Electronic Journal of Differential Equations, Vol. 2007(2007), No. 103, pp. 1–7.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

PERIODIC SOLUTIONS OF MULTISPECIES-COMPETITION PREDATOR-PREY SYSTEM WITH HOLLING'S TYPE III FUNCTIONAL RESPONSE AND PREY SUPPLEMENT

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ABSTRACT. In this paper, we consider a nonautonomous multispecies competition predator-prey system with Holling's type III functional response and prey supplement. It is proved that the system is uniformly persistent under some conditions. Furthermore, we show that the system has a unique positive periodic solution which is globally asymptotically stable.

1. INTRODUCTION

The ecological predator-prey systems with Holling's type functional response have been studied extensively by many authors [1, 2, 3, 4, 5, 7, 8, 9, 12, 13, 14, 15]. One of the most interesting questions in mathematical biology concerns the existence of positive periodic solutions for population dynamical systems [6, 11, 16, 17, 18]. For the continuous Lotka-Volterra systems, such a problem has been investigated extensively, and many skills and techniques have been developed.

The existence of positive periodic solutions for such systems can be obtained by standard techniques of bifurcation theory [5], or by theory of topological degree [11]. In fact, these methods have been widely applied to various Lotka-Volterra systems [16, 17, 18]. However, in population dynamics, in order to keep the persistence of Lotka-Volterra system, human always give some supplement of prey. To the author's knowledge, the population dynamical systems with prey supplement are seldom discussed.

The purpose of this paper is to study the asymptotic behavior of a nonautonomous multispecies predator-prey system with Holling's type III functional response and prey supplement. Moreover, the competition among predator species and among prey species is simultaneously considered. We will investigate the following nonautonomous predator-prey system of differential equations

2000 *Mathematics Subject Classification.* 92D25, 34C25.
Key words and phrases. Competition predator-prey system; periodic solution; Holling's type III functional response; global attractivity.

Submitted May 2, 2007. Published July 25, 2007.
Supported by grant 200611003 from the Youthful Teacher Research Foundation of North China Electric Power University.

$$\dot{x}_i(t) = x_i(t)\Big[b_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t) - \sum_{k=1}^{m} \frac{c_{ik}(t)x_i(t)y_k(t)}{f_{ik}(t) + x_i^2(t)}\Big] + \phi_i(t), \tag{1.1}$$

$$\dot{y}_j(t) = y_j(t)\Big[- r_j(t) + \sum_{k=1}^{n} \frac{d_{jk}(t)x_k^2(t)}{f_{jk}(t) + x_k^2(t)} - \sum_{k=1}^{m} e_{jk}(t)y_k(t)\Big]. \tag{1.2}$$

Here, $x_i(t)$ denotes the density of prey species x_i at time t, $y_j(t)$ denotes the density of predator species y_j at time t, $\phi_i(t)$ represents the supplement of prey, the functions $b_i(t)$, $r_j(t)$, $a_{ij}(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$ $(i = 1, 2, \dots, n,\ j = 1, 2, \dots, m)$ are continuous, nonnegative and $b_i(t)$, $f_{jk}(t)$, $a_{ii}(t)$, $e_{jj}(t)$ are below bounded by positive constants. In section 2, we prove the uniform persistence of the system. In section 3, we prove the existence of positive periodic solution of the system. In section 4, the sufficient conditions for the uniqueness and global attractivity of the positive periodic solution of system (1.1)–(1.2) are obtained.

2. Uniform Persistence

For the sake of simplicity, we take some notations as following:

$$X = (x_1, x_2, \dots, x_n), \quad Y = (y_1, y_2, \dots, y_m), \quad F = (X, Y).$$

It is obvious that there exists a unique solution of the system (1.1)–(1.2) corresponding to any initial value $F = (X, Y) \in R^{n+m}$. Assume that

$$\begin{aligned} F(t, F) &= (X(t, F), Y(t, F)) \\ &= (x_1(t, F), x_2(t, F), \dots, x_n(t, F), y_1(t, F), y_2(t, F), \dots, y_m(t, F)), \end{aligned}$$

and $F(0, F) = F, t > 0$.

Lemma 2.1. *If $a > 0, b > 0$ and $\frac{dx}{dt} \le (\ge) x(b - ax)$, when $t \ge 0$, for any positive initial value we have*

$$\limsup_{t\to\infty} x(t) \le \frac{b}{a} \quad (\liminf_{t\to\infty} x(t) \ge \frac{b}{a}).$$

Lemma 2.2. *Both positive and nonnegative cones of R^{n+m} are invariant with respect to system* (1.1)–(1.2).

