronted with the widespread call for radical alterations to their curriculum, I became very curious as to whether the growth I experienced as an educator as a result of my tutoring experience was unique to me or if this experience could be useful for facilitating growth within future teachers.

Research Questions

This study will explore the following research questions:

- 1) What are the teaching practices exhibited by mathematics tutors and what is the rationale for these practices?
- 2) What effects, if any, upon the participants' tutoring do the participants attribute to their participation in this study?

Significance of this Study

This study is significant in the field of mathematics education in several ways. First, this study seeks to add to the body of research knowledge on the use of tutoring experiences within secondary mathematics teacher education programs as a means of providing greater opportunity for pre-service teachers to practice the work of teaching. This study will also endeavor to examine how the experiences of university students working as mathematics tutors align with recommendations of components deemed necessary for effective mathematics instruction. Furthermore, the experiences of tutors will be leveraged to begin to address the literature gap resulting from researchers primarily focusing on tutoring experiences within a K-12 setting contemporary with the pre-service teachers' intended certification.

Definitions of Terms

Terminology used within this document and their meaning is provided in this section to ensure clarity.

Client – a student who is receiving assistance from a tutor.

Tutoring Session – A time during which a tutor is rendering aid to a client.

II. LITERATURE REVIEW

Researchers have identified that one central issue contributing to the inadequate preparation of future mathematics teachers is that mathematics teacher education programs do not provide their students with enough opportunities to practice the work of teaching that are of sufficient quality (Ball & Forzani, 2009; Grossman et al., 2009; Hollins, 2011). Thus, it is apparent that a key to improving the quality of mathematics teaching in K-12 classrooms is to improve the quality of mathematics teacher preparation through the incorporation of a greater number of high quality opportunities for PSMTs to learn how to teach mathematics by engaging in situations which afford PSMTs an authentic context in which to implement teaching theories discussed during methods coursework. This chapter begins with a presentation of the theoretical framework for this study. Then, in order to better understand what is required for effective mathematics instruction, recommendations for mathematics teaching practices will be explored. Then, to address the call for more opportunities to practice teaching, research examining the application of tutoring experiences in teacher education programs will be presented, with the goal of examining the potential displayed by tutoring activities to address the characteristics necessary for effective instruction.

Theoretical Framework

Situated cognition is the theoretical framework for this research because it a learning theory founded upon the notion that deep learning of knowledge and skills occurs when learners are taught in a context that facilitates the use of these knowledge and skills in a manner consistent with how practicing professionals would do so. Situated cognition developed in response to the pervasiveness of the assumption within many

school classrooms that treat “knowledge as an integral, self-sufficient substance, theoretically independent of the situations in which it is learned and used” (Brown, Collins, & Duguid, 1989, p. 32). Such an assumption within teacher education programs appears to be at the heart of the concerns raised by Hibert et al. (2003), Smagorinsky et al. (2003), and Grossman et al. (2009). In contrast to this disjointed view of knowledge and the contexts in which it is used, Brown et al. (1989) argue that “the activity in which knowledge is developed and deployed... is not separable from or ancillary to learning and cognition. Nor is it neutral. Rather, it is an integral part of what is learned” (p. 32). Situated cognition states that all knowledge is indexed by the contexts within which it is learned and applied,

its constituent parts index the world and so are inextricably a product of the activity and situations in which they are produced. A concept, for example, will continually evolve with each new occasion of use, because new situations, negotiations, and activities inevitably recast it in a new, more densely textured form. So a concept, like the meaning of a word, is always under construction.
(Brown et al., 1989, p.33)

Brown et al. (1989) submit that conceptual knowledge is similar to a set of tools, in that “they can only be fully understood through use, and using them entails both changing the user’s view of the world and adopting the belief system of the culture in which they are used” (p. 33). If knowledge is presented as a list of concepts which learners are expected to internalize, then “it is common for students to acquire algorithms, routines, and decontextualized definitions that they cannot use and that, therefore, lie inert” (Brown et al., 1989, p. 33). This can, unfortunately, lead to students that are able to manipulate and

apply algorithms, routines, and definitions to successfully complete classroom assignments without ever revealing to their teacher, or even themselves, that they would not know what to do when they encounter a problem within the real world. By contrast, learners who actively use tools, or knowledge, instead of just acquiring them, “build an increasingly rich implicit understanding of the world in which they use the tools and of the tools themselves” (Brown et al., 1989, p. 33) and this understanding of the knowledge and the world in which it is applied are continuously changing due to the interaction of the knowledge and the context of its use because learning is a “continuous, life-long process resulting from acting in situations” (p. 33).

Learning to use a tool requires more than could ever be distilled into any set of explicit rules because “the occasions and conditions for use arise directly out of the context of activities of each community that uses the tool, framed by the way members of that community see the world” (Brown et al., 1989, p. 33). The community in which a tool is used in conjunction with the tool itself, dictate the appropriate uses of the tool based upon the “accumulated insights” (Brown et al. 1989, p. 33) of the community. Conceptual knowledge “similarly reflect[s] the cumulative wisdom of the culture in which they are use and the insights and experience of individual” (Brown et al. 1989, p. 33) and the meaning and appropriate use of this knowledge does not rely solely upon the abstract concept, but rather “is a function of the culture and the activities in which the concept has been developed” (p. 33).

In order for students to learn to use knowledge and skills as professional practitioners use them, “a student, like an apprentice, must enter into that community and its culture” (Brown et al., 19859, p. 33) because a robust understanding of conceptual

knowledge requires students to be able to appropriately apply that knowledge, which in turn requires that students understand the community of practitioners who utilize this knowledge. Thus, “learning is...a process of enculturation” (Brown et al., 1989, p. 33). Even though the practices of a culture are generally exceedingly intricate and obscured, when people are afforded the opportunity to “observe and practice *in situ* the behavior of members of a culture, people pick up relevant jargon, imitate behavior, and gradually start to act in accordance with its norms...with great success” (Brown et al., 1989, p. 34). Typically, classrooms “deny students the chance to engage the relevant domain culture, because that culture is not in evidence” (Brown et al., 1989, p. 34). Despite students being shown the knowledge, skills, and tools of academic cultures, the culture in which students observe and practice these is that of being a student in a school. The domain culture and that of being a school student “can be unintentionally antithetical” (Brown et al., 1989, p.34) to one another because of the stark differences in how domain professionals and students use knowledge, and can lead to students that “may pass exams (a distinctive part of school cultures) but still not be able to use a domain’s conceptual tools in authentic practice” (p. 34). Thus, students need exposure to the implementation of a domain’s knowledge and tools in “authentic activity – to teachers acting as practitioners and using these tools in wrestling with problems of the world” (Brown et al., 1989, p. 34). Participation in such authentic activity can illuminate the ways in which domain professionals view the world and solve emergent problems.

Appropriate activity within a domain is shaped by the domain’s culture, and the meaning attributed to activity is the result of a negotiation between the accumulated wisdom of past members and the new activities of present members, thus “activities...

cohere in a way that is, in theory, if not always in practice, accessible to members who move within the social framework” (Brown et al., 1989, p. 34). In light of this stance, Brown et al. (1989) define authentic activities “as the ordinary practices of the culture” (p. 34). “When authentic activities are transferred to the classroom, their context is inevitably transmuted; they become classroom tasks and part of the school culture” (Brown et al., 1989, p. 34). This allows students to apply classroom procedures, such as relying on keywords presented in word problems, to the now classroom task, which leads to the learning, using, and testing of concepts to remain “hermetically sealed within the self-confirming culture of the school...[where], contrary to the aim of schooling, success within this culture often has little bearing on performance elsewhere” (Brown et al., 1989, p. 34). The transmutation of an authentic activity into a classroom activity is a result of key features of the context of the authentic activity “are often dismissed as ‘noise’ from which salient features can be abstracted for the purpose of teaching” (Brown et al., 1989, p. 34). However, the context of an activity is an incredibly complex network upon which practitioners rely for essential support and “the source of such support is often only tacitly recognized by practitioners, or even by teachers or designers of simulations” (Brown et al., 1989, p. 34), making it quite difficult to construct a classroom simulation that sufficiently imitates authentic activity because it is nearly impossible to identify all the features of the activity’s natural context that are necessary for engaging the activity like a professional would. Moreover, practitioners use the context of their activities as a means of indexing their knowledge for future use. Indexing, in this case, implies that when a practitioner employs knowledge and skills while engaging in an activity, features of the context not only provide support that guide practitioners’ decisions and problem

solving process, but these features also provide a means for practitioners to organize their knowledge around the situations in which it is appropriate to apply this knowledge. That is, the context of an activity shapes how practitioners utilize their knowledge, and the use of this knowledge shapes how practitioners approach future activities.

Brown et al. (1989) argue that in order for students to be able to learn domain knowledge, rather than simply acquire it, they must be able to engage in activities that domain practitioners would engage in, and students must be able to approach addressing the problem within these authentic activities in ways that mirror how practitioners would do so. To accomplish this, Brown et al. (1989) suggest an instructional approach which they call cognitive apprenticeship. Learning through cognitive apprenticeship entails learners engaging in authentic domain activities within an authentic culture, with gradually decreasing support and guidance from a practitioner. Cognitive apprenticeship begins with students being coached by a practitioner, initially by observing the practitioner as he/she engages with authentic problems and later by receiving in-the-moment support from the practitioner while the students struggle to make sense of and solve problems on their own. While struggling to solve problems, students are encouraged to work collectively with fellow students to share insights and approaches to addressing a task so that students can practice collaboration as a means of improving their own ability to problem solve. In addition to relying on peers for problem solving support, students are to reflect upon their own experiences as well as those of their peers as a means of developing faculty with the ability to articulate their own approaches to addressing the features of an activity as a means of contributing to the collective wisdom of the culture as well as to practice gaining insight into an activity from the experiences

of others. This process of learning through doing and reflecting upon what was done by and with colleagues allows students to develop understandings of a domain's skills, knowledge, and tools in a manner that facilitate the students' ever-adapting conception of when and how to appropriately apply their knowledge.

The Development of Proficient Mathematics Teachers

In the National Council of Teachers of Mathematics' (NCTM) publication *Principles to Actions: Ensuring mathematical success for all* (2014), effective teaching is defined to be “teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically” (p. 7). NCTM proposes eight Mathematics Teaching Practices to serve as “a framework for strengthening the teaching and learning of mathematics” (2014, p. 9), These Mathematics Teaching Practices are an articulation of eight interwoven components that NCTM has identified to be integral to effective mathematics instruction. They dictate that effective teachers of mathematics must:

- Establish mathematics goals to focus learning;
- Implement tasks that promote reasoning and problem solving;
- Use and connect mathematical representations;
- Facilitate meaningful mathematical discourse;
- Pose purposeful questions;
- Build procedural fluency from conceptual understanding;
- Support productive struggle in learning mathematics; and
- Elicit and use evidence of student thinking. (NCTM, 2014, p. 10)

Because these are intended to articulate eight interconnected features of what high-quality mathematics instruction should include, they are assumed to be appropriate for use as a guide for what mathematics teacher education programs should endeavor to instill within their students. Each aspect will be elaborated upon to provide a more clear understanding of the characteristics deemed necessary by NCTM in order for PSMT education programs to produce graduates that are competent enough to enter the workforce effectively as well as understand how to continue to grow as educators throughout their teaching career.

Establish mathematics goals to focus learning. Teachers must have a shared understanding of the mathematics that students are expected to learn and how students understanding of this mathematics develop along a learning progression. Teachers’ “goals should describe what concepts, ideas, or methods students will understand more deeply as a result of instruction and identify the mathematical practices that students are learning to use more proficiently” (NCTM, 2014, p. 12). These goals should be particularly linked to the classroom’s curriculum and students’ present learning needs. By situating mathematics learning goals within an understanding of the learning progression of students facilitates opportunities for teachers to construct explicit connections among content in order to help students see how concepts are built upon and relate to one another and to encourage students to see mathematics as a coherent and interconnected discipline. Moreover, the mathematical purpose of a lesson should be understood by the students so that students understand “how the activities contribute to and support their mathematics learning” (NCTM, 2014, p. 13), which can help students to focus on key components of a lesson and be better equipped to monitor and assesses their own

learning. Finally, a clear understanding of the goals of mathematics instruction guides “the decisions that teachers make as they plan mathematics lessons, make adjustments during instruction, and reflect after instruction on the progress that students are making toward the goals” (NCTM, 2014, p. 13), including how teachers facilitate classroom discourse, ensure the establishment of connections among mathematical content, support students in their struggles, and deciding what counts as evidence of students’ learning. NCTM (2014) provides list of four teacher actions that summarize how teachers can effectively embody the principle of using goals to focus mathematics learning. These actions are:

- Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit;
- Identifying how goals fit within a mathematics learning progression;
- Discussing and referring to the mathematical purpose of a lesson during instruction to ensure that students understand how the current work contributes to their learning; and
- Using the mathematical goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction. (NCTM, 2014, p. 16)

Implement tasks that promote reasoning and problem solving. In order to guarantee that students will be able to “engage in high-level thinking” (NCTM, 2014, p. 17) effective mathematics instruction must engage students in solving and discussing tasks that encourage the use of mathematical reasoning and problem solving, and that also allow students to approach a problem from multiple entry points and apply various solution strategies. Moreover, effective mathematics teachers must understand how to

capitalize on students' prior knowledge and experiences through an understanding of how "contexts, culture, conditions, and language can be used to create mathematical tasks" (NCTM, 2014, p. 17). The five actions that explicate how NCTM expects that effective teachers would strive to promote reasoning and problem solving within their classrooms are:

- Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding;
- Selecting tasks that provide multiple entry points through the use of varied tools and representations;
- Posing tasks on a regular basis that require a high level of cognitive demand;
- Supporting students in exploring tasks without taking over student thinking;
- Encouraging students to use varied approaches and strategies to make sense of and solve tasks. (NCTM, 2014, p. 24)

Use and connect mathematical representations. Mathematics teaching should engage students in establishing connections among various mathematical representations as a means of deepening their "understanding of mathematics concepts and procedures" (NCTM, 2014, p. 24) as well as facilitating students understanding of representations as tools to aid in problem solving: "When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving abilities" (NCTM, 2014, p. 24). Of particular importance to student learning are visual representations, which can help students to make consider and make sense of relationships among quantities as they

construct diagrams, tables, and graphs. Visual representations also help to support classroom discourse because they “leave a trace of student problem solving that can be displayed, critiqued, and discussed” (NCTM, 2014, p. 25). Using visual representations allow students to gesture to parts of a math drawing, or other representation, which can aid students “in following the reasoning of their classmates and in giving voice to their own explanations” (NCTM, 2014, pp. 25 – 26). Students’ mathematical understanding is deepened through discussions that focus on the similarities persistent among different representations and an understanding of these connections can aid students in approaching problems from multiple points of view through teachers encouraging students to utilize multiple representations in an attempt to make sense of a problem and discover a path that will lead to a solution. Teachers’ actions to help students in using and forging connections among mathematical representations are summarized as follows:

- Selecting tasks that allow students to decide which representations to use in making sense of the problems;
- Allocating substantial instructional time for students to use, discuss, and make connections among representations;
- Introducing forms of representations that can be useful to students;
- Asking students to make math drawings or use other visual supports to explain and justify their reasoning;
- Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representations;
- Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems. (NCTM, 2014, p. 29)

Facilitate meaningful mathematical discourse. Effective mathematics teaching engages students in discourse as a means of advancing the mathematical learning of the whole class. This discourse “includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication” (NCTM, 2014, p. 29). By engaging students in mathematical discourse, teachers afford students opportunities to share and clarify their ideas, generate convincing arguments to justify their understanding of how and why things work, develop proficiency with the language of mathematical ideas, as well as view situations from alternative perspectives. Creating a classroom culture that supports student learning through discourse requires teachers to how to honor the contributions of students and their thinking while maintaining a focus on mathematical ideas of the lesson. This requires that teachers be able to decide which students share their work, the order in which these students should present, and which questions should be asked in order to ensure that students forge appropriate connections among the key strategies and ideas that are the impetus for the lesson. Moreover, students also need opportunities to discuss mathematical ideas with their classmates, as well as respond to and question other students’ contributions to the classroom conversation, however teachers need to be able to facilitate this inter-student discourse in a manner that genuinely places the students in control of the conversation. Teachers’ actions that can facilitate the use of meaningful mathematical discourse as a learning tool include:

- Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations;

- Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion;
- Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches;
- Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning. (NCTM, 2014, p. 35)

Pose purposeful questions. Effective mathematics teaching depends on the use of questions that help students to justify and reflect upon their thinking during meaningful mathematical discourse. Questions permit teachers to determine what students know, make in-the-moment decisions to adapt instruction to meet students' needs, support students in establishing important mathematical connections, and aid students in asking their own questions. In order to ensure that teachers' questions are helping to advance students' learning of the mathematics two key issues must be considered: the types of questions asked and the patterns in which questions are used.

Questions are categorized, in increasing order of level of thought required to respond, as: gathering information, which require students to recall facts, definitions, or procedures (e.g. What is the formula for finding the area of a rectangle?); probing thinking, which require students to explain, elaborate, or clarify their thinking (e.g. It is still not clear how you figure out that 20 was the scale factor, so can you explain it in another way?); making the mathematics visible, which seek to get students to discuss the structure of mathematics and to make connections among mathematical concepts (e.g. In what ways might the normal distribution apply to this situation?); and encouraging reflection and justification, which seek to tease out students' deeper understanding of

their reasoning and actions and includes asking students to argue the validity of their work (e.g. How do you know that the sum of two odd numbers will always be even?). Though the types of questions vary in the level of cognitive demand that they place on students, they are all necessary within the interactions among teachers and students, because teachers need to know what information students know in order to appropriately probe the students' thinking, and knowing how students think about material is useful for helping them to see and discuss connections among content as well as provide opportunities for students to justify their strategies.

In addition to the types of questions that are asked, teachers must also be aware of the patterns in which they ask them. The Initiate-Response-Evaluate (I-R-E) pattern consists of teachers asking an information gathering questions, typically with a particular response in mind, then students respond, and then the teacher evaluates the response. Typically, this pattern of questioning leads teachers “to allocate less than five seconds for a student to respond, and to take even less time to consider the answer themselves” (NCTM, 2014, p. 37), and generally provides minimal opportunities for students to think for themselves and provides teachers with very limited access to the mathematical understandings, or lack thereof, of their students. Closely akin to the I-R-E pattern is the funneling pattern, in which teachers use a set of questions intended to guide students to a “desired procedure or conclusion, while giving limited attention to students responses that veer from the desired path” (NCTM, 2014, p. 37). Having predetermined the path for the discussion, teachers using the funneling pattern might use higher-level questions than in an I-R-E pattern, but this funneling pattern still affords students with minimal opportunities to build their own connections and understandings of mathematical

concepts. In contrast to both the I-R-E and funneling patterns, the focusing pattern of questioning consists of teachers “attending to what the students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates” (NCTM, 2014, p. 37). Use of this pattern requires that teachers be amenable to a task being explored in multiple ways, and necessitates that teachers utilize their content knowledge and knowledge of students’ learning to plan questions and identify key points that need to become apparent in the lesson. In order for teachers to design and pose purposeful questions, NCTM (2014) recommends that teachers’ emulate the following actions:

- Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking;
- Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification;
- Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion;
- Allowing sufficient wait time so that more students can formulate and offer responses. (p. 41)

Build procedural fluency from conceptual understanding. “Effective mathematics teaching focuses on the development of *both* conceptual understanding *and* procedural fluency” (NCTM, 2014, p. 42). Teaching in a manner that facilitates connections among procedures and underlying concepts can aid students in the retention of the procedures as well as help them to be better able to apply procedures in novel situations. Fluency is a complicated concept, and means that students are able to flexibly

decide which methods and strategies to apply in making sense of and solving problems, that students understand and are capable of explaining their approaches, and that students are able to efficiently produce accurate answers. Thus, “students need procedures that they can use with understanding on a broad class of problems” (NCTM, 2014, p. 44). On the journey to fluency, students also need opportunities to practice selecting and implementing strategies and procedures in order to develop their knowledge of the appropriate ways in which these are used, but “giving students too many practice problems too soon is an ineffective approach to fluency” (NCTM, 2014, p. 45). So teachers need to carefully select which tasks to provide to students and when to give them, as well as provide feedback to students that supports their progress toward learning goals. Teachers’ actions that support students in developing procedural fluency based upon conceptual understanding are summarized as:

- Providing students with opportunities to use their own reasoning strategies and methods for solving problems;
- Asking students to discuss and explain why the procedures that they are using work to solve particular problems;
- Connecting student-generated strategies and methods to more efficient procedures as appropriate;
- Using visual models to support students’ understanding of general methods;
- Providing students with opportunities for distributed practice of procedures.

(NCTM, 2014, pp. 47 – 48)

Support productive struggle in learning mathematics. Mathematics instruction that productively supports students as they struggle throughout the process of learning

mathematics must view “students’ struggles as opportunities for delving more deeply into understanding the mathematical structure or problems and relationships among mathematical ideas, instead of simply seeking correct answers” (NCTM, 2014, p. 48). When teachers perceive students’ struggles to determine the correct answer as an indicator that they have poorly instructed their students, they often decide to attempt to aid their students by stepping in to help break down the task and guide students step by step through their difficulties. “Although well intentioned, such ‘rescuing’ undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of the mathematics” (NCTM, 2014, p. 48). In order for teachers to be able to embrace instances of students struggling as teaching opportunities, they must consider, in advance, ways that students might struggle and misconceptions that might develop and plan for ways to support their students through the struggle that still allow students to deepen their understanding of the mathematics. Additionally, teachers must accept that students’ struggle is an integral part of the learning process, convey this message explicitly to their students, and provide students with time to try to work through their struggles. In order to help students feel empowered to struggle through problems they do not understand, teachers must recognize and value students’ perseverance and effort to make sense of the mathematics and provide feedback that specifically addresses individual student’s progress in these efforts. Actions that support students’ productive struggle in learning mathematics include:

- Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle;

- Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them;
- Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles;
- Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. (NCTM, 2014, p. 52)

Elicit and use evidence of student thinking. Effective mathematics instruction must elicit evidence of students' present mathematical understanding and utilize this evidence as the foundation for making instructional decisions. Identifying what counts as evidence of student thinking requires that teachers go beyond considering whether or not an answer is correct to include considerations of what students' responses reveal about their progress through learning trajectories, which describe the development of mathematical understanding over time, and identifying common struggles, errors, and misconceptions that students have and make. Interpreting student thinking in terms of learning trajectories and common reasoning patterns can help teachers to appropriately respond to students in a manner that is sensitive to their present understandings and that facilitates progress toward the learning objectives, such as by asking students to state the problem in their own words to help a struggle student or to compare strategies as a means of extending student thinking. Moreover, an awareness of common student misconceptions can help teachers to plan to address these common issues during instruction "before errors or faulty reasoning becomes consolidated and more difficult to remediate" (NCTM, 2014, p. 53). The collection of evidence of student thinking should,

to the greatest extent possible, systematic and planned. One way teachers can achieve this is by identifying particular points throughout a lesson to check on students' thinking, by planning a high-level task to tease out student thinking and reasoning, carefully constructing a funneling pattern of questions specifically designed to elicit common errors and misconceptions so as to make them visible to the class and available for discussion, or by having students discuss a question with a partner prior to a whole-class discussion. Teacher actions that support effective gathering and use of student thinking are summarized as:

- Identifying what counts as evidence of student progress toward mathematics learning goals;
- Eliciting and gathering evidence of student understanding at strategic points during instruction;
- Interpreting student thinking to assess mathematical understanding, reasoning, and methods;
- Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend;
- Reflecting on evidence of student learning to inform the planning of next instructional steps. (NCTM, 2014, p. 56)

Tutoring and its Application Within Teacher Education

To date, only two articles have been found describing projects in which researchers utilize tutoring experiences as a medium for aiding PSTs in the development of pedagogical knowledge, one in literacy education and the other in mathematics

education. Both of these projects take the form of service-learning projects that provide PSTs with opportunities to work with elementary school students.

Hart and King (2007) compared the effect of a service-learning project on a PST literacy course by comparing two groups of PSTs distributed among four sections of a course titled “Linking Literacy Assessment and Instruction” (Hart & King, 2007, p. 324). During the semester, one group participated in a service-learning experience in which they spent one hour each week working under the guidance of the university course instructor as literacy tutors for elementary students at the local community center and engaged in reflective group discussions immediately following the day’s tutoring session, while the second group spent one hour each week participating in “self-selected and independently self-directed” (Hart & King, 2007, p. 324) tutoring sessions. Through an analysis of a pre/post survey of methods of literacy assessment and of a pre/post-test of literacy content knowledge, Hart and King found that both groups showed no significant difference on any pre-assessment and both groups displayed gains on both post-assessments. However, they found that students participating in the service-learning project showed greater gains on both assessments that is statistically significant ($p < .05$) when compared against the group of students that were not involved in the service-learning project. In order to better understand why the service-learning group showed greater gains, Hart and King analyzed data from the students’ weekly reflective journals and from focus group interviews that occurred twice with each group during the semester. They found that the service-learning participants saw their work tutoring as more than just a means to practice the skills they learn in class, but rather as addressing “a community need with real consequences...[which] motivated them to take responsibility

for their learning” (p. 328). Additionally, Hart and King found that, resulting from the collaborative nature of the service-learning project and the regular group reflections, those students felt empowered to voice their opinions and suggestions regarding the tutoring program because they were taken seriously by both the instructor and the community center staff, providing a feeling of control over their learning environment. Finally, they found that a strong factor that contributed to the gains of the service-learning group was the in-the-moment feedback and guidance provided by the university instructor during tutoring sessions.

Lee and Statham (2010) conducted a study of 17 pre-service teachers, most of whom were “white, middle-class females with little exposure to diverse groups and their learning styles or issues” (p. 2) and all of whom were currently enrolled in a mathematics education course. No description was provided of the participants intended certification or of their prior coursework. The instructor of the methods course arranged for each participant to work for one hour each week tutoring for sixteen weeks, and each participant “chose to tutor in one of two after school programs that were seeking [volunteer] tutors, one in a Title 1 elementary school [meaning that the school is receiving additional funding from the U.S. Department of Education due to its high population of low-income students], [and] another in a community center that serves a diverse, predominantly low-income population” (p. 2). This served as the setting for the service-learning project. In order to address the need for reflection during such service-learning opportunities, the participants were asked to write reflective journal entries at the conclusion of each tutoring session as well a single page reflection regarding the overall experience of participating in the project. These written reflections served as the data

sources which were analyzed by Lee and Statham using a constant comparison method to extract themes. From their analysis, Lee and Statham (2010) categorized the six emergent themes described by students based upon alignment with the service-learning goals of the project: “enhancing student learning and providing a valuable service to the community” (p. 2).

Heightened participant learning, Lee and Statham (2010) submit, was facilitated through participants’ attempts to implement the NCTM process standards and to apply varied assessment techniques. During the methods course participants learned about NCTM’s five process standards and as they “engaged in tutoring, they recognized the need for and successfully applied three NCTM standards: communication, connection, and representation” (p. 3). The participants reported that they “attempted to find various ways to communicate with the children until the children understood the concept” (p. 3) as well as worked to improve the communication skills of the children by “encouraging them to explain and/or describe mathematics problems and solutions” (p. 3). The participants also testified that working with the children helped them to better understand how to “connect mathematics using games with manipulatives that the children could associate with their real life” (p. 3). Participants also gained practice with helping students with “representing mathematical concepts using manipulatives, pictures, and mathematical symbols” (p. 4). Additionally, due to the nature of the one-on-one tutoring, participants had many opportunities to enact their knowledge of using the diagnostic assessment techniques discussed during their methods course, and participants were able to discern students’ issues with specific operations, number facts, and explaining their reasoning. This also led some participants to appreciate “the importance of children

understanding mathematical concepts and not just learning formulas and rules” (p. 5).

Lee and Statham also concluded, based upon “feedback received from the coordinators of the after school programs where the participants tutored” (p. 5) that the participants had made a “significant contribution” (p. 5) to the community by lending their time and effort to increasing the number of students that could be helped by the afterschool programs and providing students with much-needed one-on-one support.

Although participant “learning about the process standards and assessment was the heart of the course, and of most importance in terms of outcomes” (p. 6), Lee and Statham (2010) found four other benefits that the participants reported to be the result of the service-learning project. Through working with many different children, participants improved their awareness of the fact that “different children learn mathematical concepts” (p. 6) differently, which led them to understand that in order to “react flexibly to various teaching situations” (p. 6) they will often need to attempt multiple approaches before the conclusion of a lesson. Another byproduct of the participants’ experience working with a multitude of children was increased comfort and skill in “working with children of diverse ages” (p. 7). Participants also underwent personal growth resulting from the service-learning experience by helping them “to see the importance of developing deeper relationships with the students with whom they were working” (p. 7), exposing the participants to the notion that a deep personal relationship with a student can improve that student’s learning. Finally, participants’ experience during the service-learning project helped them to make plans for how they might structure their own classes in the future: participants thought “more critically about teaching methods that are used in current elementary schools” (p. 7) some of which they came to feel were not very

effective, such as worksheets intended to drill the student; participants were also afforded numerous opportunities “to apply what they learned in class and [find] out what worked well and what didn’t” (p. 8); moreover, participants showed evidence that they were beginning “to think about applying new ideas in their future teaching” (p. 8), resulting directly from the combination of their course content and the one-on-one interaction with the students during the service-learning project.

Discussion. In this section, some of the results of the studies by Hart and King (207) and by Lee and Statham (210) will be examined through the lens of situated cognition. Hart and King (2007) found that PSTs participating in the service-learning component of the project had significantly greater gains in their knowledge of both literacy content and techniques of assessment. They found that service-learning participants attributed these gains to be a result the context in which they employed the information discussed during the course (i.e. the tutoring of elementary students) because the fact that the service-learning participants were going to have to be able to apply the content they learned during the course while aiding the learning of real elementary students lead these PSTs to take greater responsibility for their learning. The service-learning participants also felt that the in-the-moment guidance provided by their instructor during tutoring interactions contributed strongly to their gains in content and assessment knowledge. These results seem to indicate that fact that the service-learning participants were expected to implement course knowledge in the context of aiding actual elementary students lead service-learning participants to view this as a sufficiently authentic teaching opportunity so as to make them feel compelled to learn as much as possible during the course so that they would be able to help their elementary tutees to

the best of their abilities, and that having the guidance of their instructor while tutoring helped them to improve their understanding of the application of course knowledge.

Lee and Statham (2010) found the tutoring context of the service-learning project afforded participants opportunities to implement three of the five NCTM process standards (i.e. communication, connection, representation) because the participants were actively engaged in trying to help elementary students to develop their understanding of mathematical content. Interacting with elementary students in this context also allowed the participants to develop insight into identifying various ways in which different students think about content and to work to react flexibly to these idiosyncrasies, an important component of the context of teaching upon which teachers must rely so as to be able to ensure every student is enable to succeed. They also found that participants began to abstract from their experiences in the tutoring context ways in which they might want to structure the teaching and learning that occurs in their own classrooms once they become professional teachers.

Summary

From the research, it is clear that engaging in tutoring activities has the potential to provide PSMTs with sufficiently authentic opportunities to practice implementing methods course content by affording PSMTs with numerous opportunities to experience teaching real students on a small scale. Hence, it seems that the incorporation of tutoring activities into mathematics teacher education programs has the capability to address concerns that prospective teacher education is lacking in opportunities for quality practice of teaching (Ball and Forzani, 2009; Grossman et al., 2009). However, due to the scarcity of research investigating tutoring's application as a medium for teaching PSMTs about

mathematics pedagogy, it will be important to conduct further investigations into which of NCTM's (2014) eight Mathematics Teaching Practices can be practice and explored within the context of mathematics tutoring, as well as the extent to which mathematics tutoring activities are authentic approximations of professional mathematics teaching.

III. METHODOLOGY

The National Council of Teachers of Mathematics (2014) defines proficient teaching as being both effective and versatile. In order to be prepared for proficient teaching, NCTM recommends that effective mathematics instruction requires teachers to engage in each of eight interwoven mathematics teaching practices:

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking (NCTM, 2014, p. 10)

Researchers have identified that one issue hindering the effective preparation of future mathematics teachers is limited opportunities to engage in high quality teaching practice during teacher education programs (Ball & Forzani, 2011; Grossman et al., 2009; Hibert et al., 2003; Smagorinsky et al., 2003). Some researchers have examined the effects of tutoring activities implemented within teacher education programs on PSTs (Hart & King, 2007; Lee & Statham, 2010). Unfortunately, the research on tutoring's application to teacher education, especially in mathematics, is quite sparse, and these projects utilize tutoring of elementary students on their campuses. According to situated cognition, in order to effectively facilitate the PSMTs learning of the knowledge and skills required to become effective mathematics instructions, their learning must take place in a context

that affords them opportunities to engage in authentic teaching activities in a manner consistent with how professional mathematics teachers would do so. Thus, it is the goal of this study to contribute to the understanding of tutoring's potential within teacher education through an exploration of the experiences and activities of college students working as mathematics tutors in a university tutoring center and seeks to address the following research questions:

1) What are the teaching practices exhibited by the participants and what is the rationale for these practices?

2) What effects, if any, upon the participants' tutoring do the participants attribute to their participation in this study?

The understanding developed from the examination of practices exhibited by the participating mathematics tutors will be used to help determine if the Learning Center, and similar university tutoring centers, could serve as a medium to provide PSMTs with opportunities to engage in teaching activities that authentically approximate those of professional mathematics teachers. The understanding of the participants' experiences as a Learning Center mathematics tutors will also be used to address the extent to which the culture of the Learning Center encourages the adoption of beliefs and values that are appropriate for future mathematics educators.

Design

In order to investigate the potential for PSMTs working as mathematics tutors in a university tutoring center to serve as a way to incorporate opportunities to practice authentic mathematics teaching, this study will follow an embedded case study design. The first layer of this study is an instrumental case study, as per Cresswell (2013),

focused on a particular university tutoring center, which is referred to as the Learning Center (a pseudonym). This particular center was chosen, over another mathematics tutoring center on the same university campus, because the researcher's personal experience working for the Learning Center served as a major impetus for the creation of this study and also provided a connection with the Learning Center's administration that facilitated their cooperation with the researcher. Additionally, the Learning Center has served as a model which other university tutoring centers have endeavored to emulate. The second layer of this case study is concerned with three particular Learning Center mathematics tutors. A case study is an appropriate methodology for the current study because the researcher seeks to investigate the experiences of individuals operating within the bounded environment of the Learning Center in order to develop a rich description of the participants' tutoring practices and decision making process. From the descriptions of the individual participants' tutoring, the researcher aims to better understand what PSMTs can gain from similar experiences. Due to the highly subjective nature of interpreting lived experiences, it is necessary for the researcher to be forthcoming with any personal experience with the phenomenon under study in order to inform both the researcher and the audience of any biases that may be present within the study (Cresswell, 2013; Merriam, 2009). As discussed in Chapter 1, the researcher worked as a mathematics and physics tutor for the Learning Center during all four years of undergraduate study and chose to pursue a career in education as a direct result of this experience. The researcher believes a large portion of his teaching proficiency to be a direct consequence of his tenure at the Learning Center. This intimate relationship with the Learning Center has great potential to bias the researcher if care is not taken during

each phase of the study. The analysis of data gathered from tutors working at the Learning Center is particularly susceptible to the researcher's bias, and great care will need to be taken to ensure that conclusions drawn from the data are true to the understanding and interpretations of the participants themselves, regardless of their (mis)alignment with the researcher's personal experience.

Participants

The participants for this study were purposefully sampled from the mathematics tutors currently employed by the Learning Center. The main goal of this study is to examine the tutoring practices and rationale utilized by Learning Center mathematics tutors in order to investigate whether mathematics tutoring experiences can provide a context in which teacher education programs may afford PSMTs with opportunities to engage in authentic mathematics teaching activities.

Recruitment of participants occurred in two stages. First, the researcher, with the aid of the Learning Center's assistant director, reached out to mathematics tutors during the Fall 2015 semester via email to seek out tutors interested in volunteering. This resulted in the recruitment of a single participant. Due to acquiring but a single tutor, the researcher executed second recruitment stage during the semester following stage one. During this second stage of recruitment, the researcher gave a short presentation during the Learning Center's training day before the start of the Spring 2016 semester and offered a \$50 Visa gift card, to be delivered upon completion of the final interview, as an incentive to encourage participation. This resulted in the acquisition of four additional participants. It is important to note that none of the tutors who volunteered had teaching

experiences other than those as Learning Center mathematics tutors, nor had anyone ever been enrolled in mathematics education or teaching course work.

Data, in the form of audio recorded observations and interviews, were collected from each of the volunteers during the semester in which they were recruited. From the five Learning Center mathematics tutors who volunteered, three were chosen to serve as the subjects of a case study embedded within the Learning Center because they represented different amounts of experience working as mathematics tutors and thus might serve to illuminate any effect that time working at the Learning Center has on one's mathematics tutoring.

The name used for each of the three chosen participants is a pseudonym in order to protect their identities. Ralph is a computer science and mathematics double major in his sophomore year, and he participated during his first semester working as a Learning Center mathematics tutor. Fred is a finance and mathematics double major in his senior year and participated during his fourth semester working as a tutor for the Learning Center, however his first two semesters working at the Learning Center he served as a business tutor. Michelle is an Applied Mathematics major in her senior year, and she participated during her 5th semester working as a Learning Center mathematics tutor, the first of which she was a private tutor, and she now serves as a lead mathematics tutor for the Learning Center.

Setting

The Learning Center is housed within the library of a mid-sized public university in central Texas, employs a total of more than 50 tutors, and offers tutoring in mathematics, physics, chemistry, biology, statistics, business, economics, finance, and

writing. The tutoring lab is open to students each day of the week, except Saturday: Mondays, Tuesdays and Wednesdays from 9am – 8:30pm, Thursdays from 9am – 4:30pm, Fridays from 9am – 12pm, and Sundays from 5pm – 9pm. Additionally, the tutoring lab is closed for holidays and during final exams, in order to allow the tutors time to relax and to study for their exams, respectively. The math and science area of the tutoring lab (see Figure 1 for a diagram) consists of nine tables with approximately eight seats per table. On each table is a sign with one or more courses written on it (e.g. College Algebra), indicating where a student should sit to receive help with a specific course. Students seeking tutoring do not need an appointment, rather they need only walk in to the tutoring lab during operating hours, sign in at the front desk (so that the Learning Center can keep track of student visits), sit at an appropriate table to begin working on their materials, and raise a flag when they need assistance. This organization is utilized to compensate for the walk-in nature of student visits and the expectation that each tutor will move among the table and help anywhere they can; grouping the students by specific courses helps the tutors to identify if they may be able to aid a student in need. Additionally, mathematics and the sciences share tutoring lab space due to the fact that many tutors of science can also tutor some mathematics, and vice-versa. As indicated in Figure 1, there are white boards and chalk boards surrounding much of the mathematics and science section of the lab, and tutors also have access to several mobile white boards to use during tutoring. The Learning Center also offers additional resources to students in the form of handouts covering general topics useful for all students (e.g. anxiety management, time management, test prep and study skills, note taking suggestions) as well as content specific documents that range from fundamental topics (e.g. operations of

signed numbers, kinematics equations, stoichiometry) to more advanced (e.g. techniques of integration).

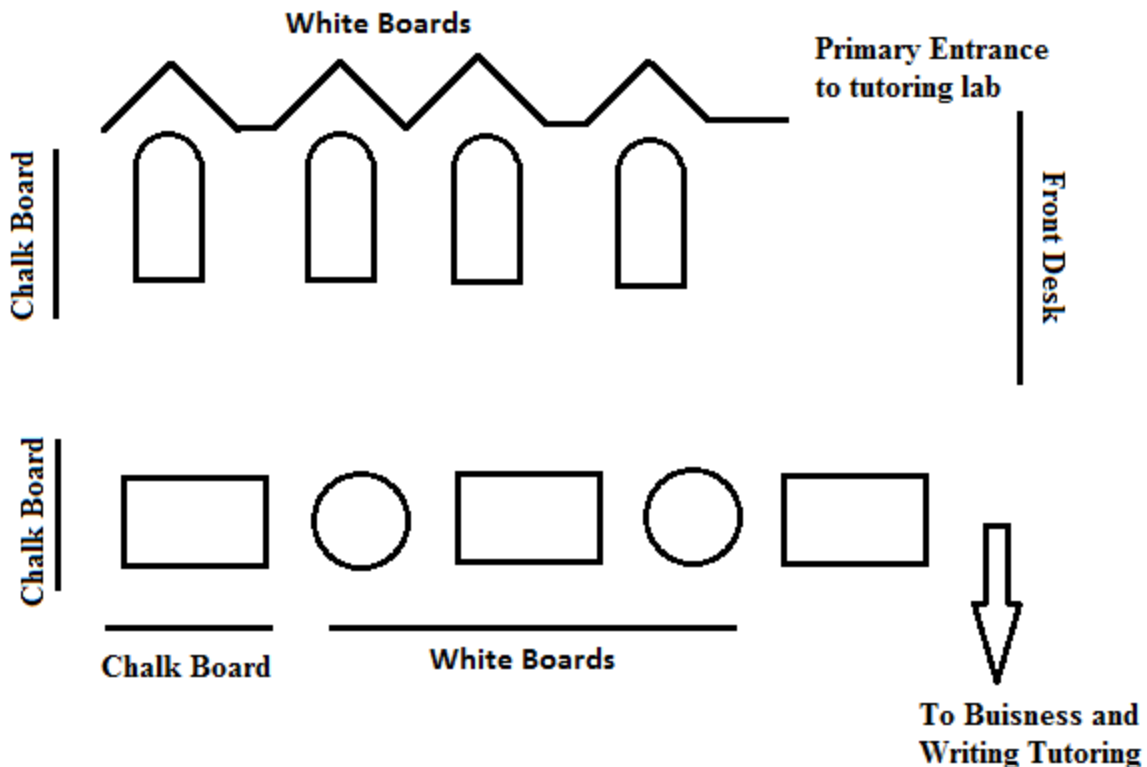


Figure 1: Diagram of the layout of the mathematics and sciences section of the Learning Center's tutoring lab.

Interviews with both the director and assistant director of the Learning Center were conducted, in addition to exploring the Learning Center's website, in order to facilitate a more robust depiction of the study's setting. From data published on the Learning Center's website, the Learning Center serves an average of 6,503 students over 25,944 visits, and of these 2,723 students over 10,883 visits were seeking assistance specifically with mathematics coursework. The Learning Center's vision and mission statements, also published on their website, are:

Mission statement: The [Learning Center] supplements instruction and provides services that support students in the development of skills necessary for their effective performance in and positive adjustment to the university learning environment.

Vision statement: [The Learning Center]’s ultimate goal is to instill confidence in Texas State students and assist them in becoming independent learners who rely on their strengths and abilities

According to [the Learning Center]’s director, “the goals of [the Learning Center] are... getting the students to become independent learner... [and to] be academically successful.” The assistant director explains that she stresses to each tutor that “the tutoring lab is focused on autonomous learning” and that each tutor’s goal should be “to work [his/herself] out of a job,” meaning that “tutors should not be out there spitting out answers, being a calculator they need to be [helping] the students with getting the answers themselves.” In order to help achieve these goals the director explains that the Learning Center tutors participate in numerous professional development opportunities:

All tutors participate in new-hire training at the beginning of fall and spring semester which involves tutors getting to know each other through ice-breakers and teambuilding activities, reviewing policies and procedures, introducing tutoring techniques, and discussing different learning styles. The lead tutors also get an opportunity to go into more depth about the content they will cover and specific, difficult situations their areas encounter. The new hire training usually takes place over two days, and each day last about eight hours. [The Learning Center] also [has] monthly trainings that take place on a Friday for two hours.

Some of the monthly trainings we have done in the past include communication skills; how to apply for jobs/graduate school, what to keep in mind when writing your résumé/CV, and things to help with interviewing; how to handle stress; becoming more mindful of diversity and culture on campus; active shooter training, in which the police department speaks about previous incidents and what to keep in mind if a situation like that happens; how to develop SMART goals (specific, measurable, attainable, realistic, time-based), to mention a few. In the fall, our tutors go to the SASP conference (Student Academic Support Programs) hosted by [the Learning Center]. Throughout the semester, each lead tutor is also required to conduct at least one content training, in which they will discuss specific chapters/concepts seen lately in their sections. The leads contribute to professional development by observing all team members and giving them feedback on their performance, tips on improvement, and acknowledgements of their unique contributions.

The assistant director further explains that even though the Learning Center's goal is for their clients to become independent, if a client "really have a gap in knowledge, [tutors] are going to have to pause and review more than [they] would have" normally; it all depends on the individual needs of each client.

It is important to note that the "learning styles" that are discussed during the Learning Center's professional development sessions as well as those that are mentioned by the participants in this study refer to students that are "visual learners," "kinesthetic learners," etc., however researchers have found "that the extant data do not provide

support for the learning-styles hypothesis” (Pashler, McDaniel, Rohrer, & Bjork, 2008, p. 116).

Data Collection

Data for this study was gathered through observations of each participant’s tutoring and interviews with each participant which focused upon asking each participant to explain the decisions they were observed making as well as their rationale for these choices. Both the observations and interviews were audio recorded.

Each participant was observed three times, each lasting for approximately one hour. While a participant was being observed he/she carried a digital audio recorder in his/her pocket to record the conversations that occurred between the participant and his/her clients. During these observations, the researcher followed the participant from client to client, always sitting either at the same table as the client or an adjacent table, and taking handwritten field notes about the interactions between the participant and clients as well as the content material discussed during the interactions. There were no instances in which a participant’s client appeared to be disturbed by the researcher’s presence, if the client even appeared to notice the researcher’s presence at all. Following each observation, the researcher reviewed the field notes and audio recording to produce a detailed record of the tutoring interactions that took place, specifically focused on identifying choices that the participant made while tutoring. Typically within one week of the corresponding observation, the participant was interviewed, for one to two and a half hours, during which the researcher asked the participant to explain why he/she made the various decisions they were observed to make. In order to help the participant to recall with the greatest detail possible, the researcher utilized written descriptions and playback

of recorded audio segments to remind the participant of the specific contexts which motivated the researcher's questions. These observations and the first two interviews were intended to provide evidence to address the first research question, which is focused on examining the participants' tutoring decisions and the rationale with which they reached those decisions. As a result of this focus, an general interview protocol was not developed for the first two interviews because their sole purpose was to encourage the participants to explain why they made the choices they did, and so while the researcher had identified specific instances from the observations about which to ask the participant, the question was, almost always, some rendition of "I saw that you did this. Why did you do that?" The participants were in complete control of how they chose to respond to the researcher's questions, and the researcher did not move on to start discussing another tutoring moment until the participant indicated he/she had finished with the conversation elicited by the previous question.

The third and final interview was the only interview that was not explicitly focused on events from tutoring observations. Instead, this interview, which occurred near the end of the semester, was intended to address the second research question and directly asked the participant to discuss the effects on their tutoring, if any, that he/she has noticed that have occurred as a result of participating in this study.

Data Analysis

Data analysis occurred in three phases throughout the execution of this study. However, before articulating the contents of these three stages of analysis, it is important to note that throughout this project the data gathered from the observations of the participants was leveraged in order to provide context, both while interviewing

participants as well as when interpreting the contents of each interview. The first phase occurred while the observation and interview data were being collected and was focused on ensuring that the researcher was correctly characterizing and interpreting the participants' rationale which they described during the interviews. This was manifest in two ways: first, when a participant shared his/her thought process underlying a decision, the researcher would often verbally paraphrase the participant's explanation to ensure that the participant was not being misinterpreted; second, after completing the first interview with each participant, the researcher had gained some insight into the participants' thinking and whenever a participant was observed to make decisions similar to ones discussed in a preceding interview, the researcher would present these situations to the participant and ask him/her to verify that the researcher's interpretation of his/her rationale was correct.

The second phase of data analysis took place after the researcher had collected all observation and interview data from each participant. During this phase, interview data, in the form of transcripts, was coded using a combination of an a priori scheme based on the eight practices of effective mathematics instruction outlined by NCTM (2014) as well as open coding. The eight practices were chosen to serve as a coding guide because this project's first research question is focused on determining the participants' tutoring practices and the rationale for these practices with the goal of comparing them to those of professional mathematics teachers. Thus, choosing to use NCTM's recommendations for what high-quality mathematics entails as a means of categorizing the thoughts and actions of the participants and provided the researcher with appropriate language for articulating their pedagogical purpose. The researcher also elected to openly code the

data because there were many segments during the interviews that appeared to be significant to understanding the participants but were not directly linked to NCTM's eight recommendations.

The third phase of data analysis occurred after the data had been coded during phase two. During this final phase, each participant's interview data was examined according to the codes applied during phase two, and the data within each code was sub-coded according to which of the teacher actions outlined by NCTM (2014) it embodied. This third phase was intended to aid the researcher in developing a more holistic understanding of how each participant embodied, or not, each of the eight recommendations for quality mathematics instruction, and also helped to organize the data in a fashion that was conducive to creating a coherent description of the context of participants' tutoring, the actions they took, and their discussion of the rationale underlying these actions.

While reporting the data collected, the researcher endeavored to ensure that the descriptions of each participant's rationale were true to the participants in two key ways. First, throughout each interview the researcher made sure to verify that the participants' explanations were being correctly interpreted by the researcher by re-voicing the explanations back to each participant and asking for verification or clarification. The second way in which the researcher strove to ensure that the presentation of the rationale for each participant's tutoring decisions was an accurate portrayal was by, as much as possible, utilizing the participants' exact words from the interviews, thereby significantly reducing the opportunities for the researcher's personal experience working as a tutor at the Learning Center to skew the interpretation and presentation of interview data.

IV. RESEARCH FINDINGS

The goal of this study is to better understand the teaching practices exhibited by mathematics tutors working in the Learning Center as well as their rationale for the choices they make while tutoring clients. The study uses case studies and qualitative methods to analyze data collected from tutor observations and interviews. This chapter describes the findings of this study through three case reports. The first eight sections correspond to the Eight Mathematics Teaching Practices described in NCTM's *Principles to Actions: Ensuring mathematical success for all* (2014) and address the first research question: What are the teaching practices exhibited by mathematics tutors and what is the rationale for these practices? The ninth section exhibits instances in which the participants are spurred to reflect on their practices as an organic result of conversations during the interviews. The tenth section addresses the second research question: What effects, if any, upon the participants' tutoring do the participants attribute to their participation in this study? The ten sections are titled as follows: establish mathematics goals to focus learning, implement tasks that promote reasoning and problem solving, use and connect mathematical representations, facilitate meaningful discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning mathematics, elicit and use evidence of student thinking, reflections during interviews, and effects upon participant's tutoring that the participant attributes to participating in this study.

The three participants for this study were chosen from among five participants observed and interviewed. The participants were chosen based on their differing amounts

of experience working at the Learning Center, and the three cases are presented in order from least experience to most experience.

The observation data presented in this chapter will include description of the mathematics content as possible, but omissions of details (e.g. the specific equation a client is working on) are the result of not having a record of said detail. Moreover, within each case there will be several pieces of observational data that are used to illustrate multiple components of the Eight Mathematics Teaching Practices. This is a consequence of both the overlapping nature of the Eight Practices and also the intricate nature of interacting with learners.

Case 1: Ralph

Ralph is a Computer Science and Mathematics double major in his sophomore year. Mathematics course Ralph has taken include Calculus 1, 2, and 3. He has never taken any teaching methods or education courses nor has he had any teaching experience prior to being employed by the Learning Center. This is his first semester working as a tutor for the Learning Center.

Teaching practice 1. Establish mathematics goals to focus learning.

Establishing clear goals that articulate the mathematics that students are learning. Ralph's primary focus during a tutoring session is to help his clients succeed on the problems they are presently struggling with, and feels that he successfully accomplishes this goal when he and the client "got a good amount of problems solved" and "actually finished an assignment" as opposed to times when "a whole tutoring session [is] just one problem" that they had to "spend a lot of time" on: "It feels good to actually get some work done." While helping a client through a problem, Ralph

endeavors to avoid discussing any unnecessary information “because sometimes you can give [clients] way too much information.” For example, during an interview Ralph says that when he is helping calculus clients

“if I started talking about the derivation of formulas and derivatives... when they just need to know a simple power rule for this [problem], and they did do the derivations a while ago, they don’t need that background information. It would just be... a distraction.”