It follows from this lemma that any solution of system (1.1)–(1.2) with nonnegative initial conditions remains nonnegative.

Definition 2.3. System (1.1)–(1.2) is said to be uniformly persistent if for any positive initial value, there exist positive constants A, B, C, D such that $0 < A \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le B < +\infty$ $(i = 1, 2, \dots, n)$ and $0 < C \le \liminf_{t\to+\infty} y_j(t) \le \limsup_{t\to+\infty} y_j(t) \le D < +\infty$ $(j = 1, 2, \dots, m)$.

Assume that $0 < f^L = \inf_{t\geq 0} f(t) \leq \sup_{t\geq 0} f(t) = f^M < +\infty$. In the following, it is convenient to assume that

$$p_i = \frac{b_i^M}{a_{ii}^L} + \sqrt{\frac{\phi_i^M}{a_{ii}^L}}, \quad q_j = \frac{1}{e_{jj}^L}(-r_j^L + \sum_{k=1}^{n} d_{jk}^M),$$
$$\alpha_i = \frac{1}{a_{ii}^M}(b_i^L - \sum_{k=1,k\neq i}^{n} a_{ik}^M p_k - \sum_{k=1}^{m} \frac{c_{ik}^M q_k p_i}{f_{ik}^L}) + \sqrt{\frac{\phi_i^L}{a_{ii}^M}},$$
$$\beta_j = \frac{1}{e_{jj}^M}(-r_j^M + \sum_{k=1}^{n} \frac{d_{jk}^L \alpha_k^2}{f_{jk}^M + p_k^2} - \sum_{k=1,k\neq j}^{m} e_{jk}^M q_k)$$

$(i = 1, 2, \dots, n;\ j = 1, 2, \dots, m)$. It is obvious that if $q_j > 0$, then $\alpha_i < p_i$, and if $q_j > 0, \alpha_i > 0$, then $\beta_j < q_j$ $(i = 1, 2, \dots, n;\ j = 1, 2, \dots, m)$. So, from now on, we suppose that $q_j > 0$ $(j = 1, 2, \dots, m)$.

Theorem 2.4. *If $\alpha_i > 0$, $\beta_j > 0$, $q_j > 0$, $(i = 1, 2, \dots, n;\ j = 1, 2, \dots, m)$, then system (1.1)–(1.2) is uniformly persistent.*

Proof. It follows from Lemma 2.2 that any solution of system (1.1)–(1.2) which has a nonnegative initial condition remains nonnegative. From equation (1.1), we have $\dot{x}_i \leq x_i(b_i^M - a_{ii}^L x_i) + \phi_i^M$. For any initial value $x_i(0) > 0$, from Lemma 2.1, we have

$$\limsup_{t\to\infty} x_i(t) \leq \frac{b_i^M}{a_{ii}^L} + \sqrt{\frac{\phi_i^M}{a_{ii}^L}} = p_i, \quad (i = 1, 2, \dots, n). \tag{2.1}$$

From (1.2), we have $\dot{y}_j \leq y_j(-r_j^L + \sum_{k=1}^{n} d_{jk}^M - e_{jj}^L y_j)$. Thus for any $y_j(0) > 0$, from Lemma 2.1, we have

$$\limsup_{t\to\infty} y_j(t) \leq \frac{1}{e_{jj}^L}(-r_j^L + \sum_{k=1}^{n} d_{jk}^M) = q_j, \quad (j = 1, 2, \dots, m). \tag{2.2}$$

For $\varepsilon_1 > 0$ small enough, there exists $t_1 > 0$ such that

$$x_k(t) < p_k + \varepsilon_1, \text{and } y_k(t) < q_k + \varepsilon_1, \tag{2.3}$$

for $t > t_1$. From (1.1) and inequality (2.3), we have

$$\dot{x}_i \geq x_i(b_i^L - \sum_{k=1,k\neq i}^{n} a_{ik}^M(p_k + \epsilon_1) - \sum_{k=1}^{m} \frac{c_{ik}^M(q_k + \epsilon_1)(p_i + \epsilon_1)}{f_{ik}^L} - a_{ii}^M x_i) + \phi_i^L, \tag{2.4}$$