Moreover, Ralph is “a little bit paranoid about telling [clients] something wrong” because “chances are they’re going to latch onto that one wrong part” which could lead to them getting something wrong on an assessment, potentially causing the client to think that Ralph “[doesn’t] know what [he’s] talking about” thus tarnishing the reputation of both Ralph and the Learning Center. Ralph is aware that the Learning Center’s goal, and therefore his job as Learning Center tutor, is to help clients “to do well in the class... [and] be able to do these problems [themselves] without tutors” and he works to accomplish this goal by striving to be a source of completely accurate content knowledge for clients, providing a clear explanation of the steps required to solve a given problem in a manner that is consistent with how clients’ professors present material. This is most clearly evinced during a conversation about a tutoring session in which Ralph is helping a student in Calculus III and he tells her that he feels “like she should have someone who knows what they are doing” and commented during an interview that “if she had asked me for help while I was taking the class... it would have been a completely different session because I would have known where to point her immediately” rather than “struggling with her...trying to remember this stuff [and] trying to rework through it”

which lead this particular interaction “to be more of a problem solving session versus a tutoring session.” Ralph’s belief that he needs to understand each facet of solving a problem so that he can confidently lead his clients is a result of Ralph trying to emulate his own learning experiences. Ralph has found that, typically, his professors will show him the information “but sometimes it just won’t sink in” so he “just need[s] to work problems... see more examples or think through the whole process [himself], even if it turns out to be the exact same way” that his professor discussed the material, “the fact that [he] went through the material [himself] made [him] able to comprehend it.” Thus, Ralph wants to mirror the manner in which, he assumes, clients’ professors explain materials to give his clients learning opportunities similar to those that work for him as well as the methods clients’ professors use to solve problems in order to ensure that his tutoring does “not contradict the professor.”

Ralph’s tutoring goals are centered around the extent to which he is knowledgeable of the content area in which his clients need help, and when he makes a mistake while answering a client’s question or when he is unable to clearly explain the solution to a problem because he is unfamiliar with the content, Ralph feels that he is doing a disservice to his clients. Interestingly, during one interview while discussing the fact that Ralph “feel[s] like [he is] not doing [his] job” when he is not “able to help them [his clients] out,” Ralph mentions, without solicitation, that

“at [the Learning Center], a lot of the students are very understanding. They know that you’ve been there before [and] that you don’t know all the answers in the world, which I feel is something people put on their professors and teachers. But

at [the Learning Center], it's a bit more relaxed because we're just students who maybe just took this class a year or so ago."

Yet, in spite of Ralph having experiences, such as this, in which his clients understand and do not begrudge him that he doesn't know all the answers, Ralph still feels that he must be able to answer all his clients' questions in order to do his job.

Identifying how the goals fit within a mathematics learning progression. Ralph shows some awareness of the connection between the learning progression of his clients and his goal to not provide clients with details or information that he considers to be irrelevant to helping his clients succeed on their present task. During one interview discussing a tutoring session in which a client tells Ralph "I have no idea what I'm doing," Ralph's initial response is "thinking I'm not going to have to correct [just] a little sign error, I'm going to have to go back and maybe explain some previous concepts before we can actually start solving the problem," so after Ralph first ensures that he understands the question the client is working on, he then needs an "understanding [of] what does the fellow student know" so that he can determine how to "best benefit them" without giving "them way too much information."

Using the mathematical goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction. Due to the nature of being a tutor for the Learning Center, Ralph is unable to actually plan specific lessons for specific content. However, Ralph has developed a general plan for deciding what help he should offer to clients who have already done some work on a problem and are still struggling to get the correct answer: first he checks that the overall process that the client has used to tackle the problem is correct, paying attention to the steps the client used without paying much

attention to the specific numbers and calculations used; if the client's process is correct, then Ralph moves to examining the specific numbers and calculations that the client used to see if he can identify any arithmetic errors that the client may have made that lead to the client's incorrect answer; if the client's overall process looks correct and Ralph is unable to locate any specific errors in the calculations, then he finally decides to work through the problem on his own with the hope that this will illuminate where the client's work went awry. This general approach that Ralph has developed is best illustrated while discussing a tutoring session in which he was helping a calculus client who was tasked with using derivatives to find the maximum value of a function:

“The general process for maximizing is you get your equations, you take the derivative, set it equal to zero, [and] find the x-value, and she [the client] had done all of that. Then she plugged back in the x-value to the original [function] to find what that max would be. It was in the process of... plugging it back in that we noticed that it got weird.”

This prompts Ralph to take a closer look at the details of the client's calculations in the final step. However, it turns out that the client's arithmetic in plugging the x-value back in was also correct, which lead Ralph to think “‘Something's wrong,’ so I [started] trying to figure out for myself if it was factoring out correctly,” which finally reveals that the client's mistake had, in fact, been that she did not correctly factor when she was trying to solve for the zeroes of her derivative. Ralph finds that this “process... works well, especially when [clients] seem to understand what they're doing, because that's when [I] don't need to review lecture, or go back and review a concept, because [they know] the concept and everything,” thus helping Ralph to ensure that he helps his clients with the

specific part(s) they are struggling with while avoiding telling the clients information that they do not need help with.

When Ralph is helping a client to solve a problem from a content area that Ralph is comfortable with, but the client does not have work for Ralph to look over, he does not have a consistent plan that he uses to guide his tutoring, but he does show evidence of using his overall tutoring goals to guide decisions that he makes during the interaction. Typically Ralph will begin a tutoring session by silently reading through the problem the client is working on in order to allow him “to make sure [he] understand[s] what [the question is] asking” and so that he can figure out how to find the solution. Determining the solution path helps Ralph to decide what information the client needs to know in order to be successful, which leads Ralph to either ask the client if they can recall a relevant fact, for example “Do you know how to find the slope?” or to simply state the relevant information required. When asked how Ralph decides between asking the client a question or simply telling them the information they need, Ralph says “I don’t know if I put too much thought into it” and elaborates that “asking questions is good” because it “makes [the tutoring session] easier for me because [the clients] will sometimes guide me in the right direction, and then in turn,... point me exactly to how I can help them.”

If Ralph determines that his client does not recall the relevant knowledge required to solve a problem, “if it’s specifically what they’re focusing on [he does not] like to skip any steps” because this knowledge is the focus of the client’s present problem; however, if the necessary knowledge comes from a prior class, such as a Calculus III client that is struggling with an algebra topic, Ralph is usually “trying to be fast-paced on algebra... trying to get the [present] content down” by maintaining the primary focus of the session

on the present course material. Ralph assumes that clients in advanced courses know the prior content that is presently required and just need a reminder. For example in the

“case with the girl working on the maximizing problem, she probably knew how to do her algebra [and] she probably knew how to factor well, so that might just be a case of her [not] getting a good night’s sleep the day before. Even though it [was] a previous concept that [caused] the error, that doesn’t always mean that they don’t know that [concept].”

Interestingly, Ralph comments that “pretty frequently” clients “show up to [the Learning Center] thinking they are having trouble in the class [but] it’s really a couple of previous concepts that weren’t covered in the class that they really needed help on.”

Additionally, Ralph will sometimes allow a client to work on their own, either on their paper or at the board, while Ralph watches over them. Ralph sees these situations in which he encourages the client to work independently to serve three purposes. First, Ralph has the opportunity to be “checking [that] they’re not making a simple math error,” which helps Ralph to gauge if the client understands how to proceed through the problem. The second purpose in “[getting] them to do the work” is that “they’re going to need to know how to do it on the test” and “the whole point [of his job] is to get them to where they don’t need [the Learning Center].” The third use that Ralph sees for getting his clients to work independently is that it gives him the opportunity to be “thinking ahead a bit” so that can identify the next step and be thinking about how to guide the client through it.

On occasions when Ralph is unfamiliar or uncomfortable with the content material of a client’s question, Ralph’s default decision is to “get them another source [of

help] before [he] tells them something wrong,” in the form of another, more experienced tutor who is better equipped to help with the present problem, due to Ralph’s focus on making sure the clients get the help they need. Ralph says that “because [the Learning Center] is fast paced, it forces me to call on other people because [the Learning Center] is there to help [the clients] out so [I] have to offer them the best service.” However, he goes on to say that even if the Learning Center isn’t busy “as long as they were asking me for help and I wasn’t able to help them, that’s the key thing” that leads Ralph “to find somebody who can.” Ralph goes on to say that it “has happened to me where I don’t understand a concept,” and there are no better suited tutors available for him to call, so

“I just try to explain it the best I can. If [the client] doesn’t understand it, I might say... ‘Don’t worry too much if you’re not understanding it now, because the first time I was trying to understand [this] I didn’t understand it too well; with more problems [and] experience, hopefully [the client] will understand [the problem].”

This suggestion that perhaps the client will come to better understand the material after doing more work on related problems stems directly from Ralph’s own experiences as a learner in which he has to work extensively on his own in order to make sense of the material discussed during class.

Ralph’s goal of helping his client to solve the problem they are presently working on serves as the motivation for his reflections upon his tutoring sessions. When Ralph has clients

“come back and [tell him] ‘You helped me out last time’ ... [or] ‘You explained things well to me’ [it] makes [him] a bit more confident in... the way [he is] specifically approaching problems with this student; [he] feel[s] like whatever

[he's] been doing with the student, since [he's] gotten positive feedback, [he] should try to keep with it because it's working well."

However, these episodes of positive feedback do "not necessarily [affect his confidence in] tutoring other students." Whenever Ralph is not able to help a client with a problem that Ralph feels like he "ought to be able to do" he is bothered by this and

"while [he's] at [the Learning Center] in the [tutoring] mode and [he has] some spare time where [he's] not helping somebody out, [he] definitely think[s]...'Maybe I need to brush up on this material' [or] 'How can I explain this better?' and [he tries] to think for [him]self, maybe I should have taken this [other] approach to tutor [this client]."

Ralph will sometimes, "but not too regularly," even go so far as to go to a more experienced tutor and ask "How would you explain this type of problem to somebody?" and then he will "try to follow that same method when [he's] doing it" the next time.

Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. There was a single instance during Ralph's observations in which Ralph makes his goal of focusing on a client's present problem known, albeit not explicitly, to the client. In the course of solving a problem, Ralph's client produced a graph of the sine function and asked Ralph if it was correct. Ralph responds with "this first part is; I'm not really checking anything else" because the problem only required a small region of the graph and "nothing else mattered after that." When asked, during the interview, if Ralph was actually only focusing on that particular region, Ralph said that if the rest "was glaringly wrong, I would have said something, probably." Ralph elaborates

that “a lot of people that come into [the Learning Center] know what they’re doing and they [ask] ‘I did all of this, can you just check my work?’ then normally [he] just glance[s] over the overall process and [will] say something like ‘I’m not checking the specific math, but the process you’ve done is perfect.’”

Teaching practice 2. Implement tasks that promote reasoning and problem solving.

Motivating students’ learning of mathematics through opportunities for exploration and solving problems that build on and extend their current mathematical understanding. Ralph says that when he is working with a client he will imagine “If I only had the math tools they have, how could they get from where they are not to solving the problem without me telling them anything that they don’t know?” because “I feel like it’s good if they can use the tools they already know, they would be able to [solve this problem] on a test without any extra learning if they just used the tools that they know.” This shows that Ralph is aware of the benefits of helping his clients to see how to leverage the content they currently know in order to extend their understanding in order to solve the problem they are presently struggling with even though, as previously discussed, Ralph does not consistently encourage his clients to explore problems independently. Moreover, this sentiment is in alignment with Ralph’s focus on making sure that he does not tell his clients any unnecessary information in the process of helping them to work through their problems. Furthermore, Ralph says that even when he “[has] more time” to work with clients, if they do not respond positively to him asking them questions (i.e. not being able to answer the questions, saying “I don’t know”) Ralph will elect to not try to help the client to think through the problem with him and instead just

tell them how he would do it because continuing to do so is “more likely to get them frustrated.” The only example present in Ralph’s observations in which Ralph explicitly attempts to help a client connect prior knowledge to the problem at hand is while he is helping a Business Calculus client who is working to construct a linear price-demand function using two given data points. In this example, after Ralph has helped walk the client through the process of constructing the equation he discusses how the client could check that this equation is correct, telling her “if you plug in x_2 , you should be able to get your y_2 , and if you plug in your x_1 you should get your y_1 .” Although Ralph does not encourage or guide the client to discover this on her own, he is attempting to show her how she can use the knowledge from her prior algebra courses to help her presently.

Selecting tasks that provide multiple entry points through the use of varied tools and representations. No evidence of this component of promoting reasoning and problem solving is present in Ralph’s observations or interviews. However, this is not unexpected because tutors do not get to select the tasks that their clients are assigned.

Posing tasks on a regular basis that require a high level of cognitive demand. Again, there is no evidence of this present in Ralph’s observations or interviews because Ralph does not get to select his clients’ tasks.

Supporting students in exploring tasks without taking over student thinking. When Ralph is helping a client with something that he does not know well he tends to be much quieter. During an interview, Ralph discusses an example (that was not a part of observational data) of a time when he was helping a client with Differential Equations, a course he has never taken, but the client was working with summations, something that Ralph does know how to deal with. Ralph says that “she was walking me through the

process” because he was “asking [her] to explain it to me in the hopes that I can understand it and then throw in my input [where] I can.” Sessions like these seem, to Ralph, to be less like tutoring sessions and more like when he and his friends meet up to study and they’re “just working together through homework problems... talking about it... bouncing ideas off of each other versus more of a tutoring session where [he’s] guiding them” through the problem.

Another situation when Ralph says that he does try to support clients’ work on a task without taking over the thinking is when a client calls Ralph over and has essentially the same question they had asked Ralph to help with before. Ralph says, during an interview, that “Normally I try not to do anything, because I’ve already explained it. I normally just point them to [where] they wrote down before all the steps... and then hopefully they’ll go back through the steps,” rather than “stating it outright again. It’s a lot more beneficial for them to go back through an example problem and work through it themselves.” It seems that Ralph has reflected on similar prior experiences which has lead him to develop this idea that the best way to help clients in this situation is to get them to utilize their notes as they try to do the work themselves.

There was only one example from Ralph’s observations in which he allows a client to explore a problem without taking over the thinking, while in the rest of his tutoring interactions Ralph is in control of the thinking. This single example in which Ralph allows a client to explore a problem on his own without Ralph immediately taking over the thinking is when Ralph is helping a calculus client who is working on integrating the function $1/x$. The client decides to re-write the function as x^{-1} and proceeds to start trying to use the power rule for integration and Ralph decides to quietly watch as the

client carries out this process, which Ralph knows is doomed to fail. During the interview, Ralph says that he decides to do this because he wants the client “to see why [the power rule] doesn’t work” in this situation. In his experience, clients “get used to using the power rule” and he wanted the client to see for himself that this “gut instinct [will] fail... [because] whether you fail or succeed, you’ve at least tried, and then you [can] learn from that.” During this example, after the client realizes that the power rule will not help to solve this problem, Ralph proceeds to take over the thinking, telling the client “I don’t know if you remember them going over this, but the derivative of $\ln(x)$ is $1/x$.” Here we have an example of Ralph’s prior experiences revealing that students often struggle with integrating $1/x$, which leads Ralph to experiment with allowing the client to work through the typical incorrect process in order to help the client see firsthand how and why the power rule is inappropriate.

During the interviews, Ralph provides some insight as to why he generally elects to take over the thinking while helping clients work through problems. When clients are in a rush to get through their work, most often because “class is in ten minutes” or “in other cases when [he has] more time” to ask clients questions and they are not responding positively to them (indicated by clients giving wrong answers, defeated “I don’t know” responses, and body language) then Ralph is “just going to help [the client] get the problem done, [by telling them] ‘Here’s how I would solve it.’” For example, during an observation in which Ralph is working with a Business Calculus client to create a price-demand equation, Ralph initially asks the client “Do you know how to find the slope?” and although the client says that she does, she starts to say something about $y=mx+b$, which was not what Ralph was wanting her to say so he “just assumed...that she

wouldn't know $(y_2 - y_1)/(x_2 - x_1)$, so [he] figure[d] just to give it to her." Another time during an interview, Ralph says "I'm really trying to explain to [the client] my thought process because I feel like a big part of math is just learning how to think in the right way."

Encouraging students to use varied approaches and strategies to make sense of and solve tasks. The only instance during the observations in which Ralph references an alternative task is while helping a Calculus III client that is working setting up a repeated integral. In this example, after Ralph has helped the client to graph the function $y=x/3$, the client then works to find the intersection between this function and $y=2$. The client correctly does so algebraically, after which Ralph says "since you drew a nice graph you might not have even had to work that out because, up one over three, now we're at $[x=]$ 3, up one over three, now we're at 6." Although Ralph does not encourage this client to use an alternative approach he does mention that another approach exists and explains how to execute it. During an interview, Ralph provides an explanation that sheds light on why it might be that there were no instances of Ralph encouraging clients to try alternative approaches to solving problems. He says that if the client has an example that was worked out by the professor, Ralph is encouraged by the Learning Center to help the client to understand how to solve the problem in that manner because that is what the professor wants from the client: "Because when the professor goes over examples in class, we're told at [the Learning Center] that's what we should cater to. We should try to match up with the professor. We don't want to tell them stuff that's against the professor or [is] a more difficult way than what the professor's teaching."

Teaching practice 3. Use and connect mathematical representations.

Selecting tasks that allow students to decide which representations to use in making sense of the problems. Due to the nature of tutoring at the Learning Center, it is not surprising that Ralph exhibited no evidence of this component of connecting mathematical representations because, as a tutor, Ralph is not in a position to select the tasks that his clients work on.

Allocating substantial instructional time for students to use, discuss, and make connections among representations. The only example of Ralph discussing a connection among representations is the time when, while helping a Calculus III client set up a repeated integral, Ralph mentions that instead of finding the intersection point of the functions $y=x/3$ and $y=2$ algebraically, the client could have also used the “nice graph” she drew to locate the intersection. In this instance, Ralph is not providing an opportunity for the client to make this connection herself, nor does he explain, in any detail, why this connection exists. This is aligned with Ralph’s focus on helping with the task at hand and, thus, trying to avoid discussing details not directly related to the present problem, especially because, in this example he is helping a client that is in Calculus III and this connection is a topic from College Algebra.

Introducing forms of representations that can be useful to students. During an interview, Ralph discusses that when he is helping Calculus I clients to understand the limit definition of the derivative (something that was not seen during any observation) he will “draw the picture [of the function] and show that the h is shrinking down to zero,” providing a visual representation of the process of transforming a secant line into the tangent line. He goes on to give another example of when he is helping calculus clients to find the maximum value of a function, he will sometimes say “Here is what a graph of

this problem would look like and this point, that's' the maximum. That's what we're looking for." Although neither of these examples surfaced during the observations, Ralph shows an awareness of the fact that producing a visual representation of a problem can be helpful to clients as they endeavor to solve problems.

Asking student to make math drawings or use other visual supports to explain and justify their reasoning. The only evidence present during Ralph's observations of him asking a client to produce a picture was when he was helping the Calculus III client who was working on the repeated integral, and needed to locate the intersection of $y=x/3$ and $y=2$. In this example, Ralph asks the client if she can draw the graph of $y=x/3$, however the client is unable to do so, so Ralph launches into telling her how to do so. During the interview Ralph explains that he "really like[s] graphs and pictures" and that he "feel[s] like one thing that makes people struggle with math is because they can't visualize it, so if [he] can draw a picture or ask [the client] to try and draw a picture" this can help clients to better visualize and make sense of the mathematics. Ralph's experience has shown him that many clients struggle to make sense of mathematics because they are unable to picture what is going on, which seems to have lead him to actively experiment with using drawings in his explanations. The success of using images in his tutoring seems to have lead him to begin to integrate this idea into the discussions and problem solving within his tutoring.

Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. While helping the Calculus III client to produce a graph of $y=x/3$, after she is unable to draw the graph herself, Ralph tells her "we're just going to start from the origin, because that's where

this example begins...Does that make sense? Because if you plugged in zero, then you would get zero so then you would have to start there.” Here, Ralph is making use of the connection between algebraic and graphical representations of functions, but in this example he does not focus the client’s attention on the basis for these connections. Then, later during this same interaction, after the client has found the intersection between $y=x/3$ and $y=2$ algebraically, Ralph mentions to her that because she drew “such a nice graph” she could use the graph to visually locate the intersection of the two functions but, again, he does not explain the basis of this connection rather he merely uses it to present an alternative approach.

Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems. During the observations and interviews, Ralph gives no evidence of attempts to examine and assess his clients’ abilities to use representations to solve problems.

Teaching practice 4. Facilitate meaningful mathematical discourse.

Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Due to Ralph’s tendency to take over the thinking while helping his clients to work through problems there were no examples during his observations of Ralph engaging a client in a conversation in which the client was sharing his/her ideas about how to think through the task at hand. However, Ralph does comment during an interview that “when [he has] been tutored, [he] feel[s] like [he is] not learning because [his] dad’s doing all the thinking... then [he] need[s] to try it on [his] own where [he is] thinking” in order to actually learn. He goes on to say that when he is being taught “a concept that [he] is not familiar with, something brand new... [he]

tend[s] to shut down” but he knows that “[he] needs to be thinking, [he] needs to be engaged...to try and learn this...but [he is] not exactly sure how to fix it.” Ralph later adds that, like his own experiences being tutored, the more familiar a client is with the material they are discussing during the tutoring session (e.g. “Oh yeah, I heard that in lecture” or “I remember working on a problem similar to this”), the more engaged they are. The only example he gives of trying to facilitate this engagement is that while he is tutoring athletes, he tries “to relate it to the sport that they play. If you can do that then they’ll most likely be more engaged [and do] more thinking with you, versus...letting you do all the thinking.” Ralph has reflected on his tutoring experiences in which the client was more engaged in the conversation and attributed this increased engagement to the level of clients’ familiarity with the content and context of a problem. This has lead Ralph to sometimes actively attempt to help his clients, particularly athletes, to better connect with the problem through relating the problem to clients’ personal life.

Selecting and sequencing approaches and solutions strategies for whole-class analysis. There were no instances of Ralph engaging more than one client at a time, so there we not opportunities to examine how Ralph decides to sequence and connect the strategies of different clients.

Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. When Ralph first approaches a client, he explains during an interview, after the client indicates what he/she needs help with, Ralph spends some time quietly “looking [at the problem] and trying to figure out what’s going on” so that he can make sure he leads the client along the correct path to the solution. For example, during an observation in which Ralph is helping a calculus client how’s

working on an integral of a trig function, Ralph silently reads the problem, figures out how to solve it, and identifies what knowledge the client needs, then announces to the client “you need to know your double angle formulas,” with no explanation as to why Ralph knows this is what is needed. However, Ralph says that whenever he is unsure of the content that his client needs help with he is “going to put more work on them... because [he is] rusty...and the student [has] been to class” so they should be more familiar with the material than Ralph. He gives, as an example, when “I’ve never confronted a Business Calculus problem before, I should know how to do [it], I just don’t know what supply, demand, product, profit” means.

Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning. There were no instances of Ralph making explicit connections among different client’s approaches because there were no instances of Ralph working with more than one client at a time, nor were there instances in which Ralph discusses how a client’s solution connects to the mathematical content being examined because there are no examples of Ralph actually allowing a client to share how he/she would solve a problem, thus there were no opportunities for Ralph to comment on how a client’s solution method aligned with the content.

Teaching practice 5. Pose purposeful questions.

Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. From the observations, it seems that Ralph primarily asks leading and funneling questions. For example, while working a Calculus III client the fraction $21/6$ is an answer. Ralph proceeds to ask “ $21/6$ can be simplified, right?” Upon discussing this moment during an interview, it turns out that Ralph

attributes this question to be an instance of Ralph being comfortable with telling his clients information that he assumes they are comfortable with, confirming that this question was in fact a leading question that was intended to carry the conversation forward. During another observed tutoring interaction, when Ralph is working with a Business Calculus client to produce a price-demand equation, after determining that linear slope is important for this problem, Ralph ask the client “Do you know how to find the slope?” in an attempt to lead the client to respond with the equation for the slope between two points.

Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. During an interview Ralph explains that, during the observed interaction with the Business Calculus client creating a price-demand equation, he asks the client “Do you know how to find the slope?” as a “kind of check, because if she had no clue how to find the slope, then [he] need[s] to go in” and talk about slope. He “wanted to just check that she [knew how to find slope], [he] didn’t want to just give it to her.” During another example in which Ralph is working with a Calculus III client, after Ralph has silently considered the problem, he asks the client “So, real quick, do you know the average height formula?” These kinds of fact recalling questions make up the bulk of the questions that Ralph asks his clients. Moreover, there are no examples of Ralph asking his clients to explain or justify their answers to his questions.

Ralph also explains that he typically won’t probe a client to see if they really understand because he places responsibility on the client to signal when something that Ralph says does not make sense to them, saying that “if I suddenly state something [that

doesn't make sense] normally [the clients] are pretty good [at asking] 'Why?'" Ralph says that he also makes use of "a little bit of body language" to help him identify when clients are not understanding him, discussing that when he is "explaining it to [the clients] and they're following and then suddenly there's something they don't understand, they pause, they don't write it down... [and] they're trying to think about it." These serve as signals to Ralph that he might need to go back over what he is trying to explain. It seems that Ralph believes that if a client does not understand something Ralph says, they will be sufficiently self-aware to be able to tell Ralph what it is they do not understand. This appears to be a result of the fact that Ralph has had experiences in which clients have done this, but has not had an experience which allowed him to explicitly see that this might not always be the case.

Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. During an interview, Ralph comments:

"Going back to when do I ask questions, I think it's subconscious to me... when I don't know the problem... or I haven't been in class recently, I really question the student to get a feel [for the problem], but other than that I don't know if it's random, or if it's past experiences, or maybe my [observations] of [more experienced tutors leads to thinking] 'Oh, well [the more experienced tutor], they would ask a question here, or they would just kind of give up and give [the client] the answer.'"

Thus, it seems that Ralph does not craft questions in-the-moment that are intentionally selected to achieve goals.

Allowing sufficient wait time so that more students can formulate and offer responses. While working with a Business Calculus client on building a price-demand equation, Ralph asks the client “Do you know how to find the slope?” the client responds that she thinks so and starts to say “ $y=$,” at which point Ralph decides that her answer is not going where he wanted so he “was quick to jump back in” to interrupt her and try “to rephrase the question as ‘Which one of these is the slope?’” indicating an equation on the client’s paper that is in slope-intercept form. While discussing, during an interview, the fact that Ralph was so quick to interrupt the client, Ralph says that “One of my reviews from the lead math tutor said that I need to give the students more time to respond when I’m asking questions. I’ve tried to do that a bit better.”

Teaching practice 6. Build procedural fluency from conceptual understanding.

Providing students with opportunities to use their own reasoning strategies and methods for solving problems. As previously discussed, there is an instance in which Ralph is working with a calculus client who is trying to integrate $1/x$ using the power rule, Ralph elects to let the client execute the strategy the client has chosen, even though Ralph knows it is doomed to fail because Ralph wants to help the client “see how it wouldn’t make sense [to use the] power rule.” After the client gets to wrong answer Ralph saw coming, Ralph proceeds to remind him that the derivative of $\ln(x)$ is $1/x$. This is the only example present in Ralph’s observations in which he allows a client to pursue a method of his own choosing, rather than simply telling the client which method to use.

Asking students to discuss and explain why the procedures that they are using work to solve particular problems. There is no evidence from the observations of Ralph

of him asking a client to explain why the procedure the client chose to use works and is appropriate. There is, however, a previously presented example of when Ralph is working with a Calculus III client on graphing $y=x/3$, in which Ralph tells the client, after she struggles to create the graph, “we’re just going to start from the origin because that’s where this example begins. Does that make sense?” and rather than ask the client to explain why it makes sense, Ralph immediately explains that it is “because when you plugged in zero, then you would get zero, so then you would have to start there.”

Connecting student-generated strategies and methods to more efficient procedures as appropriate. The only example in which Ralph allows a client to pursue his own strategy is when a calculus client wants to use the power rule to integrate $1/x$. After the client’s approach fails to yield an answer, Ralph tells the client that integrating “ $1/x^2$ is easy, but $1/x$ is a special case,” then Ralph goes on to tell the client, “I don’t know if you remember your professor going over this, but the derivative of $\ln(x)$ is $1/x$ ” so “once you know this we can go backwards” capitalizing on the fact that the derivative and integral “are opposite” operations. In this example, Ralph is trying to help the client to remember how knowing the derivative of $\ln(x)$ and the relationship between the derivative and integral operations helps to be able to integrate $1/x$.

Using visual models to support students’ understanding of general methods. There were no instances in the observations in which Ralph produced a visual model to aid in his explanation of a problem to a client.

Providing students with opportunities for distributed practice of procedures. During an interview, Ralph says he “feel[s] like the student has to work through a different example... [or] problem themselves” in order to understand how to solve

problems on their own. This belief is based on Ralph's own experiences of having to work through problems himself, even after getting help from his dad, in order for the material and procedures to make sense, even if Ralph ends up going through the exact same process that his professor or tutor did, the fact that he has done it himself is what finally made it make sense: Ralph says

“I feel like I do most of my learning outside of the lectures. I go to the lectures...[jot] down some notes...and just [try] to follow along, but I might not completely get it at the time...but then, when I'm working on the homework assignment... and I'm looking back through the textbook and trying to...explain it to myself...that's when my actual learning occurs, I feel like.”

The only example from Ralph's observations in which Ralph's client tries another example on her own is after Ralph helps a Business Calculus client to construct a price-demand equation, the client says that she will try the next part, constructing a price-supply equation, on her own. However, Ralph is not confident she will be able to do it on her own because, as discussed during an interview, Ralph was worried that the labels the client had placed on the data that were relevant to the first task might interfere with her thinking on this second part, so Ralph elects to tell her which new data points the client should use as the x-values as she works to construct the second equation.

Ralph later discusses in an interview that when he is faced with situations in which a client asks for help on a task that is, essentially, identical to some other problem that Ralph has already helped the client on, Ralph says that he typically tries to not talk the client through the problem again because he has “already explained it.” Instead, he will direct the client to the notes he/she took in class and also those from when Ralph

helped them before because he says “it’s a lot more beneficial for them to go back through an example problem and work through it themselves” because “it might force them to think” for themselves rather than Ralph “stating it outright again.” Here, we see that Ralph has reflected on his own experiences as a learner in order to help him develop ideas about how to effectively aid his clients, which has lead Ralph to feel comfortable with telling his clients how to solve problems because he expects that they, like himself, will actually learn how to solve the problems later when they are working through examples on their own.

Teaching practice 7. Support productive struggle in learning mathematics.

Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. There is no evidence with which to discuss the extent to which Ralph anticipates the struggles that his clients will have due, in part, to the fact that Ralph has no way of knowing ahead of time what tasks his clients will be asking for help with. However, during an interview, Ralph shares that from his experience working at the Learning Center it is common for clients to “show up to [the Learning Center] thinking they’re having trouble in the class [when] it’s really a couple of previous concepts that weren’t covered in the class that they really [need] help on” and he explains that his response to this is to remind his clients of the prior information that they are needing to recall. There were two examples of this common issue present in Ralph’s observation: a calculus client working on maximizing a function using the first derivative but she was unable to find the correct answer because she struggled to factor correctly while trying to solve for the zeroes of the derivative and a

Calculus III client who was struggling to find a directional derivative of a function because he was struggling to recall implicit differentiation from Calculus I.

Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. When Ralph does let a client work on all or part of a problem on his/her own, he tends to very quickly step back in to guide or take over the thinking. For example, as previously mentioned, after Ralph helps a Business Calculus client to create a price-demand equation, the client says that she will try the next part, creating a price-supply equation, on her own. However, before Ralph leaves he elects to tell her what the new x -values should be for this new equation in order to “make sure [she got] set up properly” because he doesn’t “want [her] to suddenly start on the wrong foot again.” Another time, Ralph is helping a Calculus III client who is struggling to graph $y=x/3$. When Ralph asks her if she can sketch the graph, as soon as the client begins to struggle, Ralph steps back in, telling her “we’re going to start from the origin” and to “run three and rise one.”

The only example from Ralph’s observations in which he allows a client to work all the way through a process without stepping in is when a calculus client is trying to incorrectly integrate $1/x$ by using the power rule. In this example, Ralph knows that this method will fail, but Ralph decides to let the client work through it anyway because he wants the client to see for himself that this “gut instinct [will] fail.” After the client is finished trying this wrong path, Ralph steps back in and reminds the client of the appropriate method for integrating $1/x$, reminding the client to that “the derivative of $\ln(x)$ is $1/x$.” During the interview discussing this example, Ralph comments that “whether you fail or succeed, you’ve at least tried, and then you [can] learn from that”

which shows that Ralph sees potential benefit to allowing clients to struggle, even if the path the client has chosen is fated to fail, because it can provide an opportunity for the client to learn from the experience. Ralph also discusses during an interview that when a client asks him to help with a problem that is similar to something that he has already helped them with, he tends to try to not do anything except direct the client to the notes they have and encourages the client to try to work through it because “it’s a lot more beneficial for them to go back through an example problem and work it themselves” rather than Ralph just telling them how to do it again: “the more [the clients] do it [themselves] the better.”

Helping students to realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Ralph says, during an interview, that he feels “like there’s definitely a balance between information you’re just straight up [telling the client] versus information you really try to fight for the student to answer” themselves because if “you’re just asking questions that they don’t know the answer to... it’s more likely to get them frustrated” and Ralph does not “want to get the student frustrated.” If a client is not able to answer the questions that Ralph is asking them, he says that he will say things like “this is complicated stuff you’re trying to do” so “you know what, it’s okay. Here’s what we need to do.” This is “just a way to make [the client] feel better, and to remind them [that this] is a tricky concept” they are working on. Sometimes Ralph will tell clients “I’ve had these same struggles... when I was working through this type of stuff,” which he thinks helps his clients to “think... ‘Oh, he’s not this freak out here just to solve my homework. He’s a guy who’s been through what [I’m going] through’” in order to help his clients “be more inclined to

respond positively and listen” because Ralph went through these same struggles but still ended up successful enough in the courses to be a Learning Center math tutor.

However, when Ralph is “a bit confused on a question, [he] won’t necessarily say ‘I’m confused’ because [he] want[s] to come off like [he] know[s] what [he is] doing.” Ralph tries to make his clients feel okay about being confused by telling them that it is, in fact, okay to struggle and even tries to empathize by explaining that he too struggled, but he doesn’t want to reveal when he, as a tutor, is confused by a problem because he is worried he will lose credibility with the client.

Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. Although there are no examples from Ralph’s observations in which he explicitly praises a client for the efforts or perseverance, he says that often times “when [he] see[s] [clients] really trying [he] will always say ‘Good job! Keep up the good work!’ [or] something like that.”

Teaching practice 8. Elicit and use evidence of student thinking.

Identifying what counts of evidence of student progress toward mathematics learning goals. Throughout the observations and interviews, Ralph never discusses how he decides whether or not his clients are progressing towards reaching learning goals. This might be a result of the fact that Ralph’s goals for his tutoring sessions are focused on making sure that he is able to show the student how to solve the problem.

Eliciting and gathering evidence of student understanding at strategic points during instruction. During an interview, Ralph says that the way he checks whether or not his explanations are making sense to his clients is that “sometimes [the clients] just sound uncertain, even though [when he asks] ‘Did that make sense?’ they [respond]

‘Yeah?’” which leads Ralph to think that maybe he should try something else to help the client to understand. When asked if there are any other ways that he checks whether or not his clients understand his explanations Ralph says “No, not really. Not that I can think of.” Ralph’s reflections on some of his prior experiences has helped him determine some ways in which clients reveal that they do not understand what Ralph has explained, without always explicitly stating it.

Interpreting student thinking to assess mathematical understanding, reasoning, and methods. While helping a Business Calculus client working on building a price-demand equation, after Ralph has determined that the client needs to be able to find the slope between two points in order to solve this problem, he asks the client “Do you know how to find the slope?” to which the client responds “I think so” and proceeds to start talking about $y=mx+b$, which is related but not the answer Ralph was looking for. This leads Ralph, as revealed during the interview, to “assume... that she would know $(y_2-y_1)/(x_2-x_1)$, so [he decided to] just give it to her.” During another observed tutoring interaction while Ralph is helping a Calculus III client with a repeated integral, the client starts talking through how to evaluate the integral saying “So, if I’m gonna go $dx...$ (pause).” Ralph interprets this pause as an indication that she is unsure of how to proceed because, he explains during an interview, “if [the client is] the one who’s... directing the conversation and talking and then they pause, [he] feel[s] like that’s an asking question type of thing, [and] that they want [him] to step in” and help.

During an interview discussing a tutoring session in which Ralph is helping a calculus student who has already worked through a problem in which she is asked to find

the maximum volume for a rectangular box, but she was unsure of her answer because the numbers weren't making sense, Ralph shares that

“if the student has already identified that a solution doesn't make sense, then that's good. That shows that they're thinking, because sometimes you're given clues or you can estimate your answer. An...example [is] if you're finding the pH of an acidic solution and you get a pH of 12, well that's basic, so you know something must have been wrong [because] you want it less than 7... In this specific problem, a weird answer would've been like a negative volume... [or] if you have a negative height, that wouldn't make any sense.”

Thus, because the client had worked through this problem already and was concerned that her answer was incorrect because it didn't make sense when she interpreted her answer in the context of the problem, that indicates to Ralph that “she understood the process perfectly... so [he's] just looking for something minimal” in her work that lead to the wrong result.

Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. While helping a Business Calculus client working on building a price-demand equation, after Ralph helps her to construct the equation Ralph discusses how she could check that her equation is correct: “If you plug in x_2 you should get your y_2 , and if you plug in your x_1 you should get your y_1 .” The client responds “So I'm going to check that by...(pause)” indicating that she did not follow what Ralph just said, which leads Ralph to respond “By plugging in a point that [the problem] gave you,” which is an explanation that is similar to the first one he gave, but expressed differently. During the interview, Ralph comments that “because I [had]

just told it to her, I [thought] ‘Maybe I shouldn’t tell it to her in the way [again]’ so I tried changing it up” as a way of “trying to get it to sink in.” Here is an example of Ralph reflecting in the moment realizing that because the client didn’t seem to understand what he was saying the first time perhaps Ralph should try to say it in another way rather than just repeating the same explanation.

During another observed tutoring interaction when Ralph is helping a Calculus III client with a repeated integral, the client tries to talk through the execution of the integral saying “So, if I’m gonna go dx ... (pause)” Ralph decides that this is an indication that the client is unsure of how to proceed and is asking for help, so he steps in and tells her “So remember, dx means you’re going to be shooting that way. What’s the first function we hit?” as a means of helping guide the client through the problem. She is able to correctly answer this guiding question, and Ralph continues to guide her through the rest of the problem.

When Ralph decides that a client does not understand how to proceed through solving a problem, he will sometimes try to rephrase his explanations hoping that will help the information to “sink in” or he will step in to take over the thinking and either tell the client directly what needs to be done or ask questions that lead them along.

Reflecting on student learning to inform the planning of next instructional steps. During an interview, Ralph discusses that there have been times when a client has returned to the Learning Center to inform Ralph that he was very helpful to him/her. This leads Ralph to be “a bit more confident in...the way [he’s] specifically approaching problems with this student because... [he] feel[s] like whatever [he has] been doing with the student... [he] should try to keep [doing] it because it’s working well.”

Reflections during interviews.

While discussing a time when Ralph is working with a Business Calculus student and the fact that Ralph chose to tell the client the formula for finding the slope between two points, Ralph comments that “maybe I should’ve asked her” what the formula was instead of just tell her, because “maybe it would have got her to think, and got back in her memory... [of] a lecture where the professor might have pointed to average slopes. That would’ve been a good thought process to go back... and try to get it herself...because if she’s on a test, she’s not going to have anybody telling her” the formula. At the Learning Center “we know... the end goal is to do well in the class...to be able to do these problems...without tutors. So that would be [a] benefit of trying to get her to think.”

While being interviewed about a moment during an observation in which Ralph is helping a Calculus III client with a repeated integral, when the client is leading the discussion talking through how she would approach the problem she pauses briefly in her discussion and Ralph, in-the-moment, interprets this as a sign that the client is asking him to step in and help her out. While Ralph is explaining that “when [he asks a] question, and then there’s a pause, that’s [the client] thinking, but when they’re the one who’s...directing the conversation and talking, and then they pause, [he] feel[s] like that’s” the client’s way of signaling they want him to step in and help, he goes on to comment “but maybe I shouldn’t do that. Thinking about it now, maybe, even if they’re directing the conversation, they’re trying to get their thoughts together.”

While discussing an instance during an observation in which Ralph is struggling to help a Calculus III client because he feels like she should have someone who knows what they are doing rather than Ralph “struggling with her [while] trying to remember

this” content, when I (the researcher) comment that it seems that Ralph feels like tutoring sessions go well when he doesn’t have to struggle through the content, Ralph responds “Personally, I don’t know which one’s more beneficial. It might be more beneficial if we’re bouncing ideas [off of each other] because maybe that’s engaging her a lot more...versus if I’m really confident and I’m like ‘Here’s the way to do it.’”

Effects upon Ralph’s tutoring that Ralph attributes to be the result of his participation in this study.

When Ralph is asked, during the final interview “how, if at all, did participating in this project have any sort of [effect] on your tutoring?” Ralph responds

“I’m not sure if it actually appeared in my tutoring, but [participating] definitely made me more self-conscious. Not in a bad way, but I’m actually just more aware of myself. When I’m tutoring, or helping out, or answering a question I’m more aware of what I’m doing because of these interviews because we’ve been analyzing every little moment.”

He also says that

“in-the-moment it’s more like I’m thinking ‘Maybe I should wait a bit longer before I say something,’ or ‘Maybe I need to jump in now.’ Because... one of the things we talked about was me jumping in and what times I would do that, versus me being patient and trying to let the student work things out.”

And he gives the following summarizing comment:

“Because we were talking about how it was kind of subconscious almost and so just doing these interviews analyzing everything, I’m thinking about that. I don’t

know if somebody I was tutoring would be like, 'Whoa, you changed,' but definitely on the conscious level, I'm more aware of what I'm doing."

Summary.

From the data, we see that Ralph's tutoring is guided by his goal of making sure that he provides accurate content knowledge to his clients and uses these goals to guide his reflections on the effectiveness of his tutoring. Ralph strives to aid his clients' reasoning and problem solving by, whenever possible, endeavoring to explain how to solve a problem using only the content knowledge his clients presently possess. Ralph does try to elicit clients' thinking in order to determine what content clients are and are not familiar with to help him decide how to best explain a problem's solution. Ralph shows little evidence of helping his clients to see connections among mathematical representations or of helping clients to develop their procedural fluency from their conceptual understanding. Throughout the vast majority of Ralph's tutoring observations, Ralph maintains primary control over the thinking and reasoning of each tutoring session, and Ralph is seen to primarily make use of simple fact-recall questions in a leading or funneling fashion. He also shows very little evidence of helping clients to productively struggle through material.

Case 2: Fred

Fred is a Finance and Mathematics double major in his senior year. This study takes place during Fred's fourth semester working at the Learning Center. Additionally, when Fred started working at the Learning Center he was a business tutor for his first two semesters— during which he assisted clients with accounting, economics, and finance

courses – and then he, and then became a mathematics tutor. Mathematics courses Fred has taken include Calculus 1, 2, and 3, Introduction to Advanced Mathematics, Analysis 1 and 2, Linear Algebra, Topology, and Modern Algebra. Fred has never taken any teaching or education courses, nor has he had any teaching experience prior to being employed by the learning center.

Teaching practice 1. Establish mathematics goals to focus learning.

Establishing clear goals that articulate the mathematics that students are learning. Fred’s primary goal is for his clients to be successful on their own. During an interview, Fred explains that he tries to help his clients to better understand the content as well as how and when to apply that knowledge because clients will eventually reach a point where “you can’t just memorize a formula” and be successful. In order to help achieve this, Fred explains that he strives to use vocabulary terms while tutoring “because [he] want[s] to imprint consistency” between how the client’s professor talks about the content and how Fred does, and he “will even ask [his] students, ‘Well how does your professor teach?’ or ‘What does he call this?’ or ‘What notation does he use?’” He says that he does this because he is “just trying to reinforce whatever [the client’s] professor is doing.” To Fred, understanding the vocabulary and definitions is “absolutely pivotal” to understanding and doing well in mathematics. This conceptualization leads Fred to actively work to incorporate opportunities into his tutoring sessions that are intended to help his clients to better understand the abstract vocabulary and definitions involved in a task.

Fred says that he see his tutoring as a kind of “boot camp” helping to prepare clients for class and assessments. He doesn’t like to shy away from using vocabulary in

an attempt to not intimidate his clients because he can't "tell everyone that it is [all] going to be alright... then they [go out] and all get slaughtered... there is a fine line" between meeting the clients where they are and donning rose-colored glasses by avoiding the use of vocabulary that might scare the clients. Fred goes on to say that he sees himself as a bridge between his clients' and their professors. He works to establish an "intermediary connection between what is happening in lecture and their [present] understanding [through his] use [of] vocabulary, but then [he] also incorporates some brief explanations of how that word is defined...so they can go back to lecture and understand what the professor is talking about."

Identifying how the goals fit within a mathematics learning progression.

Although Fred makes no explicit comments about how his tutoring goals align with clients' learning progressions, he does discuss his focus on helping the clients to be able to understand that they won't always be able to "just memorize a formula" and be successful, so Fred wants to help his clients to better understand how and when to apply the content knowledge their professors are discussing.

Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Throughout Fred's observations and interviews, there are no instances of Fred discussing his goals with his clients or how the clients' current work is contributing to their learning.

Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction. In order to help his clients to be successful on their own, Fred explains during an interview that he tries "to get [his

clients] to do as much [of the work] as [he] can” because this helps him to identify what the client knows and what the client is struggling with. During an interview Fred gives, as an example,

“Let’s say you’re doing a typical algebra problem and once you’ve set it up you’re just solving for x . [Some] tutor[s] will walk the student through the whole problem, and I’m like ‘Dude, just let them try it and then if they don’t know how to do something they’re going to ask you.’”

Additionally, Fred says that when he is finished helping a client, he will “tell [the client] ‘It seems like you’re having trouble with this’” hence [he is] implying pretty strongly ‘You should work on this.’”

Fred further explains that another way he helps clients to be successful on their own is after he is finished tutoring a client, Fred’s discussion of the process involved in solving the problem at hand will sometime extent to a discussion about other related information that is not directly used on the present task. For example, while helping a client to identify the domain of a rational function, after they have determined that $x = -3$ is the only value not included in the domain and Fred has helped the client to write the domain in interval notation, he includes a brief discussion of “If we were to include $x = -3$ ” in the domain, then they would use a bracket instead of a parenthesis to write the interval. When asked about including this discussion of the other case during an interview, Fred says that he “wish[es]... [he could] answer one question... [and] not have to answer it again” for that same client. So he tries to “tutor in a way where... [he] only [has] to answer four questions instead of sixteen,” which, in the context of this example, Fred means that because this client’s question is focused on finding and writing the

domain of a function, Fred would rather explain the other case of how to write an interval that does include a specific value now even though it is not relevant to the present problem's solution in the hopes that when the client encounters another domain problem she will know how to write the interval no matter which case arises instead of having to ask for help because now the problem is slightly different from the one Fred helped with before. Fred's experiences have helped him to conclude that if he extends his discussion of the process involved in a task to include other related situations, he can help clients to develop a more complete understanding of the general process and better enable his clients to be successful on future tasks that utilize the same general ideas.

Fred explains that another way he strives to help his clients be successful on their own is by explicitly using vocabulary while discussing content in order to help reinforce its meaning by using the vocabulary in context, but then also providing the definition, if need be, to help clients recognize and be more comfortable with the vocabulary when they encounter it during lecture or assessments. As opposed to what Fred says he sees some other tutors doing which is using words like "stuff" and "things" in place of more specific language in order to avoid using words that might intimidate the client. Fred explains during an interview, that he tries to translate the mathematics "without losing the true meaning of what's going on" by, for example, explaining to calculus clients that an integral is a "statement [that] is asking a question... It's saying 'What do I need to take the derivative of to get this integrand?'" Fred goes on to give a caveat though, discussing that while he strives to maintain consistency between the approach he uses while tutoring a client and the way the client's professor approaches material and problem solving, sometimes he will elect to "throw everything out the window...[when] it seems like [the]

professor is confusing” the client because “if [he] explain[s] it this [other] way and [the client does] well on [the] test, then [Fred] might as well go this way.” Although Fred strives to explain material in ways that are consistent with his clients’ professors, his experiences have shown him that this is not always helpful to the clients, leading him to understand that there will be times when he needs to abandon this goal and work to provide an explanation that helps the clients, even if it is different from how their professors discuss it.

In order to help Fred ensure that he is addressing his client’s needs, he endeavors to fully understand the client’s question before he starts explaining things to them. Fred says, during an interview, that there used to be times when he would assume that he knew what a client wanted, then launch into a long discussion, only to find out at the end that that wasn’t the client’s question. He says he “made that mistake for a long time and then was finally like ‘I just need to ask people what they want.’” He says that the frequency with which he misinterprets a client’s question has “drop[ped] pretty considerably” and says that he “think[s] it’s [because he] started asking [clients] ‘What is your question?’” Moreover, Fred is cautious about how deeply into theory he goes while tutoring to ensure that he doesn’t lose the client’s attention or interest, explaining that he used to go into great detail, showing clients derivations, but eventually realized that he’s “actually wasting [his] time” because clients typically are not interested in them and they are not going to help the client understand the task at hand. He still believes that clients should know these things, but has shifted his tutoring priorities. He says he does ask clients if they are interested in this information, but almost no one says “yes.” Fred, through reflecting on numerous prior tutoring experiences, has found that when he assumes he

knows a client's question he too often ends up providing an explanation that either addresses something other than what the client is struggling with or he goes into too great of detail about the material, both of which cause his clients to not get much, if anything, from these explanations. Thus, Fred has decided that he should take the extra few moments required to make sure that he understands precisely what his clients are asking and the appropriate depth in which to explain.

In the event that Fred encounters a client with a problem on which Fred does not know how to do, Fred says during an interview, that he openly admits this to the client, telling them "I don't remember this so I'm not going to bother explaining it to you, but I will give you all of these resources" as a means of helping the client to help his/herself.

Teaching practice 2. Implement tasks that promote reasoning and problem solving.

Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. There are no instances during Fred's observations or discussions during his interviews that speak to whether or not Fred attempts to motivate his clients' learning through opportunities to explore content.

Selecting tasks that provide multiple entry points through the use of varied tools and representations. There are no examples of Fred selecting tasks that afford his clients opportunities to approach solving the problem from multiple perspectives.

Posing tasks on a regular basis that require a high-level of cognitive demand. There are no examples of Fred posing any tasks to his clients.

Supporting students in exploring tasks without taking other student thinking.

While tutoring a Business Calculus client who is working through a definite integral, Fred asks the client to go to the board and work out the problem. After the client finishes, Fred verbalizes the steps that the client just worked through. During the interview, Fred explains that this student

“has the solution [manual] in front of him, and I’m trying to talk to him but he keeps looking at the solution... [and] he wouldn’t stop and listen to me... He just wants me to give him an answer but [that’s] not how calculus works... So I got him up on the board, instead of writing [myself]... I needed him to do the problem himself, so I had him write it, and then I verbalized it. So I was getting him to do at least half the work by himself instead of me doing both and him just listening.”