$(i = 1, 2, \dots, n)$ for $t > t_1$. From the above inequality, for any initial value $x_i(0) > 0$, using Lemma 2.1, we get

$$\liminf_{t\to+\infty} x_i(t) \geq \frac{1}{a_{ii}^M}(b_i^L - \sum_{k=1,k\neq i}^{n} a_{ik}^M(p_k + \epsilon_1) - \sum_{k=1}^{m} \frac{c_{ik}^M(q_k + \epsilon_1)(p_i + \epsilon_1)}{f_{ik}^L}) + \sqrt{\frac{\phi_i^L}{a_{ii}^M}},$$

$(i = 1, 2, \dots, n)$. Let $\epsilon_1 \to 0$, we have

$$\liminf_{t\to+\infty} x_i(t) \geq \frac{1}{a_{ii}^M}(b_i^L - \sum_{k=1,k\neq i}^{n} a_{ik}^M p_k - \sum_{k=1}^{m} \frac{c_{ik}^M q_k p_i}{f_{ik}^L}) + \sqrt{\frac{\phi_i^L}{a_{ii}^M}} = \alpha_i, \tag{2.5}$$

$(i = 1, 2, \dots, n)$. For $\epsilon_2 > 0$ small enough, there exists $t_2 > t_1$ such that

$$x_k(t) > \alpha_k - \epsilon_2 > 0, \quad (k = 1, 2, \dots, n), \tag{2.6}$$

for $t > t_2$. Using equation (1.2) and inequalities (2.3) and (2.6), we have

$$\dot{y}_j \geq y_j(-r_j^M + \sum_{k=1}^{n} \frac{d_{jk}^L(\alpha_k - \epsilon_2)^2}{f_{jk}^M + (p_k + \epsilon_1)^2} - \sum_{k=1,k\neq j}^{m} e_{jk}^M(q_k + \epsilon_1) - e_{jj}^M y_j), \quad t > t_2.$$

So, for any initial value $y_j(0) > 0$, from Lemma 2.1, we obtain

$$\liminf_{t\to+\infty} y_j(t) \geq \frac{1}{e_{jj}^M}(-r_j^M + \sum_{k=1}^{n} \frac{d_{jk}^L(\alpha_k - \epsilon_2)^2}{f_{jk}^M + (p_k + \epsilon_1)^2} - \sum_{k=1,k\neq j}^{m} e_{jk}^M(q_k + \epsilon_1)),$$

$(j = 1, 2, \dots, m)$, by the arbitrariness of ϵ_1 and ϵ_2, we have

$$\liminf_{t\to+\infty} y_j(t) \geq \frac{1}{e_{jj}^M}(-r_j^M + \sum_{k=1}^{n} \frac{d_{jk}^L\alpha_k^2}{f_{jk}^M + p_k^2} - \sum_{k=1,k\neq j}^{m} e_{jk}^M q_k) = \beta_j, \tag{2.7}$$

$(j = 1, 2, \dots, m)$. Then, from (2.1), (2.5) and (2.2), (2.7), we conclude that system (1.1)–(1.2) is uniformly persistent. □

Assume $K_0 = \{F = (X, Y) \in R_+^{n+m} : \alpha_i^\star \leq x_i(t) \leq p_i^\star (i = 1, 2, \dots, n), \beta_j^\star \leq y_j(t) \leq q_j^\star (j = 1, 2, \dots, m)\}$, where

$$0 < \alpha_i^\star < \alpha_i, \quad p_i^\star > p_i, \quad 0 < \beta_j^\star < \beta_j, \quad q_j^\star > q_j. \tag{2.8}$$

Corollary 2.5. *K_0 is an invariant set with respect to system (1.1)–(1.2) and is ultimately bounded area of the solution of (1.1)–(1.2).*

Proof. Since $\alpha_i \leq \liminf_{t\to+\infty} x_i(t) \leq \limsup_{t\to+\infty} x_i(t) \leq p_i$, then there exists $t_1 > 0$ such that $\alpha_i^\star \leq x_i(t) \leq P_i^\star$ for $t \geq t_1$.

By analogy with x_i, we know that there exists $t_2 > t_1$, such that $\beta_j^\star \leq y_j(t) \leq q_j^\star$ for $t \geq t_2$. It is obvious that $0 < \alpha_i^\star < p_i^\star$ and $0 < \beta_j^\star < q_j^\star$ are independent of any positive solution of system (1.1)–(1.2). So for any solution $F = (x_1(t), \dots, x_n(t), y_1(t), \dots, y_m(t))$ with positive initial value, we have

$$(x_1(t), \dots, x_n(t); y_1(t), \dots, y_m(t)) \in K_0$$

for $t \geq t_2$. □

3. Existence of Positive Periodic Solution

In this section, we assume that all the coefficient functions are ω-periodic ($\omega > 0$), continuous and nonnegative, and $b_i(t), r_j(t),\ a_{ii}(t), e_{jj}(t)$ are positive functions, then system (1.1)–(1.2) will be a ω period system.