During another tutoring session, Fred is helping a client who is doing test corrections and is working on finding the domain of a rational function. The client tells Fred “I want to write the steps as we go, so what should I write?” So Fred starts to discuss that from “the start of [the course]...one thing we know is that [polynomials’] domain is the whole real line”, and that it is often easier to find where a function is undefined rather than where it is defined. Fred goes on to say “here, we have a rational function, where does that cause issue?... long story short, this function is not defined when the denominator is zero.” He then asks “Does that sit with you okay?” to which she replies “I want to write out the steps as you go, so what should I write?” Then Fred gives a caveat that these steps “might not be universally applicable.” Then, rather than give a step-by-step guide to finding the domain, he instead focuses his discussion on checking

for x-values that cause a zero denominator and those that lead to the even root of a negative number. During the interview, Fred says that “when she used the words ‘steps’ [he] was concerned because... not every problem is going to say ‘Find the domain. Find the range.’ [Then] if a student remembers steps and they get a problem that just asks for one particular thing they’re going to write out each step rather than know which step is important... [or] the students will just not recognize problems that don’t have the steps” so Fred decides instead to focus his discussion on the general ideas that are required to find where a function is undefined because these ideas are more generally applicable, and thus more useful. Fred’s experiences with his tutoring clients has lead him to realize that although clients might ask for a step-by-step description of what to do, this is often not the best way to help clients because it can lead clients to focus too intently on the exact steps Fred lays out, which sometimes results in clients not being able to solve related problems that require slight alterations to the outlined steps or to not be able to address questions that are focused on only a subsection of the entire process.

Encouraging students to use varied approaches and strategies to make sense of and solve tasks. There are no examples from observations of Fred in which he encourages a client to use multiple approaches to solving a task, nor are there instances in the interviews where Fred discusses times when he does this.

Teaching practice 3. Use and connect mathematical representations.

Selecting tasks that allow students to decide which representations to use in making sense of the problems. There are no examples from Fred selecting tasks to give to his clients with the goal of allowing the client to decide which representation to use to help make sense of the problem.

Allocating substantial instructional time for students to use, discuss, and make connections among representations. There are no instances of Fred spending time allowing his clients to explore connections among representations.

Introducing forms of representations that can be useful to students. While helping a client who is working to find the domain of a rational function, after they have determined that the domain is everything except for $x=-3$ Fred asks the client “How do we express it in interval notation?” to which the client responds “-3 to infinity,” which are the correct bounds for one part of the answer but the client gives no indication as to recalling the other half of the interval nor does she reference the interval markings that need to be associated with the bounds. Then, as Fred moves to the board he asks “Are you a visual person?” to which she responds “yes” so Fred proceeds to sketch a number line marked with a -3, and he uses this drawing to show the two sides of the interval that are needed. During the interview Fred says that “if she had said ‘No’... [he is] sure [he] could have found some [other] creative way of talking about it” but he says that he does “not think [he has] ever talked to someone [who] says ‘No, I’m [an] audio [learner]’ or ‘I’m verbal’” so it seems that “most people are visual” learners.

While helping a client that is tasked with describing the end behavior of a polynomial function, after discussing that the client only needs to look at the leading term to determine the end behavior, Fred brings up the parent function x^2 and tells the client that the “left and right [sides] are up,” then he mentions the function x^3 and asks the client “What’s the end behavior here?” and the client is able to correctly state that the left side is down and the right side is up. During the interview, Fred says that “when explain[ing] end behavior... every single time [he has] used that analogy...as opposed to trying to

explain to students numerically” by investigating the kinds of results produced when negative and positive values are input into the leading term. Fred explains that each time he helps clients with examining the end behavior of functions, he uses this same analogy, so it seems that each time Fred has deployed this method of explaining function end behavior, Fred has determined it to be sufficiently successful and thus has not found a need to modify his method of tutoring this topic.

Asking students to make math drawings or use other visual supports to explain and justify their reasoning. There are no examples of Fred asking his clients to produce math drawings.

Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. During an interview, Fred explains that it is fairly common for calculus clients to be able to execute the mathematics but they don’t actually understand what it is they are doing. For example, the “chain rule: some people know how to do the chain rule, and they have no idea what the chain rule actually is” but they need to understand this because “they’re probably going to get a chain rule problem that is [very] long and they haven’t memorized it.” To help clients that are struggling with definitions he will try to help the client “break [it] down” but he is “always cautious about [doing this] because when you abstract each word in the definition is important...but it doesn’t hurt to just give the student a basic idea of the definition to start off with.” If it turns out that the client “doesn’t really [understand] the idea[s]” of the definition he will “do an example and relate it to the definition, [asking]... ‘Does this example fit the definition? Does it not?’” Fred does this “because when [clients] see the definition it’s just floating up in the air, [so he] give[s]

examples to bring them back to the ground.” Fred’s experiences have shown him that clients struggle with understanding the abstract features of definitions and processes, even if they are able to correctly implement them. Thus, Fred works to weave opportunities into his tutoring sessions in which he can specifically discuss with his clients the abstract underlying ideas that can aid clients in better understanding how and when to apply definitions and processes while solving problems.

While Fred is helping a calculus client who is working on finding the integral of a function using u -substitution, the client is struggling to determine the integral of u^{-2} , Fred says this is something the client “is quite familiar with” and shifts to talking about x^{-2} , which leads to the client being able to recall how to evaluate the integral using the power rule. During the interview Fred explains that “for some reason...people will know how to do an integral using x ... [but if] you substitute x for the variable u they just don’t know what’s going on. So [he] switch[es] to x all the time.” He does say that he is “concerned that it will confuse the students, but [he] think[s] it’s still worth it if it helps them understand a little more.” Here Fred explains that even though switching the variables in a function might confuse his clients, his experience has shown him that many clients struggle with applying calculus concepts to functions described using variables other than x so he frequently will change the variables in a problem to be x ’s in order to help his clients to better understand how to tackle the present problem.

While helping a client working on determining the end behavior of a polynomial, after Fred has discussed the parent functions x^2 and x^3 , Fred then shifts back to focusing on the client’s problem and asks “What does this power tell us?” and the client responds “whether it opens up or down” which, while true, is not the specific to the problem at

hand, so Fred says “let’s look at the parent function with similar attributes” and asks “Is our power even or odd? So what’s our end behavior?” and the client is able to correctly determine the end behavior of the polynomial. In this example, Fred is using the parent functions x^2 and x^3 and also makes use of the connection between the parity of the exponent on the leading term and the parity of x^2 and x^3 as a guide to help the client to determine the end behavior of the polynomial, but Fred never explicitly states that all polynomials with even exponent on the leading term behave similarly to x^2 and those with odd exponent are similar to x^3 . During the interview, Fred attributes his lack of discussion of this connection to the fact even though he “understand[s] a health amount about polynomials” in his “experience with mathematics, polynomials haven’t really been discussed in depth [so he] kind of explain[s] [them] from a distance.”

Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems. Fred provides no evidence of attempting to assess clients’ abilities to utilize representations to help solve problems.

Teaching practice 4. Facilitate meaningful mathematical discourse.

Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. During an interview Fred says “when I’m tutoring I kind of view it as a conversation because when you first start off [working as a tutor] you get into the habit of thinking ‘I know this very well’ so I can just tell a student ‘You do this because I say you do it because I know it’s right.’” Now that he has started “to think of it as a conversation” he approaches tutoring thinking “this is what I’ve got and I’m trying to help you [the client] with this problem, but if you don’t agree with me just tell me or if your professor uses a different word let’s talk about how these

definitions differ” because “if you [and the client] are not talking about the same problem, it’s hard to conclude the same things.” Fred wants to engage the client as a participant in a conversation with him as a way to help ensure that the way Fred is talking about the problem aligns with how the professor discusses the material and that it makes sense to the client.

During an observation while Fred is helping a calculus client Fred asks the client “What is the derivative of $\log_{10}(x)$?” the client begins to talk about $\ln(x)$, then pauses, so Fred starts to step in, saying “Well we know...go ahead” stopping himself and putting the client back in the lead of this conversation. During the interview, Fred explains that “too often [when] tutors start talking, they end up just doing the whole problem for the student because the student doesn’t want to interrupt... [so] I try to get them to explain most of it and then I’ll explain it [because]...to often they’ll just sit there and you’ll explain the whole thing [and] I wanted her to give as much effort by herself as she could.” So Fred is trying to get the client to lead the conversation until she is unsure of how to proceed, at which point he will step in and start helping. As Fred gained more concrete tutoring experiences upon which to reflect, he found that just telling clients what to do is not as beneficial to their learning as when he endeavors to engage clients in a conversation because when clients are engaged in a discussion with Fred, Fred is better able to make sure that he and his clients are on the same page throughout the tutoring session.

However, Fred does say that while “very rarely I just do a problem for a student... sometimes we’re slammed or I’m sleep deprived and I’m just like ‘This is how do you do the problem.’”

Selecting and sequencing approaches and solutions strategies for whole-class analysis and discussion. There were no instances in which Fred engaged multiple clients so as to afford him the opportunity to decide how to sequence a discussion of the various strategies they employed.

Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. During an observation, Fred approaches a calculus client who is working on a problem that has a DX out in front of an equation, and Fred asks the client “What do you have out front?” and the client explains that this is how his professor writes the derivative operator. During the interview, Fred explains that he had never seen this notation for a derivative, so this was an instance in which he genuinely didn’t know what the problem was asking so he asked the client to explain it to him. When asked if he ever asks clients questions like “What’s going on here?” even though he already knows what the problem is asking, Fred replies “I do. And in fact to be honest, I wish I asked it more often because that’s probably one of the best... questions to lead with [because it] is testing [the client’s] understanding. Especially with defining conditions because that’s what most [important]: if you don’t know the definition you’ve got no strength. So I do actually do a healthy amount” of asking these kinds of questions. Thus, Fred will place his clients in charge of the conversation in order to help identify what they do and do not understand about the task at hand, allowing Fred to better determine how to best help the client.

Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning. There were no instances of Fred making

connections among multiple clients' approaches because there were no examples of Fred engaging more than one client at a time.

Teaching practice 5. Pose purposeful questions.

Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. During an observation, Fred is helping a client who is working on taking the derivative of $\log(r(w))$. When Fred first approaches the client, she is having trouble with the derivative of the logarithmic function. Initially Fred asks her “what is $\frac{d}{dx} \log_{10} x$?” to see what she could recall. She is unable to recall, so Fred reminds her of the rule for taking the derivative of logarithms. Then, having helped her with the part she was struggling with, Fred goes back to asking questions “So what is that going to imply for the right hand side?” During the interview, Fred explains that this client “is one of [his] regulars for calculus so [he] knows that her implicit differentiation is pretty good [so because] she said at the beginning... ‘I’m having trouble with the log’...after [they] had discussed how to differentiate implicitly $\log_{10} x$ then [he told her] ‘Okay, now you're just doing stuff you’ve done before.’” He didn’t just leave the client to work on her own, but rather her “wanted to see if she could do” the rest on her own. Thus, when clients are not able to answer Fred’s guiding questions he will step in and talk the client through the information they are missing, but as soon as he helps the client through the particular spot they are struggling with he goes back to questioning the client to help ensure the client does as much of the work as possible.

Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. During an observation in which Fred is helping a calculus client who is working on a problem that has a D_X out in

front of a function, Fred asks the client “What do you have out front?” and the client explains that this is the notation his professor uses for the derivative. During the interview, Fred explains that he had never seen this notation before, so he needed the client to explain this to him. Fred goes on to comment that he does ask clients this kind of question even when he does understand the material they are working on “because that’s probably one of the best ways to lead” into a tutoring session because “it’s kind of like troubleshooting in computer science. [He’s] worked with students and they know how to go from the beginning to the end [of a problem], and then [Fred] ask[s] them a very simple definition ‘What’s the derivative?’ or ‘How’s this method done?’” and the clients are unable to answer.

Another example of Fred probing his clients thinking occurs during an observation in which Fred is tutoring a client that is working on test corrections and needs help finding the domain of a rational function. After identifying what the client is working on, Fred asks “What’s the first thing you think? The first step that you would take?” The client responds “set it equal to zero” to which Fred says “You kind know what you’re doing, but we need to know the right order and why we’re doing it because not every problem will look just like this.” During the interview, Fred explains that this initial “What are you thinking?” question is intended to get client engaged in the process and to help Fred identify what the client knows so that he can better address her needs. Fred has found, through reflecting on other tutoring experiences, that getting his clients engaged in the conversation through explaining their thoughts about how to approach a problem allows Fred to better gauge clients’ present understandings and struggles.

Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. During an interview, Fred discusses that when he is tutoring calculus clients working on an integral, and gives as an example $\int x^2 dx$. He says that he

“will put that off to the side and [he] will say [to the client] ‘Okay. Well that statement is asking a question in a way. It’s saying “What do I need to take the derivative of...to get this integrand of x^2 ?” then they [the clients] don’t do anything because they’re [confused]... [so he] would say ‘Well what do we know about the power rule?’ [and] the all write x^3 [to which he replies] ‘Does that work?...are you sure? Take a moment.’ Then they [will say] ‘Oh, that’s $3x^2$ ’ [which leads Fred to respond] ‘Is there a three here? No, [so] how do you get rid of that?’”

During the interview, Fred shares that he likes to explain integration in this manner to help the clients get used to translating from the symbols into a description of what the symbols are really trying to convey “without trying to lose the true meaning of what’s going on. Then after that, [he] will let them do it on their own because [now] they have some peace of mind when they’re” working it for themselves.

In this example described by Fred, Fred is using questions to help the client to better see/understand what is actually meant by an integral as well as how knowing that the integral is really asking a question about derivatives can help the client to determine the answer. However, the questioning pattern described here is certainly a funneling pattern, but Fred does seem to be trying to keep the client engaged and thinking through the process with him.

Allowing sufficient wait time so that more students can formulate and offer responses. There were no instances in which Fred asked a client a question and then did not give the client time to respond before he stepped back in with another question or to take over the thinking.

Teaching practice 6. Build procedural fluency from conceptual understanding.

Providing students with opportunities to use their own reasoning strategies and methods for solving problems. During an observation in which Fred is helping a Business Calculus client who is working on a definite integral, Fred asks the client to work out the integral on the white board, and while the client is working Fred offers up some help, telling the client “we’re going to keep out constants out front” as well as explaining that after finding the integral, the client no longer needs to write the integration symbol. During the interview, Fred explains that he has worked with this client before and the client typically has the solution manual in front of him, “but [Fred does] not want him to look at the solution, but still he would not stop and listen to [Fred].” Fred says that he could tell that the client “just wants [Fred] to give him an answer but that’s not how calculus works...so [Fred] got him up on the board, instead of [Fred] writing.” This way, Fred “was getting [the client] to do at least half of the work himself instead of [Fred] doing [everything] and [him] just listening.”

During another portion of the interview, Fred says “very rarely do I just do a problem for a student. I try to get them to do as much as I can...[until] they give you those puppy eyes” when they reach a point where they don’t know how to continue. At which point, he will step in and render aid. Fred’s experiences have led Fred to believe

that the best way he can help his clients is to encourage them to do as much of the work as possible and only stepping in to guide his clients when they absolutely need it.

Asking students to discuss and explain why the procedures that they are using work to solve particular problems. There are no examples from Fred's observations in which he asks a client to explain why a procedure works to solve a problem, and Fred makes no references to doing this during his interviews.

Connecting student-generated strategies and methods to more efficient procedures as appropriate. There are no examples of Fred connecting the strategies employed by his clients to other, potentially more efficient, procedures.

Using visual models to support students' understanding of general methods. There are no instances in which Fred explicitly uses a visual model to help a client to understand a general method for solving a problem.

Providing students with opportunities for distributed practice of procedures. During the observation in which Fred is helping a Business Calculus client to work through a definite integral, as the session wraps up Fred tells the client "your integrals are good, it's just evaluating that you need to work on." During the interview, Fred explains that often, especially with new clients, he will "tell them 'It seems like you're having trouble with this' hence implying pretty strongly 'You should work on this.'" Other times he will "say 'Try the next problem' which [he] thinks is kind of risky as a tutor because it's not [his] job to tell people what problem they should do...but [he does] not care...because it is better than just letting the students sit there mindlessly."

Additionally, while Fred is discussing his approach to explaining that an integral is really "asking... 'What do you need to take the derivative of to get this integrand?'"

Fred explains that after he had gone through a discussion like this with a client, he “will let them do it on their own [so] they [will] have some peace of mind when they’re taking their” test or quiz.

Teaching practice 7. Support productive struggle in learning mathematics.

Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. There is no discussion during Fred’s interviews in which he reveals evidence that he attempts to anticipate places where clients may struggle and tries to prepared to support them through that struggle.

Giving students time to struggle with tasks, and asking questions that scaffold students’ thinking without stepping in to do the work for them. During an interview, Fred explains that a typical way in which he helps clients that are struggling with integration is to tell them that the integral is a “statement [that] is asking a question... it says ‘What do I need to take the derivative of...to get this integrand of [for example] x^2 ?’” In his experience “all [clients are able to] write x^3 ,” so he asks “‘Does that work?...Are you sure?’” and if the clients struggle to see if x^3 works, he asks “‘What is the derivative of this?’ ‘Oh, that’s $3x^2$.’” So Fred asks “‘Is there a three here?’ ‘No.’ ‘How do you get rid of that?’” Fred explains that he does this to help translate integral problems “without...[losing] the true meaning of what’s going on.” Here, Fred explains that he tries to guides the clients’ thinking by asking questions and waiting for the clients to respond, rather than just telling them what to do.

Fred also discusses that when he approaches a client who tells him “‘I haven’t been to class’ [or] ‘I haven’t read my book’” his response is to tell them “‘read the

chapter and then I'll help you...you don't have to read it word for word just skim through it' because [he] need[s] something to work with." Although this is not an example in which Fred is helping a student to struggle through a particular problem, Fred does want this clients to have at least attempted to try to understand the material from class on their own before he steps in to help them so that Fred can focus on the portions of the material that his clients are struggling to make sense of themselves.

While discussing, during an interview, the fact when clients pause during an explanation they are giving, sometimes Fred decides to step in and help guide them while other times he remains silent and allows them to work, Fred explains that

"the funny thing about pauses is people say, 'Oh, pauses. They provide so much information.' That sounds like total crap because there's literally hardly any information... That's what's scary about if someone pauses. How do you even know there's a single thought running through their mind? So I try and let students do as much of the work themselves because I think that's the best way to do it. However, I rely on body language a lot in terms of how I read people" to help decide if the client is still thinking or if they are just waiting for Fred to step in.

Helpings students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. During an observation in which Fred is helping a client who is working in taking the derivative of a logarithmic function that requires implicit differentiation and the application of the chain rule, Fred tells her "Let's write out the original...function even if it seems to be tedious." During the interview, Fred says that he "say[s] that often" and explains that this

is his way of trying to empathize with the client by saying “You might think this is tedious, but we’re going to do it because it is good for you.”

Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. During an interview, Fred says that “it’s good to tell [clients] what they’re doing well on because then you give them direction, ‘I’m not saying you have to try the whole problem, but why don’t you try this algebraic exercise.’” During the observations, there is an instance in which, while helping a Business Calculus client who is working on a definite integral, Fred tells the client that he is struggling with “is just the algebra aspect so far your calculus has been just fine.”

Teaching practice 8. Elicit and use evidence of student thinking.

Identifying what counts as evidence of student progress toward mathematics learning goals. Fred provides no indications of what he deems to be evidence that a client is making progress toward a learning goal.

Eliciting and gathering evidence of student understanding at strategic points during instruction. During an observation while Fred is working with a calculus client who is working on a problem that has a D_x in front of an equation, Fred asks the client “What do you have out front?” to which the client explains that this is his professor’s way of notating the derivative operator. During an interview, Fred explains that, in this case he asked this question because Fred himself had never seen this notation before and so he needed the client to tell him what it meant. However, Fred goes on to say that he does pose this kind of question to clients even when he does know what’s going on “and in fact, to be honest, [he] wish[es] [he] asked it more often because that’s probably one of the best...questions to lead with [because it] is testing their understanding, especially with

defining conditions because... if you don't know the definition you've got no strength.” Fred goes on to explain that these kinds of questions can “also catch big problems... kind of like troubleshooting in computer science.” For example, Fred says that “some people know how to do the chain rule [but] they have no idea what the chain rule actually is, and it's good to catch that because...they're probably going to get a chain rule problem that's [very] long [that] they haven't memorized” how to do. Fred uses these questions to help him differentiate between clients who understand the definitions, processes, and/or content from those that are able to execute the procedures without understanding how and why to apply the ideas on their own.

During an observation when Fred is working with a client who is working on test corrections and trying to find the domain of a function, when Fred first approaches the client he asks her “What's the first think that you think? The first step that you would take?” During the interview, Fred explains that this is another example of him trying to determine what the client presently knows about the problem to help Fred provide better, more personalized aid to the client. Later during this same observation, after Fred has helped the client to determine the single x -value that is not included in the domain, he asks her “How do we express it in interval notation?” Fred explains during the interview that because he had helped the client to find the domain, her initial question, now that they are on the next phase of the problem, Fred wants to examine what it is the client knows about this portion of the task.

Interpreting student thinking to assess mathematical understanding, reasoning, and methods. During an interview, Fred says that “more often than not students... if you make them feel comfortable... are forward with you” about saying when they are not

understanding some part of a problem or an explanation that Fred has given. Fred says that maybe it helps to say “key phrases like ‘Stop me if I lose you,’ ‘Stop me if I stop making sense,’[or] ‘Tell me if this approach isn’t working, [and] we’ll try something else’ but [he has] to start with something.” Fred says that he wishes he could tell ahead of time if a client will respond well to an explanation, but instead he tries “to read body language” and if a client is one of “the timid ones, [he] like[s] to sit next to them instead of work[ing] on the board because they don’t want to be embarrassed.” Fred’s reflections on prior experiences working with different clients has lead him to believe that whenever clients feel comfortable with him, they are more willing to speak up when they do not understand.

During an observation while Fred is helping a client take the derivative of a logarithm that requires the application of the chain rule as well as implicit differentiation, the client explains that she is having trouble with the derivative of the logarithm part. After Fred talks her through how to take the derivative of a logarithm Fred tells her “Okay, now you’re just doing stuff you’ve done before” and has the client continue to work through the rest of the problem on her own while he watches. During the interview, Fred explains that he “discussed how to differentiate implicitly log of a function” because that was the part the client was struggling with, but then he passed control of the problem back to the client because he “wanted to see if she could it” herself.

Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. During an interview, Fred explains that while he is tutoring clients “[he] need[s] that personal interaction to make sure that [he] care[s] about [the clients’] interest.” He explains that this personal interest and interaction

with his clients is important “if you do the same monotonous tasks...all of a sudden... you're no longer interested in [the clients'] final answer, you're just interested in answering it. And why is that a problem? [He] think[s] it's a huge problem because people are very complicated and very distinct, and so if you view everyone as the same...not only are you not answering these people's [particular] problems, but you are [also] doing a terrible job because [tutoring] requires you to be like a technician: you need go in and... find the issue, and you need to be precise about it... [otherwise] it hurts the client [because] your questions become more vague and ambiguous instead of being very precise.” Fred goes on to explain that “by precise, [he does] not just mean how you state the question, [he] mean[s] precise in taking into consideration the student: How much do they know? What do you think they need to know? What the actual question is? You want to dig in and... find where in solving this equation they don't understand.” Fred has found from his experiences, that his tutoring sessions are more helpful to his clients whenever Fred is able to specifically tailor a tutoring session to the present understandings and struggles of his client and so he actively works to integrate this idea into his tutoring practices.

During an observation when Fred is working with a Business Calculus client that is working on evaluating a definite integral and has the solution manual with him, Fred decides to get the client to go to the board and work on the problem up there. During the interview, Fred explains that he has worked with this client before and that Fred does “not want him to look at the solution, but he still would stop and listen to [Fred].” Fred also comments that he “could tell... [that the client] just want[s] [Fred] to give him an answer but that's not how calculus works...so [Fred] got him up on the board” because

Fred “needed him to do the problem himself.” As a result of the fact that this client has the solution manual in front of him and because Fred has interpreted that the client just wants Fred to hand him an answer, Fred decides to get the client to go to the board to work so that the client will “do at least half the work himself” instead of just listening to Fred talk through it. While this is not an example of Fred deciding to ask questions of the client, Fred is getting the client to go to the board to work so that Fred can get the client to do some thinking so that Fred can better aid him.

While helping a client that is working to determine the domain of a rational function, Fred has finished guiding her through how find the single x -value that is not included in the domain, Fred asks the client “How do we express it in interval notation?” to which the client responds “-3 to infinity,” which, while a significant portion of the answer, is only discussing one side of the interval and makes no mention of which interval markings need to accompany the -3 and the infinity. So Fred asks the client “Are you a visual person?” “Yes” responds the client, then Fred sketches a number line on the white board and marks -3 on the line, then proceeds to use this number line to discuss that there is a section of the interval on the left of -3 and one on the right of -3. During the interview, Fred explains that he chose to draw this number line to help the client to see “where the chunks of the intervals are” in order to aid her in correctly writing the interval.

During an interview, Fred explains that while tutoring clients, his “main goal is to [determine] what are [the clients] learning in class because [he is] just supposed to be coaching [the clients]. He is “not a lecturer, [he is] just trying to reinforce whatever [the client’s] professor is doing.” However, there are times when it “seems like [the] professor

is confusing” the client so Fred will sometimes decide to “throw everything out the window” and explain the information to the clients in a different way, “but [he]...[worries] about that because the last thing [he] want[s] to do is undermine the professor or take [the client] off course... but there are some cases where a student just learns differently,” and “if [he] explain[s] it this [other] way and [the client does] well on [his/her] test, then [Fred] might as well go this [other] way.”

Reflecting on evidence of student learning to inform the planning of next instructional steps. During an interview, Fred explains that, as a student, he “love[d] that [he] could...[derive] all the present value formulas...[using his] math background and calculus...[and] start[ing] off with parameters” so when he was first working as a business tutor, he would tell clients “Oh, you took calculus...well this is how you get this, and this is the math implication” but based on the fact that after he finished explaining to clients how to derive the relationships “the student[s] [would] just be like ‘Whew’ [lead him to] realize [he’s] actually...wasting [his] time [because] they’re never going to use this...it’s not that [he] gave up, [he] just saw [that] it’s better to focus on what they need to know.” So now Fred ask[s] them if they want [him] to [explain the derivation] but...rarely [does] anyone answer ‘Yes.’” Although Fred believes that an understanding of the underlying theories is key to truly understanding mathematics, his tutoring experiences have lead him to realize that many clients are not interested in these deeper connections. Thus Fred’s reflections on these experiences lead him to modify is conceptualization of his default approach to a tutoring session to focus less on the deep mathematical theories, but he will still engage interested clients in such discussions.

During an interview, Fred discusses that there have been times when he has been working with a client and is “going into depth with [an] explanation and then [the clients] stop [him] at the end [and] they say ‘Well, that wasn’t my question.’” He explains that he doesn’t mind “getting a problem wrong in a certain aspect, but answering the wrong question, that’s what burns [him], and so [he] like[s] to remember those moments...so when someone asks the same question again” he can stop and make sure that he understands what the client is asking so as to avoid making the same mistake.

Reflections during interviews.

During an interview while discussing the fact that Fred will sometimes ask clients “What’s going on here?” in order to get the client to talk about the problem and provide Fred with some insight as to what the client already knows about the problem Fred comments “to be honest, I wish I asked it more often because that’s probably one of the best ways to lead” into a tutoring session. He goes on to say that “when I do ask [this question] I’m like ‘Why don’t I ask that question more?’”

Another time during an interview while discussing the fact that Fred thinks it is important that he use definitions while tutoring his clients in order to help them become familiar with their meanings and applications, in response to a comment that before participating in this study no one has ever sat Fred down and asked him “Why are definitions so important to you?” Fred says that our conversations about his emphasis on definitions are “great because I don’t think I’d ask myself these questions” and later, during another conversation about him using definitions to explain how to find the domain of a function Fred has an epiphanic moment, exclaiming “I guess I really am a definition person [chuckles].”

Effects upon Fred's tutoring that Fred attributes to be the result of his participation in this study.

One change that Fred attributes to his participation in this project is that his level of involvement with the clients has "started to increase." He explains "I'm definitely more involved. I make sure that when they ask a question, I don't just say, 'Okay. I know what their question is.' I try and dig out what do they know first, a little more than I used to. Then when I'm done, then I try and get that validation more now, 'Okay. Do you have this?' And almost all of the time, I will suggest like, 'Okay, I want you to work on this now.' So certain things that I used to do, but it was just like I was a zombie, so to speak."

Fred also says,

I think fundamentally when you do something-- if it's even being in a relationship because it's kind of like a client, even though that the person changes, if I stay the same, there's a relationship there. When you have a relationship or in a task, especially both, when you do that over time you become placid. "This is boring." When you have someone come in, like, "All right, can't you see there's some problems here?" Which you didn't say, but when you pointed things or even got me to question-- bring about questions to myself, like, "Yeah, I could improve that."

Fred adds further that participating has help him to reflect more on his tutoring, saying that he is "definitely one of those individuals who reflects on most of the things in [his] life... but even provided that, there are plenty of things that [he] just didn't consider or could have considered and didn't." Moreover, Fred says that participating has "brought about a lot of [personal] constructive criticism...[such as] 'it's probably good that I'm

doing this. Maybe I should do more of it.' reinforcing the good and fixing things that could be worked on so to speak, like a tune-up."

Summary.

We see that Fred's tutoring is guided by his goal to help his clients be successful on their own through helping clients to make sense of abstract mathematical concepts via specific examples as well as trying to get his clients to do as much of the work as possible. Fred works to promote his clients' reasoning and problem solving abilities by posing questions as a means of guiding clients' thinking as they work through problems. Fred helps his clients to see connections among mathematical representations by utilizing visual representations to help some clients as well as highlighting that the particular variable used in an expression has no bearing on computations done on or with that expression. Fred works to position his clients as contributors to the conversation by endeavoring to have clients explain as much of a problem as they can. Fred's use of questions to help guide his clients through solving problems helps Fred to develop his clients' procedural fluency from their conceptual understanding by productively supporting his clients as they struggle through problems. Throughout his tutoring, Fred relies on information he gathers, primarily through questioning, about his clients' thinking in order to guide his tutoring sessions.

Case 3: Michelle

Michelle is an Applied Mathematics major in her senior year. Michelle is in her 5th semester working as a mathematics tutor at the Learning Center. During Michelle's first semester at the Learning Center she worked as a private tutor, then she became a lab tutor starting during her second semester, and now serves as the lead mathematics tutor.

The duties of the lead mathematics tutor, in addition to tutoring clients, include: observing mathematics tutors annually and offering feedback to help the tutor to improve his/her tutoring, serving as a model tutor for new tutors to observe as a means of aiding new tutors as they are learning to tutor, and planning and facilitating semesterly professional development workshops for mathematics tutors which focuses upon content that clients typically struggle with and is expected to start manifesting in the tutoring lab based upon the typically pacing of the university's mathematics courses. Mathematics courses Michelle has taken include Calculus 1, 2, and 3, Discrete Mathematics 1, Introduction to Probability and Statistics, Differential Equations, Introduction to Advanced Mathematics, Linear Algebra, and Analysis 1. Michelle has never taken any teaching or education courses nor has she had any teaching experiences before being employed by the Learning Center.

Teaching practice 1. Establish mathematics goals to focus learning.

Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Michelle strives to help her clients be able to succeed in their mathematics endeavors, saying during an interview, "I want everybody else to not necessarily like math, but [to] get through their class and be like 'Yeah, I totally aced that test.' I want everybody else to feel confident about" their ability to succeed in math. Michelle explains that she sometimes let's her clients work on their own while she watches over them so that she

"can validate what they're doing... especially when they're writing on their own or they're telling you what we think we should do...it's [her] trying to instill that

confidence in them so they can do it on their own, because they came to use for assistance and I want them to never have to come back to me.”

Michelle goes on to give an example, explaining that

“normally when students come in with exam reviews... [she] will just hit the high points with them... like for example, the four step process [for] the difference quotient [she] will normally [say] ‘Here are the steps, here’s what your product should be, work it out on your own and I’ll come back.’”

Another way that Michelle says she endeavors to help students to be successful on their own is during the times when a client is “trying to pull out their calculators and they don’t need to” so she will tell them “You don’t need your calculator to solve this.” She goes on to explain that she “remind[s] them ‘You’re not allowed that on your test, so why are you using it now?’” and that even though the client came to “the tutoring lab...[she is] trying to prepare [the client] for what’s coming, so [she] want[s] [the client] to have those skills off the bat and not use the crutch anymore.”

While tutoring, Michelle wants the focus of the session to be upon the present content material of the course her client is working in. She explains, during an interview, that she tends to skip over the algebra

“with the upper level [clients], [but] obviously not at the algebra table. [However] if [an upper level client] stops [her] and they say ‘How did you do that?’ [she does] not have a problem going back, but [she does] make a point to say ‘Oh you learned this earlier. This should be review. If it’s not then let me grab you this worksheet we have about it’ and that...is the point when [she] start[s] telling them

instead of asking questions because it should be a review and then [she] will just try to review them quickly so [the client] can master those harder concepts.”

Identifying how the goals fit within a mathematics learning progression. There were no instances in which Michelle indicated that she identified how her goals fit within a learning progression.

Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. There are no examples from Michelle’s observations in which she explicitly discusses the goals of her tutoring with her client.

Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction. While discussing during an interview the fact that Michelle will sometimes ask clients what they think they should do and asks them questions about what should happen next, while other times she will tell the client is supposed to happen next, Michelle explains that “usually, if it’s like a ‘How do we start this problem?’ [she] will definitely ask them [questions] because most of the time they have a better idea of that than [her] at the moment.” Michelle adds that she “tend[s] to [tell clients what to do] mostly with just the algebra part of [the problem]... in the upper-level math where they should have a grasp on it.” In an observation, there is a time when Michelle is working with a calculus client and at some point the client needs to find the common denominator between some fractions. Michelle explains that “finding a common denominator... sounded like it wasn’t terrible, but something he just need a refresher on. [Also], I distinctly remember at that moment, there were more flags going up, and so I needed to speed up a little, but I don’t think I lost him on it...because he had a grasp on

way harder stuff and so I figured if it was just simplifying, we could handle it.” Later, Michelle explains that she knows that “not all of the time, but some professors are not judging your arithmetic. They’re judging what they’re teaching you: calculus, the algebra, whatever.”

Teaching practice 2. Implement tasks that promote reasoning and problem solving.

Motivating students’ learning of mathematics through opportunities for exploring and solving problems that build on their current mathematical understanding. Michelle tries to help calculus clients to utilize their knowledge of basic derivatives to help them to do more complicated ones. During an interview, Michelle says that when she is helping clients with calculus “a lot of times they’ll be like, ‘I don’t know how to do this [derivative] problem.’ So she will write ‘I don’t know the derivative of this.’ [Then she] will write an easier derivative and [she] will [say] ‘You know this one. Why is it different when it’s just a different number?’ and then they [will say] ‘Oh. I just hit a wall because it didn’t look familiar.’”

While working with a client who is struggling to multiply x by $x^{1/2}$, Michelle responds by asking him “What would you do if I gave you x^2 times x^3 ?” During the interview, Michelle explains that she chose to ask the client this question “because [she] know[s] he can do that. [She] always go[es] back to something that [she] know[s] they’ve seen before...[so] then they make the connect that ‘I’m just adding these numbers’ or whatever.”

During an observation a client is not following how Michelle simplified $2x^2/x$, so Michelle elects to rewrite the expression as $2xx/x$. During the interview, Michelle

explains that this “is something new [she has] started doing” and in this case the client “has just shown [her] he could cancel stuff with factoring and so [she] knew he’d be able to see it if it was all multiplying.” By reflecting on her prior experiences, Michelle has found that she can better aid her clients in understanding how and when to apply their knowledge by connecting a seemingly unfamiliar task with one that Michelle knows the client knows how to do, because this allows Michelle to highlight the similarities between what the client is comfortable with and the present task.

Selecting tasks that provide multiple entry points through the use of varied tools and representations. There is no evidence of Michelle selecting tasks that would allow a client to use one of several approaches to solving it.

Posing tasks on a regular basis that require a high level of cognitive demand. There is no evidence of Michelle regularly posing tasks the clients, high-demand or otherwise.

Supporting students in exploring tasks without taking over student thinking. One way that Michelle endeavors to support her clients in their problem solving without taking control of the situation is to utilize a different example that is simpler but still requires the application of the same knowledge and skills. This is evinced in a previously mentioned example of when Michelle is working with a client who is struggling to multiply x by $x^{1/2}$, Michelle responds by asking him “What would you do if I gave you x^2 times x^3 ?” During the interview, Michelle explains that she chose to ask the client this question “because [she] know[s] he can do that. [She] always go[es] back to something that [she] know[s] they’ve seen before...[so] then they make the connect that ‘I’m just adding these numbers’ or whatever.” Another example, which has also been discussed

prior to this, occurs during an observation when a client does not follow Michelle simplified $2x^2/x$, so Michelle elects to rewrite the expression as $2xx/x$. During the interview, Michelle explains that this “is something new [she has] started doing” and in this case the client “has just shown [her] he could cancel stuff with factoring and so [she] knew he’d be able to see it if it was all multiplying.”

Another way Michelle attempts to support her clients without taking over the problem solving is by working out an example for her client and then asking the client to try to solve the next part of the problem on their own. For example during an observation while Michelle is working with a client to determine the horizontal asymptote of a function by dividing each term of the numerator and denominator by the highest degree exponent term from the denominator, Michelle works out what the numerator will look like after this division, then asks the client to tell her what the denominator should look like. During the interview, Michelle says that she did this because she “know[s] that [she] work[s] better when [she has] an example” and by giving the client an example to follow she hoped he would “be easily able to tell [her] what [the denominator] is supposed to be.” She further explains that this is a way she tries “to keep them involved and not [try] to take over the whole thing” herself. Here, we see that Michelle has reflected on her own experiences as a learner and because she is better able to see how to solve problems when she has an example to guide her thinking, she elects to provide similar support to her clients.

Michelle, however, does not always attempt to guide her clients through problems. There are times when Michelle elects to just tell clients what to do in order to solve a problem and her decision to tell clients is based on her experience working with

individual clients or particular groups of clients, as well as her own comfort with the material in question. For example, during an observation in which Michelle is helping Business Calculus client who is working on a question asking about where a function is continuous, the first thing Michelle does it tell the client that this “is just a fancy way of asking ‘When do we divide by zero?’.” During the interview, Michelle explains that she chose to do this because “not everybody, but most of the time those business kids need the ‘What exactly am I trying to find?’ so by telling them this keyword means this, they’ll remember that.” Michelle goes on to say that another reason she will sometimes opt to just tell a client something is when she does “not have a super strong hold on it [her]self. [When] it’s just one of those things that [she has] accepted about math” and as an example she says that when people ask about why dividing by zero leads to vertical asymptotes in a graph “what [she] end[s] up telling people is that when you divide it by zero, it’s an infinity idea and so that’s why it keeps going either way.” Michelle explains that in these situations she is “less confident...and [she tries] not to show that” and will either “try to grab [another tutor] before [she] can make a fool of [her]self” or she will say “‘Can you just trust me on this one?’” and more often than not, her client will just trust her.

Encouraging students to use varied approaches and strategies to make sense of and solve tasks. There are no examples of Michelle encouraging a client to implement multiple strategies to assist in understanding or solving a problem.

Teaching practice 3. Use and connect mathematical representations.

Selecting tasks that allow students to decide which representations to use in making sense of the problems. There are no examples from Michelle’s observations in

which she selects a task for a client with the purpose of allowing the client to choose a representation to use to aid in his/her sense making.

Allocating substantial instructional time for students to use, discuss, and make connections among representations. While there are no examples of Michelle taking time during a tutoring session to discuss the connections among different representations with her clients, during an interview Michelle says that “math for [her] in [her] head is a bunch of building blocks...[that are] all connected” and discovering connections for herself “is really exciting [because] then [she] get[s] to show [her] students... these connections too and hopefully make those connections last for them.”

Introducing forms of representations that can be useful to students. There are no instances during Michelle’s observations in which Michelle introduces a representational form to a client.

Asking students to make math drawings or use other visual supports to explain and justify their reasoning. There are no examples from Michelle’s observations of Michelle asking a client to produce a visual representation to aid the client in explaining their reasoning.

Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. While tutoring her clients, Michelle will attempt to help clients struggling with a problem by giving them an example that is similar in structure to their present problem and requires the application of similar skills, but is less complicated looking, in order to draw clients’ attention to how to use their ability to solve the simpler example to solve the present problem. For example, during an interview Michelle discusses that when she is helping a calculus

client who is struggling to find the derivative of a function, she “will write briefly, ‘I don’t know the derivative of this.’ [Then she] will write an easier derivative and [she] will [say] ‘You know this one. Why is it different when it’s just a different number?’ and then [the client will say] ‘Oh. I just hit a wall because it didn’t look familiar.’” An example from Michelle’s observations occurs while she is working with a client who is struggling to multiply x by $x^{1/2}$, Michelle responds by asking him “What would you do if I gave you x^2 times x^3 ?” During the interview, Michelle explains that she chose to ask the client this question “because [she] know[s] he can do that. [She] always go[es] back to something that [she] know[s] they’ve seen before...[so] then they make the connection that ‘I’m just adding these numbers’ or whatever.” A second example from Michelle’s observation occurs when a client does not follow how Michelle simplified the expression $2x^2/x$, Michelle decides to rewrite it as $2xx/x$ and explains, during the interview, that this “is something new [she has] started doing” and in this case the client “has just shown [her] he could cancel stuff with factoring and so [she] knew he’d be able to see it if it was all multiplying.”

In addition to discussing related examples, Michelle will also try to draw clients’ attention to key features of solving a problem that are implied by the vocabulary used within the problem. For example, while helping a Business Calculus client working on a question about deciding where a function is continuous, the first thing Michelle does is tell him that “This is a fancy way of asking when are we discontinuous...just a fancy way of asking when do we divide by zero, most of the time.” Michelle explains during the interview that this “is Business Calculus where...[and] those kids need the direct

answer...[they] need the ‘What exactly am I trying to find?’ so by telling them this keyword means this, they’ll remember that.”

Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems. There are no examples from Michelle’s observations of her endeavoring to assess her clients’ abilities to use representations to aid their problem solving.

Teaching practice 4. Facilitate meaningful mathematical discourse.

Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches using varied representations. During an interview discussing a time when the client starts to talk then stops because she wants Michelle to start explaining and Michelle responds with “No, no, you need to keep talking”, Michelle explains that she tries “to make [her] kids keep talking and say whatever they think, that way [she] can gauge what knowledge they have at the moment and where [she] can go from there: if [she] need[s] to correct them or [if it’s] like ‘You’ve already got it, you don’t need me.’” Michelle’s experiences have enabled her to see that she can better aid her clients if she encourages them to explain, as much as possible, what they are thinking about the problem at hand because she is better able to determine what, if any, part of the problem they are struggling with as well as what related knowledge the clients already understand.

During another observation in which Michelle is working with a client to find the vertical asymptotes of a graph, after Michelle has finished helping the client to factor the denominator of the function being examined, Michelle turns to the client and asks “‘What were we supposed to be doing?’” During the interview, Michelle explains that she does this “all the time [and] it’s to remind them, ‘Okay, we got on this little side path of

factoring [and now] we need to get back to what we were trying to do at first...so it's [her way of] making sure [they] knew what [they] were doing and that [they] weren't lost in the factoring world."

Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. During the single instance from Michelle's observation in which she is working with multiple clients who are all working on the same Business Calculus exam review, Michelle does not engage the clients in a discussion of the different solution strategies used by each client and none of the clients actually mention the approach he/she chose to use.

Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. During the observation in which Michelle is working with approximately six Business Calculus students who are working on applying L'Hospital's rule to determine a limit, Michelle is primarily engaged with the one client who initially asked the question, and there are no instances during this tutoring session in which she asks a client to explain him/herself to another client. The only time during this session that Michelle explicitly tries to engage the whole group of clients is when $\ln(1)$ shows up in the work and Michelle asks "What's $\ln(1)$? Anybody?" but no one answers and Michelle ends up telling them that it is zero.

Although there is only the one example of Michelle working with multiple students at a single time, she does offer several comments during her interviews that speak to her endeavors to position students as leaders in the conversation. During one interview, Michelle explains that she likes to use collective pronouns while talking with her clients "because then we're a team and we're trying to get through this together."

Michelle goes on to say that she “make[s] it a point to [say] ‘Oh, we got this wrong here. Oh, we missed a step.’ But then when [the client] gets it right it’s definitely ‘You did it right!’ so collectively we’re working on it, but you finished it...That way they feel like they’re in control.” Michelle goes on to say that she likes to ask her clients questions as a means of getting them involved in the tutoring session

“because [she does] not want it to become just another lecture for them. Because [she] feel[s] like in lecture the professor’s in control. The students can ask questions, but the professor dictates where it’s going to go. In a tutoring session it’s about what the student needs, and [she] want[s] them to know that if they have a problem, if [they] need to sidetrack somewhere, then [they] can go there... [She] want[s] [the clients] to know they can ask questions and that they dictate what knowledge [Michelle] get[s] to share.”

Michelle further comments that while she is tutoring she strives to be “consciously... aware that [she is] not a lecturer, [she is] interacting and most of the time [she] tend[s] to look at the student and if they give [her] those looks like ‘Oh I know.’ Then [she] would [say] ‘What do you think?’ or if they give [her] the look that they don’t know, [she is] still going to ask because that’s going to make them stop and realize that [she is] not lecturing them.”

Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning. Michelle provides no evidence during her observations or interviews of attempting to connect clients’ approaches and reasoning to instructional goals.

Teaching practice 5. Pose purposeful questions.

Advancing student understanding by asking questions that build on, but do not take over or funnel student thinking. Michelle strives to help her clients understand how to solve problems by giving them examples that require the same skills and understandings, but are simpler in appearance, in order to emphasize to the client that they have the requisite knowledge and ability, but were put-off by the unfamiliarity of the present task. For example, Michelle explains during an interview that when a client says they don't know how to take the derivative of a function, she "will write an easier [function] and [she] will [say] 'You know this one. Why is it different when it's just a different number?'" and then [the client says] 'Oh, I just hit a wall because it didn't look familiar.'" Furthermore, during an observation in which a client is working on multiplying two terms together, one with x to a fractional power, the client is unsure of how to combine the exponents in the product of the terms. Michelle decides to ask him "What would you do if I gave you x^2 times x^3 ?" During the interview Michelle explains that she elected to ask about this other example "because [she] know[s] he can do that. [She] always go[es] back to something that [she] know[s] they've seen before, and [the present problem] is just a little bit more complicated... so if they can tell me that then [she] will [say] 'Okay, let's apply what you just did to this'" [so] then they make the connection that 'I'm just adding these numbers.'" Another example is when Michelle is working with a client who does not see how Michelle simplified $2x^2/x$, Michelle decides to rewrite the expression as $2xx/x$ because the client had "just shown [her] that he could cancel stuff with factoring and so [she] knew he'd be able to see it if it was all multiplying."

Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Michelle explains that she likes to start tutoring sessions by asking her clients “What’s our first thought?” because she “want[s] to go with what [the client] thinks and then maybe alter it if [she does] not think it’s right.” Michelle goes on to explain that throughout the tutoring session, “if [a client] give[s] [her] those looks like “Oh I know.” Then [Michelle] will [say] ‘What do you think?’ or [even] if they give [her] the look that they don’t know, [she is] still going to ask because that’s going to make them stop and realize [she is] not lecturing them.”

An example in which Michelle probes the thoughts of her client occurs while working with a client on a problem that is asking him to find where a function is continuous and the client has already done some work and found the x -value that makes the numerator zero, Michelle tells him, “Continuous is the same this as asking ‘When do I divide by zero, most of the time.’ So how did you figure out...why did you say three?” while indicating the value he found that makes the numerator zero. The client responds “you can’t do three minus three [divided] into five.” To which Michelle replies “Oh yes you can. I can have zero on top.” During the interview Michelle explains that she pursued this line of questioning “because [she] saw that he was on the right track because he knew zero couldn’t be somewhere...so it was [her] trying to get him to say ‘Oh, it has to be on the bottom.’ He had the right idea, just in the wrong spot.” She “wanted him to know that he did something almost right.”

Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. During an observation when

Michelle is helping a client who is working to find the vertical asymptotes for a graph which leads Michelle and the client to spend a time working through how to factor the denominator of the function, after they finish factoring Michelle asks the client “What were we supposed to be doing?” During the interview Michelle explains that this is to help “remind them ‘Okay, we got on this little side path of factoring [and now] we need to get back to what we were trying to do at first.’” This is her way of “making sure [the client] knew what [he was] doing and that [he was] not lost in the factoring world,” by highlighting that this side work of factoring is just a small piece of the larger process of solving this problem.

Allowing sufficient wait time so that more students can formulate and offer responses. There are no instances in which Michelle poses a question to a client and does give the client lots of time to think and respond before Michelle attempts to rephrase the question or start trying to guide the client. Additionally, during an observation in which Michelle is working with a client that is in Discrete Mathematics and is working on a proof, there is a noticeable increase in the amount of time that Michelle is waiting to attempt to rephrase her questions or further scaffold the client’s thinking. During the interview, Michelle explains that there are two reasons for this. First, she says there is “a lot of silence between these questions because [she] was trying to remember what [she] was supposed to do...because [she] remembers being very scared when he walked in [and telling him] ‘I’m not very good at proofs, but we’ll get through it.’” The second reason is because the student is in a proofs course “it’s fine if [clients] need to think about it... [because] the logic behind it should be [the focus] at [this] point.” Throughout this observation, whenever Michelle would step in and help guide the client through a tough

spot, such as reminding him about what intersection of sets means, Michelle would guide just long enough until he got “little traction under his feet” and then she would switch back to asking him questions and letting him lead the discussion. Until it reached a point when the client could no longer figure out how to progress, at which point Michelle lead him through the remaining steps and started to ask questions like “Do you understand this?” “Are you okay with this? Do I need to show a different example?” Michelle explains that she did this “because [she] knew, judging by the beginning of [this session] that it was hard to grasp for him and so [she] didn’t want to keep going if he didn’t feel solid on [it].”

Teaching practice 6. Build procedural fluency from conceptual understanding.

Providing students with opportunities to use their own reasoning strategies and methods for solving problems. During an observation in which Michelle is tutoring a calculus working on the derivative of a rational function, the client wants to rewrite the function as a product of the numerator with the denominator to the power -1, Michelle tells him “You’re not wrong” and does not suggest that the client use the quotient rule until after the client asks her “What’s easier?” Although the client ends up following Michelle’s suggestion and uses the quotient rule to finish out the problem, Michelle explains during the interview that she didn’t want to tell the client he was wrong “because he wasn’t.”

Asking students to discuss and explain why the procedures they are using work to solve particular problems. Although there is not an example of a time when Michelle asks a client to explain why the procedure he/she is using is appropriate for solving a

particular problem, there is an instance in which Michelle asks a client to explain the reasoning he used to arrive at an answer that Michelle knows is incorrect in an attempt to have the client remind himself of the correct procedure. While assisting a client that is reviewing a quiz problem asking him to determine where a function is continuous and the client already has some work written down in which he has found the x -value that makes the numerator zero, Michelle ask his “How did you figure out...why did you say 3?” and so the client explains “because you can’t do 3-3 [divided] into 5” to which Michelle replies “Yes I can. I can zero on top” Then the client says “You can’t have it on bottom.” During the interview, Michelle explains that she “saw [that] he was on the right track because he knew zero couldn’t be somewhere...so [this] was [her] trying to get him to say ‘Oh, it has to be on the bottom.’ He had the right idea, just in the wrong spot.”

Connecting student-generated strategies and methods to more efficient procedures as appropriate. While working with a calculus client who is taking the derivative of a rational function and the client wants to rewrite the function and use the product rule, Michelle tells him “You’re not wrong” which prompts the client to ask “What’s easier?” at which point Michelle suggests using the quotient rule. During the interview, Michelle explains that she didn’t advocate for him to use the quotient rule from the start “because sometimes [she has] seen it where it’s easier to use the product rule” and she didn’t want to tell the client he was wrong “because he wasn’t, [she] just wanted him to see that sometimes the path that looks harder isn’t necessarily.”

Another example occurs while she is working with the client who has incorrectly factored the quadratic $2x^2+5x-12$ as $(2x+3)(x-4)$, after Michelle walks him through checking his factoring and shows that the client’s factors lead to a $-5x$ instead of the $+5x$

they started with, the client wants to start over and try to factor again. Michelle tells him that “this means the numbers are right, but my signs are wrong” and suggests they just switch the signs on the 3 and 4. During the interview Michelle explains that “was [trying to] save him [some] time because he was right, almost... so [she] wanted him to know [that he] knew the right numbers, it’s just [the] signs were wrong.”