Lemma 3.1 ([10])**.** *Suppose that a continuous operator U maps a closed bounded convex set $\Omega \subset R^n$ into itself. Then Ω contains at least one fixed point of U; that is, there exists at least one $z \in \Omega$ for which $Uz = z$ holds.*

Theorem 3.2. *If system (1.1)–(1.2) satisfies conditions (2.8), then system (1.1)–(1.2) at least have a positive ω period solution in R^+.*

Proof. Firstly we define a Poincaré mapping $P : R^{n+m} \to R^{n+m}$, i.e. $Px_0 = x(\omega + t_0; t_0; x_0)$, here $x_0 = (x_1(t_0), x_2(t_0), \dots, x_n(t_0); y_1(t_0), y_2(t_0), \dots, y_m(t_0)) \in R_+^{n+m}$. It is easy to know, if P has a fixed point $x^\star \in R_+^{n+m}$, it is equivalent that system (1.1)–(1.2) at least have a ω period positive solution. For K_0 mentioned above, it is obvious that $K_0 \subset R_+^{n+m}$ is a bounded closed convex set and a positive invariant set with system M_0.

Since the solution is continuous with the initial value, if $x_0 \in K_0$, then P is continuous with x_0 in K_0 and P maps K_0 into itself. That is to say that if $x_0 \in K_0$, then $x(t + t_0; t_0, x_0) \in K_0 (t \geq t_0)$. Let $t = \omega$, we have $x(\omega + t_0; t_0, x_0) \in K_0$, i.e. $PK_0 \subset K_0$. By Lemma 3.1 we know that P at least have a fixed point $x^\star$, i.e. $Px^\star = x^\star$, and then the corresponding system (1.1)–(1.2) at least have a ω positive period solution. □

4. Global Asymptotic Stability and Uniqueness of the Positive Period Solution

Without loss of generality, we assume $m \leq n, c_{ik} = 0$, $k = m+1, \dots, n$, $i = 1, 2, \dots, n$, let $p = \max_{1 \leq i \leq n}\{p_i\}$, $\alpha = \min_{1 \leq i \leq n}\{\alpha_i\}$, $q = \max_{1 \leq j \leq m}\{q_j\}$, and $\beta = \min_{1 \leq j \leq m}\{\beta_j\}$.

In the following, we assume that

(A) $q_j > 0$, $\alpha_i > 0$ and $\beta_j > 0$, $(i = 1, 2, \dots, n; j = 1, 2, \dots, m)$.
(B)

$$\alpha = \min\big\{a_{ii}(t) - \sum_{k=1,\, k\neq i}^{n} (a_{ik}(t) + \frac{c_{ik}(t)(f_{ik}(t) + p^2)q}{f_{ik}^2(t)}) - \sum_{j=1}^{m} \frac{2d_{ji}(t)p}{f_{ji}(t)},$$
$$\frac{\alpha c_{jj}(t)}{f_{jj}(t) + p^2} - \sum_{k=1,\, k\neq j}^{n} \frac{\alpha c_{jk}(t)}{f_{jk}(t) + p^2} - \sum_{k=1}^{m} e_{jk}(t)\big\} > 0,$$

$(i = 1, 2, \dots, n; j = 1, 2, \dots, m)$.

Lemma 4.1 ([1]). *Let g be a nonnegative function defined on $[0, +\infty)$ such that g is integrable on $[0, +\infty)$ and is uniformly continuous on $[0, +\infty)$. Then*

$$\lim_{t \to +\infty} g(t) = 0.$$

Theorem 4.2. *If system (1.1)–(1.2) satisfies the conditions (A) and (B), then system (1.1)–(1.2) has a unique positive period solution which is globally asymptotically stable.*