Using visual models to support students’ understanding of general methods.

The only example from Michelle’s observations in which she makes use of a visual model to aid in her explanation of a solution occurs while working with a group of Business Calculus clients who are taking the limit of $\ln(x)$ as x approaches 0, in order to help explain that this limit is negative infinity, Michelle sketches a graph of $\ln(x)$ and indicates the region of the graph that represents the values of $\ln(x)$ as x gets closer and closer to zero.

Providing students with opportunities for distributed practice of procedures.

Michelle explains that “normally when students come in with exam reviews, they tend to want to rush through it [so she] will try to just hit the high points with them...like, for example, the four step process [for] the difference quotient [she] will normally [say] ‘Here are the steps, here’s what your [answer] should be, work it out on your own and I’ll come back [and check your work].’”

While working with a calculus client who is taking the derivative of a rational function and initially wants to use the product rule, but after asking Michelle “What’s easier?” decides to follow her advice and apply the quotient rule. Michelle proceeds to allow the client to work on his own while she watches over him. During the interview, Michelle explains that she does this “so [she] can validate what [her clients] are doing.”

She says that she “tend[s] to... [do this] especially when they’re writing on their own or they’re telling [het] what [they] think [they] should do [because she] is trying to instill that confidence in them so they can do it on their own.”

While working with a client to find the horizontal asymptotes of a function by evaluating the limit as x approaches infinity, Michelle first shows the client how to divide each term in the numerator by the highest degree exponent term from the denominator, and then asks him to work out what the denominator is supposed to be. During the interview Michelle explains that she does this because she “know[s] that [she (Michelle)] work[s] better when [she] has an example in front of [her]” and that she is “trying to keep him involved [in the problem] and not trying to take over the whole thing.”

Teaching practice 7. Support productive struggle in learning mathematics.

Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. There is no evidence from Michelle’s interviews that indicates that she attempts to anticipate what her clients will struggle with so that she can be prepared to support them productively as they struggle to solve a problem.

Giving students time to struggle with tasks, and asking questions that scaffold students’ thinking without stepping in to do the work for them. One way in which Michelle attempts to allow her clients to struggle with a task that she discusses is that “normally when students come in with exam reviews... [she] will try to just hit the high points with them” by reminding them of the steps they need to take then telling the client “work it out on your own and I’ll come back.”

An example of Michelle allowing a client to work on his own without Michelle intervening occurs when she is working with a client who wants to take the derivative of a rational function by rewriting it and using the product rule, but then, after asking for Michelle's suggestion on "What's easier?" decides to employ the quotient rule, Michelle watches over him while he works it out himself. Michelle explains during the interview that she does this with clients so she "can validate what they're doing" by telling them when they are doing parts correctly, "especially when they're writing on their own or they're telling you what [they] think we should do... it's trying to instill that confidence in them so they can do it on they're own."

Another way that Michelle works to allow clients to struggle through making sense of a problem on their own is by asking them how to solve a related example that requires a similar skill set to solve. For example, while working with a client that cannot recall how to multiply two terms that each have x raised to an exponent, one of which is a fractional exponent, Michelle elects to ask him "What would you do if I gave you x^2 times x^3 ?" During the interview, Michelle explains that she chose to ask the client this question "because [she] know[s] he can do that. [She] always go[es] back to something that [she] know[s] they've seen before...[so] then they make the connect that 'I'm just adding these numbers' or whatever."

Moreover, Michelle comments during an interview says that whenever she is finished helping a client "if [she is] worried about them not being able to do it without [her], [she] will pick a problem out of their book [or], depending on the subject [she] can make one up... and [she] will say 'You work on this, I'm going to go answer another question and I will come back and check and see how you did.'"

Helping students realize that confusion and errors are a natural part of learning by facilitating discussions on mistakes, misconceptions, and struggles.

Michelle discusses during an interview that “a lot of times, especially with math students, it’s always like they’re onstage, especially when they’re talking to us, [they are] like ‘I don’t want to be wrong.’ Nobody wants to be wrong, and so I try to make my kids keep talking and say whatever they think. That way I can gauge what knowledge they have at the moment and where I can go from there [and to see] if I need to correct them or I’m like ‘You’ve already got it, you don’t need me.’”

When asked during an interview about what she does when a client is resistant to talking Michelle says “usually I’ll start [by saying], ‘Okay, what are you thinking? It’s okay to be wrong. I’m wrong all the time.’... And then I can say ‘Remember when I did this wrong?’” Michelle goes on to say that she will sometimes tell clients “‘We can be wrong together’ [to] try to make them feel okay about being wrong in this space, that it’s okay. It’s like ‘We’re going to be wrong here, we’ll get it all out here and the we can be right on the test.’ So it’s just trying to make them feel safer and comfortable at [the Learning Center] and learning.”

Michelle goes on to explain explains that she likes to use collective pronouns with her clients because “we’re a team and we’re trying to get through this together...[and] I make it a point to [say] ‘Oh, we got this wrong here. Oh, we missed a step.’ But then when they get it right it’s definitely ‘You did it!’ So collectively, we’re working on it, but you finished it. It takes off the pressure, I think.”

Finally, while discussing during an interview that Michelle has gotten more comfortable with being wrong in front of her clients and with not knowing how to answer

every question, Michelle comments that working at the Learning Center has “definitely allowed me to be able to ask questions in class, for sure. That was a big, big problem for me. It’s about the whole fear of, “Does anybody else have this question?’ But now I know...it’s okay if nobody else did, at least I got what I needed... And that’s something I try to tell the kids, ‘Ask your professor during class. It’s okay. You’re not the only person with this question...and if everybody is like, ‘Oh, I got this,’ [they can say] ‘Well, it stumped one of the tutors at [the Learning Center], so it’s okay. At least two of us had this question.’”

Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. While wrapping up a tutoring session with a Calculus III client during which the client’s struggles were with concepts from algebra (i.e. multiplying terms together each with x raised to powers), Michelle tells the client “The good news is you’re doing the Cal III right.” During the interview, Michelle explains that she wanted the client to know “You’re doing the harder parts right, so don’t get so down on yourself because you’re doing the baby steps wrong. It’s been a while since you’ve done those. It’s okay that you don’t remember” because, in her experience “some professors are not judging your arithmetic, they’re judging what they’re teaching you.” During another observation while working with a client who is trying to find where a rational function is continuous, but ended up finding the x -value that makes the numerator, rather than the denominator, zero, Michelle asks him “Why did you say 3?” which helps lead the client to realizing that he needs to find the x -value that makes the denominator zero. During the interview, Michelle explains that she didn’t want to just tell him he was wrong “because [she] saw he was on the right track because he knew zero

couldn't be somewhere... [she] wanted him to know he did something almost right and then [realize] 'Oh, but we needed to do this on the other side.'" During an interview, Michelle says "I try to be really happy and excited that they got it right. That's a big deal!"

Teaching practice 8. Elicit and use evidence of student thinking.

Identifying what counts as evidence of student progress toward mathematics learning goals. Michelle make use of body language as a means of helping her to gauge if her clients are understanding her explanations, and has even realized that some body language that might appear to indicate client understanding, doesn't actually indicate anything useful or sometimes even the precise opposite. Michelle comments during an interview that clients "give [her] a lot of non-verbal cues," and she particularly mentions that clients will nod their heads "which [she has determined] doesn't mean anything...because [she] nod[s] her head all the time... in [her Analysis] class and [she thinks] 'Yeah, I totally get what you're saying at this moment, but if you're about to ask me what you just said, I couldn't repeat it back.'"

Eliciting and gathering evidence of student understanding at strategic points during instruction. Michelle discussed during an interview that whenever she "look[s] at the student...if they give [her] those looks like, 'Oh I know,' then [she] would [ask] 'What do you think?'" Or if they give [her] the look that they don't know, [she is] still going to ask because that is going to make them stop and relies [she is] not lecturing them." There is an instance during an observation in which Michelle is tutoring a client who is working on a Linear Algebra problem asking about the domain and codomain of a transformation. The client begins to start speaking then quickly stops and seems to want

Michelle to talk about the problem, which prompts Michelle to say “No, no, you need to keep talking.” During the interview Michelle explains that she tries “to make [her] kids keep talking and say whatever they think, that way [she] can gauge what knowledge they have at the moment and where [she] can go from there.”

While working with a Business Calculus client who is working to find when a rational function is continuous, after Michelle helps the client to realize that he needs to find the values that make the denominator equal zero, she makes the client factor the denominator and solve for the roots. During the interview, Michelle explains that she chose to have him do the work because her “first step is to always try and get them to do it themselves, and [this client] had other work on his page show he knew how to do it, and so [she] wanted him to show [her] he could do it and that he wasn’t just copying from somebody else. And then if he couldn’t [do it], [she] would have done up to the board and...show[n] him.”

Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Michelle explains during an interview that “students this semester are very verbal with me and so they will automatically tell me if they don’t get it or if they do... but if they’re not telling me anything, that’s usually a sign they don’t understand.” She also says that “whenever I ask them questions and they just stare at me and I end up having to breakdown the question further” is another indication that a client does not understand. While discussing Michelle’s deduction that clients nodding their head while she is explaining things “doesn’t [actually] mean anything,” Michelle explains that when this happens she has “to rephrase questions [to] make sure that [she is] still interacting

with them and that they're understanding what [she is] saying without actually asking 'Do you understand?'"

Michelle also uses the level of content of her client's course work as well as evidence of faculty with more advanced skills and procedures to aid her in deciding what material her client presently understands. While discussing during an interview how Michelle chooses between asking clients what they are thinking and what step should come next or just telling the client what to do next, Michelle explains that she tends to just tell clients things "mostly with just the algebra part of [the problem] and [when the client is] in [an] upper-level math where they should have a grasp on it." For example, she mentions a time when she is working with a calculus client that needs to add two fractions: "finding a common denominator, that sounded like it wasn't terrible, but something he just needed a refresher on...because he had a grasp on way harder stuff and so I figured it was just simplifying, we could handle it."

Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. During an interview, Michelle says that her experience during her semester working as a private tutor "help[s] [her] pick up on what people need." She explains that her "first three clients [were] all completely different: one student was very much there to learn [and] needed [her], one of them was like 'You're just here to check my work.', and the other one totally just didn't want to be there at all but knew [he/she] needed [Michelle]." Michelle goes on to discuss that this helps her when she interacts with a client for the first time, commenting that "it's almost like autopilot, 'Okay this is the slacker student and I need to get on to them immediately' [or] 'This is the one student that needs to work hard.'" This helps Michelle to "go into the

mode [she] need[s] to [in order] to make sure that they're working." In Michelle's experience "students [are] all one of [these] three types, with little nuances in between: the slacker will respond to [her] getting on to them... the good students definitely need encouragement, and the in-betweens you take care of."

Michelle also explains during an interview that she knows that she is "a visual and a physical learner...[and that she has] to write down what [people] are saying or" she will forget it. Michelle says that knowing this about herself helps with her tutoring because

"that's why [she] use[s] the board all the time, and that's why [she tries] to make [the clients] write while [she is] writing. Because [she] know[s] that if [she] can engage them in two of the three [learning styles], then [she is] getting through to at least one of them... and if not, then most of the time the kids that need something [else] will [say], 'Hold on, I need to write this down.', 'Hold on, can you say that again?', [or] 'Hold on, can I come up to the board with you?'"

Michelle works to scaffold her client's thinking by asking them about related but simpler examples that require the same skills as a task they are presently struggling with. For example, Michelle discusses during an interview that whenever a calculus client doesn't know how to take the derivative of a function she "will write an easier derivative and [she] will [say], 'You know this one. Why is it different when it's just a different number?' and then [the client says], 'Oh, I just hit a wall because it didn't look familiar.'" Another example occurs while Michelle is working with a calculus client that is struggling to multiply two terms that each have x raised to an exponent, one of which is a fractional power, Michelle decides to ask him "What would you do if I gave you x^2 times x^3 ?" which the client was able to correctly answer, allowing Michelle to highlight that he

is “just adding these numbers.” Michelle explains during the interview that she “always go[es] back to something that [she] know[s] they’ve seen before, and ours is just a little bit more complicated...[and] then [she] will [say], ‘Okay, let’s apply what you just did to this.’” A third example occurs when Michelle’s client does not follow how Michelle simplifies the expression $2x^2/x$, so Michelle decides to rewrite it as $2xx/x$, and explains, during the interview, that she does this “because he’d just shown [her] he could cancel stuff with factoring, and so [she] knew he’d be able to see it if it was all multiplying.”

Michelle also tries to probe clients thinking in order to help them to realize for themselves that they have made a mistake in a process. For example, during an observation in which Michelle is working with a Business Calculus client who is trying to determine when a rational function is continuous, and has already done some work to determine the x -value that will make the numerator of the function equal zero, Michelle asks the client “How did you figure out...why did you say 3?” to which the client replies “because you can’t do 3-3 [divided] into 5” and Michelle responds “Yes I can. I can have zero on top,” which leads the client to realize that he needed to find when the denominator is zero. During the interview, Michelle explains that she chose to ask him about how he got his answer, rather than just tell him that he needed to find when the denominator is zero “because [she] wanted him to know that he did something almost right and then [say himself] ‘Oh, but we needed to do this on the other side.’”

Reflecting on evidence of student learning to inform the planning of next instructional steps. Michelle discusses during an interview that sometimes she decides to tell clients what the next step is, rather than asking them what they think “because asking [them] a lot of questions... doesn’t always work” because she can tell, using body

language, that “sometimes they get overwhelmed with too many questions. They feel like they’re on the spot, and sometimes it’s better and easier to just [say], ‘Okay, this is what we do next,’... sometimes [it] works [but] it doesn’t always.” Michelle goes on to explain that she knows that deciding to just tell a client the next step didn’t work or wasn’t helpful to the client “if they come back and they don’t know the next step after that. Usually, [she thinks] ‘I should have asked [the client] what we do next. I should have laid out some groundwork.’” Which leads her to either “start [the problem] over and [say] ‘Okay, let’s go step-by-step and have you tell me what we need to do,’ or [she] will [say], ‘Okay let’s finish this one with a lot more help, and [then] we’ll work on [this skill] again.’”

Michelle also explains during an interview that “if I am worried about [the clients] not being to do [the problem] without me, I will pick a problem out of their book...[or] make up up...and I’ll say, ‘You work on this, I’m going to go answer another question and I will come back and check and see how you did.’”

Reflections during interviews.

While discussing the observation in which Michelle is tutoring a Linear Algebra client working on a problem about the domain and codomain of a transformation and the fact that when the client stops and wants Michelle to talk, Michelle replies ‘No, no, you need to keep talking.’ Michelle explains that when she was still new to tutoring, she would have “probably just... let her speak and then just continue with what I thought she was going to say, [telling her] the correct information.” Michelle goes on to say “I’m sure she would have gotten something out of it, but I think by her speaking she gets more, or feels more confident...because that reinforces that she knows what she’s talking about.”

Michelle also comments during an interview that after she was observed by “one of the supervisors” Michelle was told that she is “very good, but [she] tend[s] to lecture” so Michelle is aware that she “really need[s] to get out of that.” However, Michelle says there are still times when “I just start talking...mostly because I’m on a roll [and] I have to stop myself and remember that I’m trying to show somebody how to do this on their own.”

Effects upon Michelle’s tutoring that Michelle attributes to be the result of her participation in this study.

Michelle says that participating in this project has “definitely made me think about why I’m saying what I am, in terms of just talking to a client, trying to establish a rapport, and then with the content as well, [thinking] ‘Why am I saying this? Is this going to help them?’ ...in the moment... And then, I reflect on them. It’s like, ‘That really worked for this person.’ I think about it a lot more than I used to...[participating in this study] made me think about what type of tutoring it was.”

Michelle goes on to say that “in our first interview, I realized that I really wanted my students to be confident, and so I’ve been trying to pursue that in all of my endeavors” and commenting that this aspect of her tutoring was not something that she was consciously aware of before participating in the study.

Michelle also says that participating in this study has resulted in her “considering joining [a] PhD program” in mathematics education.

Summary.

Michelle's tutoring is guided by her goal of helping her clients to be confident in their own mathematical abilities by focusing in on the general processes involved in solving problems. Michelle strives to help her clients develop their reasoning and problem solving abilities as well as their procedural knowledge by often explaining how to solve one problem through the examination of a similar but simpler example and highlighting the connections between the problem at hand and the simpler example. Throughout Michelle's tutoring, she is constantly positioning her clients as leaders in the conversation by asking clients what they think should be done next and by encouraging them to talk as much as possible in order to help Michelle to better understand what her clients know and to use this knowledge to guide her tutoring decisions. This positioning is also Michelle's way productively supporting her clients as they struggle through problems because getting the clients to speak their minds helps her to determine how to best scaffold her clients' thinking.

V. DISCUSSION AND IMPLICATIONS FOR TEACHER EDUCATION

This was a qualitative embedded case study examining the practices and rationale exhibited by mathematics tutors working in the Learning Center. Participants of this study were three mathematics tutors with differing amounts of experience working at the Learning Center (i.e. Ralph is in his first semester, Fred in his fourth, and Michelle in her fifth). Two research questions guided this study:

- 1) What are the teaching practices exhibited by the participants and what is the rationale for these practices?
- 2) What effects, if any, upon the participants' tutoring do the participants attribute to their participation in this study?

To answer these questions I conducted three one-hour observations of each participant's tutoring in the Learning Center lab, and conducted three interviews during the Spring of 2016, lasting between one and two and a half hours each, during which I asked the participants to discuss and explain why they chose to do the things they were observed doing while tutoring. Ralph was observed on March 31st, April 7th, and April 21st, and he was interviewed on April 1st, April 22nd, and April 29th; Fred was observed on March 23rd, April 13th, and April 20th, and interviewed on April 11th, April 22nd, and April 28th; Michelle was observed on March 17th, March 28th, and April 13th, and interviewed on March 22nd, April 8th, and April 28th. The findings from these data sources were presented in Chapter IV and were primarily organized using the Eight Mathematical Teaching Practices and the associated actions of teachers that embody each practice (NCTM, 2014); two other elements were also used to organize the data: participant reflections and effects perceived by the participants to have resulted from their participation. In this

chapter, I present a discussion of these findings through the lens of situated cognition theory, which purports that learning is a process of enculturation that is facilitated by learners participating in authentic activities that afford them opportunities to think, behave, and problem solve like professionals in the field which they are learning (Brown et al., 1989). I also present recommendations for future research into the implementation of tutoring activities within mathematics teacher education programs.

Discussion of Findings

Situated cognition explains that deep learning of knowledge and skills is a process of assimilating into the culture of professional practitioners which is facilitated by learners engaging in activities in which professionals would engage within a context that allows learners to utilize appropriate knowledge and skills in a manner that is consistent with how professionals would do so (Brown et al., 1989). Brown et al. (1989) explain that it is important for learners to engage with the professional culture because this affords learners the opportunities to observe how professionals examine and solve problems and attempt to emulate professionals' behavior. Additionally, it is important that learning activities occur within authentic contexts because the context of activities provide important tacit information that professionals use to guide their decisions; moreover, the context in which knowledge and skills are employed serve to index learners' experiences in ways that help to define appropriate usage of knowledge and skills (Brown et al., 1989).

Thus, because the culture which learners engage with is integral to ensuring that the learners develop problem solving skills appropriate to the discipline which they are learning, the culture of the Learning Center will be discussed as will its alignment with

the culture of professional mathematics teachers in order to examine the extent to which assimilation into the Learning Center's culture might be beneficial for future mathematics teachers. Then, participants' data will be synthesized to provide a discussion of how the participants leverage the context of their tutoring sessions to guide their in-the-moment decisions as well as how the context of their experiences is used to index their prior experiences resulting in their development of standard initial approaches to solving common issues that arise while tutoring.

The culture of the Learning Center

The tutors who work at the Learning Center are expressly encouraged to help their clients work toward becoming autonomous learners who are capable of working through problems on their own, and each of Ralph, Fred, and Michelle display evidence of assimilating this aspect culture, albeit to different extents. While there were some instances during Ralph's observations in which he allowed his client to work through some part of a problem by his/herself and Ralph explicitly mentions that he does this because "the end goal is [for clients] to do well in the class [and] be able to do these problems [themselves] without tutors," during the majority of Ralph's observed tutoring Ralph is in control of the session, often solving the problem himself while the client follows along. This is not surprising considering that this is Ralph's first semester working at the Learning Center and that, as Ralph explains, the majority of his learning occurs while he is struggling to make sense of course material after his professors have shown him how to solve some example problems. In contrast, while there are instances of both Fred and Michelle electing to just tell clients what to do next while solving a problem, the majority of their tutoring sessions consist of asking their clients "What's

going on here?” or “What do you think we should do next?” or “What is this derivative?” and it isn’t until after the client unsuccessfully answers Fred and Michelle’s guiding questions or expressly gives up that Fred and Michelle decide to step in and either ask more pointed guiding questions, provide a related but less complex example, or do some of the work for the client. However, even when it occurs that Fred or Michelle decide to step in and take over the thinking, once they have helped walk the client through the specific piece with which the client was struggling they shift back to probing the client for what should be done next, returning control of the thinking to the client until the next time the client reaches a stumbling block. Based on the fact that Ralph is aware that the Learning Center encourages tutors to make their clients do the work themselves, but he displays minimal evidence of actually realizing this goal while both Fred and Michelle extensively embody this aspect of the Learning Center’s culture, it appears that Fred and Michelle’s experiences working for the Learning Center have lead them to believe that clients learn better whenever they are, to the fullest extent possible, encouraged to work problems themselves.

The Learning Center’s emphasis on helping clients to become autonomous learners leads each of Ralph, Fred, and Michelle to focus upon making sure their clients understand the general process involved in solving the task at hand, but again to varying degrees. Each of Fred, Ralph, and Michelle explain that, when looking over a client’s work, they first focus on checking that the client is applying the correct overall process for solving the present problem without paying much attention to the specific calculations that are involved in order to determine the extent of the client’s understanding of the process required to solve the problem. Then, if the general process seems correct, they

will proceed to take a closer look at the specific details of the client's work to search for where the client's mistakes and misunderstandings lie. However, this is extent to which Ralph emphasizes the general process whilst tutoring his clients. Fred on the other hand, when he can, likes to take the opportunity to discuss the process beyond its application to the present problem so as to provide a more holistic explanation of applying the process (e.g. when aiding a client in writing an interval that does not include $x = -3$, Fred takes a moment to discuss how the client would notate the interval if it did include that value). Additionally, Fred will explain the process of solving a specific problem in terms of the general process applied to the task at hand (e.g. explaining to his calculus clients that an integral is actually asking "What [function] do I need to take the derivative of... to get this integrand?") He does this in an attempt to enable his clients to be able to apply the process to the whole range of problems that are related to the content of the present problem without needing to seek assistance on a later problem because it has been ever so slightly modified. Michelle emphasizes the importance of her clients' understanding of the general process by utilizing examples that require the same knowledge and skills as the task with which her client is presently struggling but are qualitatively simpler than the present task in order to draw her clients' attention to the key features of a task that indicate which process to apply (e.g. explaining how to multiply terms with fractional exponents via examining how the client would multiply terms with integer exponents; explaining the simplification of the division of exponents by writing the exponents as repeated multiplication). She does this in order to highlight how her clients can use the knowledge and skills they have already demonstrated an understanding of to solve problems that, at first, do not appear to be familiar. Once again, Ralph displays evidence

of attempting to incorporate the Learning Center's goal of fostering the independence of his clients into his tutoring practice by making sure to check his clients' understanding of the process necessary to solve a particular problem but he still maintains primary control over the majority of each tutoring session thus preventing him from fully actualizing this goal. On the other hand, Fred and Michelle, as a result of their extensive tutoring experience, have found ways in which to weave a discussion of the general process involved in solving a class of problems (e.g. integrating a function, multiplying exponential terms) into their tutoring practices while still helping their clients to understand how to solve the problem at hand. Because neither Michelle nor Fred have had any teaching education or experiences outside of the tutoring interactions and professional development opportunities within the Learning Center, it seems likely that over time Ralph, and by extension other brand new Learning Center mathematics tutors, will gradually improve his ability to help clients develop a more holistic understanding of the general problem solving process through additional opportunities to both observe more experienced tutors doing so and attempt to emulate those tutors during his own sessions.

Another key component of the Learning Center's vision is to "instill confidence" in each client that seeks aid from the Learning Center tutors. Interestingly, Michelle is the only participant who explicitly mentions this as component of her rationale for tutoring decisions she makes. She explains that "especially when [clients] are writing on their own or they're telling [her] what [they] think [they] should do" she likes to watch them work and listen to what they have to say so she "can validate what they're doing" and is "trying to instill that confidence in them so they can do it on their own." It is unlikely that Ralph

and Fred are unconcerned with helping their clients to feel confident in their own abilities, particularly because they both mention wanting to help their clients to be successful on their own which, inherently, requires clients to have confidence in their own abilities, but the facilitation of this confidence does not appear to be something that Ralph or Fred consciously strive to accomplish while tutoring. Michelle constantly strives to make her clients feel like they (i.e. Michelle and her client) are on a team working to solve the problem together through her insistence on using collective pronouns while engaging her clients in discourse (e.g. “What were we supposed to be doing?”). While each of Ralph and Fred are not explicitly working to facilitate their clients’ confidence, the reasons for this appear to be distinct. While tutoring, Ralph is focused on making sure he gets the content correct because he is concerned with maintaining his own image as a knowledgeable tutor and is still building his confidence in himself as a tutor and this likely contributes strongly to his lack of explicit focus on the confidence of his clients. Thus, it is possible that as Ralph gains more tutoring experience, allowing him to build confidence in his own ability to tutor, he will be able to shift increasing amounts of his focus onto helping his clients to become more self-confident. On the other hand Fred comments during an interview that “teaching is not really something [he] enjoy[s];” he takes his job at the Learning Center seriously and endeavors to do the best he can to help his clients, but he does not revel in the experience in the same exuberant manner Michelle does. Thus, there appears to be an affective component to that is contributing to this distinction between Michelle and Fred. However, because Fred is committed to helping his clients as much as possible, perhaps he might be more willing to actively pursue

affecting his clients' confidence after participating in a professional development focused on the benefits this might have to clients and ways in which to positively affect it.

Another salient feature of the Learning Center's culture is the encouragement of tutors to rely upon one another to work toward helping their clients to be successful. Whenever Michelle is faced with content that she is unsure of, if there is another tutor available who she knows is more knowledgeable in this area, she will recruit the other tutor's aid, such as when she is working with a Linear Algebra client and decides to enlist Fred's assistance. Moreover, Michelle says that "if I have time, I try to watch what they're doing and learn from it...that way the next time that question comes up, I'm better prepared for it...[because] watching somebody definitely helps me remind myself of what I'm supposed to do." Ralph also says that whenever he calls in another, better equipped tutor to help his clients if time permits "[he] will usually hang out" and watch the other tutor's tutoring in order "to learn... from them, especially if it's a subject [he has] taken." Although there are no instances during Fred's observations of him enlisting the aid of a fellow tutor, he does comment that he enjoys being able to help his fellow tutors. This aspect of the Learning Center culture is very likely a byproduct of the fact that new tutors, as Ralph explains, are required to observe the lead tutor for their discipline, an experience that provides Ralph insight into the ways in which the more experienced tutors interact with and explain things to clients, which he then later endeavors to emulate while he is helping his own clients. As a result of the Learning Center's requirement that new tutors observe more experienced tutors with the explicit goal of using these observations as a means for aiding new tutors to improve their tutoring, Learning Center tutors are, practically from their first day on the job,

enculturated into the belief that they can and should strive to emulate the effective practices employed by their fellow tutors. Moreover, by establishing this norm early into the tutors' experiences, tutors feel comfortable with and empowered to seek the assistance of their peers whenever they encounter a client to whom they cannot render aid.

Context of tutoring session's role in tutors' actions.

Context of tutoring guides participants' decisions. Each of Ralph, Fred and Michelle ask their clients questions, albeit of varying levels of demand, during a tutoring session in order to determine what the client presently knows in relation to the task at hand so they can know what prior knowledge is available for use during the tutoring session. While Ralph's interest in clients' present knowledge stops at wanting to make sure his explanations only include "the tools they already know," Michelle often takes this knowledge further by attempting to leverage the knowledge she knows her clients know to aid them with problems that, at first, appear unfamiliar (e.g. using the client's ability to multiply terms with integer exponents to showcase how the client should multiply terms with fractional exponents). Additionally, all three participants strive to gather evidence of how clients' professors present material so they can attempt to provide explanations that are in alignment with the professor, a practice that Ralph says is explicitly encouraged by the Learning Center. Fred, however, mentions that if it seems that the way a professor explains things isn't helping a client to understand, he will "throw everything out the window" and try to explain in a different way. Questioning learners as a means of identifying their present knowledge and understanding is an essential skill for becoming an effective educator, mathematics or otherwise (NCTM,

2014). Were it not for the participants' tutoring experiences it is highly probable that none of them would have any understanding of how to effectively tease out an individual's present understanding of mathematics content through the posing of pointed questions.

Ralph and Michelle both explain that, based on the content level of a client's coursework, they will skip over or move quickly through steps that are couched in content from prior or lower-level course work (e.g. while working with calculus clients, they both tend to assume the client has a grasp on the algebra and will, thus, default to glossing over the algebraic steps). Michelle also mentions that there are times when she assumes a client knows how to do something because she sees evidence of the client's ability with that skill present in other work that the client has done on his/her paper.

While Fred is helping a calculus client who has the solution manual in front of him and seems to refuse to want to pay attention to what Fred is trying to say, Fred decides to ask the client to come to the board to work it out in order to get the client to stop relying on the solution manual and actually become engaged in the conversation with Fred.

Although it is difficult to conclude that other tutors would make the same decisions if they were presented with the same situations as Ralph, Fred, and Michelle, it is clear that Learning Center mathematics tutors rely heavily upon the context of tutoring session, which is primarily dictated by the particular client whom they are assisting, to aid them in making in-the-moment decisions as to the best way to engage their present client in a manner which is sensitive to the client's particular needs. Much like the participants in Lee and Statham's (2010) study, by virtue of the fact that Learning Center

tutors have opportunities to see the various ways different learners think and their different instructional needs, Learning Center mathematics tutors develop their ability to react flexibly to these idiosyncrasies.

Context of tutoring indexes participants' experiences. Each of Ralph, Fred, and Michelle provide rationale for decisions they made with regard to client engagement that are the result of prior, related experiences with other clients. Ralph says that whenever clients are more familiar with the content or context of a problem, the more likely they are to be engaged in the conversation rather than allowing him to do “all the thinking.” He explains that he works to leverage this whenever he is tutoring athletes because if he can “relate [the problem or material] to the sport that they play...then they’ll most likely be more engaged.” Fred has found that if he takes control of a tutoring session and starts to do all the talking, clients will almost never interrupt him to interject what it is they know or are thinking, so he tries to get the clients to explain as much of the problem as he can in order to get the client to put in as much effort as possible. Michelle has had a similar experience with her clients and, as a result, will tell clients “No, no, you need to keep talking” so that she can gauge what they do and do not understand. Additionally, Michelle’s experience as a private tutor provided her with a provides her with a means of categorizing clients as being one of three types: students that are there to learn, students that just need someone to verify that their work is correct, and “slacker” students who don’t want to get help but have realized they need help. This categorization results in “almost like an autopilot” approach to her tutoring in that she knows that the “slacker” students need her to immediately get them working on problems themselves, students

there to learn “need encouragement,” and the one’s in-between need some mixture of both.

Each of the three participants also use prior experiences of interpreting clients’ body language as a guide for how to interpret that of their present clients. Ralph has found that whenever a client is following along with his explanation and writing notes on their paper, if “they pause [and] they don’t write [anything] down” this is most often an indication that they do not understand whatever it is that Ralph just explained. Ralph has also found that sometimes, even when a client responds affirmatively to his asking “Did that make sense?” their intonation and facial expressions sometimes indicate the exact opposite, which leads Ralph to consider trying another approach or explanation to help his client to understand. Fred explains that he tries to read the body language of his clients and if a client appears to be one of “the timid ones,” Fred will generally sit with the client at the table rather than work at the board “because [these clients] don’t want to be embarrassed.” Michelle discusses that her clients “give [her] a lot of non-verbal cues” that she uses to gauge the clients’ understanding, and she has found that, typically, when a client is nodding their head while she is explaining things it usually “doesn’t mean anything” with regard to whether or not they actually understand what she is saying and will be able to solve this problem on their own once she leaves. The fact that each of the participants rely heavily upon clients’ non-verbal cues is quite interesting because none of the participants mentioned that the Learning Center trains them or discusses with them the importance of body language, nor did the director or assistance director mention providing professional development which focus on this. However, body language is an important component of human communication. Thus, it seems that merely by virtue of

tutoring requiring interaction with other human beings the participants utilize the body language of their clients to provide them with evidence with which to make inferences about clients' thinking and emotional state.

Moreover, Ralph, Fred, and Michelle have each developed standard responses to many different issues that commonly arise while assisting clients. When a client asks Ralph for help on something that he has already helped them with, he says that his default action is to direct the client to the notes they have from the last time Ralph helped, as well as those they took in class because, in his experience, "it's a lot more beneficial for them to go back through an example problem and work it themselves" rather than Ralph just explaining how to do it again. Fred has found that calculus clients often struggle to recognize how to execute an integral whenever the function is described using a variable other than x , so he will very often switch the variable to be x 's to present the problem in a form the clients are "quite familiar with." Additionally, it has been Fred's experience that clients often struggle to make sense of definitions because they are so abstract, so he sometimes tries to help his clients to translate the meanings of definitions by going through examples with the client and examining whether or not the definition applies to the example. Whenever Fred encounters a client that has not been to class or has not read the textbook, Fred will tell them "read the chapter and then I'll help you" because he needs the client to have some knowledge and understanding of the content so that Fred has "something to work with." When Fred is helping a client that is reviewing an exam he tends to focus very heavily on the overall process involved in solving a problem because that will more helpful to the client than solving any particular problem. Fred is wary whenever a client asks him to explicate the specific steps to solving any problem

because he is concerned that the client will not be able to identify problems that are only asking about a small subsection of the process or not be able to “recognize problems that don’t have the steps,” and so when this happens he instead tries to focus his discussion on the general ideas involved in the process. When Michelle is helping calculus clients that do not know how to take the derivative of a function because it doesn’t look familiar, she explains that her default action is to write a related but simpler example that uses the same rule and tell the client “You know this one. Why is it different when it’s just a different number?” When Michelle has spent a while working with a client on a small subsection of a process, she will often ask the client “What were we supposed to be doing?” as a way of reminding the client that all the work they have been doing is small detour on the path to the solution rather than an end unto itself. When clients are working on exam review and asks Michelle for help, she typically will remind them of the general process to solve the problem and she might even tell them what the answer is supposed to be, but then she will tell the client “work it out on your own and I’ll come back” because they are about to have an exam and need to be able to work through it on their own. If, at the end of a session with a client Michelle is concerned that the client would not be able to solve the problem on his/her own, Michelle “will pick a problem out of [the] book... [or] make one up” and tell the client to work through it by his/herself and she will come back to check on their work.

Finally, Fred’s experience working with clients in more advanced courses explicitly affects the manner in which he tutors clients in lower content levels and he is the only one of the three participants to exhibit this. Fred says, “I think calculus brings students, for the first time, to seeing that you...can’t just memorize a formula. They’re

kind of forced in to... having to understand the ideas...so when I tutor lower level math, I don't just say 'Well that's just the way it is,' absolutely not."

Again, it would be inappropriate to conclude that the experiences of all Learning Center mathematics tutors would have identical outcomes, however the evidence herein clearly indicates the existence of some common threads to the different ways in which tutors' experiences are indexed by the context of these experiences. It is apparent that Learning Center mathematics tutors make use of clients' body language as a source of insight into the minds of their clients and that they each develop standard responses to issues that are frequently raised by clients. It is quite reasonable to conclude that as Learning Center mathematics tutors continue to interact with clients – both new and returning – their ability to interpret non-verbal communication will improve through continued practice with varied degrees of success. Moreover, as these tutors engage in more tutoring sessions the breadth of the issues that they have to deal with which arise from clients' different needs will lead to the development of additional standard responses to common issues as well as the refinement of existing responses.

Summary. It is clear that the culture of the Learning Center has an effect upon the decisions that each of Ralph, Fred, and Michelle make while tutoring clients because the rationale that each provide for the various decisions they were observed making align strongly with the goals explicated in the mission and vision of the Learning Center. Moreover, each of the three participants provide clear evidence that their past experiences working with clients often serves as a guide for how they respond to the wide range of issues facing them as they work to assist their present client.

Unsurprisingly, Ralph's adherence to the culture of the Learning Center appears to still be in a fledgling state, as evinced by, for example, the fact that while Ralph mentions the importance of allowing clients to work on their own in order to foster their ability to work autonomously, his actions throughout the observations infrequently align with this goal. Additionally, Ralph provided the fewest explanations for his actions that were the result of his experiences tutoring previous clients. Both his surface level adherence to the Learning Center culture and relative small number of decisions based on past experience are easily attributed to the fact that the semester in which this study takes place is Ralph's very first semester working as a tutor. However, at the conclusion of this study, Ralph says that he has become more aware of what he is doing in-the-moment due to me asking him to explain why he made choices that he did.

While it is not altogether surprising that Ralph's participation had this effect on him, it is quite interesting that both Fred and Michelle, tutors with quite a bit more experience than Ralph, made similar claims that their participation in this study lead to them to do more thinking in-the-moment about what they are going to do and say next. I had expected that the more veteran tutors would have been more consciously aware in-the-moment of why they were making the tutoring decisions they were by virtue of the fact that they had both were more adept at engaging their clients and had both developed responses to many common issues facing their clients. On that note, it was quite surprising to hear that Michelle was not even aware of the extent to which she strives to help her clients feel more confident in their own abilities until she participated in this study.

Assimilating into the culture of the Learning Center leads tutors to believe that clients are better served by tutors encouraging them to work through problems on their own as much they can rather than tutors merely telling clients what steps to take to find an answer. Tutors also learn to strongly emphasize to clients the importance of understanding the general process involved in solving various classes of problems as a means of helping clients to move toward becoming independent learners. Enculturating into the Learning Center also facilitates tutors learning to utilize fellow tutors as both an in-the-moment resource to better aid a client and as a source of knowledge of effective tutoring practices to integrate into their own tutoring. Moreover, the context of the tutoring sessions within the Learning Center provide tutors with opportunities to engage with different clients each with different needs and thus requiring tutors to be able to develop their ability to identify and meet these individual needs through the use of questions as a means of identifying clients' present understandings and particular struggles to aid in deciding how to best meet their needs as well as utilize body language as a source of insight into clients' minds. Additionally, the context of these tutoring sessions facilitates tutors development of standard initial responses to issues that repeatedly arise across many different clients.

Thus, it appears that these tutoring experiences are an effective learning experience for the participants and to help illuminate why, I turn to experiential learning theory (Kolb, 2015). Kolb (2015) says that learning occurs when learners are able to engage in each stage of the learning cycle – concrete experience, reflective observation, abstract conceptualization, and active experimentation – and that this cycle forms a learning spiral as learners engage in other new experiences and, through reflecting on

these new experiences, leads learners to evaluate and possibly modify their related abstractions. From that data presented in this study, we see that each participant engages in numerous concrete experiences in which they get to interact with clients and attempt to help clients improve their understanding of mathematical content. We also see that each participant engages, to varying degree, in reflecting upon these tutoring experiences with a particular emphasis on evaluating how well they were able to help the client to better understand the problem at hand. These reflections lead the participants to abstract from their tutoring experiences practices that they find to be helpful to clients, which they then go on to apply during subsequent, similar tutoring experiences. Additionally, due to the collaborative nature of tutoring in the Learning Center, the participants also discuss that they will observe other tutors' practices and extract from these observations other practices which they endeavor to integrate into their own tutoring. Two participants, Ralph and Michelle, also illustrate that they reflect upon their own experiences as learners to help guide some of their tutoring decisions. Hence, working as a tutor for the Learning Center affords tutors with dozens, if not hundreds, of passes around Kolb's experiential learning spiral as tutors engage with each new client and each new problem and strive to do the best job they can of helping each client to better understand material.

Implications for Teacher Education

The purpose of investigating the experiences of the Learning Center mathematics tutors was to be able to make an informed argument about whether or not incorporating such experiences into mathematics teacher education has potential to help alleviate the fragmentation of many pre-service mathematics teachers' understanding of teaching theory and practice that is the result of their learning about teaching practices and

methodologies being greatly separated from their opportunities to actually put this knowledge into practice (Hibert et al., 2003; Smagorinsky et al., 2003; Grossman et al., 2009). As such, this section will begin by discussing the compatibility of the culture of the Learning Center with that of mathematics teachers, and then examine the extent to which the tutoring activities taking place at the Learning Center mirror the authentic activities undertaken by professional mathematics teachers.

Compatibility of the Learning Center’s culture with that of mathematics teachers. Because learning is a process of enculturation, in order for mathematics teacher education programs to benefit from incorporating tutoring experiences at the Learning Center, or similar tutoring centers, the culture of the Learning Center must be very closely aligned to those of mathematics teachers so that the views and beliefs PSMTs develop and adopt as they assimilate into the culture of the tutoring center are beneficial to PSMTs as they transition into professional teaching. The Learning Center’s vision includes a focus on helping clients to become “independent learners,” a fact that each of Ralph, Fred, and Michelle are expressly aware of and, to varying degrees, explicitly striving to achieve as they assist their clients. In the pursuit of this goal, each of the three participants have developed a focus upon ensuring that their clients understand the general process of solving problems rather than being fixated entirely on particular examples. Moreover, the Learning Center’s vision specifically includes an emphasis on helping clients to “rely on their [own] strengths and abilities.” While only Michelle made particular mention of this being a component of her tutoring, the Learning Center is a place wherein tutors are expected to show clients that they are capable of succeeding in mathematics on their own. Helping learners towards autonomy through the development

of their self-confidence, facilitated by a focus on general procedures and processes is directly aligned with NCTM's guiding principle that effective teachers "engages students in meaningful learning...that promote their ability to make sense of mathematical ideas and reason mathematically," (NCTM, p.5) because students need to feel empowered that they are capable of using their own knowledge and understandings to think and reason about mathematics and to be taught in a manner that eventually enables them to do so independently.

Additionally, Learning Center tutors form a community in which everyone is committed to helping the clients succeed and, thus, tutors feel comfortable seeking the aid of their colleagues whenever they are unable to assist a client. However, the importance of this community does not stop there, because not only are tutors willing and able to defer to other, better equipped tutors when necessary, the Learning Center explicitly encourages tutors, particularly new tutors, to use the opportunities to observe other tutors' tutoring as a way of learning more about being a Learning Center tutor. NCTM's professionalism principle states that "in an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics" (NCTM, p.5).

While there are likely other features inherent in the culture of mathematics teachers that are not incorporated into the culture of the Learning Center mathematics tutors, it is clear that the values of the Learning Center tutoring are aligned with at least a subsection of that of mathematics teachers. Equally important, the values encouraged by the Learning Center do not run counter to those that NCTM purports are necessary for

effective, high-quality mathematics instruction. Thus, the beliefs and values PSMTs would adopt by assimilating into the Learning Center's tutoring culture would remain relevant and useful as the PSMTs moved into professional teaching.

However, it is important to note that the levels of autonomy expected of the adult learners who are the clients of the Learning Center could differ substantially from that of K-12 students. Moreover ways in which to appropriately engage adult learners are likely different from those appropriate for K-12 students. Thus, it will be important that PSMTs who engage in tutoring activities, such as those described herein, be given opportunities to explicitly discuss these differences between adult and K-12 learners in order to identify which aspects of their tutoring experiences will be most useful as they transition into professional mathematics teaching.

Learning Center mathematics tutoring compared to the authentic activities of mathematics teachers. In addition to ensuring that the beliefs and values instilled by enculturating into the Learning Center tutoring will be useful to PSMT's as they become professional mathematics teachers, in order for mathematics teacher education programs to derive educational benefits from the inclusion of tutoring experiences at the Learning Center it must be the case that the tutoring activities that take place in the Learning Center are adequate approximations of the authentic activities that mathematics teaching professionals engage in. As evinced by the data presented in Chapter 4, each of the participants, including Ralph who is in his very first semester tutoring, are having experiences and providing rationale that address some aspect of each of the Eight Mathematics Teaching Practices NCTM's advocates are integral to high-quality mathematics teaching and learning. From the data, we can see that each participant is

guided by goals which, to varying degree, align with the Learning Center's goals; tutors work to encourage clients' reasoning and problem solving by working to build upon clients' current knowledge and tutors have opportunities to support students without taking over their thinking; tutors are afforded opportunities to aid their clients in using and forging connections between mathematical representations focusing clients' attention on key features of the mathematics; tutoring is replete with occasions in which tutors can meaningfully engage clients in discourse about mathematics by positioning clients as primary contributors to the discussion; tutoring sessions are inundated with occasions in which tutors can pose pointed questions to their clients; tutors have opportunities to aid clients in developing procedural fluency by allowing clients to apply their own reasoning to solving problems as and to work independently on problems; tutors are exposed to copious instances of clients struggling to make sense of mathematics and thus have ample opportunity to work to support clients productively through this struggle as well as to feel more comfortable whilst embroiled in these struggles; and successful tutoring, as the Learning Center envisions it, is predicated heavily upon tutors eliciting and utilizing clients' thinking so as to determine the best course of action to aid each client in advancing toward self-confident, autonomous learning. Thus, if PSMTs were engaged in tutoring activities like those that take place at the Learning Center, there would be ample opportunities for them to practice engaging learners in ways that align with the NCTM's vision of high-quality mathematics instruction.

In the interest of a fair and balanced comparison of the observed tutoring activities and the authentic activities that comprise professional mathematics teaching, it must be noted that there are several sub-components of these Eight Practices that are not

evident in the participant's observed actions and rationale. However, the majority of these missing sub-components fall into two categories: engaging multiple learners at once and planning instruction ahead of time. While, based on Michelle's experiences as well as my own, tutoring at the Learning Center does occasionally include opportunities for a tutor to engage multiple clients working on the same problems and material, it is evident from the aggregation of the data presented herein that the majority of the interactions Learning Center tutors have is with individual clients. Moreover, it is not surprising that there is little opportunity to plan instruction ahead of time because the very nature of tutoring is to be reactive to the present needs of many learners learning dramatically different material at a moment's notice. That being said, these features of professional mathematics instruction are, in my opinion, best suited to be a focus of the learning of PSMTs during their student teaching practicums, when they have sustained opportunities to be engaged with a consistent group of learners – thus affording PSMTs the necessary conditions so as to be able to regularly engage multiple learners in conversation – learning from a single curriculum – granting PSMT's the ability to foresee upcoming struggle because they will be able to know the upcoming material.

Summary. The evidence presented herein clearly suggest that there is much that PSMTs can learn about the high-quality mathematics instruction NCTM encourages mathematics teacher education programs to strive to facilitate. By enculturating into the Learning Center's tutoring culture, PSMTs would develop values that are right at home in a mathematics classroom focused on promoting students' abilities to think and reason mathematically and would learn the important role that collaborating with and seeking assistance from one's fellow teachers can play in ensuring learners' success. Additionally,

if PSMTs were to engage in mathematics tutoring activities at the Learning Center with the support of one or more mathematics teacher educators, the PSMTs would be privy to many different experiences of engaging learners with which teacher educators could facilitate discussion of practices that are useful to and healthy for helping mathematics learners as well as employ contextualized examples to showcase when PSMTs could make alternative choices, which the PSMTs could then endeavor to employ during later tutoring activities. Moreover, if the multiple PSMTs are engaged in tutoring activities as part of the same course, then there would be opportunities for the PSTMs to engage in discourse comparing and contrasting different decisions made by their classmates, and the PSMTs would each have experiences that would permit them to learn vicariously through the experiences of their peers.

A final note on the potential benefits of providing PSMTs with tutoring activities is derived from my own experience working at the Learning Center as well as Michelle's. At the onset of this study, Michelle was planning to pursue a graduate degree in applied mathematics and had never considered becoming a teacher. However, as a result of participating in this study wherein she was merely asked to explain why it was she made the choices she did, she began to consider pursuing a graduate degree in mathematics education and ended up applying to and being accepted into a doctoral program in Mathematics Education. In a similar fashion, when I first became a Learning Center tutor, I was convinced I would go on to pursue a graduate program in mathematics or physics, but after years of working at the Learning Center I realized that I actually love teaching and also elected to pursue a PhD in Mathematics Education. For Michelle and I, it was not until after we started tutoring that we realized our passions for teaching, thus another

benefit PSMTs might derive from participating in tutoring activities as part of their teacher education program is that they might be able to affirm for themselves that teaching is, in fact, a career they wish to pursue or, possibly more importantly, realize earlier in their lives that teaching is not the career for them.

Suggestions for future research

In order for this research to blossom into widespread modifications to mathematics teacher education, it must be possible for the ideas explored herein to be widely applicable and easily implemented throughout the many different teacher education programs that work tirelessly toward the goal of providing quality education to the youth of our nation and world. As a result, research must be conducted to elucidate where, when, and how such tutoring activities should be incorporated into teacher education.

Research into the experiences of other tutors, both at the Learning Center and in tutoring facilities across a wide variety of colleges and universities must be conducted so that we can examine the extent to which the experiences of Ralph, Fred, and Michelle align with those of other tutors. If it is the case that a wide array of tutoring facilities, representing different visions of what mathematics tutoring should look like afford their tutors with experiences that permit them opportunities to engage in each of NCTM's Eight Principles, this will go a long way toward facilitating the widespread implementation of this project's vision. However, if it is the case that there tutoring centers in which tutors are not engaging in significant portions of the Eight Principles, then further research into why such discrepancies exist will be necessary to an investigation of how teacher education programs interested in employing such tutoring

activities might be able to compensate for any short falling of the tutoring facility on their campus.

Additionally, future research must be conducted to investigate in which course or courses within teacher education programs such tutoring experiences should be deployed, and what this implementation should look like, so as to facilitate quality learning among the participating PSMTs. However, it is important to note that the content level of the mathematics with which PSMTs would engage learners within the Learning Center, and very likely other university tutoring centers, is appropriate for PSMTs seeking secondary mathematics certification, and possibly some middle school programs, but not for elementary school certification. It is my hypothesis that tutoring experiences such as these might be best leveraged during PSMTs' methods coursework so as to allow them opportunities to practice implementing teaching tools and practices while they are learning about them in class.

This kind of research will also afford opportunities to examine the effects such experiences have on PSMTs' learning of mathematics teacher methods and allow for comparison to students in more traditional teacher education programs. Moreover, it will be of paramount interests to illuminate what differences, if any, exist between PSMTs who had such tutoring experiences and those from traditional programs during student teaching practicum as well as the transition into professional mathematics instruction.

REFERENCES

- Ball, D. L. & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, 60 (5), 497-511.
- Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, 18 (1), 32-42.
- Creswell, J. W. (2013). *Qualitative Inquiry and Research Design*. 3rd Ed., Sage Publications: Thousand Oaks, CA.
- Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: theory and practice*, 15 (2), 273-289.
- Hart, S. M. and King, J. R. (2007). Service learning and literacy tutoring: Academic impact on pre-service teachers. *Teaching and Teacher Education*, 23, 323 – 338.
- Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An “experiment” model for teaching and teacher education. *Journal of Mathematics Teacher Education*, 6, 201 – 222.
- Hollins, E. R. (2011). Teacher preparation for quality teaching. *Journal of Teacher Education*, 62 (4), 395 – 407.
- Kolb, D. A. (2015). *Experiential learning: Experience as the source of learning and development* (2nd ed.). NJ: Pearson Education.
- Lee, K. & Statham, A. (2010). The impacts of a service-learning experience on pre-service mathematics teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, 2, 1 – 10.

- Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.
- National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.
- Pashler, H., McDaniel, M., Rohrer, D., & Bjork, R. (2008). Learning styles: Concepts and evidence. *Psychological Science in the Public Interest*, 9 (3), 105 – 119.
- Smagorinsky, P., Cook, L. S., & Johnson, T. S. (2003). The twisting path of concept development in learning to teach. *Teachers College Record*, 105 (8), 1399 – 1436.