Proof. Let $G(t) = (u_1(t), u_2(t), \dots, u_n(t), v_1(t), v_2(t), \dots, v_m(t)) \in R_+^{n+m}$ be a positive periodic solution obtained in the proof of Theorem 3.2, and let $F(t) = (x_1(t), x_2(t), \dots, x_n(t), y_1(t), y_2(t), \dots, y_m(t)) \in R_+^{n+m}$ be a solution of (1.1)–(1.2) with $F(0) > 0$. Since solution of system (1.1)–(1.2) remains positive, we can set

$$U_i(t) = \ln u_i(t), \quad X_i(t) = \ln x_i(t) \quad (i = 1, 2, \dots, n),$$
$$V_j(t) = \ln v_j(t), \quad Y_j(t) = \ln y_j(t) \quad (j = 1, 2, \dots, m).$$

Consider a Lyapunov function $V(t) = \sum_{i=1}^{n} |U_i(t) - X_i(t)| + \sum_{j=1}^{m} |V_j(t) - Y_j(t)|$. Now we calculate and estimate the upper right derivative of $V(t)$ along the solution of system (1.1)–(1.2):

$$\begin{aligned}
&D^+V(t)\\
&\le \sum_{i=1}^n \frac{U_i(t)-X_i(t)}{|U_i(t)-X_i(t)|}(\dot U_i(t)-\dot X_i(t)) + \sum_{j=1}^m \frac{V_j(t)-Y_j(t)}{|V_j(t)-Y_j(t)|}(\dot V_j(t)-\dot Y_j(t))\\
&\le \sum_{i=1}^n \frac{U_i(t)-X_i(t)}{|U_i(t)-X_i(t)|}[-\sum_{k=1}^n a_{ik}(t)(e^{U_k(t)}-e^{X_k(t)}) - \sum_{k=1}^m \frac{c_{ik}(t)u_i(t)(e^{V_k(t)}-e^{Y_k(t)})}{f_{ik}(t)+u_i^2(t)}\\
&\quad + \sum_{k=1}^m \frac{c_{ik}(t)y_k(t)(-f_{ik}(t)+x_i(t)u_i(t))(e^{U_i(t)}-e^{X_i(t)})}{(f_{ik}(t)+u_i^2(t))(f_{ik}(t)+x_i^2(t))}]\\
&\quad + \sum_{j=1}^m \frac{V_j(t)-Y_j(t)}{|V_j(t)-Y_j(t)|}[-\sum_{k=1}^m e_{jk}(t)(e^{V_k(t)}-e^{Y_k(t)})\\
&\quad + \sum_{k=1}^n \frac{d_{jk}(t)f_{jk}(t)(u_k(t)+x_k(t))(e^{U_k(t)}-e^{X_k(t)})}{(f_{jk}(t)+u_k^2(t))(f_{jk}(t)+x_k^2(t))}]\\
&\le \sum_{i=1}^n [\sum_{k=1,k\ne i}^n (a_{ik}(t)+\frac{c_{ik}(t)(f_{ik}(t)+p^2)q}{f_{ik}^2(t)}) + \sum_{j=1}^m \frac{2d_{ji}(t)p}{f_{ji}(t)} - a_{ii}(t)]|e^{U_i(t)}-e^{X_i(t)}|\\
&\quad + \sum_{j=1}^m [-\frac{\alpha c_{jj}(t)}{f_{jj}(t)+p^2} + \sum_{k=1,k\ne j}^n \frac{\alpha c_{jk}(t)}{f_{jk}(t)+p^2} + \sum_{k=1}^m e_{jk}(t)]|e^{V_j(t)}-e^{Y_j(t)}|\\
&\le -\alpha[\sum_{i=1}^n |u_i(t)-x_i(t)| + \sum_{j=1}^m |v_j(t)-y_j(t)|].
\end{aligned}$$

An integration of the above inequality leads to

$$V(t)+\alpha\int_0^t [\sum_{i=1}^n |u_i(t)-x_i(t)| + \sum_{j=1}^m |v_j(t)-y_j(t)|] < V(0) < +\infty.$$

Then

$$\limsup_{t\to+\infty}\int_0^t [\sum_{i=1}^n |u_i(t)-x_i(t)| + \sum_{j=1}^m |v_j(t)-y_j(t)|] \le \frac{V(0)}{\alpha} < +\infty.$$

Thus by Lemma 4.1 we have

$$\begin{gathered}
\lim_{t\to+\infty} |u_i(t)-x_i(t)| = 0 \quad (i=1,2,\dots,n);\\
\lim_{t\to+\infty} |v_j(t)-y_j(t)| = 0 \ (j=1,2,\dots,m).
\end{gathered}$$

This implies that the positive ω periodic solution $G(t)$ is globally asymptotically stable, and then it is unique. □

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